



## WORK, ENERGY AND POWER

CBSE Class XI Physics - Chapter 5

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📖 Comprehensive Study Material with Complete Derivations

### 1. INTRODUCTION

In physics, the terms work, energy, and power have very specific and precise meanings. While in everyday language these terms are used loosely, in physics they represent well-defined physical quantities with mathematical expressions. This chapter explores these fundamental concepts that are essential for understanding mechanical processes.

#### 1.1 Scalar Product (Dot Product)

Before studying work, we need to understand the scalar product of vectors, as work is defined using this mathematical operation.

**Definition:**

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta = AB \cos \theta$$

where  $\theta$  is the angle between vectors  $\vec{A}$  and  $\vec{B}$



**Derivation: Properties of Scalar Product**

### Property 1: Commutative Law

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

**Proof:**

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\vec{B} \cdot \vec{A} = BA \cos \theta$$

Since  $AB = BA$  and  $\cos \theta$  is same from either direction,

$$\therefore \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

### Property 2: Distributive Law

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

**Proof:**

$$\text{Let } \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{C} = C_x \hat{i} + C_y \hat{j} + C_z \hat{k}$$

$$\text{Then } \vec{A} \cdot (\vec{B} + \vec{C}) = A_x(B_x + C_x) + A_y(B_y + C_y) + A_z(B_z + C_z)$$

$$= (A_x B_x + A_y B_y + A_z B_z) + (A_x C_x + A_y C_y + A_z C_z)$$

$$= \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

**For Unit Vectors:**

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1 \text{ (since } \cos 0^\circ = 1)$$

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0 \text{ (since } \cos 90^\circ = 0)$$

**Component Form:**

$$\text{If } \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \text{ and } \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\text{Then: } \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

## 2. WORK

## 2.1 Definition of Work

Work is said to be done by a force when the body is displaced actually through some distance in the direction of the applied force.

**Mathematical Definition:**

$$W = \vec{F} \cdot \vec{d} = Fd \cos \theta$$

where:

- $F$  = magnitude of force
- $d$  = magnitude of displacement
- $\theta$  = angle between force and displacement vectors

**Important Points about Work:**

1. Work is a **scalar quantity**
2. SI Unit: Joule (J) = Newton  $\times$  meter (N·m)
3. Dimension:  $[ML^2T^{-2}]$
4. Work can be positive, negative, or zero
5. Work depends on **displacement**, not distance

## 2.2 Nature of Work (Sign Conventions)

Angle $\theta$	$\cos \theta$	Nature of Work	Example
$0^\circ \leq \theta < 90^\circ$	Positive	Positive Work	Pushing a box forward
$\theta = 90^\circ$	Zero	Zero Work	Carrying a bag horizontally
$90^\circ < \theta \leq 180^\circ$	Negative	Negative Work	Friction opposing motion

## 2.3 Work Done by Variable Force

When force varies with position, we need to use integration to calculate work done.

## Derivation: Work by Variable Force

### Step 1: Consider small displacement

For a small displacement  $\Delta x$ , if force is approximately constant  $F(x)$ :

$$\Delta W \approx F(x) \Delta x$$

### Step 2: Sum over all small displacements

Total work from  $x_i$  to  $x_f$ :

$$W = \sum F(x) \Delta x \text{ (summing over all intervals)}$$

### Step 3: Take limit as $\Delta x \rightarrow 0$

As  $\Delta x \rightarrow 0$ , the sum becomes an integral:

$$W = \int_{x_i}^{x_f} F(x) dx$$

### Graphical Interpretation:

The work done equals the **area under the F vs x curve** between  $x_i$  and  $x_f$

## 3. KINETIC ENERGY

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### 3.1 Definition

Kinetic energy is the energy possessed by a body by virtue of its motion.

$$K = \frac{1}{2}mv^2$$

- Always positive or zero
- Scalar quantity
- SI Unit: Joule (J)
- Dimension:  $[ML^2T^{-2}]$

## Derivation: Work-Energy Theorem

### Step 1: Start with kinematic equation

For constant acceleration  $a$ , we have:

$$v^2 - u^2 = 2as$$

where  $u$  = initial velocity,  $v$  = final velocity,  $s$  = displacement

### Step 2: Multiply both sides by $m/2$

$$(m/2)(v^2 - u^2) = (m/2)(2as)$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas$$

### Step 3: Apply Newton's Second Law

From  $F = ma$ , we have  $ma = F$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = Fs$$

### Step 4: Recognize work and kinetic energy

$$W = Fs \text{ (work done)}$$

$$K_f = \frac{1}{2}mv^2 \text{ (final kinetic energy)}$$

$$K_i = \frac{1}{2}mu^2 \text{ (initial kinetic energy)}$$

### Final Result - Work-Energy Theorem:

$$K_f - K_i = W_{\text{net}}$$

or

$$\Delta K = W_{\text{net}}$$

*The change in kinetic energy of a body is equal to the net work done on it.*

## Derivation: Work-Energy Theorem for Variable Force

### Step 1: Time rate of change of KE

$$dK/dt = d/dt (1/2mv^2) = m v (dv/dt)$$

Since  $dv/dt = a$  (acceleration),

$$dK/dt = m v a$$

### Step 2: Apply Newton's Second Law

$$F = ma$$

$$\text{Therefore: } dK/dt = v F$$

### Step 3: Express in terms of displacement

Since  $v = dx/dt$ ,

$$dK/dt = F (dx/dt)$$

$$dK = F dx$$

### Step 4: Integrate from initial to final position

$$\int_{K_i}^{K_f} dK = \int_{x_i}^{x_f} F dx$$

$$K_f - K_i = \int_{x_i}^{x_f} F dx$$

**Final Result:**

$$\Delta K = W$$


This proves the work-energy theorem for variable forces.

## 4. POTENTIAL ENERGY

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### 4.1 Definition

Potential energy is the energy possessed by a body by virtue of its position or configuration.

 **Derivation: Relation between Force and Potential Energy**

### Step 1: Define potential energy change

For a conservative force  $F$ , the change in potential energy is defined as:

$$\Delta V = -W = -\int F \, dx$$

(Negative sign indicates PE increases when work is done against the force)

### Step 2: For infinitesimal displacement

$$dV = -F \, dx$$

### Step 3: Solve for force

$$\mathbf{F} = -dV/d\mathbf{x} \text{ (in one dimension)}$$

This shows that force is the negative gradient of potential energy.

### In three dimensions:

$$\vec{\mathbf{F}} = -\nabla V = -(\partial V/\partial x \hat{\mathbf{i}} + \partial V/\partial y \hat{\mathbf{j}} + \partial V/\partial z \hat{\mathbf{k}})$$

## 4.2 Gravitational Potential Energy

### Derivation: Gravitational Potential Energy (mgh)

#### Step 1: Consider work against gravity

Gravitational force:  $F_g = -mg$  (downward, taking up as positive)

To lift an object, applied force:  $F_{app} = +mg$  (upward)

#### Step 2: Calculate work done

Work done in lifting from height  $h_1$  to  $h_2$ :

$$W = \int_{h_1}^{h_2} F_{app} \, dh$$

$$W = \int_{h_1}^{h_2} mg \, dh$$

$$W = mg(h_2 - h_1)$$

#### Step 3: Define potential energy

Taking  $h_1 = 0$  as reference (ground level) and  $h_2 = h$ :

$$V(h) = mgh$$

This is the gravitational potential energy at height  $h$ .

**Verification:**

$$F = -dV/dh = -d(mgh)/dh = -mg \checkmark$$

(Negative sign indicates downward force)

### 4.3 Elastic Potential Energy (Spring)

#### Derivation: Elastic Potential Energy of a Spring

**Step 1: Hooke's Law**

For an ideal spring:

$$F_{\text{spring}} = -kx$$

where  $k$  = spring constant,  $x$  = displacement from equilibrium

(Negative sign: restoring force opposes displacement)

**Step 2: Work done in stretching/compressing**

To stretch/compress the spring, we must apply external force:

$$F_{\text{ext}} = +kx \text{ (opposing the restoring force)}$$

Work done in displacing from 0 to  $x$ :

$$W = \int_0^x F_{\text{ext}} dx'$$

$$W = \int_0^x kx' dx'$$

**Step 3: Evaluate the integral**

$$W = k \int_0^x x' dx'$$

$$W = k [x'^2/2]_0^x$$

$$W = k(x^2/2 - 0)$$

$$W = \frac{1}{2}kx^2$$

#### Step 4: Define elastic potential energy

This work is stored as elastic potential energy:

$$V(x) = \frac{1}{2}kx^2$$

#### Verification:

$$F = -dV/dx = -d(\frac{1}{2}kx^2)/dx = -kx \quad \checkmark$$

This confirms Hooke's Law.

#### For displacement from $x_1$ to $x_2$ :

$$W = \frac{1}{2}k(x_2^2 - x_1^2)$$

$$\Delta V = \frac{1}{2}k(x_2^2 - x_1^2)$$

## 5. CONSERVATION OF MECHANICAL ENERGY

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### Derivation: Law of Conservation of Mechanical Energy

#### Step 1: Apply Work-Energy Theorem

For a body moving under conservative force:

$$W = \Delta K = K_f - K_i$$

#### Step 2: Express work in terms of potential energy

For conservative force:

$$W = -\Delta V = -(V_f - V_i) = V_i - V_f$$

#### Step 3: Equate the two expressions

$$K_f - K_i = V_i - V_f$$

$$K_f + V_f = K_i + V_i$$

#### Final Result:

$$E = K + V = \text{constant}$$

where E is the total mechanical energy.

*The total mechanical energy remains conserved for conservative forces.*

### Conditions for Conservation of Mechanical Energy:

1. Only conservative forces should act on the system
2. No non-conservative forces (friction, air resistance) should be present
3. No external work should be done on the system
4. System should be isolated from external influences

## 5.1 Conservative and Non-Conservative Forces

Conservative Forces	Non-Conservative Forces
Work done is path-independent	Work done is path-dependent
Work done in closed path = 0	Work done in closed path $\neq 0$
Can define potential energy	Cannot define potential energy
Examples: Gravity, Spring force, Electrostatic force	Examples: Friction, Air resistance, Viscous force

## 6. POWER

### 6.1 Definition

Power is defined as the rate at which work is done or the rate at which energy is transferred.

**Average Power:**

$$P_{\text{avg}} = W/t$$

**Instantaneous Power:**

$$P = dW/dt = \vec{F} \cdot \vec{v} = Fv \cos \theta$$

- Scalar quantity
- SI Unit: Watt (W) = J/s
- Dimension:  $[ML^2T^{-3}]$
- 1 hp = 746 W

## 7. COLLISIONS

### 7.1 Elastic Collisions in One Dimension

#### Derivation: Velocities After Elastic Collision (1D)

**Given:**

- Mass  $m_1$  moving with velocity  $v_{1i}$
- Mass  $m_2$  initially at rest ( $v_{2i} = 0$ )
- After collision: velocities are  $v_{1f}$  and  $v_{2f}$
- Collision is elastic

**Step 1: Apply Conservation of Momentum**

$$m_1v_{1i} + m_2(0) = m_1v_{1f} + m_2v_{2f}$$

$$m_1v_{1i} = m_1v_{1f} + m_2v_{2f} \dots (1)$$

**Step 2: Apply Conservation of Kinetic Energy**

(For elastic collision)

$$\frac{1}{2}m_1v_{1i}^2 + 0 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

$$m_1v_{1i}^2 = m_1v_{1f}^2 + m_2v_{2f}^2 \dots (2)$$

**Step 3: Rearrange equation (1)**

$$m_1(v_{1i} - v_{1f}) = m_2v_{2f} \dots (3)$$

**Step 4: Rearrange equation (2)**

$$m_1(v_{1i}^2 - v_{1f}^2) = m_2v_{2f}^2$$
$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2v_{2f}^2 \dots (4)$$

**Step 5: Divide equation (4) by equation (3)**

$$(v_{1i} + v_{1f}) = v_{2f}$$
$$v_{1i} = v_{2f} - v_{1f} \dots (5)$$

**Step 6: Substitute (5) in (1)**

$$m_1(v_{2f} - v_{1f}) = m_1v_{1f} + m_2v_{2f}$$
$$m_1v_{2f} - m_1v_{1f} = m_1v_{1f} + m_2v_{2f}$$
$$m_1v_{2f} - m_2v_{2f} = 2m_1v_{1f}$$
$$v_{2f}(m_1 - m_2) = 2m_1v_{1f}$$

**Step 7: Final velocities**

After algebraic manipulation:

$$v_{1f} = [(m_1 - m_2)/(m_1 + m_2)] v_{1i}$$
$$v_{2f} = [2m_1/(m_1 + m_2)] v_{1i}$$

**Special Cases:****Case 1: Equal masses ( $m_1 = m_2$ )**

$$v_{1f} = 0, v_{2f} = v_{1i}$$

(First mass stops, second moves with initial velocity)

**Case 2:  $m_1 \gg m_2$** 

$$v_{1f} \approx v_{1i}, v_{2f} \approx 2v_{1i}$$

(Heavy mass continues almost unchanged, light mass moves faster)

**Case 3:  $m_1 \ll m_2$** 

$$v_{1f} \approx -v_{1i}, v_{2f} \approx 0$$

(Light mass bounces back, heavy mass barely moves)

## 7.2 Coefficient of Restitution

**Coefficient of Restitution (e):**

$$e = (v_{2f} - v_{1f}) / (v_{1i} - v_{2i})$$

- For perfectly elastic collision:  $e = 1$
- For perfectly inelastic collision:  $e = 0$
- For real collisions:  $0 < e < 1$

## 8. IMPORTANT FORMULAS SUMMARY

### Complete Formula Sheet

<b>Scalar Product</b>	$\vec{A} \cdot \vec{B} = AB \cos \theta = A_x B_x + A_y B_y + A_z B_z$
<b>Work (constant force)</b>	$W = \vec{F} \cdot \vec{d} = Fd \cos \theta$
<b>Work (variable force)</b>	$W = \int F(x) dx$
<b>Kinetic Energy</b>	$K = \frac{1}{2}mv^2$
<b>Work-Energy Theorem</b>	$W_{\text{net}} = \Delta K = K_f - K_i$
<b>Gravitational PE</b>	$V = mgh$
<b>Elastic PE (spring)</b>	$V = \frac{1}{2}kx^2$
<b>Force from PE</b>	$F = -dV/dx$
<b>Conservation of Energy</b>	$K + V = \text{constant} \quad (E = \text{constant})$
<b>Power (average)</b>	$P_{\text{avg}} = W/t$

**Power (instantaneous)**

$$P = dW/dt = \vec{F} \cdot \vec{v} = Fv \cos \theta$$

**Elastic collision  
( $v_{1f}$ )**

$$v_{1f} = [(m_1 - m_2) / (m_1 + m_2)] v_{1i}$$

**Elastic collision  
( $v_{2f}$ )**

$$v_{2f} = [2m_1 / (m_1 + m_2)] v_{1i}$$

## 9. CASE STUDY BASED QUESTIONS

### Case Study 1: Hydroelectric Power Plant

#### Scenario:

A hydroelectric power plant is located at the base of a dam. Water falls from a height of 100 m into the turbines. The power plant has an efficiency of 80% in converting the potential energy of water into electrical energy. The density of water is  $1000 \text{ kg/m}^3$  and  $g = 10 \text{ m/s}^2$ .

#### Questions:

1. Calculate the potential energy of  $1 \text{ m}^3$  of water at the top of the dam.
2. If water flows at a rate of  $500 \text{ m}^3/\text{minute}$ , what is the total power available from the falling water?
3. What is the actual electrical power generated by the plant?
4. If the plant operates for 10 hours, how much electrical energy is produced in kWh?

## Case Study 2: Roller Coaster Physics

### Scenario:

A roller coaster car of mass 500 kg starts from rest at point A, which is 50 m above the ground. It moves along a frictionless track and passes through point B at ground level, then rises to point C which is 30 m above the ground. Assume  $g = 10 \text{ m/s}^2$  and air resistance is negligible.

### Questions:

1. What is the total mechanical energy of the system at point A?
2. Calculate the speed of the car at point B (ground level).
3. What is the speed of the car at point C (30 m height)?
4. If 10% of mechanical energy is lost due to friction between B and C, what will be the actual speed at C?
5. Explain why the principle of conservation of mechanical energy is applicable in this case (except for part 4).

## Case Study 3: Spring-Mass System in Automobile Suspension

### Scenario:

An automobile suspension system uses a spring with spring constant  $k = 5 \times 10^4 \text{ N/m}$ . When a car of mass 1200 kg is placed on the spring system (4 springs, one for each wheel), the springs compress. Each spring supports one-fourth of the car's weight.

### Questions:

1. Calculate the force supported by each spring when the car is at rest.

2. Find the compression in each spring when the car is at equilibrium position.
3. Calculate the elastic potential energy stored in each spring.
4. If the car goes over a bump and one spring is further compressed by an additional 5 cm, what is the extra potential energy stored?
5. When the spring returns to equilibrium, what form of energy does this potential energy convert to?

### **Case Study 4: Space Probe Mission**

#### **Scenario:**

A space probe of mass 800 kg is launched from Earth's surface. To escape Earth's gravitational field, it needs to reach an altitude where gravitational force becomes negligible. At the surface, gravitational PE is taken as zero. At height  $h = 1000$  km, the probe's speed is measured as 5 km/s. The initial speed at launch was 11 km/s. Take  $g = 10 \text{ m/s}^2$  (average value).

#### **Questions:**

1. Calculate the initial kinetic energy of the probe at launch.
2. Find the kinetic energy at height  $h = 1000$  km.
3. What is the gravitational potential energy at  $h = 1000$  km?
4. Verify the conservation of mechanical energy between launch and height  $h$ .
5. If there were no energy losses, what would be the minimum launch speed required to just reach infinite distance (escape velocity)?

## Case Study 5: Collision in Traffic Accident Analysis

### Scenario:

A traffic accident occurs when a car A of mass 1200 kg moving at 72 km/h collides head-on with car B of mass 800 kg moving at 54 km/h in the opposite direction. After the collision, both cars stick together and move in the direction of car A.

### Questions:

1. Convert the speeds to m/s.
2. Calculate the total momentum before collision (take direction of car A as positive).
3. Find the common velocity after collision using conservation of momentum.
4. Calculate the kinetic energy before and after collision.
5. How much kinetic energy is lost in the collision? Where does this energy go?
6. What type of collision is this (elastic/inelastic/perfectly inelastic)?

## 10. EXPECTED EXAM QUESTIONS (HIGH PROBABILITY)

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### Very Short Answer Questions (1 Mark)

1. Define the SI unit of work and power.
2. Can kinetic energy of a body be negative? Justify.
3. State whether work done by friction is positive or negative when a body slides down an inclined plane.
4. What is the angle between force and displacement when work done is maximum?

5. Name the physical quantity whose SI unit is J/s.
6. What is the dimensional formula of power?
7. State the work-energy theorem in one sentence.
8. When is the work done by a force said to be zero?
9. Define coefficient of restitution.
10. What is the value of coefficient of restitution for a perfectly elastic collision?

### Short Answer Questions (2-3 Marks)

11. Prove that in an elastic collision between two bodies of equal mass (one at rest), the moving body comes to rest and the stationary body moves with the initial velocity of the moving body.
12. Show that spring force is a conservative force.
13. A ball is dropped from a height  $h$  on a floor. If coefficient of restitution is  $e$ , derive an expression for the height attained after  $n$ th bounce.
14. Explain why the total mechanical energy of a freely falling body remains constant.
15. A light body and a heavy body have equal kinetic energy. Which one has greater momentum? Prove your answer.
16. Show that for small oscillations of a pendulum, mechanical energy remains conserved.
17. Distinguish between conservative and non-conservative forces with two examples each.
18. A man carries a load on his head and walks on a horizontal road. Is any work done by him on the load? Explain.
19. Derive the relation between momentum and kinetic energy.
20. Prove that power  $P = \vec{F} \cdot \vec{v}$  for instantaneous power.

### Long Answer Questions (5 Marks)

21. **Derive the work-energy theorem for a variable force and discuss its significance.**
22. **Derive an expression for the elastic potential energy stored in a stretched spring. Also prove that spring force is conservative.**
23. **State and prove the principle of conservation of mechanical energy. Give two situations where it is not applicable.**
24. **Derive expressions for final velocities in an elastic collision in one dimension when one body is initially at rest. Discuss special cases.**

25. Define power. Derive  $P = \vec{F} \cdot \vec{v}$  and explain how power varies with time for a body accelerating uniformly from rest.

### Numerical Problems (High Priority)

26. A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction  $\mu = 0.1$ . Compute: (a) work done by applied force in 10 s, (b) work done by friction in 10 s, (c) work done by net force in 10 s, (d) change in KE in 10 s. **[NCERT]**
27. A pump on ground floor can pump water to fill a tank of volume  $30 \text{ m}^3$  in 15 minutes. If the tank is 40 m above ground and pump efficiency is 30%, calculate the electric power consumed. **[NCERT]**
28. A bullet of mass 50 g is fired with speed 200 m/s on a soft plywood. It emerges with 10% of its initial kinetic energy. Find the emergent speed. **[NCERT]**
29. A block of mass 1 kg moving at 2 m/s enters a rough patch from  $x = 0.10 \text{ m}$  to  $x = 2.01 \text{ m}$  where retarding force  $F = -k/x$  with  $k = 0.5 \text{ J}$ . Find the final kinetic energy and speed. **[NCERT]**
30. A bob of mass  $m$  is suspended by a string of length  $L$ . It is given horizontal velocity  $v_0$  at lowest point such that it completes a semi-circle with string becoming slack at topmost point. Find: (a)  $v_0$ , (b) speed at mid-point, (c) ratio of KE at mid-point to top point. **[NCERT]**
31. A spring of spring constant  $k = 5 \times 10^3 \text{ N/m}$  is initially stretched by 5 cm. Find the work required to further stretch it by another 5 cm. **[Common Board Question]**
32. A particle of mass 0.5 kg travels in a straight line with velocity  $v = ax^{3/2}$  where  $a = 5 \text{ m}^{-1/2} \text{ s}^{-1}$ . What is the work done by net force during displacement from  $x = 0$  to  $x = 2 \text{ m}$ ? **[NCERT]**
33. An elevator can carry maximum load of 1800 kg (elevator + passengers) moving up with constant speed 2 m/s. If frictional force is 4000 N, find the minimum power delivered by motor in watts and hp. **[NCERT]**
34. A raindrop of mass 1.00 g falling from height 1.00 km hits ground with speed 50.0 m/s. (a) What is work done by gravitational force? (b) What is work done by resistive force? (Take  $g = 10 \text{ m/s}^2$ ) **[NCERT]**
35. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of same mass moving with speed  $V$ . If collision is elastic, find the final velocities. **[NCERT]**

## Graph-Based Questions

36. A graph of kinetic energy vs position is given for a particle. The kinetic energy decreases linearly from  $K_0$  at  $x = 0$  to zero at  $x = x_0$ . (a) What is the nature of force acting? (b) Find the force as a function of position. (c) Calculate work done by the force.
37. Force vs displacement graph shows  $F$  varying linearly from  $F_0$  at  $x = 0$  to  $2F_0$  at  $x = x_0$ . Find: (a) Work done, (b) If the body starts from rest and has mass  $m$ , find its speed at  $x_0$ .
38. A  $F$ - $x$  graph shows a variable force acting on a body. The graph consists of: (i)  $F = 10$  N from  $x = 0$  to  $x = 4$  m, (ii)  $F$  decreases linearly from 10 N to 0 from  $x = 4$  m to  $x = 8$  m. Find total work done.

## Assertion-Reason Questions

39. **Assertion (A):** The work done by friction is always negative.  
**Reason (R):** Friction always opposes motion.
40. **Assertion (A):** In an elastic collision of two billiard balls, the total KE is conserved during the short time of collision when they are in contact.  
**Reason (R):** Energy spent against deformation during collision is recovered during separation.
41. **Assertion (A):** A weightlifter holding a 150 kg mass steadily on his shoulder for 30 s does no work on the load during this time.  
**Reason (R):** Work is done only when there is displacement in the direction of force.
42. **Assertion (A):** Power of a body is always positive.  
**Reason (R):** Power  $P = \vec{F} \cdot \vec{v}$  can be negative if force and velocity are in opposite directions.
43. **Assertion (A):** The total mechanical energy of a system is always conserved.  
**Reason (R):** Mechanical energy is conserved only when conservative forces act on the system.

## 11. ANSWERS TO CASE STUDIES

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### Case Study 1: Hydroelectric Power Plant

1. PE of 1 m<sup>3</sup> water =  $mgh = (1000 \text{ kg})(10 \text{ m/s}^2)(100 \text{ m}) = \mathbf{10^6 \text{ J} = 1 \text{ MJ}}$
2. Flow rate =  $500 \text{ m}^3/\text{min} = 500/60 \text{ m}^3/\text{s} = 8.33 \text{ m}^3/\text{s}$   
Mass flow rate =  $8333.33 \text{ kg/s}$   
Power = (mass flow rate)  $\times g \times h = 8333.33 \times 10 \times 100 = \mathbf{8.33 \text{ MW}}$
3. Electrical power =  $0.80 \times 8.33 \text{ MW} = \mathbf{6.66 \text{ MW}}$
4. Energy in 10 hours =  $6.66 \text{ MW} \times 10 \text{ h} = \mathbf{66.6 \text{ MWh} = 66,600 \text{ kWh}}$

### Case Study 2: Roller Coaster

1. At A:  $E = PE = mgh = 500 \times 10 \times 50 = \mathbf{250,000 \text{ J} = 250 \text{ kJ}}$
2. At B:  $E = KE = \frac{1}{2}mv^2$   
 $250,000 = \frac{1}{2}(500)v^2$   
 $v = \mathbf{31.62 \text{ m/s} \approx 32 \text{ m/s}}$
3. At C:  $E = KE + PE$   
 $250,000 = \frac{1}{2}(500)v^2 + 500 \times 10 \times 30$   
 $\frac{1}{2}(500)v^2 = 100,000$   
 $v = \mathbf{20 \text{ m/s}}$
4. Energy at C with loss =  $250,000 - 0.10 \times 250,000 = 225,000 \text{ J}$   
 $225,000 = \frac{1}{2}(500)v^2 + 150,000$   
 $v = \mathbf{12.25 \text{ m/s}}$
5. Conservation applies because only gravity acts (conservative force) and track is frictionless.

### Case Study 3: Automobile Suspension

1. Force per spring =  $mg/4 = 1200 \times 10 / 4 = \mathbf{3000 \text{ N}}$
2.  $F = kx$ , so  $x = F/k = 3000 / (5 \times 10^4) = \mathbf{0.06 \text{ m} = 6 \text{ cm}}$
3.  $PE = \frac{1}{2}kx^2 = \frac{1}{2} \times 5 \times 10^4 \times (0.06)^2 = \mathbf{90 \text{ J}}$

4. Extra compression = 5 cm = 0.05 m

Total compression = 0.11 m

$$\text{Extra PE} = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2} \times 5 \times 10^4 \times (0.11^2 - 0.06^2) = \mathbf{212.5 \text{ J}}$$

5. Converts to kinetic energy (oscillation) and eventually to heat due to damping.

#### Case Study 4: Space Probe

1. Initial KE =  $\frac{1}{2} \times 800 \times (11000)^2 = \mathbf{48.4 \times 10^9 \text{ J} = 48.4 \text{ GJ}}$

2. KE at 1000 km =  $\frac{1}{2} \times 800 \times (5000)^2 = \mathbf{10 \times 10^9 \text{ J} = 10 \text{ GJ}}$

3. PE at 1000 km =  $mgh = 800 \times 10 \times 10^6 = \mathbf{8 \times 10^9 \text{ J} = 8 \text{ GJ}}$

4. Initial E = 48.4 GJ

Final E = 10 GJ + 8 GJ = 18 GJ

Loss = 30.4 GJ (due to air resistance, gravitational work against varying g, etc.)

5. For escape: KE  $\geq$  PE at infinity

Minimum  $v_{\text{escape}} = \sqrt{2gR}$  where R = Earth radius

$$v_{\text{escape}} \approx \mathbf{11.2 \text{ km/s}}$$

#### Case Study 5: Traffic Collision

1.  $v_A = 72 \text{ km/h} = 20 \text{ m/s}$ ,  $v_B = 54 \text{ km/h} = 15 \text{ m/s}$

2.  $p_{\text{initial}} = m_A v_A - m_B v_B = 1200(20) - 800(15) = \mathbf{12,000 \text{ kg}\cdot\text{m/s}}$

3.  $p_{\text{final}} = (m_A + m_B)v_f$

$$12,000 = 2000 \times v_f$$

$$v_f = \mathbf{6 \text{ m/s}}$$

4.  $KE_{\text{before}} = \frac{1}{2}(1200)(20)^2 + \frac{1}{2}(800)(15)^2 = 240,000 + 90,000 = \mathbf{330 \text{ kJ}}$

$$KE_{\text{after}} = \frac{1}{2}(2000)(6)^2 = \mathbf{36 \text{ kJ}}$$

5. Loss = 330 - 36 = **294 kJ**

Energy goes into: deformation, heat, sound, internal energy

6. **Perfectly inelastic collision** (bodies stick together)

## 12. TIPS FOR SCORING MAXIMUM MARKS

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### Derivation Writing Tips:

1. **Always start with "To Derive:"** or "To Prove:" clearly stating what needs to be derived
2. **Write all assumptions** (e.g., "Assuming no friction," "Taking upward direction as positive")
3. **Number each step** - this helps in partial marking
4. **Show all mathematical steps** - don't skip intermediate steps
5. **Box or highlight the final result** using different ink color or underlining
6. **Draw diagrams wherever applicable** (e.g., for collision problems, free body diagrams)
7. **State units in final answer** if numerical
8. **For vector derivations**, clearly indicate vector notation with arrows

### Common Mistakes in Board Exams:

- X Not converting km/h to m/s (very common mistake)
- X Forgetting negative sign in work done by friction
- X Using conservation of energy where friction is present
- X Confusing mass and weight (writing kg instead of N for force)
- X Not squaring velocity in KE formula (writing  $mv$  instead of  $\frac{1}{2}mv^2$ )
- X Taking  $g = 9.8 \text{ m/s}^2$  when problem says use  $g = 10 \text{ m/s}^2$
- X Forgetting to multiply by  $\cos \theta$  in work formula
- X Not checking if collision is elastic before applying KE conservation
- X Mixing up initial and final values in momentum conservation
- X Writing kWh as unit of power (it's energy!)

### Last Minute Revision Checklist:

Topic	Status
Scalar product properties and calculation	<input type="checkbox"/>
Work done - sign conventions	<input type="checkbox"/>
Work-energy theorem derivation	<input type="checkbox"/>
Work by variable force (integration)	<input type="checkbox"/>
Gravitational PE derivation	<input type="checkbox"/>
Spring PE derivation ( $\frac{1}{2}kx^2$ )	<input type="checkbox"/>
Force from PE ( $F = -dV/dx$ )	<input type="checkbox"/>
Conservation of mechanical energy	<input type="checkbox"/>
Conservative vs non-conservative forces	<input type="checkbox"/>
Power formulas ( $P = W/t$ and $P = F \cdot v$ )	<input type="checkbox"/>
Elastic collision formulas derivation	<input type="checkbox"/>
Special cases in collisions ( $m_1 = m_2$ , etc.)	<input type="checkbox"/>
NCERT numerical problems (Ex 5.2, 5.7, 5.8, 5.9, 5.11)	<input type="checkbox"/>
All formula derivations practiced	<input type="checkbox"/>

Topic	Status
Units and dimensions of all quantities	<input type="checkbox"/>

## 13. IMPORTANT DIAGRAMS TO PRACTICE

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**These diagrams are frequently asked in exams and carry marks:**

1. Work done by variable force (F vs x graph with shaded area)
2. Spring force vs displacement graph
3. PE vs position graph for spring (parabola)
4. Energy conservation in pendulum (showing KE and PE at different points)
5. Elastic collision before and after diagram (1D)
6. Roller coaster showing energy at different points
7. Free body diagram for block on rough incline
8. Power vs time graph for uniformly accelerating body

## 14. QUICK REFERENCE - IMPORTANT POINTS

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**Golden Rules for Exam:**

- **Work:** Scalar, can be +ve/-ve/zero,  $W = F \cdot d \cdot \cos \theta$
- **KE:** Always  $\geq 0$ , scalar,  $K = \frac{1}{2}mv^2$
- **PE:** Can be -ve depending on reference, scalar
- **Conservative Force:** PE can be defined, work is path-independent
- **Non-conservative:** PE cannot be defined, work is path-dependent
- **Elastic Collision:** Both momentum AND KE conserved

- **Inelastic:** Only momentum conserved, KE decreases
- **Power:** Rate of doing work, NOT amount of work
- **1 kWh:** Unit of ENERGY = 3.6 MJ (not power!)
- **1 hp:** Unit of power = 746 W



## Study Material

Comprehensive CBSE Physics Study Material

This material includes all NCERT derivations, case studies, and expected exam questions

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*For educational purposes only. Study responsibly and practice regularly.*

WFL LOVE