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GRAVITATION

CBSE Class XI Physics - Chapter 7

📍 Raipur, Chhattisgarh

Comprehensive Study Material with Complete Derivations

1. INTRODUCTION

Gravitation is the natural phenomenon by which all physical objects with mass or energy attract one another. It is one of the four fundamental forces of nature. This chapter deals with:

- Newton's Universal Law of Gravitation
- Kepler's Laws of Planetary Motion
- Acceleration due to gravity and its variation
- Gravitational Potential Energy
- Escape Velocity and Orbital Motion
- Earth Satellites and their applications

Historical Background:

Galileo (1564-1642): Recognized that all bodies, irrespective of their masses, are accelerated towards the earth with a constant acceleration.

Tycho Brahe (1546-1601): Made precise astronomical observations of planetary positions.

Johannes Kepler (1571-1640): Analyzed Tycho Brahe's data and formulated three laws of planetary motion.

Isaac Newton (1642-1727): Proposed the Universal Law of Gravitation that explained both terrestrial and celestial phenomena.

2. KEPLER'S LAWS OF PLANETARY MOTION

2.1 Law of Orbits (First Law)

Statement: All planets move in elliptical orbits with the Sun situated at one of the foci of the ellipse.

An ellipse is a closed curve where the sum of distances from any point on the ellipse to two fixed points (foci) is constant. For a circle, both foci merge into one point at the center.

- **Perihelion:** The point on the orbit closest to the Sun
- **Aphelion:** The point on the orbit farthest from the Sun
- **Semi-major axis:** Half the distance between perihelion and aphelion

2.2 Law of Areas (Second Law)

Statement: The line that joins any planet to the sun sweeps equal areas in equal intervals of time.

$$\Delta A/\Delta t = \text{constant}$$

Derivation 1: Kepler's Second Law from Conservation of Angular Momentum

Step 1: Consider a planet of mass m at position **r** with velocity **v** with respect to the Sun.

Step 2: The area swept by the radius vector in time Δt is:

$$\Delta A = \frac{1}{2} |\mathbf{r} \times \mathbf{v}\Delta t|$$

Step 3: Therefore, the areal velocity is:

$$\Delta A/\Delta t = \frac{1}{2} |\mathbf{r} \times \mathbf{v}| = \frac{1}{2} |\mathbf{r} \times \mathbf{p}|/m$$

where $\mathbf{p} = m\mathbf{v}$ is the linear momentum.

Step 4: Angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

$$\Delta A/\Delta t = L/(2m)$$

Step 5: Since gravitational force is a central force (directed along **r**), the torque on the planet is zero.

$$\tau = \mathbf{r} \times \mathbf{F} = 0$$

Therefore, angular momentum **L** is conserved.

Conclusion: Since L and m are constants, ΔA/Δt = constant.

This is Kepler's Second Law of Areas.

2.3 Law of Periods (Third Law)

Statement: The square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the elliptical orbit.

$$T^2 \propto a^3$$

$$T^2/a^3 = \text{constant} = 4\pi^2/(GM_S)$$

where M_S is the mass of the Sun and a is the semi-major axis.

3. UNIVERSAL LAW OF GRAVITATION

Newton's Universal Law of Gravitation:

Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

$$F = G\ (m_1 m_2)/r^2$$

In vector form:

$$\mathbf{F} = -G\ (m_1 m_2)/r^2\ \hat{\mathbf{r}} = -G\ (m_1 m_2)/r^3\ \mathbf{r}$$

where:

- G = Universal gravitational constant = 6.67×10^{-11} N m^2 kg^{-2}
- $\hat{\mathbf{r}}$ = unit vector from m_1 to m_2
- The negative sign indicates attractive force

Properties of Gravitational Force:

1. **Universal:** Acts between any two objects with mass
2. **Always Attractive:** Never repulsive
3. **Central Force:** Acts along the line joining centers of masses
4. **Inverse Square Law:** $F \propto 1/r^2$
5. **Conservative Force:** Work done is path-independent
6. **Independent of Medium:** Acts even in vacuum
7. **Long Range Force:** Extends to infinity
8. **Weakest Force:** Weakest among four fundamental forces

3.1 Principle of Superposition

If we have a system of n particles, the total gravitational force on particle i is the vector sum of forces due to all other particles:

$$F_i = F_{i1} + F_{i2} + \dots + F_{in} = \Sigma F_{ij}$$

Each force acts independently and is not affected by the presence of other bodies.

4. THE GRAVITATIONAL CONSTANT (G)

4.1 Cavendish Experiment (1798)

Henry Cavendish was the first to determine the value of G experimentally using a torsion balance.

Experimental Setup:

- Two small lead spheres (m) attached to ends of a light rod
- Rod suspended by a thin wire (torsion wire)
- Two large lead spheres (M) brought near the small ones
- Gravitational attraction causes the rod to twist
- Angle of twist (θ) is measured

Derivation 2: Calculation of G from Cavendish Experiment

Step 1: Gravitational force between large sphere (M) and nearby small sphere (m) at distance d:

$$F = G(Mm)/d^2$$

Step 2: Torque due to gravitational forces (if L is length of rod):

$$\tau_{grav} = F \times L = G(Mm)L/d^2$$

Step 3: Restoring torque of the wire:

$$\tau_{rest} = \kappa\theta$$

where κ is the torsion constant (measured independently) and θ is angle of twist.

Step 4: At equilibrium:

$$G(Mm)L/d^2 = \kappa\theta$$

Step 5: Solving for G:

$$G = \kappa\theta d^2/(MmL)$$

Conclusion: By measuring θ, and knowing κ, d, M, m, and L, Cavendish calculated:

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

5. ACCELERATION DUE TO GRAVITY (g)

5.1 Acceleration at Earth's Surface

Derivation 3: Relation between g and G

Step 1: Consider a body of mass m on Earth's surface. Gravitational force:

$$F = G(M_E m)/R_E^2$$

where M_E = mass of Earth, R_E = radius of Earth.

Step 2: This force also equals weight of the body:

$$F = mg$$

Step 3: Equating both expressions:

$$mg = G(M_E m)/R_E^2$$

Conclusion:

$$g = GM_E/R_E^2$$

Standard value: $g = 9.8 \text{ m/s}^2$ (or approximately 10 m/s^2)

5.2 Variation of g with Height

Derivation 4: Acceleration at Height h above Earth's Surface

Step 1: At height h above Earth's surface, distance from center = $(R_E + h)$

$$g(h) = GM_E/(R_E + h)^2$$

Step 2: Rewriting:

$$g(h) = GM_E/R_E^2 \times 1/(1 + h/R_E)^2$$

Step 3: Since $g = GM_E/R_E^2$:

$$g(h) = g/(1 + h/R_E)^2$$

Step 4: Using binomial expansion for $h << R_E$:

$$(1 + h/R_E)^{-2} \approx 1 - 2h/R_E$$

Conclusion:

$$g(h) \approx g(1 - 2h/R_E) \text{ for } h << R_E$$

Note: g decreases with height.

5.3 Variation of g with Depth

Derivation 5: Acceleration at Depth d below Earth's Surface

Step 1: At depth d below surface, assume Earth is uniform sphere of density ρ.

Only the inner sphere of radius (R_E - d) contributes to gravitational force.

The outer shell of thickness d exerts zero net force (from shell theorem).

Step 2: Mass of inner sphere:

$$M_s = (4/3)\pi(R_E - d)^3\rho$$

Step 3: Acceleration at depth d:

$$g(d) = GM_s/(R_E - d)^2 = G \times (4/3)\pi(R_E - d)^3\rho/(R_E - d)^2$$

$$g(d) = (4/3)\pi G(R_E - d)\rho$$

Step 4: Since $g = (4/3)\pi G R_E \rho$ at surface:

$$g(d) = g(R_E - d)/R_E$$

Conclusion:

$$g(d) = g(1 - d/R_E)$$

Note: g decreases linearly with depth and becomes zero at Earth's center.

Important Points about g:

- g is maximum at Earth's surface
- g decreases whether we go up or down from surface
- At center of Earth: g = 0
- At infinity: g = 0
- g varies with latitude (equatorial bulge effect)
- g varies with rotation of Earth

6. GRAVITATIONAL POTENTIAL ENERGY

6.1 Potential Energy Near Earth's Surface

For points close to Earth's surface ($h << R_E$), gravitational force can be considered constant (= mg).

Work done in lifting a mass m from height h_1 to h_2:

$$W = mg(h_2 - h_1)$$

Potential Energy at height h:

$$U(h) = mgh + U_0$$

Usually we take $U_0 = 0$ at ground level (h = 0).

6.2 General Expression for Gravitational Potential Energy

Derivation 6: Gravitational Potential Energy at Distance r

Step 1: Consider moving a mass m from distance r_1 to r_2 from Earth's center.

Gravitational force at distance r:

$$F = GM_E m/r^2$$

Step 2: Work done against gravity:

$$W = \int_{r_1}^{r_2} F\,dr = \int_{r_1}^{r_2} (GM_E m/r^2)\,dr$$

Step 3: Integrating:

$$W = GM_E m \int_{r_1}^{r_2} (1/r^2)\,dr = GM_E m\,[-1/r]_{r_1}^{r_2}$$

$$W = GM_E m\,(1/r_1 - 1/r_2)$$

Step 4: This work done equals change in potential energy:

$$U(r_2) - U(r_1) = -GM_E m\,(1/r_1 - 1/r_2)$$

Step 5: Taking reference point at infinity ($U(\infty) = 0$):

$$U(r) - 0 = -GM_E m(1/\infty - 1/r)$$

Conclusion:

$$U(r) = -GM_E m/r$$

Note: Potential energy is negative (attractive force). Zero at infinity.

General Formula for Two Masses:

Gravitational potential energy between two masses m_1 and m_2 separated by distance r:

$$U = -Gm_1m_2/r$$

6.3 Gravitational Potential

Gravitational potential at a point is the potential energy per unit mass:

$$V = U/m = -GM/r$$

Unit: J/kg or m^2/s^2

7. ESCAPE SPEED

Derivation 7: Escape Velocity from Earth's Surface

Concept: Escape speed is the minimum speed required to project a body from Earth's surface so that it escapes Earth's gravitational field and never returns.

Step 1: Total mechanical energy at Earth's surface ($r = R_E$):

$$E_{initial} = KE + PE = (1/2)mv_e^2 - GM_Em/R_E$$

Step 2: At infinity ($r = \infty$), for minimum escape speed:

$$E_{final} = 0 + 0 = 0$$

(Both KE and PE are zero)

Step 3: By conservation of energy:

$$E_{initial} = E_{final}$$

$$(1/2)mv_e^2 - GM_Em/R_E = 0$$

Step 4: Solving for v_e:

$$(1/2)mv_e^2 = GM_Em/R_E$$

$$v_e^2 = 2GM_E/R_E$$

Step 5: Since $g = GM_E/R_E^2$:

$$v_e^2 = 2gR_E$$

Conclusion:

$$v_e = \sqrt{(2GM_E/R_E)} = \sqrt{(2gR_E)}$$

Substituting $g = 9.8\ m/s^2$ and $R_E = 6.4 \times 10^6$ m:

$$v_e = 11.2\ km/s$$

Important Points about Escape Speed:

- Independent of mass of the projectile
- Independent of direction of projection

- Depends only on mass and radius of the planet
- For Moon: $v_e \approx 2.3$ km/s (Moon has no atmosphere due to low escape speed)
- Escape speed from Sun's surface ≈ 618 km/s

8. EARTH SATELLITES

8.1 Orbital Velocity

Derivation 8: Orbital Velocity of Satellite

Step 1: Consider a satellite of mass m orbiting Earth at height h above surface.

Orbital radius: $r = R_E + h$

Step 2: Centripetal force required for circular motion:

$$F_c = mv^2/r$$

Step 3: This is provided by gravitational force:

$$F_g = GM_Em/r^2$$

Step 4: Equating $F_c = F_g$:

$$mv^2/r = GM_Em/r^2$$

Step 5: Solving for v:

$$v^2 = GM_E/r$$

Conclusion:

$$v = \sqrt{(GM_E/r)} = \sqrt{(GM_E/(R_E + h))}$$

For satellite close to Earth's surface ($h \approx 0$):

$$v_0 = \sqrt{(gR_E)} \approx 7.9 \text{ km/s}$$

8.2 Time Period of Satellite

Derivation 9: Time Period of Satellite (Kepler's Third Law)

Step 1: Orbital speed of satellite:

$$v = \sqrt{(GM_E/r)}$$

Step 2: Distance traveled in one complete orbit:

$$\text{Distance} = 2\pi r$$

Step 3: Time period T:

$$T = \text{Distance/Speed} = 2\pi r/v$$

Step 4: Substituting $v = \sqrt{(GM_E/r)}$:

$$T = 2\pi r/\sqrt{(GM_E/r)} = 2\pi r\sqrt{(r/GM_E)}$$

$$T = 2\pi\sqrt{(r^3/GM_E)}$$

Step 5: Squaring both sides:

$$T^2 = (4\pi^2/GM_E)r^3$$

Conclusion:

$$T^2 = (4\pi^2/GM_E)r^3 = kr^3 \text{ (where } k = 4\pi^2/GM_E)$$

This is Kepler's Third Law for satellites.

For satellite close to Earth's surface ($r \approx R_E$):

$$T_0 = 2\pi\sqrt{(R_E/g)} \approx 85 \text{ minutes}$$

8.3 Geostationary Satellite

A geostationary satellite appears stationary relative to Earth because its orbital period equals Earth's rotation period (24 hours).

Conditions for Geostationary Orbit:

- Time period T = 24 hours
- Must orbit in equatorial plane
- Direction of rotation same as Earth's rotation
- Orbital height h ≈ 36,000 km above Earth's surface

9. ENERGY OF AN ORBITING SATELLITE

Derivation 10: Total Energy of Satellite

Step 1: Kinetic Energy of satellite in orbit:

$$KE = (1/2)mv^2 = (1/2)m(GM_E/r) = GM_Em/(2r)$$

Step 2: Potential Energy of satellite at distance r:

$$PE = -GM_Em/r$$

Step 3: Total Mechanical Energy:

$$E = KE + PE = GM_Em/(2r) - GM_Em/r$$

$$E = GM_Em/(2r) - 2GM_Em/(2r)$$

Conclusion:

$$E = -GM_Em/(2r)$$

Important Relations:

$$E = -KE = PE/2$$

$$|PE| = 2|KE|$$

Key Points about Satellite Energy:

- Total energy is negative (bound system)
- KE is positive, PE is negative, $|PE| = 2|KE|$

- As orbital radius increases, KE decreases, PE increases (becomes less negative)
- Total energy increases (becomes less negative) with increasing r
- To move satellite to higher orbit, energy must be supplied

10. IMPORTANT FORMULAS - QUICK REFERENCE

Physical Quantity	Formula	Remarks
Universal Law of Gravitation	$F = Gm_1m_2/r^2$	$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
Acceleration due to gravity	$g = GM_E/R_E^2$	$g = 9.8$ m/s$^2 \approx 10$ m/s^2
g at height h	$g(h) = g(1 - 2h/R_E)$	For $h << R_E$
g at depth d	$g(d) = g(1 - d/R_E)$	Linear decrease with depth
Gravitational Potential Energy	$U = -GMm/r$	Zero at infinity
Gravitational Potential	$V = -GM/r$	Energy per unit mass
Escape Velocity	$v_e = \sqrt{(2gR_E)}$	$v_e = 11.2$ km/s for Earth
Orbital Velocity	$v = \sqrt{(GM_E/r)}$	$v_0 = 7.9$ km/s (near surface)
Time Period	$T = 2\pi\sqrt{(r^3/GM_E)}$	$T_0 = 85$ min (near surface)
Kepler's Third Law	$T^2 = kr^3$	$k = 4\pi^2/GM$
KE of Satellite	$KE = GM_Em/(2r)$	Always positive
PE of Satellite	$PE = -GM_Em/r$	Always negative

Physical Quantity	Formula	Remarks
Total Energy	$E = -GM_Em/(2r)$	E = -KE = PE/2
Relation	$v_e = \sqrt{2}\, v_0$	Escape speed = √2 × orbital speed

11. CASE STUDIES WITH QUESTIONS

Case Study 1: Mars Mission

India's Mars Orbiter Mission (Mangalyaan) was launched on November 5, 2013, and successfully entered Mars orbit on September 24, 2014. The spacecraft had a mass of 1,350 kg. Mars has a mass of 6.4×10^{23} kg and radius of 3.4×10^6 m. The orbiter was placed in an elliptical orbit with periapsis (closest point) at 421.7 km and apoapsis (farthest point) at 76,993.6 km from Mars surface.

Question 1: What is the escape velocity from Mars surface?

(a) 3.2 km/s **(b)** 5.0 km/s **(c)** 7.2 km/s **(d)** 11.2 km/s

Question 2: The acceleration due to gravity on Mars surface is approximately:

(a) 2.5 m/s^2 **(b)** 3.7 m/s^2 **(c)** 5.2 m/s^2 **(d)** 9.8 m/s^2

Question 3: According to Kepler's second law, when the orbiter is at periapsis:

(a) Its speed is minimum **(b)** Its speed is maximum
(c) Its speed is same as at apoapsis **(d)** Its angular momentum is zero

Solutions - Case Study 1:

Answer 1: (b) 5.0 km/s

$v_e = \sqrt{(2GM/R)} = \sqrt{(2 \times 6.67 \times 10^{-11} \times 6.4 \times 10^{23} / 3.4 \times 10^6)} \approx 5.0$ km/s

Answer 2: (b) 3.7 m/s²

$g = GM/R^2 = (6.67 \times 10^{-11} \times 6.4 \times 10^{23}) / (3.4 \times 10^6)^2 \approx 3.7$ m/s²

Answer 3: (b) Its speed is maximum

From Kepler's second law (conservation of angular momentum), when r is minimum (periapsis), v must be maximum to keep L = mvr constant.

Case Study 2: Geostationary Satellite

Communication satellites are placed in geostationary orbits so they appear stationary relative to a point on Earth's surface. These satellites have a period of 24 hours and orbit at a height of approximately 36,000 km above the equator. They are crucial for telecommunications, weather monitoring, and broadcasting. Given: $G = 6.67 \times 10^{-11}$ N m² kg⁻², $M_E = 6 \times 10^{24}$ kg, $R_E = 6.4 \times 10^6$ m.

Question 1: The orbital radius of a geostationary satellite from Earth's center is approximately:

(a) 6,400 km **(b)** 12,800 km **(c)** 36,000 km **(d)** 42,400 km

Question 2: The orbital velocity of a geostationary satellite is approximately:

(a) 1.9 km/s **(b)** 3.1 km/s **(c)** 5.4 km/s **(d)** 7.9 km/s

Question 3: Which of the following is NOT a requirement for a geostationary satellite?

(a) Must orbit in equatorial plane
(b) Must rotate in same direction as Earth
(c) Must have period of 24 hours
(d) Must have circular polar orbit

Solutions - Case Study 2:

Answer 1: (d) 42,400 km

Using $T^2 = (4\pi^2/GM)r^3$ with T = 24 hours = 86,400 s:
$r^3 = GMT^2/(4\pi^2) \rightarrow r \approx 4.24 \times 10^7$ m = 42,400 km

Answer 2: (b) 3.1 km/s

$v = \sqrt{(GM/r)} = \sqrt{(6.67 \times 10^{-11} \times 6 \times 10^{24} / 4.24 \times 10^7)} \approx 3.1$ km/s

Answer 3: (d) Must have circular polar orbit

A geostationary satellite must orbit in the equatorial plane, not a polar orbit.

Case Study 3: Variation of g with Altitude

Mount Everest, at 8,849 m above sea level, is Earth's highest peak. The International Space Station (ISS) orbits at approximately 408 km altitude. Scientists have measured that the value of g decreases with altitude. At sea level, g = 9.8 m/s^2. Earth's radius $R_E = 6.4 \times 10^6$ m.

Question 1: The value of g at the summit of Mount Everest is approximately:

(a) 9.77 m/s^2 **(b)** 9.50 m/s^2 **(c)** 8.90 m/s^2 **(d)** 7.80 m/s^2

Question 2: At the altitude of ISS (408 km), g is approximately what fraction of its value at Earth's surface?

(a) 0.50 **(b)** 0.75 **(c)** 0.88 **(d)** 0.95

Question 3: If Earth's radius were to increase by 1% while mass remains constant, g would:

(a) Increase by 1% **(b)** Decrease by 1%
(c) Increase by 2% **(d)** Decrease by 2%

Solutions - Case Study 3:

Answer 1: (a) 9.77 m/s^2

$g(h) = g(1 - 2h/R_E) = 9.8(1 - 2 \times 8849/6.4 \times 10^6) \approx 9.77\ m/s^2$

Answer 2: (c) 0.88

$g(h)/g = (R_E/(R_E + h))^2 = (6400/(6400 + 408))^2 \approx 0.88$

Answer 3: (d) Decrease by 2%

Since $g \propto 1/R^2$, if R increases by 1%, g decreases by approximately 2% (using calculus: $dg/g = -2dR/R$)

Case Study 4: Black Holes

A black hole is a region of spacetime where gravity is so strong that nothing, not even light, can escape. The escape velocity from a black hole's "event horizon" equals the speed of light $c = 3 \times 10^8$ m/s. The radius of the event horizon is called the Schwarzschild radius. For a black hole with mass M, the Schwarzschild radius $R_S = 2GM/c^2$.

Question 1: The Schwarzschild radius of a black hole with mass equal to Sun's mass (2×10^{30} kg) is approximately:

(a) 1 km **(b)** 3 km **(c)** 10 km **(d)** 30 km

Question 2: If a star of mass M collapses to form a black hole, its Schwarzschild radius is:

(a) Directly proportional to M **(b)** Inversely proportional to M
(c) Proportional to M^2 **(d)** Independent of M

Question 3: Why can't light escape from a black hole?

(a) Light is absorbed by the black hole
(b) Escape velocity exceeds speed of light
(c) Light has no mass
(d) Black holes don't emit photons

Solutions - Case Study 4:

Answer 1: (b) 3 km

$R_S = 2GM/c^2 = (2 \times 6.67 \times 10^{-11} \times 2 \times 10^{30})/(3 \times 10^8)^2 \approx 3000$ m = 3 km

Answer 2: (a) Directly proportional to M

From $R_S = 2GM/c^2$, we see $R_S \propto M$

Answer 3: (b) Escape velocity exceeds speed of light

At the event horizon, escape velocity equals c. Inside the event horizon, escape velocity > c, so even light cannot escape.

Case Study 5: Binary Star System

Many stars exist in binary systems where two stars orbit their common center of mass. Consider a binary system with stars of masses $M_1 = 4M_\odot$ and $M_2 = 2M_\odot$ (where $M_\odot$ is solar mass) separated by distance $d = 10^{12}$ m. They complete one orbit in time T. The stars orbit in circular paths around their common center of mass with different radii r_1 and r_2.

Question 1: The ratio of orbital radii r_1/r_2 is:

(a) 1:2 **(b)** 2:1 **(c)** 1:4 **(d)** 4:1

Question 2: The distance r_1 of star 1 from the center of mass is:

(a) $(1/3) \times 10^{12}$ m **(b)** $(1/2) \times 10^{12}$ m
(c) $(2/3) \times 10^{12}$ m **(d)** $(3/4) \times 10^{12}$ m

Question 3: Both stars have the same:

(a) Linear velocity **(b)** Angular velocity
(c) Centripetal acceleration **(d)** Kinetic energy

Solutions - Case Study 5:

Answer 1: (a) 1:2

For center of mass: $M_1r_1 = M_2r_2$
$r_1/r_2 = M_2/M_1 = 2M_\odot/4M_\odot = 1/2$

Answer 2: (a) (1/3) × 10^{12} m

Since $r_1 + r_2 = d$ and $r_1/r_2 = 1/2$:
$r_1 = d/(1 + M_1/M_2) = 10^{12}/(1 + 2) = (1/3) \times 10^{12}$ m

Answer 3: (b) Angular velocity

Both stars complete one orbit in the same time T, so they have the same angular velocity $\omega = 2\pi/T$. Linear velocities are different ($v = \omega r$).

12. EXPECTED QUESTIONS FOR BOARD EXAMS

Very Short Answer Questions (1 Mark)

1. State Newton's Universal Law of Gravitation.
2. Write the dimensional formula of universal gravitational constant G.
3. What is the value of g at the center of Earth?
4. Define escape velocity.
5. What is a geostationary satellite?
6. State Kepler's second law of planetary motion.
7. At what height above Earth's surface does g become g/2?
8. Why does the Moon have no atmosphere?
9. What is the relation between escape velocity and orbital velocity?
10. Define gravitational potential.

Short Answer Questions (2-3 Marks)

1. Derive the relation between g and G.
2. Show that acceleration due to gravity decreases with height.
3. Explain why gravitational potential energy is negative.
4. Derive the expression for escape velocity.
5. Calculate the orbital velocity of a satellite revolving close to Earth's surface.
6. State and explain Kepler's law of periods.
7. What is the principle of superposition? Explain with reference to gravitational force.
8. Why is gravitational force called a conservative force?
9. Derive an expression for acceleration due to gravity at depth d below Earth's surface.
10. Compare the variation of g with height and depth.

Long Answer Questions (5 Marks)

1. Derive an expression for gravitational potential energy of a body at height h from Earth's surface. Hence show that U = mgh for small heights.
2. State and derive Kepler's second law from conservation of angular momentum.
3. Derive the expression for orbital velocity and time period of a satellite. Hence deduce Kepler's third law.
4. Describe Cavendish's experiment to determine the value of universal gravitational constant G.
5. Derive expressions for kinetic energy, potential energy and total energy of an orbiting satellite. Show that E = -KE = PE/2.

Numerical Problems

1. Calculate the value of acceleration due to gravity at a height equal to half the radius of Earth. (Given: $g = 9.8\ m/s^2$)
2. At what depth below Earth's surface will the value of g be half its value at the surface? ($R_E = 6400$ km)
3. Calculate the escape velocity for a rocket from Earth's surface. ($g = 10\ m/s^2$, $R_E = 6.4 \times 10^6$ m)

4. A satellite orbits Earth at height 400 km. Calculate its orbital velocity and time period. (Given: R_E = 6400 km, g = 9.8 m/s²)
5. Calculate the height at which a satellite must orbit to appear stationary above a point on equator. (T = 24 hours, R_E = 6400 km)
6. Two stars of masses M and 2M are at distance d apart. Find the position of null point where gravitational field is zero.
7. A body weighs 63 N on Earth's surface. What is its weight at height $h = R_E/2$?
8. Calculate the ratio of orbital velocities of two satellites at heights R_E and $2R_E$ from Earth's surface.
9. A satellite is orbiting at height $h = R_E$. Calculate its kinetic energy, potential energy and total energy if mass = 500 kg.
10. Calculate the energy required to lift a satellite of mass 1000 kg from Earth's surface to height $2R_E$.

13. EXAM PREPARATION TIPS

Key Points to Remember for Scoring Maximum Marks:

1. **All derivations are important:** Practice writing complete derivations for:
    - Relation between g and G
    - Variation of g with height and depth
    - Escape velocity
    - Orbital velocity and time period
    - Energy of satellite
2. **Draw diagrams wherever required:** Especially for:
    - Kepler's laws
    - Variation of g
    - Satellite motion
    - Cavendish experiment

3. **Write formulas clearly:** Always box or highlight final formulas
4. **State assumptions:** Mention assumptions like "Earth is uniform sphere", "height $h << R_E$", etc.
5. **Check units:** Always write correct SI units in numerical problems
6. **Show all steps:** Never skip intermediate steps in derivations and numericals

Common Mistakes to Avoid:

- ✗ Confusing g (acceleration) with G (gravitational constant)
- ✗ Using wrong formula for height vs depth variation of g
- ✗ Forgetting negative sign in potential energy $U = -GMm/r$
- ✗ Confusing escape velocity with orbital velocity
- ✗ Using R_E instead of $(R_E + h)$ in satellite problems
- ✗ Writing weight in kg instead of Newton
- ✗ Mixing up KE, PE and total energy relations for satellites
- ✗ Not converting units (km to m, hours to seconds)
- ✗ Forgetting factor of 2 in escape velocity $v_e = \sqrt{2}\, v_0$
- ✗ Writing g as constant everywhere (it varies!)

Formulae to Memorize:

1. $F = Gm_1m_2/r^2$	2. $g = GM_E/R_E^2$
3. $g(h) = g(1 - 2h/R_E)$	4. $g(d) = g(1 - d/R_E)$
5. $U = -GMm/r$	6. $v_e = \sqrt{(2gR_E)}$
7. $v = \sqrt{(GM_E/r)}$	8. $T^2 = (4\pi^2/GM)r^3$

9. KE = GMm/(2r)	10. E = -GMm/(2r)

Quick Revision Checklist:

Topic	Done ✓
Newton's Universal Law of Gravitation	☐
All three Kepler's Laws with derivation of second law	☐
Relation between g and G derivation	☐
Variation of g with height (complete derivation)	☐
Variation of g with depth (complete derivation)	☐
Gravitational potential energy derivation	☐
Escape velocity derivation	☐
Orbital velocity and time period derivation	☐
Energy of satellite (KE, PE, Total) derivation	☐
Cavendish experiment description	☐
All numerical problems from NCERT	☐

Topic	Done ✓
Practice case study questions	☐

14. IMPORTANT CONSTANTS AND DATA

Constant/Data	Symbol	Value
Universal Gravitational Constant	G	6.67×10^{-11} N m² kg⁻²
Acceleration due to gravity	g	9.8 m/s² (≈ 10 m/s²)
Mass of Earth	M_E	6×10^{24} kg
Radius of Earth	R_E	6.4×10^{6} m (6400 km)
Mass of Sun	M_S	2×10^{30} kg
Mass of Moon	M_M	7.4×10^{22} kg
Radius of Moon	R_M	1.74×10^{6} m
Earth-Sun distance	-	1.5×10^{11} m (1 AU)
Earth-Moon distance	-	3.84×10^{8} m
Escape velocity from Earth	v_e	11.2 km/s
Orbital velocity (near surface)	v_0	7.9 km/s
Period of satellite (near surface)	T_0	85 minutes
Height of geostationary satellite	h	36,000 km

📚 Study Material

Comprehensive CBSE Physics Study Material

This material includes all NCERT derivations, case studies, and expected exam questions

For educational purposes only. Study responsibly and practice regularly.