📚 CHAPTER 8

MECHANICAL PROPERTIES OF SOLIDS

Math Love Institute - Raipur, Chhattisgarh

Complete Study Material with Concepts, Formulas & Examples

🎯 1. INTRODUCTION

Understanding Solid Bodies

Key Idea: While we often think of solid bodies as perfectly rigid, in reality, all bodies can be deformed when sufficient external force is applied. Even a steel bar will change shape under large forces.

🔑 Key Definitions

1. Elasticity: The property by virtue of which a body tends to regain its original size and shape when the applied force is removed.

2. Elastic Deformation: The deformation that disappears when the deforming force is removed. The body returns to its original shape.

3. Plasticity: The property of materials that get permanently deformed and do not return to their original shape when force is removed (e.g., putty, mud).

4. Plastic Deformation: Permanent deformation that remains even after the force is removed.

⚠️ **Important Note:** The elastic behavior of materials plays a crucial role in engineering design - buildings, bridges, automobiles, aircraft, etc. all require knowledge of elastic properties of materials like steel, concrete, etc.

📊 2. STRESS AND STRAIN

2.1 Stress (σ)

Definition: When a deforming force is applied on a body, a restoring force is developed inside the body. The restoring force per unit area is called **stress** .

Stress (σ) = F/A

Where:

- F = Applied force (in Newton)
- A = Cross-sectional area (in m^2)

SI Unit: N m^{-2} or Pascal (Pa)

Dimensional Formula: $[ML^{-1}T^{-2}]$

Types of Stress

Type of Stress	Description	Example
1. Tensile Stress	Restoring force per unit area when a body is stretched. Force applied perpendicular to cross-section, pulling the body.	Stretching a wire or rod
2. Compressive Stress	Restoring force per unit area when a body is compressed. Force applied perpendicular to cross-section, pushing the body.	Compression of a pillar

Type of Stress	Description	Example
3. Shearing Stress (Tangential Stress)	Restoring force per unit area when force is applied parallel to the surface, causing relative displacement between opposite faces.	Cutting with scissors, pushing a book sideways
4. Hydraulic Stress (Volume Stress)	Force per unit area applied perpendicular at every point on the surface of a body (usually by a fluid). Equal to hydraulic pressure.	Object submerged in fluid under high pressure

2.2 Strain (ε)

> **Definition:** Strain is the fractional change in dimension (length, volume, or shape) of a body due to the applied stress.
> **Important:** Strain is a **dimensionless quantity** (pure number with no units).

Types of Strain:

1. Longitudinal Strain (ε) = ΔL/L

- ΔL = Change in length
- L = Original length

(Associated with tensile or compressive

stress)

2. Shearing Strain = Δx/L = tan θ ≈ θ

- Δx = Relative displacement of faces
- L = Length (height) of body
- θ = Angular displacement (in radians, small angles)

3. Volume Strain = ΔV/V

- ΔV = Change in volume
- V = Original volume

(Associated with hydraulic stress)

📝 **Note:** For small angles, $\tan \theta \approx \theta$ (when θ is in radians). For example, if θ = 10°, there is only 1% difference between θ and tan θ.

⚖️ 3. HOOKE'S LAW

Hooke's Law

Statement: Within the elastic limit, stress is directly proportional to strain.

```
Stress ∝ Strain

Stress = k × Strain

Where:
• k = Proportionality constant
• k is called Modulus of Elasticity

General Form:
Modulus of Elasticity = Stress / Strain
```

⚠️ **Important Points:**

- Hooke's law is an **empirical law** (based on experiments)
- Valid for **most materials** but not all
- Valid only for **small deformations**
- Valid only within the **elastic limit**
- Some materials do not exhibit this linear relationship (e.g., rubber, biological tissues)

📈 4. STRESS-STRAIN CURVE

> **Purpose:** A stress-strain graph shows the relationship between stress and strain for a material under tensile (or compressive/shear) stress. It helps us understand how materials deform under increasing loads.

Key Regions in Stress-Strain Curve

Region/Point	Description	Behavior
O to A (Proportional Region)	Stress is directly proportional to strain. Hooke's law is valid in this region.	Elastic behavior - body returns to original shape when load is removed
A to B (Elastic Region)	Stress and strain are not proportional, but body still returns to original shape when load is removed.	Still elastic behavior
Point B (Yield Point/Elastic Limit)	Maximum stress up to which material remains elastic. Beyond this, permanent deformation occurs.	Yield Strength (σ_y) - critical point
B to D (Plastic Region)	Strain increases rapidly even for small increase in stress. Material shows permanent set.	Plastic deformation - permanent change

Region/Point	Description	Behavior
Point D (Ultimate Tensile Strength)	Maximum stress the material can withstand.	Ultimate Strength (σ_u)
Point E (Fracture Point)	Point at which material breaks/fractures.	Breaking point

Brittle Materials: If points D and E are close together, the material is **brittle** (breaks soon after reaching ultimate strength).
Examples: Glass, cast iron, concrete

Ductile Materials: If points D and E are far apart, the material is **ductile** (can be drawn into wires, undergoes large plastic deformation before breaking).
Examples: Copper, aluminum, mild steel

Elastomers

Materials like rubber, elastic tissue of aorta can be stretched to several times their original length and still return to original shape. These are called **elastomers**.
Characteristics:

- Very large elastic region
- Do not obey Hooke's law over most of the elastic region

- No well-defined plastic region

🔬 5. ELASTIC MODULI

Definition: The ratio of stress to strain within the elastic limit is called the **Modulus of Elasticity**. It is a characteristic property of the material.
There are three types of elastic moduli corresponding to three types of deformations.

5.1 Young's Modulus (Y)

Young's Modulus (Y) = Tensile (or Compressive) Stress / Longitudinal Strain

$$Y = \sigma/\varepsilon = (F/A)/(\Delta L/L) = (F \times L)/(A \times \Delta L)$$

Where:
- F = Applied force
- A = Cross-sectional area
- L = Original length
- ΔL = Change in length

SI Unit: N m^{-2} or Pascal (Pa)

Practical Unit: GPa (GigaPascal) = 10^9 Pa

Key Points about Young's Modulus:

- Applies to **solids only** (both tensile and compressive stress)
- Metals have **large Young's moduli** (require large force for small deformation)
- Larger Y means material is **more elastic** (stiffer, resists deformation)
- Steel is more elastic than copper, brass, and aluminum

Material	Density ρ (kg m^{-3})	Young's Modulus Y (10^9 N m^{-2})	Ultimate Strength (10^6 N m^{-2})	Yield Strength (10^6 N m^{-2})
Aluminum	2710	70	110	95
Copper	8890	110	400	200
Iron (wrought)	7800-7900	190	330	170
Steel	7860	200	400	250
Glass	2190	65	50	—

Material	Density ρ (kg m^{-3})	Young's Modulus Y (10^9 N m^{-2})	Ultimate Strength (10^6 N m^{-2})	Yield Strength (10^6 N m^{-2})
Concrete	2320	30	40	—
Wood	525	13	50	—
Bone	1900	9.4	170	—
Polystyrene	1050	3	48	—

5.2 Shear Modulus or Modulus of Rigidity (G)

Shear Modulus (G) = Shearing Stress / Shearing Strain

G = (F/A)/(Δx/L) = (F × L)/(A × Δx)

Or,

G = (F/A)/θ = F/(A × θ)

Where:

- F = Applied tangential force
- A = Area parallel to force

- Δx = Relative displacement
- L = Original length (height)
- θ = Angular displacement (in radians)

Also: $\sigma_s = G \times \theta$

SI Unit: $N\ m^{-2}$ or Pa

Key Points about Shear Modulus:

- Applies to **solids only** (liquids and gases cannot sustain shearing stress)
- Generally **less than Young's modulus**
- For most materials: **G ≈ Y/3**

Material	Shear Modulus G ($10^9\ N\ m^{-2}$)
Aluminum	25
Brass	36
Copper	42
Glass	23
Iron	70
Lead	5.6

Material	Shear Modulus G (10^9 N m^{-2})
Nickel	77
Steel	84
Tungsten	150
Wood	10

5.3 Bulk Modulus (B)

Bulk Modulus (B) = Hydraulic Stress / Volume Strain

$B = -p/(\Delta V/V)$

Where:
- p = Hydraulic pressure (stress)
- ΔV = Change in volume
- V = Original volume
- Negative sign indicates volume decreases when pressure increases

SI Unit: N m^{-2} or Pa

Compressibility (k) = $1/B = -(1/\Delta p)$ ×

(ΔV/V)

```
(Fractional change in volume per unit
increase in pressure)
```

Key Points about Bulk Modulus:

- Applies to **solids, liquids, and gases**
- **Solids** have highest bulk modulus (least compressible)
- **Gases** have lowest bulk modulus (most compressible)
- Gases are about **a million times more compressible** than solids
- For equilibrium, B is always **positive**

Material	Bulk Modulus B (10^9 N m^{-2})
SOLIDS	
Aluminum	72
Brass	61
Copper	140
Glass	37
Iron	100
Nickel	260

Material	Bulk Modulus B (10^9 N m^{-2})
Steel	160
LIQUIDS	
Water	2.2
Ethanol	0.9
Carbon disulphide	1.56
Glycerine	4.76
Mercury	25
GASES	
Air (at STP)	1.0×10^{-4}

📝 **Why are solids least compressible?**

The incompressibility of solids is due to tight coupling between neighboring atoms. In liquids, molecules are also bound but not as strongly as in solids. In gases, molecules are very poorly coupled to their neighbors.

📐 5.4 POISSON'S RATIO (σ or μ)

Observation: When a wire is stretched, it becomes thinner. When compressed, it becomes thicker. This is lateral strain.
Poisson's Discovery: Within elastic limit, lateral strain is directly proportional to longitudinal strain.

Poisson's Ratio = Lateral Strain / Longitudinal Strain

$$\sigma \text{ or } \mu = (\Delta d/d)/(\Delta L/L) = (\Delta d/\Delta L) \times (L/d)$$

Where:
- d = Original diameter
- Δd = Change in diameter (contraction)
- L = Original length
- ΔL = Change in length (elongation)

Nature: Pure number (dimensionless)
Typical Values:
- Steel: 0.28 to 0.30
- Aluminum alloys: ~0.33

Key Points:

- Poisson's ratio is a **pure number** (no dimensions or units)
- Value depends only on **nature of material**

- Theoretical range: **-1 to 0.5**
- For most materials: **0 to 0.5**

⚡ 5.5 ELASTIC POTENTIAL ENERGY IN A STRETCHED WIRE

Concept: When a wire is stretched, work is done against inter-atomic forces. This work is stored as **elastic potential energy** in the wire.

Work Done in Stretching Wire:

$W = (YA/2L) \times l^2$

Or,

$W = (1/2) \times Y \times (l/L)^2 \times AL$

$W = (1/2) \times$ Young's modulus $\times$ strain^2 $\times$ volume

W = (1/2) × stress × strain × volume

Elastic Potential Energy per unit volume:

u = (1/2) × σ × ε

Where:

- Y = Young's modulus
- A = Cross-sectional area
- L = Original length
- l = Elongation
- σ = Stress
- ε = Strain

Physical Meaning: The energy stored is proportional to the square of the strain. This is similar to elastic potential energy in a spring ($U = \frac{1}{2}kx^2$).

📋 SUMMARY TABLE: STRESS, STRAIN & ELASTIC MODULI

Type	Stress	Strain	Chang in Shape
Tensile/Compressive	F/A (perpendicular to opposite faces)	ΔL/L (Longitudinal)	Yes
Shearing	F/A (parallel to opposite surfaces)	θ or Δx/L (Pure shear)	Yes
Hydraulic	p (pressure, perpendicular everywhere)	ΔV/V (Volume change)	No

✏️ SOLVED EXAMPLES

Example 1: Calculating Stress, Elongation & Strain

Problem: A structural steel rod has a radius of 10 mm and a length of 1.0 m. A 100 kN force stretches it along its length. Calculate (a) stress, (b) elongation, and (c) strain on the rod. Young's modulus of structural steel is 2.0×10^{11} N m^{-2}.

Solution:

Given:

- $r = 10$ mm $= 10 \times 10^{-3}$ m $= 0.01$ m
- $L = 1.0$ m
- $F = 100$ kN $= 100 \times 10^{3}$ N
- $Y = 2.0 \times 10^{11}$ N m^{-2}

(a) Stress:

Stress = $F/A = F/(\pi r^2)$

$= (100 \times 10^3 \text{ N}) / (3.14 \times (10 \times 10^{-3})^2 \text{ m}^2)$

$= (100 \times 10^3) / (3.14 \times 10^{-4})$

= 3.18×10^8 N m^{-2}

(b) Elongation:

From $Y = (F/A) \times (L/\Delta L)$

$\Delta L = (F/A) \times L / Y$

$\Delta L = (3.18 \times 10^8 \times 1) / (2.0 \times 10^{11})$

= 1.59×10^{-3} m = 1.59 mm

(c) Strain:

Strain = $\Delta L/L = (1.59 \times 10^{-3}) / 1$

= 1.59×10^{-3} = 0.16%

Example 2: Two Wires Connected End to End

Problem: A copper wire of length 2.2 m and a steel wire of length 1.6 m, both of diameter 3.0 mm, are connected end to end. When stretched by a load, the net elongation is found to be 0.70 mm. Obtain the load applied.

Given:

- L_C = 2.2 m (copper)
- L_S = 1.6 m (steel)
- d = 3.0 mm = 3.0×10^{-3} m (same for both)
- $\Delta L_C + \Delta L_S$ = 0.70 mm = 7.0×10^{-4} m
- $Y_C = 1.1 \times 10^{11}$ N m^{-2} (from table)
- $Y_S = 2.0 \times 10^{11}$ N m^{-2} (from table)

Solution:

Since wires are connected, tension W is same in both.

$W/A = Y_C \times (\Delta L_C/L_C) = Y_S \times (\Delta L_S/L_S)$

$\Delta L_C/\Delta L_S = (Y_S/Y_C) \times (L_C/L_S)$

$= (2.0 \times 10^{11})/(1.1 \times 10^{11}) \times (2.2/1.6) = 2.5$

Also, $\Delta L_C + \Delta L_S = 7.0 \times 10^{-4}$ m

Solving: $\Delta L_C = 5.0 \times 10^{-4}$ m, $\Delta L_S = 2.0 \times 10^{-4}$ m

Now, $W = (A \times Y_C \times \Delta L_C)/L_C$

$A = \pi(1.5 \times 10^{-3})^2$ m^2

$W = \pi(1.5 \times 10^{-3})^2 \times [(5.0 \times 10^{-4} \times 1.1 \times 10^{11})/2.2]$

$W = 1.8 \times 10^2$ N = 180 N

Example 3: Human Pyramid - Bone Compression

Problem: In a human pyramid in a circus, the entire weight of the balanced group is supported by the legs of a performer lying on his back. The combined mass of all persons performing the act and tables, plaques etc. is 280 kg. The mass of the performer at the bottom is 60 kg. Each thighbone (femur) has length 50 cm and effective radius 2.0 cm. Determine the amount by which each thighbone gets compressed.

Given:

- Total mass = 280 kg
- Mass of bottom performer = 60 kg
- Mass supported by legs = 280 - 60 = 220 kg
- L = 0.5 m
- r = 2.0 cm = 0.02 m
- $Y_{bone} = 9.4 \times 10^9$ N m^{-2} (from table)

Solution:

Weight supported = 220 × 9.8 = 2156 N

Weight per thighbone = 2156/2 = 1078 N

$A = \pi r^2 = \pi \times (0.02)^2 = 1.26 \times 10^{-3}$ m^2

$\Delta L = (F \times L)/(Y \times A)$

$= (1078 \times 0.5)/(9.4 \times 10^9 \times 1.26 \times 10^{-3})$

$= 4.55 \times 10^{-5}$ m $= 4.55 \times 10^{-3}$ cm

Fractional compression = $\Delta L/L$ = 0.000091 = **0.0091%**

This is a very small change!

Example 4: Shear Modulus - Lead Slab

Problem: A square lead slab of side 50 cm and thickness 10 cm is subject to a shearing force (on its narrow face) of 9.0×10^4 N. The lower edge is riveted to the floor. How much will the upper edge be displaced?

Given:

- Side = 50 cm = 0.5 m

- Thickness = 10 cm = 0.1 m
- $F = 9.0 \times 10^4$ N
- $A = 0.5 \times 0.1 = 0.05\ m^2$
- L = 0.5 m (height)
- $G_{lead} = 5.6 \times 10^9\ N\ m^{-2}$ (from table)

Solution:

Shearing stress = $F/A = (9.0 \times 10^4)/0.05 = 1.8 \times 10^6\ N\ m^{-2}$

From $G = (F/A)/(\Delta x/L)$

$\Delta x = (F/A) \times L / G$

$= (1.8 \times 10^6 \times 0.5) / (5.6 \times 10^9)$

$= 1.6 \times 10^{-4}$ m = 0.16 mm

Example 5: Bulk Modulus - Water at Depth

Problem: The average depth of Indian Ocean is about 3000 m. Calculate the fractional compression, $\Delta V/V$, of water at the bottom of the ocean, given that the bulk modulus of water is $2.2 \times 10^9\ N\ m^{-2}$. (Take $g = 10\ m\ s^{-2}$)

Given:

- h = 3000 m
- $\rho_{water} = 1000\ kg\ m^{-3}$
- $g = 10\ m\ s^{-2}$
- $B = 2.2 \times 10^9\ N\ m^{-2}$

Solution:

Pressure at depth h: $p = h\rho g$

$= 3000 \times 1000 \times 10$

$= 3 \times 10^7\ N\ m^{-2}$

From $B = -p/(\Delta V/V)$

$\Delta V/V = p/B = (3 \times 10^7)/(2.2 \times 10^9)$

$= 1.36 \times 10^{-2} = 1.36\%$

Water is compressed by 1.36% at this depth.

🏗 6. APPLICATIONS OF ELASTIC BEHAVIOR

6.1 Design of Cranes and Lifting Equipment

Problem: How thick should a steel rope be for a crane with lifting capacity of 10 tonnes?

Solution Approach:

- Load $W = 10$ tonnes $= 10^4$ kg
- Yield strength of mild steel: $\sigma_y = 300 \times 10^6\ N\ m^{-2}$
- For safety, extension should not exceed elastic limit

Area required:

$A \geq W/\sigma_y = Mg/\sigma_y$

$= (10^4 \times 9.8)/(300 \times 10^6)$

$= 3.3 \times 10^{-4}\ m^2$

Radius required: $r \approx 1$ cm

Safety Factor: Usually 10× safety margin is provided, so $r \approx 3$ cm is used.

Why Braided Rope? Instead of single thick wire, multiple thin wires are braided together for flexibility and strength.

6.2 Beam Design for Buildings and Bridges

```
Beam Bending Formula:

δ = Wl³/(4bd³Y)

Where:
• δ = Sagging (bending) at center
• W = Load at center
• l = Length of beam
• b = Breadth
• d = Depth
• Y = Young's modulus
```

Design Principles:

- **Material:** Use material with large Y (steel preferred)
- **Dimensions:** $\delta \propto d^{-3}$ and $\delta \propto b^{-1}$, so increasing depth is more effective than breadth
- **Length:** Keep span l as small as possible
- **Shape:** I-beam cross-section provides large load-bearing surface and depth while reducing weight

Why I-Beam Shape?

The I-shaped cross-section is commonly used because:

- Provides large depth (d) to prevent bending
- Reduces weight without sacrificing strength
- Prevents buckling (twisting)
- Large load-bearing surface at top and bottom
- Cost-effective design

6.3 Maximum Height of Mountains

Question: Why is the maximum height of a mountain on Earth ~10 km?

Analysis:

At the base of a mountain of height h:

- Force per unit area = $h\rho g$ (where ρ = density of rock)
- This creates shearing stress
- Rocks will flow if stress exceeds elastic limit

Calculation:

For typical rock: elastic limit = 30×10^7 N m^{-2}

$h\rho g = 30 \times 10^7$

$h = 30 \times 10^7 / (3 \times 10^3 \times 10)$

$h = 10$ km

This is why Mount Everest (8.85 km) is close to maximum possible height!

6.4 Column and Pillar Design

Design Features:

- **Rounded ends:** Support less load, can buckle
- **Distributed ends:** Support more load, prevent buckling
- Wider base prevents toppling
- Hollow cylinders often used (same strength, less material)

⚠️ IMPORTANT POINTS TO REMEMBER

Critical Concepts

1. About Tension in Wire:
For a wire suspended from ceiling with weight W at bottom, tension at any cross-section is **W**, not 2W (even though ceiling exerts equal and opposite force). Therefore, tensile stress = W/A.

2. Hooke's Law Validity:
Valid only in the **linear part** of stress-strain curve (region O to A). Not valid beyond proportional limit.

3. Applicability of Moduli:

- **Young's and Shear moduli:** Relevant for **solids only**
- **Bulk modulus:** Relevant for **solids, liquids, and gases**

4. Elasticity Misconception:
In daily life, we think materials that stretch more are "more elastic" - this is **wrong**! A material that stretches **less** for a given load is **more elastic** (higher Young's modulus means more elastic).

5. Metals vs Elastomers:
Metals have **larger** Young's modulus than alloys and elastomers. Large Y means material requires large force to produce small changes in length (more rigid, more elastic).

6. Stress is Not a Vector:
Unlike force, stress cannot be assigned a specific direction. Force acting on a portion of body has definite direction, but stress is defined per unit area.

7. Multiple Elastic Constants:
A deforming force in one direction can produce strains in other directions (e.g., Poisson effect). Complete description requires multiple elastic constants, not just one.

📝 PRACTICE QUESTIONS

Numerical Problems

Q1. A steel wire of length 4.7 m and cross-sectional area 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area 4.0×10^{-5} m^2 under a given load. What is the ratio of Young's modulus of steel to that of copper?

Q2. Two wires of diameter 0.25 cm, one made of steel and the other of brass are loaded as shown. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. A mass of 4 kg is attached to steel wire and 6 kg to brass wire. Compute the elongations of the steel and brass wires. ($Y_{steel} = 2.0 \times 10^{11}$ Pa, $Y_{brass} = 0.91 \times 10^{11}$ Pa)

Q3. The edge of an aluminum cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminum is 25 GPa. What is the vertical deflection of this face?

Q4. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. ($Y_{steel} = 2.0 \times 10^{11}$ Pa)

Q5. A steel cable with radius 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 N m^{-2}, what is the maximum load the cable can support?

Q6. Compute the bulk modulus of water from the following data: Initial volume = 100.0 L, Pressure increase = 100.0 atm (1 atm =

1.013×10^5 Pa), Final volume = 100.5 L. Compare with bulk modulus of air.

Q7. What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03×10^3 kg m^{-3}? ($B_{water} = 2.2 \times 10^9$ Pa)

Q8. Compute the fractional change in volume of a glass slab when subjected to a hydraulic pressure of 10 atm. ($B_{glass} = 37 \times 10^9$ Pa)

Q9. How much should the pressure on a liter of water be changed to compress it by 0.10%?

Q10. A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2. Calculate the elongation of the wire when the mass is at the lowest point. ($Y_{steel} = 2.0 \times 10^{11}$ Pa)

Conceptual Questions

Q11. Why is Young's modulus of rubber less than that of steel? What does it signify?

Q12. The stretching of a coil spring is determined by its shear modulus. Explain.

Q13. Why are gas cylinders made of steel and not glass even though glass has higher bulk modulus?

Q14. A wire can sustain a weight of 10 kg without exceeding its elastic limit. Can this same wire be used to sustain a weight of 20 kg? If yes, how?

Q15. Two wires A and B have the same length and area of cross-section. But Young's modulus of A is two times the Young's modulus of B. Which wire will have more elongation when same force is applied?

📖 QUICK FORMULA REFERENCE

All Important Formulas at a Glance

1. STRESS & STRAIN:

- Stress (σ) = F/A
- Longitudinal Strain (ε) = $\Delta L/L$
- Shearing Strain = $\Delta x/L = \tan\theta \approx \theta$
- Volume Strain = $\Delta V/V$

2. YOUNG'S MODULUS:

- $Y = \sigma/\varepsilon = (F/A)/(\Delta L/L)$
- $Y = (F \times L)/(A \times \Delta L)$
- $\Delta L = (F \times L)/(A \times Y)$

3. SHEAR MODULUS:

- $G = (F/A)/(\Delta x/L) = (F \times L)/(A \times \Delta x)$
- $G = (F/A)/\theta = F/(A \times \theta)$
- $\sigma_s = G \times \theta$
- $G \approx Y/3$ (for most materials)

4. BULK MODULUS:

- $B = -p/(\Delta V/V)$
- Compressibility: $k = 1/B = -(1/\Delta p) \times (\Delta V/V)$

5. POISSON'S RATIO:

- σ or μ = (Lateral Strain)/(Longitudinal Strain)
- $\sigma = (\Delta d/d)/(\Delta L/L) = (\Delta d/\Delta L) \times (L/d)$

6. ELASTIC POTENTIAL ENERGY:

- $U = (1/2) \times Y \times (l/L)^2 \times AL$
- u (per unit volume) $= (1/2) \times \sigma \times \varepsilon$
- $u = (1/2) \times Y \times \varepsilon^2$

7. BEAM BENDING:

- $\delta = Wl^3/(4bd^3Y)$

8. GENERAL RELATIONSHIPS:

- Stress = Modulus × Strain
- Modulus = Stress/Strain
- Strain = Stress/Modulus

🧠 MEMORY TIPS & TRICKS

Easy Ways to Remember

1. Remember Y, G, B:

Young's modulus = **Y**anking (pulling/pushing)

Grind (shear) modulus = **G**rinding/sliding

Bulk modulus = **B**ottom of ocean (pressure all around)

2. Units Remember:

All moduli have same unit: **Pa (Pascal)** or N m^{-2}

Strain has **NO unit** (it's a ratio)

3. Elastic vs Plastic:

Elastic = Like **E**lastic band (returns back)

Plastic = Like **P**utty (permanent change)

4. More Elastic Material:

Higher Young's modulus = **MORE elastic** (stiffer, resists deformation)

Steel > Copper > Aluminum > Brass

5. Stress-Strain Curve Points:

A = **A**mazing elastic limit starts

B = **B**oundary of elasticity (yield point)

D = **D**angerous limit (ultimate strength)

E = **E**nd (fracture)

6. Compressibility Order:

Gases >> Liquids >> Solids

(Gases most compressible, solids least)

📚 END OF STUDY MATERIAL 📚

Chapter 8: Mechanical Properties of Solids

Complete Coverage with Concepts, Formulas, Examples & Practice Questions

Math Love Institute

Raipur, Chhattisgarh

For Best Results: Study → Practice → Revise → Excel!