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CHAPTER 9

MECHANICAL PROPERTIES OF FLUIDS

Complete Study Material with Concepts, Derivations & Examples

CBSE Class XI Physics | 2025-26

1. INTRODUCTION

In this chapter, we shall study some common physical properties of liquids and gases. Liquids and gases can flow and are therefore, called **fluids**. It is this property that distinguishes liquids and gases from solids in a basic way.

🔑 Key Understanding

What are Fluids?

- Fluids are substances that can **flow**
- Both **liquids and gases** are fluids
- Unlike solids, fluids have **no definite shape** of their own
- Fluids take the shape of their container

1.1 Difference Between Solids, Liquids and Gases

Property	Solid	Liquid	Gas
Shape	Definite shape	No definite shape (takes container shape)	No definite shape
Volume	Fixed volume	Fixed volume	Fills entire volume of container
Compressibility	Very low	Very low	High
Shear Stress Resistance	Very high	Very low (about million times smaller)	Very low
Molecular Arrangement	Tightly coupled	Moderately coupled	Poorly coupled

📝 **Important Note**

Why are fluids important?

- Fluids are everywhere around us
- Earth has an envelope of air and two-thirds of its surface is covered with water
- Every mammalian body constitutes mostly of water
- All processes in living beings including plants are mediated by fluids

2. PRESSURE

A sharp needle when pressed against our skin pierces it. Our skin, however, remains intact when a blunt object with a wider contact area is pressed against it with the same force. This everyday experience convinces us that both the force and its coverage area are important.

2.1 Definition of Pressure

Average Pressure:

$$P_{av} = F/A$$

Where:

- **F** = Normal force exerted by fluid (in Newton)
- **A** = Area on which force acts (in m^2)
- P_{av} = Average pressure (in $N\ m^{-2}$ or Pa)

Pressure at a Point:

$$P = \lim_{\Delta A \to 0} (\Delta F/\Delta A)$$

Key Properties of Pressure:

- Pressure is a **scalar quantity**
- It is the component of force **normal to the area**
- **SI Unit:** $N\ m^{-2}$ or Pascal (Pa)
- **Dimensions:** $[ML^{-1}T^{-2}]$
- **Named after:** Blaise Pascal (1623-1662)

2.2 Common Units of Pressure

Unit	Value in Pascal (Pa)	Usage
1 atmosphere (atm)	1.013×10^5 Pa	Atmospheric pressure at sea level
1 bar	10^5 Pa	Meteorology
1 millibar	10^2 Pa	Weather reports
1 torr	133 Pa	Medicine, physiology
1 mm of Hg	133 Pa	Blood pressure measurement

2.3 Density

Density Formula:

$$\rho = m/V$$

Where:

- **ρ** = Density (Greek letter rho)
- **m** = Mass of fluid
- **V** = Volume occupied
- **SI Unit:** kg m^{-3}
- **Dimensions:** $[ML^{-3}]$

Relative Density

Relative density = Density of substance / Density of water at 4°C

- It is a **dimensionless** quantity

- Density of water at 4°C = 1.0×10^3 kg m^{-3}
- Example: Relative density of aluminum = 2.7
- So, density of aluminum = 2.7×10^3 kg m^{-3}

2.4 Densities of Common Fluids

Fluid	Density (kg m^{-3})	State
Water	1.00×10^3	Liquid (at STP)
Sea water	1.03×10^3	Liquid
Mercury	13.6×10^3	Liquid
Ethyl alcohol	0.806×10^3	Liquid
Whole blood	1.06×10^3	Liquid
Air	1.29	Gas (at STP)
Oxygen	1.43	Gas (at STP)
Hydrogen	9.0×10^{-2}	Gas (at STP)

** STP = Standard Temperature (0°C) and 1 atm Pressure*

3. PASCAL'S LAW

Pascal's Law Statement:

"The pressure in a fluid at rest is the same at all points if they are at the same height. A change in pressure applied to an enclosed fluid is

> **transmitted undiminished to every point of the fluid and the walls of the containing vessel."**

3.1 Proof of Pascal's Law

Consider a small prismatic element ABC-DEF in the interior of a fluid at rest. Let the fluid exert pressures P_a, P_b, and P_c on the three faces with areas A_a, A_b, and A_c.

Derivation:

For equilibrium:

$F_b \sin \theta = F_c$

$F_b \cos \theta = F_a$

From geometry:

$A_b \sin \theta = A_c$

$A_b \cos \theta = A_a$

Therefore:

$F_b/A_b = F_c/A_c = F_a/A_a$

Or, $\mathbf{P_b = P_c = P_a}$

📝 **Important Conclusions:**

- Pressure is exerted **equally in all directions** in a fluid at rest
- Pressure is **not a vector** quantity (no specific direction)
- Force against any area is always **normal (perpendicular)** to that area
- In a horizontal plane, pressure is **same at all points**

3.2 Variation of Pressure with Depth

Consider a cylindrical element of fluid with base area A and height h in a container. Let P_1 be the pressure at top and P_2 at bottom.

Derivation of Pressure-Depth Relation:

For equilibrium in vertical direction:

$(P_2 - P_1) A = mg$

Mass of fluid element:

$m = \rho V = \rho A h$

Substituting:

$(P_2 - P_1) A = \rho A h g$

$P_2 - P_1 = \rho gh$

Pressure at Depth h:

$$P = P_a + \rho gh$$

Where:

- **P** = Absolute pressure at depth h
- **P_a** = Atmospheric pressure
- **ρ** = Density of fluid
- **g** = Acceleration due to gravity
- **h** = Depth below surface

Gauge Pressure:

$$P_{gauge} = P - P_a = \rho gh$$

Gauge pressure is the **excess pressure** over atmospheric pressure

🔍 Hydrostatic Paradox

Consider three vessels A, B, and C of different shapes connected at the bottom by a horizontal pipe. When filled with water, the level in all three vessels is the same, though they hold different amounts of water.

Reason: Water at the bottom has the same pressure below each vessel because pressure depends only on **height (depth)** and not on the shape or volume of the container.

3.3 Atmospheric Pressure and Barometer

The pressure of the atmosphere at any point is equal to the weight of a column of air of unit cross-sectional area extending from that point to the top of the atmosphere.

Mercury Barometer Formula:

$$P_a = \rho g h$$

Where:

- P_a = Atmospheric pressure
- ρ = Density of mercury = 13.6×10^3 kg m^{-3}
- h = Height of mercury column ≈ 76 cm at sea level

Calculation:

$P_a = 13.6 \times 10^3 \times 9.8 \times 0.76 \approx 1.013 \times 10^5$ Pa = 1 atm

Open Tube Manometer

Used to measure gauge pressure of a gas in a container.

- One end open to atmosphere
- Other end connected to gas container
- Contains liquid (oil or mercury)
- **Gauge pressure = $\rho g h$** (h = height difference)

3.4 Applications of Pascal's Law

Hydraulic Machines

Hydraulic lift, hydraulic brakes, and hydraulic press work on Pascal's law. Fluids are used for transmitting pressure.

Hydraulic Lift Principle:

Two pistons of areas A_1 (small) and A_2 (large) connected by liquid.

$$F_1/A_1 = F_2/A_2$$

Therefore:

$$F_2 = F_1 \times (A_2/A_1)$$

Mechanical Advantage = A_2/A_1

A small force on small piston produces large force on large piston!

Example 1: Hydraulic Lift

Problem: In a hydraulic lift, the piston radii are 5 cm and 15 cm. What force is required on the small piston to lift a car of mass 1350 kg?

Solution:

Given:

- $r_1 = 5$ cm $= 5 \times 10^{-2}$ m
- $r_2 = 15$ cm $= 15 \times 10^{-2}$ m
- Mass of car = 1350 kg
- $g = 9.8$ m s^{-2}

Calculation:

$F_2 = mg = 1350 \times 9.8 = 13{,}230$ N

$A_1 = \pi r_1^2 = \pi(5 \times 10^{-2})^2\ m^2$

$A_2 = \pi r_2^2 = \pi(15 \times 10^{-2})^2\ m^2$

$F_1 = F_2 \times (A_1/A_2) = 13{,}230 \times (5^2/15^2)$

$F_1 = 13{,}230 \times (1/9) =$ **$1470\ N \approx 1.5 \times 10^3\ N$**

Pressure needed:

$P = F_1/A_1 = 1470/(\pi \times 25 \times 10^{-4})$

$P =$ **$1.87 \times 10^5\ Pa \approx 1.85\ atm$**

4. STREAMLINE FLOW

The study of fluids in motion is called **fluid dynamics**. When water flows from a tap slowly, the flow is smooth. When the speed increases, it becomes turbulent.

4.1 Steady Flow

Steady Flow Definition:

The flow of a fluid is said to be **steady** if at any given point, the velocity of each passing fluid particle remains constant in time.

- Velocity at different points in space may be different
- But at any particular point, all particles have the same velocity
- Each particle follows a smooth path
- Paths of particles do not cross each other

4.2 Streamlines

Streamline Definition:

The path taken by a fluid particle under steady flow is called a **streamline**.

It is defined as a curve whose tangent at any point is in the direction of the fluid velocity at that point.

Important: No two streamlines can cross each other (if they did, fluid would have two velocities at that point, which is impossible in steady flow).

4.3 Equation of Continuity

Consider fluid flowing through a pipe of varying cross-section. At three points P, R, and Q with areas A_P, A_R, and A_Q and velocities v_P, v_R, and v_Q:

Derivation of Equation of Continuity:

Mass flowing through P in time Δt:

$\Delta m_P = \rho_P A_P v_P \Delta t$

Similarly at R and Q:

$\Delta m_R = \rho_R A_R v_R \Delta t$

$\Delta m_Q = \rho_Q A_Q v_Q \Delta t$

By conservation of mass:

$\Delta m_P = \Delta m_R = \Delta m_Q$

$\boldsymbol{\rho_P A_P v_P = \rho_R A_R v_R = \rho_Q A_Q v_Q}$

Equation of Continuity:

For incompressible fluids (ρ = constant):

$$Av = \text{constant}$$

Or,

$$A_1 v_1 = A_2 v_2$$

Where:

- **A** = Area of cross-section
- **v** = Velocity of fluid
- **Av** = Volume flux or flow rate (constant throughout pipe)

Key Conclusions:

- Where area is **smaller**, velocity is **larger**
- Where streamlines are **closely spaced**, velocity is **higher**
- This explains why water shoots out faster from narrower sections of a pipe

4.4 Laminar vs Turbulent Flow

Property	Laminar Flow	Turbulent Flow
Speed	Low speeds (below critical speed)	High speeds (above critical speed)
Path	Smooth, parallel layers	Chaotic, irregular motion
Streamlines	Well-defined, don't cross	Form eddies and vortices
Example	Slow-flowing stream	"White water rapids"

5. BERNOULLI'S PRINCIPLE

Bernoulli's equation relates pressure, velocity, and height for a fluid in steady flow. It is based on conservation of energy.

5.1 Derivation of Bernoulli's Equation

Derivation Using Work-Energy Theorem:

Consider fluid flowing through a pipe of varying cross-section at different heights.

Work done on fluid element:

$W = (P_1 - P_2)\,\Delta V$

Change in kinetic energy:

$\Delta KE = \frac{1}{2}\rho\Delta V(v_2^2 - v_1^2)$

Change in potential energy:

$\Delta PE = \rho g \Delta V(h_2 - h_1)$

By work-energy theorem:

$(P_1 - P_2)\Delta V = \frac{1}{2}\rho \Delta V(v_2^2 - v_1^2) + \rho g \Delta V(h_2 - h_1)$

Dividing by ΔV and rearranging:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Bernoulli's Equation:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

In words:

Along a streamline, the sum of:

- **P** = Pressure
- **$\frac{1}{2}\rho v^2$** = Kinetic energy per unit volume
- **ρgh** = Potential energy per unit volume

remains **constant**

⚠️ Assumptions for Bernoulli's Equation:

1. Fluid is **incompressible** (ρ = constant)

2. Flow is **steady** (not turbulent)
3. Flow is **non-viscous** (no internal friction)
4. Flow is along a **streamline**

5.2 Applications of Bernoulli's Principle

Application 1: Speed of Efflux (Torricelli's Law)

Consider a tank with a small hole at height y_1 from bottom. Surface is at height y_2. Taking $y_2 - y_1 = h$:

Torricelli's Law:

If tank is open to atmosphere ($P = P_a$):

$$v = \sqrt{(2gh)}$$

This is the same as velocity of a freely falling body!

If container is under pressure P:

$$v = \sqrt{[2gh + 2(P - P_a)/\rho]}$$

Example 2: Water Tank

Problem: A water tank is 20 m high. There is a hole at the bottom. What is the speed of water coming out?

Solution:

Using Torricelli's law:

$v = \sqrt{(2gh)} = \sqrt{(2 \times 9.8 \times 20)}$

v = √392 = **19.8 m/s ≈ 20 m/s**

Application 2: Dynamic Lift

Dynamic lift is the force that acts on a body (airplane wing, spinning ball) by virtue of its motion through a fluid.

🔍 Magnus Effect

When a ball spins while moving through air:

1. Spinning ball drags air along with it
2. Air velocity above ball = (wind speed) + (drag from spin)
3. Air velocity below ball = (wind speed) - (drag from spin)
4. By Bernoulli: Higher velocity → Lower pressure
5. Pressure below > Pressure above
6. **Net upward force** (dynamic lift)

This is why spinning cricket/tennis balls deviate from parabolic trajectory!

Application 3: Aerofoil (Aircraft Wing)

Aircraft wings are shaped so that air flows faster over the top surface than the bottom:

Lift Force on Wing:

By Bernoulli's equation:

$P_{below} + \frac{1}{2}\rho v_{below}^2 = P_{above} + \frac{1}{2}\rho v_{above}^2$

Since $v_{above} > v_{below}$:

$\mathbf{P_{below} > P_{above}}$

$$\text{Lift Force} = (P_{below} - P_{above}) \times A$$

Where A = wing area

Example 3: Aircraft Lift

Problem: A Boeing aircraft has wing area 500 m^2. Flow speeds are 70 m/s (top) and 63 m/s (bottom). Density of air = 1.3 kg/m^3. Find the lift.

Solution:

Using Bernoulli:

$\Delta P = \frac{1}{2}\rho(v_{top}^2 - v_{bottom}^2)$

$\Delta P = \frac{1}{2} \times 1.3 \times (70^2 - 63^2)$

$\Delta P = 0.65 \times (4900 - 3969) = 0.65 \times 931$

$\Delta P = 605.15$ Pa

Lift Force:

$F = \Delta P \times A = 605.15 \times 500$

F = $\mathbf{3.03 \times 10^5}$ **N**

6. VISCOSITY

Most fluids are not ideal and offer some resistance to motion. This internal friction is called **viscosity**.

6.1 Understanding Viscosity

Consider a fluid between two glass plates. Bottom plate is fixed, top plate moves with velocity v:

- Fluid layer in contact with bottom plate is **stationary**
- Fluid layer in contact with top plate moves with velocity **v**
- Intermediate layers have **velocities increasing uniformly** from 0 to v
- Each layer experiences forces from adjacent layers

> **Key Difference: Solids vs Fluids**
>
> **Solids:** Shear stress is proportional to **shear strain**
>
> **Fluids:** Shear stress is proportional to **rate of shear strain**

6.2 Coefficient of Viscosity

Viscosity Formula:

$$\eta = (F/A)/(v/l) = (F \times l)/(A \times v)$$

Where:

- **η** = Coefficient of viscosity (eta)
- **F** = Tangential force between layers
- **A** = Area of layer
- **v** = Velocity of top plate
- **l** = Distance between plates
- **v/l** = Velocity gradient (rate of shear strain)

SI Unit: poiseuille (Pl) or N s m^{-2} or Pa s

Dimensions: $[ML^{-1}T^{-1}]$

6.3 Viscosity of Common Fluids

Fluid	Temperature (°C)	Viscosity (mPl)
Water	20	1.0
	100	0.3
Blood	37	2.7
Machine Oil	16	113
	38	34
Glycerine	20	830
Honey	—	~2000
Air	0	0.017
	40	0.019

📝 **Temperature Effect on Viscosity**

- **Liquids:** Viscosity **decreases** with temperature (molecules become more mobile)
- **Gases:** Viscosity **increases** with temperature (increased random motion)

6.4 Stokes' Law

When a sphere moves through a viscous fluid, it experiences a viscous drag force.

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Stokes' Law:
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$$F = 6\pi\eta a v$$

Where:

- **F** = Viscous drag force
- **η** = Coefficient of viscosity of fluid
- **a** = Radius of sphere
- **v** = Velocity of sphere

Key Point: Drag force is proportional to velocity (not v^2)

6.5 Terminal Velocity

When a sphere falls through a viscous medium:

1. Initially, it **accelerates** due to gravity
2. As velocity increases, **viscous drag increases**
3. Eventually: Weight = Viscous drag + Buoyant force
4. Net force = 0, acceleration = 0
5. Sphere moves with **constant velocity** = Terminal velocity

Terminal Velocity:

$$v_t = 2a^2(\rho - \sigma)g \,/\, (9\eta)$$

Where:

- v_t = Terminal velocity
- **a** = Radius of sphere
- **ρ** = Density of sphere
- **σ** = Density of fluid
- **g** = Acceleration due to gravity
- **η** = Coefficient of viscosity

Key Observations:

- $v_t \propto a^2$ (depends on square of radius)
- $v_t \propto 1/\eta$ (inversely proportional to viscosity)

Example 4: Copper Ball in Oil

Problem: A copper ball of radius 2.0 mm falls through oil at 20°C with terminal velocity 6.5 cm/s. Density of oil = 1.5×10^3 kg/m^3, density of copper = 8.9×10^3 kg/m^3. Find viscosity of oil.

Solution:

Given:

- $v_t = 6.5$ cm/s $= 6.5 \times 10^{-2}$ m/s
- $a = 2.0$ mm $= 2.0 \times 10^{-3}$ m
- $\rho = 8.9 \times 10^3$ kg/m^3
- $\sigma = 1.5 \times 10^3$ kg/m^3
- $g = 9.8$ m/s^2

Using terminal velocity formula:

$\eta = 2a^2(\rho - \sigma)g / (9v_t)$

$\eta = [2 \times (2 \times 10^{-3})^2 \times (8.9 - 1.5) \times 10^3 \times 9.8] / (9 \times 6.5 \times 10^{-2})$

$\eta = [2 \times 4 \times 10^{-6} \times 7.4 \times 10^3 \times 9.8] / 0.585$

$\eta = 0.579 / 0.585$

$\eta =$ **0.99 kg m^{-1} s^{-1} ≈ 1.0 Pl**

7. SURFACE TENSION

Surface tension is a property of the free surface of liquids that makes the surface behave like a stretched elastic membrane.

7.1 Understanding Surface Tension

Observations:

- Oil and water do not mix
- Water wets glass but mercury does not
- Oil rises up a cotton wick despite gravity
- Small insects can walk on water
- Water drops are spherical
- Soap bubbles are stable

7.2 Molecular Explanation

Why Does Surface Tension Exist?

Molecule inside liquid:

- Surrounded by molecules on all sides
- Net attractive force = 0 (forces balance)
- Has certain negative potential energy

Molecule at surface:

- Surrounded by molecules only on lower half
- Net attractive force pulls it **inward**
- Has **higher** potential energy than inside molecules
- Approximately half the energy needed to remove it completely

Result: Liquid tends to minimize surface area (to minimize energy). Surface behaves like a stretched membrane!

7.3 Surface Energy and Surface Tension

Surface Tension Definition:

Method 1 - As Force per Unit Length:

$$S = F / L$$

Force per unit length acting in plane of surface

Method 2 - As Energy per Unit Area:

$$S = E / A$$

Energy required to increase surface area by unit amount

Both definitions are equivalent!

SI Unit: N/m or J/m^2

Dimensions: $[MT^{-2}]$

Deriving Surface Tension Formula:

Consider a liquid film on a wire frame with movable bar of length l:

When bar is moved by distance d:

Work done = F × d

Increase in area = 2ld (film has two surfaces)

Increase in energy = S × 2ld

By energy conservation:

F × d = S × 2ld

S = F/(2l)

7.4 Surface Tension Values

Liquid	Temperature (°C)	Surface Tension (N/m)	Heat of Vaporization (kJ/mol)
Helium	-270	0.000239	0.115
Oxygen	-183	0.0132	7.1
Ethanol	20	0.0227	40.6
Water	20	0.0727	44.16
Mercury	20	0.4355	63.2

📝 Temperature Effect

Like viscosity, surface tension of liquids generally **decreases with temperature**.

Reason: At higher temperatures, molecules have more kinetic energy and are less tightly bound.

7.5 Angle of Contact

Angle of Contact Definition:

The angle between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid is called the **angle of contact (θ)**.

Force Balance at Contact:

$$S_{la} \cos \theta + S_{sl} = S_{sa}$$

Where:

- S_{la} = Surface tension of liquid-air interface
- S_{sl} = Surface tension of solid-liquid interface
- S_{sa} = Surface tension of solid-air interface
- θ = Angle of contact

Type	Angle θ	Behavior	Example
Acute angle ($\theta < 90°$)	Small	Liquid **wets** the solid Liquid spreads	Water on glass Water on plastic
Obtuse angle ($\theta > 90°$)	Large	Liquid **does not wet** Forms droplets	Water on lotus leaf Mercury on glass

Applications:

- **Wetting agents** (soaps, detergents): Reduce θ so liquid penetrates better

- **Waterproofing agents:** Increase θ so water forms droplets and rolls off

7.6 Drops and Bubbles

Free liquid drops and bubbles are spherical because sphere has minimum surface area for given volume.

Excess Pressure in Drops and Bubbles:

1. Liquid Drop (one surface):

$$P_{inside} - P_{outside} = 2S/r$$

2. Air Bubble in Liquid (one surface):

$$P_{inside} - P_{outside} = 2S/r$$

3. Soap Bubble (two surfaces):

$$P_{inside} - P_{outside} = 4S/r$$

Where:

- **S** = Surface tension
- **r** = Radius of drop/bubble

Key Points:

- **Convex side** always has **higher pressure**
- Smaller radius → Higher pressure difference
- Soap bubble has **two** surfaces (inside and outside)

- This is why you need to blow harder to start a bubble than to continue blowing it!

7.7 Capillary Rise

When a narrow tube (capillary) is dipped in water, water rises in it. This is **capillary rise**.

Derivation of Capillary Rise Formula:

Pressure difference across curved surface:

$P_{inside} - P_{outside} = 2S/r = (2S/a) \cos θ$

(where $r = a/\cos θ$, a = radius of capillary)

For equilibrium:

$P_{atmospheric} = P_{at\ meniscus} + hρg$

Combining:

$hρg = (2S \cos θ)/a$

```
Capillary Rise Formula:
              h = (2S cos θ)/(ρga)
```

Where:

- **h** = Height of liquid rise
- **S** = Surface tension
- **θ** = Angle of contact
- **ρ** = Density of liquid
- **g** = Acceleration due to gravity
- **a** = Radius of capillary tube

Key Observations:

- h ∝ 1/a (smaller tube → higher rise)
- For acute θ: cos θ > 0, so h > 0 (rise)
- For obtuse θ: cos θ < 0, so h < 0 (depression)

Example 5: Capillary Rise of Water

Problem: Calculate capillary rise of water in a tube of radius 0.05 cm. Surface tension of water = 0.073 N/m, θ = 0° (water completely wets glass).

Solution:

Given:

- $a = 0.05$ cm $= 5 \times 10^{-4}$ m
- $S = 0.073$ N/m
- $\theta = 0°$, so $\cos\theta = 1$
- $\rho = 10^3$ kg/m^3
- $g = 9.8$ m/s^2

Calculation:

$h = (2S \cos\theta)/(\rho g a)$

$h = (2 \times 0.073 \times 1)/(10^3 \times 9.8 \times 5 \times 10^{-4})$

$h = 0.146 / 4.9$

h = **0.0298 m ≈ 3 cm**

Applications of Capillarity:

- Sap rise in plants and trees
- Oil rising in lamp wicks
- Water absorption by towels and sponges
- Ink spreading on blotting paper

8. IMPORTANT FORMULAS - QUICK REFERENCE

All Key Formulas at a Glance:

1. PRESSURE & DENSITY:

- Pressure: $P = F/A$
- Density: $\rho = m/V$
- Pressure at depth: $P = P_a + \rho gh$
- Gauge pressure: $P_g = \rho gh$

2. FLUID FLOW:

- Equation of continuity: $A_1v_1 = A_2v_2$
- Bernoulli's equation: $P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$
- Torricelli's law: $v = \sqrt{(2gh)}$

3. HYDRAULIC MACHINES:

- Pascal's law: $F_1/A_1 = F_2/A_2$
- Mechanical advantage: $F_2/F_1 = A_2/A_1$

4. VISCOSITY:

- Viscosity: $\eta = (F/A)/(v/l)$
- Stokes' law: $F = 6\pi\eta av$
- Terminal velocity: $v_t = 2a^2(\rho-\sigma)g/(9\eta)$

5. SURFACE TENSION:

- Surface tension: $S = F/l = E/A$
- Angle of contact: $S_{la} \cos\theta + S_{sl} = S_{sa}$
- Excess pressure in drop: $\Delta P = 2S/r$
- Excess pressure in bubble: $\Delta P = 4S/r$
- Capillary rise: $h = (2S \cos\theta)/(\rho g a)$

9. MEMORY TIPS & TRICKS

🧠 Easy Ways to Remember

1. Pressure-Depth Relation:

$P = P_a + \rho gh$ → "Papa added rho-g-h"

2. Continuity Equation:

Av = constant → "Area × velocity stays constant"

Remember: Narrow pipe → Fast flow (like squeezing garden hose)

3. Bernoulli's Equation:

$P + \frac{1}{2}\rho v^2 + \rho gh = C$

Think: **P**ressure + **K**inetic + **P**otential = Constant

Higher velocity → Lower pressure (airplane wings!)

4. Viscosity vs Temperature:

Liquids: Viscosity **L**owers with temperature

Gases: Viscosity **G**oes up with temperature

5. Drop vs Bubble:

Drop = **D**ouble S (ΔP = **2**S/r)

Bubble = **B**oth sides × 2 (ΔP = **4**S/r)

6. Capillary Rise:

Smaller tube = Higher rise (like straws - thin one drinks better!)

Acute angle = Water **climbs** up

Obtuse angle = Mercury **goes** down

10. COMMON MISTAKES TO AVOID

⚠️ Don't Make These Mistakes!

1. ✗ Confusing **absolute pressure** with **gauge pressure**
   ✓ Remember: Absolute = Atmospheric + Gauge
2. ✗ Using Bernoulli's equation for **turbulent flow**
   ✓ Only valid for steady, non-viscous flow
3. ✗ Forgetting that fluids have **two surfaces** in soap bubbles
   ✓ Bubble: $\Delta P = 4S/r$, Drop: $\Delta P = 2S/r$
4. ✗ Thinking pressure is a **vector**
   ✓ Pressure is a scalar quantity

5. ✗ Using wrong formula for **drop vs bubble**
   ✓ Count the surfaces carefully!
6. ✗ Forgetting **cos θ** in capillary rise
   ✓ $h = (2S \cos \theta)/(\rho g a)$
7. ✗ Confusing **laminar** vs **turbulent** flow
   ✓ Low speed = laminar, High speed = turbulent
8. ✗ Not converting units (cm → m, mm Hg → Pa)
   ✓ Always use SI units in calculations

11. SOLVED PROBLEMS

Problem 1: Swimmer Under Water

Question: What is the pressure on a swimmer 10 m below the surface of a lake?

Solution:

Given:

- $h = 10$ m
- $\rho = 1000$ kg/m^3 (water)
- $g = 10$ m/s^2
- $P_a = 1.01 \times 10^5$ Pa

Using: $P = P_a + \rho g h$

$P = 1.01 \times 10^5 + 1000 \times 10 \times 10$

$P = 1.01 \times 10^5 + 1.0 \times 10^5$

P = **2.01×10^5 Pa ≈ 2 atm**

Interpretation: 100% increase in pressure! At 1 km depth, pressure would be 100 atm!

Problem 2: Spray Pump

Question: A spray pump has a tube with cross-section 8.0 cm^2 and 40 fine holes each of diameter 1.0 mm. If liquid flows at 1.5 m/min inside tube, what is ejection speed through holes?

Solution:

Given:

- $A_1 = 8.0\ cm^2 = 8.0 \times 10^{-4}\ m^2$
- $v_1 = 1.5\ m/min = 1.5/60 = 0.025\ m/s$
- Number of holes = 40
- Diameter of each hole = 1.0 mm = 1.0×10^{-3} m

Total area of holes:

$A_2 = 40 \times \pi(0.5 \times 10^{-3})^2 = 40 \times \pi \times 0.25 \times 10^{-6}$

$A_2 = 3.14 \times 10^{-5}\ m^2$

By equation of continuity:

$A_1v_1 = A_2v_2$

$v_2 = (A_1v_1)/A_2 = (8.0 \times 10^{-4} \times 0.025)/(3.14 \times 10^{-5})$

$v_2 =$ **0.637 m/s ≈ 0.64 m/s**

Problem 3: Soap Bubble Excess Pressure

Question: What is the excess pressure inside a soap bubble of radius 5.00 mm? Surface tension of soap solution = 2.50×10^{-2} N/m. If bubble is at depth 40 cm in soap solution (relative density 1.20), what is pressure inside bubble?

Solution:

Part (a) - Excess Pressure:

For soap bubble (two surfaces):

$\Delta P = 4S/r = (4 \times 2.50 \times 10^{-2})/(5.0 \times 10^{-3})$

$\Delta P = 0.1/(5.0 \times 10^{-3})$ = **20 Pa**

Part (b) - Total Pressure:

At depth h = 40 cm = 0.4 m:

$\rho = 1.20 \times 10^3$ kg/m^3

Pressure outside bubble:

$P_{outside} = P_a + \rho gh$

$P_{outside} = 1.01 \times 10^5 + 1.20 \times 10^3 \times 9.8 \times 0.4$

$P_{outside} = 1.01 \times 10^5 + 4704$

$P_{outside} = 1.057 \times 10^5$ Pa

Pressure inside bubble:

$P_{inside} = P_{outside} + \Delta P$

$P_{inside} = 1.057 \times 10^5 + 20$

P_{inside} = **1.059×10^5 Pa**

12. PRACTICE QUESTIONS

Numerical Problems:

1. The two femurs each of cross-sectional area 10 cm^2 support the upper body mass of 40 kg. Estimate average pressure sustained by femurs.
2. At what depth in an ocean will the gauge pressure be 103 atm? (Density of sea water = 1.03×10^3 kg/m^3)
3. A vertical offshore structure is built to withstand maximum stress of 10^9 Pa. Is it suitable for ocean depth 3 km?
4. A U-tube contains water and spirit separated by mercury. Mercury columns are level with 10 cm water in one arm and 12.5 cm spirit in other. Find specific gravity of spirit.
5. Glycerine flows through horizontal tube of length 1.5 m, radius 1.0 cm. Amount collected = 4.0×10^{-3} kg/s. Find pressure difference. ($\rho = 1.3 \times 10^3$ kg/m^3, $\eta = 0.83$ Pa s)
6. Calculate terminal velocity of copper ball (r = 2 mm) in oil. $\rho_{copper} = 8.9 \times 10^3$ kg/m^3, $\rho_{oil} = 1.5 \times 10^3$ kg/m^3, $\eta = 0.99$ kg/(m·s)
7. Mercury drop of radius 3 mm at room temperature. Surface tension = 4.65×10^{-1} N/m. Find excess pressure.
8. Lower end of capillary tube (d = 2 mm) is 8 cm below water surface. What pressure needed to blow hemispherical bubble? ($S = 7.3 \times 10^{-2}$ N/m)

Conceptual Questions:

1. Why is blood pressure greater at feet than at brain in humans?
2. Why does atmospheric pressure decrease to half at ~6 km height though atmosphere extends beyond 100 km?
3. Why is hydrostatic pressure a scalar even though pressure is force/area?
4. Explain why angle of contact of mercury with glass is obtuse while water-glass is acute.
5. Why does surface tension generally decrease with temperature?

6. Can Bernoulli's equation describe flow through river rapids? Explain.
7. Why does viscosity of liquids decrease but gases increase with temperature?
8. Explain Magnus effect with example of spinning cricket ball.
9. Why are drops and bubbles spherical?
10. Derive the relation for capillary rise in a tube.

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