CHAPTER 13: OSCILLATIONS

Class 11 CBSE Physics

Academic Year: 2025-26

Prepared by: Math Love Institute, Raipur

EaaS - Education as a Service

📚 13.1 INTRODUCTION

In our daily life we come across various kinds of motions. We have learnt about rectilinear motion, projectile motion, uniform circular motion, and orbital motion of planets. In this chapter, we study **oscillatory motion**.

🌟 What is Oscillatory Motion?

Oscillatory motion is a type of periodic motion where the object moves to and fro about a mean position.

Examples of Oscillatory Motion:

- ✓ Pendulum of a wall clock
- ✓ A child on a swing
- ✓ A boat tossing up and down in a river
- ✓ Piston in a steam engine
- ✓ Vibrating strings in musical instruments (sitar, guitar, violin)
- ✓ Membranes in drums
- ✓ Diaphragms in telephones and speakers

💡 Importance of Oscillatory Motion

The study of oscillatory motion is basic to physics:

- Musical instruments produce sound through vibrations
- Sound propagates through vibrations of air molecules
- In solids, atoms vibrate about equilibrium positions (average energy $\propto$ temperature)
- AC power supply voltage oscillates

📚 13.2 PERIODIC AND OSCILLATORY MOTIONS

Periodic Motion: A motion that repeats itself at regular intervals of time

🎯 Period and Frequency

Concept	Symbol	Definition	SI Unit
Period	T	Smallest interval of time after which motion repeats	second (s)
Frequency	ν	Number of repetitions per unit time	hertz (Hz) = s^{-1}

Relationship between Period and Frequency:

$$\nu = 1/T$$

1 hertz = 1 Hz = 1 oscillation per second = 1 s^{-1}

🔑 Key Distinction:

- **Every oscillatory motion is periodic**, but **not every periodic motion is oscillatory**
- Example: Circular motion is periodic but not oscillatory
- Oscillatory motion requires movement to and fro about a mean position

💡 Oscillations vs Vibrations

There is no significant difference between oscillations and vibrations:

- **Oscillations:** Usually refers to low frequency (e.g., branch of a tree)
- **Vibrations:** Usually refers to high frequency (e.g., string of musical instrument)

📝 Example 13.1: Human Heart Beat

Question: On average, a human heart beats 75 times in a minute. Calculate its frequency and period.

Solution:

Step 1: Beat frequency = 75 beats / 1 minute

= 75 / 60 seconds

= 1.25 s^{-1}

= **1.25 Hz**

Step 2: Time period T = 1/ν

= 1 / 1.25 s^{-1}

= **0.8 s**

📊 Displacement in Periodic Motion

Displacement refers to change with time of any physical property under consideration. It can be represented by mathematical functions of time.

Simple Periodic Function:

$f(t) = A \cos \omega t$

Period $T = 2\pi/\omega$

The function is periodic: $f(t) = f(t + T)$

📚 13.3 SIMPLE HARMONIC MOTION (SHM)

Simple Harmonic Motion (SHM):

A particle executes SHM if its displacement from origin varies with time as:

$$x(t) = A \cos(\omega t + \varphi)$$

🎯 Key Parameters of SHM

Parameter	Symbol	Description
Amplitude	A	Maximum displacement from mean position
Angular Frequency	ω	$\omega = 2\pi/T = 2\pi\nu$ (in rad/s)
Phase	$\omega t + \varphi$	Time-dependent quantity determining state of motion
Phase Constant	φ	Value of phase at $t = 0$ (initial phase)

🌟 Characteristics of SHM:

- ✓ Displacement varies sinusoidally with time
- ✓ Amplitude A is fixed for a given SHM
- ✓ Displacement varies between +A and -A
- ✓ Angular frequency $\omega = 2\pi/T$
- ✓ Period T is independent of amplitude

📝 Example 13.2: Identifying Periodic Functions

Question: Which of the following represent (a) simple harmonic and (b) periodic but not simple harmonic motion?

(1) $\sin \omega t + \cos \omega t$

(2) $\sin^2 \omega t$

Answer:

(1) $\sin \omega t + \cos \omega t$

$= \sin \omega t + \sin(\pi/2 - \omega t)$

$= 2 \cos(\pi/4) \sin(\omega t - \pi/4)$

$= \sqrt{2} \sin(\omega t - \pi/4)$

This is SHM with period $T = 2\pi/\omega$ and phase angle $-\pi/4$

(2) $\sin^2 \omega t$

$= \frac{1}{2} - \frac{1}{2} \cos 2\omega t$

This is periodic with period $T = \pi/\omega$

It represents harmonic motion with equilibrium at ½ instead of zero

📚 13.4 SHM AND UNIFORM CIRCULAR MOTION

🔄 Connection Between SHM and Circular Motion

Key Insight: The projection of uniform circular motion on a diameter of the circle follows simple harmonic motion.

Setup:

- Particle P moves uniformly on circle of radius A
- Angular speed = ω
- Initial angle with x-axis = φ

Projection on x-axis:

$$x(t) = A \cos(\omega t + \varphi)$$

This is the defining equation of SHM!

💡 Important Note:

Despite this connection, the **force** acting on a particle in linear SHM is very different from the centripetal force in uniform circular motion.

📚 13.5 VELOCITY AND ACCELERATION IN SHM

🚀 Velocity in SHM

Given displacement: $x(t) = A\cos(\omega t + \varphi)$

Velocity: $v(t) = dx/dt$

$$v(t) = -\omega A \sin(\omega t + \varphi)$$

Maximum velocity: $v_{max} = \omega A$

⚡ Acceleration in SHM

Acceleration: $a(t) = dv/dt$

$$a(t) = -\omega^2 A \cos(\omega t + \varphi) = -\omega^2 x(t)$$

Maximum acceleration: $a_{max} = \omega^2 A$

🔑 Key Properties:

- ✓ Acceleration is proportional to displacement: **$a = -\omega^2 x$**
- ✓ Acceleration is always directed toward mean position
- ✓ For $x > 0$, $a < 0$ (acceleration toward center)
- ✓ For $x < 0$, $a > 0$ (acceleration toward center)
- ✓ Velocity and displacement differ in phase by $\pi/2$
- ✓ Acceleration and displacement differ in phase by π

📝 Example 13.5: SHM Calculations

Question: A body oscillates with SHM according to: $x = 5\cos[2\pi t + \pi/4]$ (SI units)

At $t = 1.5$ s, calculate: (a) displacement (b) speed (c) acceleration

Solution:

Angular frequency $\omega = 2\pi\ s^{-1}$, Period $T = 1$ s

(a) Displacement at t = 1.5 s:

$x = 5\cos[2\pi \times 1.5 + \pi/4]$

$= 5\cos[3\pi + \pi/4]$

$= -5 \times 0.707$

= **-3.535 m**

(b) Speed:

$v = -\omega A \sin(\omega t + \varphi)$

$= -(2\pi) \times 5 \times \sin[3\pi + \pi/4]$

$= 10\pi \times 0.707$

= **22 m/s**

(c) Acceleration:

$a = -\omega^2 x$

$= -(2\pi)^2 \times (-3.535)$

= **140 m/s²**

📚 13.6 FORCE LAW FOR SIMPLE HARMONIC MOTION

🎯 Force in SHM

Using Newton's Second Law:

$F = ma = m(-\omega^2 x)$

$$F(t) = -kx(t)$$

where $k = m\omega^2$

or

$$\omega = \sqrt{(k/m)}$$

🌟 Characteristics of Force in SHM:

- ✓ Force is proportional to displacement: $F \propto -x$
- ✓ Force is always directed toward mean position (restoring force)
- ✓ Negative sign indicates force opposes displacement
- ✓ This is called **Hooke's Law**

Time Period of SHM:

$$T = 2\pi/\omega = 2\pi\sqrt{(m/k)}$$

📝 Example 13.6: Two Springs System

Question: Two identical springs of constant k are attached to a block of mass m (one on each side). When displaced, show it executes SHM and find the period.

Solution:

Step 1: When block is displaced by x to the right:

- Left spring elongates by x: Force $F_1 = -kx$ (pulling left)
- Right spring compresses by x: Force $F_2 = -kx$ (pushing left)

Step 2: Net force:

$F = F_1 + F_2 = -kx + (-kx) = \mathbf{-2kx}$

Step 3: This is SHM with effective spring constant $k_{eff} = 2k$

Period:

$$T = 2\pi\sqrt{(m/2k)}$$

📚 13.7 ENERGY IN SIMPLE HARMONIC MOTION

⚡ Kinetic Energy in SHM

Kinetic Energy:

$K = \frac{1}{2}mv^2$

$= \frac{1}{2}m[\omega A \sin(\omega t + \varphi)]^2$

$$K = \frac{1}{2}kA^2 \sin^2(\omega t + \varphi)$$

(since $k = m\omega^2$)

Note: KE is zero at extreme positions and maximum at mean position

🔋 Potential Energy in SHM

Potential Energy:

For conservative force $F = -kx$

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \varphi)$$

Note: PE is maximum at extreme positions and zero at mean position

💫 Total Energy in SHM

Total Energy:

$E = K + U$

$= \frac{1}{2}kA^2 \sin^2(\omega t + \varphi) + \frac{1}{2}kA^2 \cos^2(\omega t + \varphi)$

$= \frac{1}{2}kA^2[\sin^2(\omega t + \varphi) + \cos^2(\omega t + \varphi)]$

$$E = \frac{1}{2}kA^2 = \text{constant}$$

Total energy is independent of time!

🔑 Energy Conservation in SHM:

- ✓ Total mechanical energy remains constant
- ✓ Energy continuously transforms between K and U
- ✓ At mean position ($x = 0$): $E = K$ (all kinetic)
- ✓ At extremes ($x = \pm A$): $E = U$ (all potential)
- ✓ Both K and U repeat with period $T/2$

📝 Example 13.7: Energy Calculations

Question: A 1 kg block attached to spring (k = 50 N/m) is pulled 10 cm from equilibrium and released. Calculate K, U, and E when block is 5 cm from mean position.

Solution:

Step 1: Angular frequency

$\omega = \sqrt{(k/m)} = \sqrt{(50/1)} = 7.07$ rad/s

Step 2: Displacement equation

$x(t) = 0.1 \cos(7.07t)$

Step 3: When x = 5 cm = 0.05 m:

$0.05 = 0.1 \cos(7.07t)$

$\cos(7.07t) = 0.5$, $\sin(7.07t) = \sqrt{3}/2 = 0.866$

Step 4: Velocity at x = 5 cm:

$v = 0.1 \times 7.07 \times 0.866 = 0.61$ m/s

Step 5: Kinetic Energy:

$K = ½mv^2 = ½ \times 1 \times (0.61)^2 =$ **0.19 J**

Step 6: Potential Energy:

$U = ½kx^2 = ½ \times 50 \times (0.05)^2 =$ **0.0625 J**

Step 7: Total Energy:

E = K + U = **0.25 J**

Verification: $E = ½kA^2 = ½ \times 50 \times (0.1)^2 = 0.25$ J ✓

📚 13.8 THE SIMPLE PENDULUM

🕰️ Historical Note

Galileo measured the periods of a swinging chandelier in a church by his pulse beats. He observed that the motion was periodic.

🎯 Simple Pendulum Setup

Components:

- Small bob of mass m
- Inextensible massless string of length L
- Fixed rigid support
- Bob oscillates in a plane about vertical line

Derivation of Period

Forces on bob:

- Tension T along string
- Weight mg vertically downward

Step 1: Let θ = angle with vertical

Restoring torque $\tau = -L(mg \sin \theta)$

Step 2: Using rotational motion:

$I\alpha = -mgL \sin \theta$

where I = moment of inertia, α = angular acceleration

Step 3: For small θ (in radians): $\sin \theta \approx \theta$

$$\alpha = -(mgL/I)\theta$$

Step 4: This is equation for SHM with:

$\omega = \sqrt{mgL/I}$

Step 5: For simple pendulum, $I = mL^2$

$\omega = \sqrt{mgL/mL^2} = \sqrt{g/L}$

Final Result:

$$T = 2\pi\sqrt{L/g}$$

🔑 Key Points About Simple Pendulum:

- ✓ Period depends only on length and g
- ✓ Period is independent of mass of bob
- ✓ Period is independent of amplitude (for small angles)
- ✓ SHM approximation valid for angles < 20°

θ (degrees)	θ (radians)	sin θ
0	0	0
5	0.087	0.087
10	0.174	0.174
15	0.262	0.259
20	0.349	0.342

Note: *For $\theta \leq 20°$, $\sin\theta \approx \theta$ (in radians) - approximation is good!*

📝 Example 13.8: Seconds Pendulum

Question: What is the length of a simple pendulum which ticks seconds?

Solution:

A pendulum that ticks seconds has period $T = 2$ s

From formula: $T = 2\pi\sqrt{(L/g)}$

Rearranging:

$L = gT^2/(4\pi^2)$

Substituting: $g = 9.8$ m/s², $T = 2$ s

$L = (9.8 \times 4)/(4\pi^2)$

$L = 39.2/(39.48)$

$L \approx 1$ m

🎯 SUMMARY

📌 Key Concepts

Concept	Formula/Description
Periodic Motion	Motion that repeats at regular intervals
Period T	Time for one complete oscillation
Frequency v	$v = 1/T$ (in Hz)
SHM Displacement	$x(t) = A\cos(\omega t + \varphi)$
Angular Frequency	$\omega = 2\pi/T = 2\pi v$
Velocity in SHM	$v(t) = -\omega A\sin(\omega t + \varphi)$
Acceleration in SHM	$a(t) = -\omega^2 x(t)$
Force Law	$F = -kx$
ω for Spring	$\omega = \sqrt{(k/m)}$
Period for Spring	$T = 2\pi\sqrt{(m/k)}$
Total Energy	$E = \frac{1}{2}kA^2$ = constant
Simple Pendulum Period	$T = 2\pi\sqrt{(L/g)}$

🎓 IMPORTANT EXAM QUESTIONS

Section A: Multiple Choice Questions (1 mark)

Q1. Which of the following is NOT an example of SHM?

(a) Motion of simple pendulum for small amplitude ✓
(b) Motion of a ball bouncing between two walls
(c) Vibration of guitar string
(d) Mass attached to spring

Answer: (b) - Ball bouncing involves non-elastic collisions

Q2. The acceleration of a particle executing SHM is:

(a) Constant
(b) Maximum at mean position
(c) Maximum at extreme position ✓
(d) Always in direction of motion

Answer: (c) - $a = -\omega^2 x$, maximum when $|x| = A$

Q3. The period of SHM depends on:

(a) Amplitude only
(b) Initial phase only
(c) Mass and spring constant ✓
(d) Both amplitude and phase

Answer: (c) - $T = 2\pi\sqrt{(m/k)}$, independent of A and φ

Section B: Short Answer Questions (2-3 marks)

Q4. Distinguish between periodic motion and oscillatory motion with examples.

Answer:

- **Periodic Motion:** Motion that repeats after regular intervals. Example: Revolution of Earth around Sun, rotation of Earth
- **Oscillatory Motion:** To and fro motion about mean position. Example: Pendulum, mass-spring system
- **Key:** Every oscillatory motion is periodic, but not all periodic motions are oscillatory (e.g., circular motion is periodic but not oscillatory)

Q5. What is the phase difference between displacement and velocity in SHM?

Answer:

The phase difference between displacement and velocity in SHM is **$\pi/2$ radians (90°)**.

- When displacement is maximum (at extreme positions), velocity is zero
- When displacement is zero (at mean position), velocity is maximum
- Mathematically: $x = A\cos(\omega t + \varphi)$, $v = -\omega A\sin(\omega t + \varphi) = \omega A\cos(\omega t + \varphi + \pi/2)$

Section C: Long Answer Questions (5 marks)

Q6. Derive expression for time period of simple pendulum.

Answer: [Complete derivation as shown in section 13.8]

Key Steps:

1. Identify forces: Tension T and Weight mg
2. Calculate restoring torque: $\tau = -L(mg \sin \theta)$
3. Apply rotational equation: $I\alpha = -mgL \sin \theta$
4. Use small angle approximation: $\sin \theta \approx \theta$
5. Show equation has form of SHM: $\alpha = -\omega^2\theta$
6. Find $\omega = \sqrt{(g/L)}$
7. Calculate period: $T = 2\pi/\omega = 2\pi\sqrt{(L/g)}$

🏆 FINAL SUCCESS TIPS

🌟 Master These Concepts:

- ✨ SHM is characterized by $F = -kx$ (restoring force proportional to displacement)
- ✨ Displacement, velocity, and acceleration all vary sinusoidally
- ✨ Energy oscillates between kinetic and potential, but total remains constant
- ✨ Period is independent of amplitude in SHM
- ✨ Connection between circular motion and SHM is fundamental
- ✨ Simple pendulum formula $T = 2\pi\sqrt{L/g}$ is frequently tested
- ✨ Phase relationships between x, v, and a must be clear

💪 KEY TO SUCCESS:

"Understanding SHM Opens Doors to Waves, Sound, and Modern Physics!"

📚 Must Practice:

- ☑ All derivations (velocity, acceleration, energy, simple pendulum)
- ☑ 15 numerical problems on spring-mass systems
- ☑ 10 problems on simple pendulum
- ☑ Phase relationship problems
- ☑ Energy conservation problems
- ☑ All NCERT examples and exercises

📚 Study Material Information

This comprehensive study material on **Oscillations** has been prepared following the latest CBSE curriculum and examination pattern for Class 11 Physics. The content includes detailed explanations of periodic and oscillatory motions, simple harmonic motion, relationship with circular motion, velocity and acceleration in SHM, force law, energy conservation, simple pendulum, complete derivations from NCERT, and practice questions aligned with current board exam format.

Key Features of This Material:

- Complete chapter coverage with crystal-clear concepts
- Period, frequency, amplitude, phase explained thoroughly
- Connection between SHM and uniform circular motion
- Step-by-step derivations with clear explanations
- Force law and Hooke's law applications
- Energy conservation in SHM with examples
- Simple pendulum complete derivation
- All NCERT solved examples
- Multiple choice and descriptive practice questions
- Formula summary and important concepts
- Exam-focused tips and strategies

EaaS - Education as a Service Model
For latest updates and more resources, visit: **www.mathlove.in**

Disclaimer: This material is prepared as a comprehensive study aid for Class 11 students. While every effort has been made to ensure accuracy and alignment with CBSE curriculum, students are advised to refer to their NCERT textbooks and official CBSE guidelines for examination preparation. This material covers Chapter 13: Oscillations from Class 11 Physics NCERT textbook (Reprint 2025-26).