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REAL NUMBERS

🌟 INTERESTING MATHEMATICAL FACTS

Did You Know?

🔢 **The number 2 is special!** It's the only even prime number. All other prime numbers are odd. This makes 2 unique in the entire world of mathematics!

🎯 **Prime numbers never end!** Greek mathematician Euclid proved over 2,300 years ago that there are infinitely many prime numbers. No matter how large a number you find, there's always a bigger prime!

🧮 **The Fundamental Theorem of Arithmetic** was known to ancient mathematicians, but Carl Friedrich Gauss gave the first rigorous proof in 1801. He was just 24 years old!

⭐ **Largest known prime number (as of 2024):** Has over 24 million digits! If you printed it, it would fill about 9,000 pages.

🔐 **Your online security depends on this chapter!** Banking encryption, credit card transactions, and internet security all use prime factorization. Without it, online shopping wouldn't be safe!

💡 **The √2 story:** Ancient Greeks discovered √2 was irrational around 500 BC. Legend says the mathematician who proved it was thrown into the sea because it challenged their belief that everything could be expressed as a fraction!

📋 CHAPTER OVERVIEW

Chapter	Real Numbers
Weightage	6 Marks (as per CBSE marking scheme)
1 Mark Questions	1
2 Mark Questions	1
3 Mark Questions	1

Topics Covered:

1. Euclid's Division Lemma
2. Fundamental Theorem of Arithmetic
3. HCF and LCM using Prime Factorization
4. Proving Irrationality of Numbers
5. Decimal Expansion of Rational Numbers

🎯 SECTION 1: THE FUNDAMENTAL THEOREM OF ARITHMETIC

Key Concept:

Fundamental Theorem of Arithmetic (FTA):
Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

What This Means:

Any composite number has ONE and ONLY ONE prime factorization (order doesn't matter).

Example:
• $12 = 2 \times 2 \times 3 = 2^2 \times 3$
• $12 = 3 \times 2 \times 2$ (same factorization, different order)

Prime Factorization Method:

Steps:

1. Divide the number by the smallest prime (2)
2. Continue dividing by 2 until you can't divide evenly
3. Move to next prime (3, 5, 7, 11...)
4. Continue until quotient becomes 1

Example: Factorize 96

```
96 = 2 × 48
   = 2 × 2 × 24
   = 2 × 2 × 2 × 12
   = 2 × 2 × 2 × 2 × 6
   = 2 × 2 × 2 × 2 × 2 × 3
   = 2⁵ × 3
```


💡 TIPS & TRICKS

✓ **Quick Divisibility Check:**

- **By 2:** Last digit is even
- **By 3:** Sum of digits divisible by 3
- **By 5:** Last digit is 0 or 5
- **By 11:** Difference of sum of alternate digits is 0 or divisible by 11

✓ **For Large Numbers:** Use factor tree method - it's faster and less error-prone

✓ **Always write in ascending order:** $p_1 \leq p_2 \leq p_3$... for uniqueness

⚠️ COMMON MISTAKES TO AVOID

❌ **Mistake 1:** Forgetting to write answer in exponential form

- Wrong: $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$
- Correct: $96 = 2^5 \times 3$

❌ **Mistake 2:** Not checking if all factors are prime

- $36 = 4 \times 9$ ❌ (4 and 9 are not prime)
- $36 = 2^2 \times 3^2$ ✓

❌ **Mistake 3:** Missing factors in large numbers

- Always double-check by multiplying back

🎯 SECTION 2: HCF AND LCM USING PRIME FACTORIZATION

Key Formulas:

HCF = Product of **smallest power** of each **common prime factor**

LCM = Product of **greatest power** of each **prime factor** (common + uncommon)

Important Relationship:
HCF(a, b) × LCM(a, b) = a × b
[For TWO numbers only]

Detailed Example:

Find HCF and LCM of 12 and 18

Step 1: Prime factorize both numbers

- $12 = 2^2 \times 3^1$
- $18 = 2^1 \times 3^2$

Step 2: Find HCF

- Common prime factors: 2 and 3
- Smallest powers: 2^1 and 3^1
- **HCF = $2^1 \times 3^1 = 6$**

Step 3: Find LCM

- All prime factors: 2 and 3
- Greatest powers: 2^2 and 3^2
- **LCM = $2^2 \times 3^2 = 4 \times 9 = 36$**

Step 4: Verify

- HCF × LCM = 6 × 36 = 216
- 12 × 18 = 216 ✓

💡 SHORTCUT FOR LCM

If you know HCF, use:

$$LCM(a, b) = (a \times b) / HCF(a, b)$$

Example: HCF(96, 404) = 4

```
LCM(96, 404) = (96 × 404) / 4 = 9696
```

⚠️ COMMON MISTAKES

❌ **Mistake 1:** Using the formula HCF × LCM = a × b × c for three numbers

- This formula works ONLY for two numbers!

❌ **Mistake 2:** Taking highest power for HCF or lowest for LCM

- HCF → Smallest power of common factors
- LCM → Greatest power of all factors

❌ **Mistake 3:** Forgetting uncommon prime factors in LCM

- LCM must include ALL prime factors from both numbers

🎯 SECTION 3: PROVING IRRATIONALITY

Key Theorems:

Theorem 1.2: If p is a prime number and p divides a^2, then p divides a.

Theorem 1.3: $\sqrt{2}$ is irrational (and similarly $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{11}$... for all primes)

Standard Proof Structure (Proof by Contradiction):

To Prove: $\sqrt{p}$ is irrational (where p is prime)

Steps:

1. **Assume opposite:** Assume $\sqrt{p}$ is rational
2. **Express as fraction:** $\sqrt{p} = a/b$ (where a, b are coprime)
3. **Square both sides:** $p = a^2/b^2 \rightarrow pb^2 = a^2$
4. **Show p divides a^2:** Since $pb^2 = a^2$, p divides a^2
5. **Use Theorem 1.2:** If p divides a^2, then p divides a
6. **Substitute a = pc:** $pb^2 = (pc)^2 \rightarrow pb^2 = p^2c^2 \rightarrow b^2 = pc^2$
7. **Show p divides b:** Since $b^2 = pc^2$, p divides b^2 and therefore p divides b
8. **Find contradiction:** Both a and b are divisible by p (contradicts coprime assumption)
9. **Conclude:** Our assumption was wrong, $\sqrt{p}$ is irrational

💡 PROOF WRITING TIPS

✓ **Always start with:** "Let us assume, to the contrary, that √p is rational"

✓ **State coprime clearly:** "where a and b are coprime (no common factor except 1)"

✓ **Show both divisible:** Prove p divides both a AND b for contradiction

✓ **State contradiction:** "This contradicts the fact that a and b are coprime"

✓ **Conclude properly:** "Therefore, our assumption was wrong and √p is irrational"

Proving Sum/Product is Irrational:

Type 1: Sum with rational (a + √b)

Prove: 5 + √3 is irrational

```
Assume 5 + √3 = a/b (rational)
→ √3 = a/b - 5 = (a - 5b)/b (rational)
But √3 is irrational → Contradiction!
```

Type 2: Product with rational (a√b)

Prove: 3√2 is irrational

```
Assume 3√2 = a/b (rational)
→ √2 = a/(3b) (rational)
But √2 is irrational → Contradiction!
```

📝 SECTION 4: PREVIOUS YEARS' BOARD QUESTIONS

1 MARK QUESTIONS

Q1. Express 140 as a product of its prime factors. (2023)

Solution:

```
140 = 2 × 70
    = 2 × 2 × 35
    = 2 × 2 × 5 × 7
    = 2² × 5 × 7
```

Q2. Can 6^n end with the digit 0 for any natural number n? Give reason. (2022)

Solution:

No. For a number to end in 0, it must be divisible by 10 = 2 × 5.

Prime factorization of $6^n = (2 \times 3)^n = 2^n \times 3^n$

Since there's no factor of 5, 6^n cannot end with 0.

2 MARK QUESTIONS

Q3. Find HCF and LCM of 26 and 91 and verify that HCF × LCM = Product of two numbers. (2023)

Solution:

```
26 = 2 × 13
91 = 7 × 13

HCF = 13 (common factor)
LCM = 2 × 7 × 13 = 182 (all factors with highest power)

Verification:
HCF × LCM = 13 × 182 = 2366
26 × 91 = 2366 ✓
```

Q4. Prove that √5 is irrational. (2024)

Solution:

Let us assume √5 is rational.

So √5 = a/b where a, b are coprime integers, b ≠ 0

Squaring: $5 = a^2/b^2$

→ $5b^2 = a^2$

→ 5 divides a^2 → 5 divides a

Let a = 5c

→ $5b^2 = (5c)^2 = 25c^2$

→ $b^2 = 5c^2$

→ 5 divides b^2 → 5 divides b

So both a and b are divisible by 5.

This contradicts our assumption that a and b are coprime.

Therefore, √5 is irrational.

3 MARK QUESTIONS

Q5. Find HCF and LCM of 12, 15 and 21 by prime factorisation method. (2023)

Solution:

```
12 = 2² × 3
15 = 3 × 5
21 = 3 × 7

HCF = 3¹ = 3 (smallest power of common factor)
LCM = 2² × 3 × 5 × 7 = 420 (greatest power of each factor)
```

Q6. Prove that 3 + 2√5 is irrational. (2024)

Solution:

Let us assume 3 + 2√5 is rational.

So 3 + 2√5 = a/b where a, b are integers, b ≠ 0

```
→ 2√5 = a/b - 3
→ 2√5 = (a - 3b)/b
→ √5 = (a - 3b)/(2b)
```

Since a, b are integers, (a - 3b)/(2b) is rational.

This means √5 is rational.

But this contradicts the fact that √5 is irrational.

Therefore, our assumption was wrong. Hence, 3 + 2√5 is irrational.

💯 EXAM STRATEGY & TIME MANAGEMENT

Question Type	Time	Strategy
1 Mark	30 sec - 1 min	Direct application of formula, Quick factorization, Yes/No with reason
2 Mark	2-3 minutes	Show working clearly, State formulas used, Verify answer when asked
3 Mark	4-5 minutes	Complete step-by-step solution, For proofs: Follow standard structure, Don't skip steps

Time Allocation for This Chapter:

- Total marks: 6
- Recommended time: 10-12 minutes
- Check answers: 2 minutes

📚 IMPORTANT FORMULAS - QUICK REFERENCE

1. HCF × LCM = a × b (for two numbers only)

2. HCF = Product of smallest power of common prime factors

3. LCM = Product of greatest power of all prime factors

4. If p is prime and $p|a^2$, then $p|a$

5. For number to end in 0: Must have factors 2 and 5

6. Euclid's Division Lemma: $a = bq + r$, where $0 \leq r < b$

🚀 LAST MINUTE REVISION CHECKLIST

Theory:

- ☐ Define Fundamental Theorem of Arithmetic
- ☐ State Theorem 1.2 (p divides a^2 → p divides a)
- ☐ Know structure of irrationality proofs
- ☐ Remember: HCF × LCM = a × b (two numbers only)

Methods:

- ☐ Prime factorization using factor tree
- ☐ Finding HCF and LCM from prime factors
- ☐ Euclid's division algorithm
- ☐ Proof by contradiction structure

Common Questions:

- ☐ Can n^n end with 0? (Check for factors 2 and 5)
- ☐ Prove √p is irrational (standard proof)
- ☐ Find HCF and LCM and verify relationship
- ☐ Show a + b√c is irrational

📞 NEED HELP?

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