AREAS RELATED TO CIRCLES

CHAPTER 11 - CLASS 10 MATHEMATICS

CBSE Board Exam 2025-26

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🎽 COMPREHENSIVE STUDY MATERIAL - SECTORS AND SEGMENTS 🎽

🎯 CHAPTER AT A GLANCE

Chapter Overview:

- **Chapter Name:** Areas Related to Circles
- **Main Topics:** Sector, Segment, Arc Length, Area Calculations
- **Weightage in Board Exam:** 8-10 marks
- **Difficulty Level:** Medium to High
- **Question Types:** 2 marks, 3 marks, 5 marks
- **Prerequisites:** Circle properties, Trigonometry, Area of triangle
- **Total NCERT Exercise Questions:** 14 questions (Exercise 11.1)
- **Time Required:** 12-15 days for complete mastery
- **Practice Problems Needed:** Minimum 80-100 problems

📖 INTRODUCTION

Real-Life Applications:

This chapter deals with the areas of sectors and segments of circles. These concepts have numerous real-life applications:

- 🎡 **Design of circular parks and gardens** with sector-shaped flower beds
- 🍕 **Pizza slices** - each slice is a sector of the circular pizza
- ⏰ **Clock face areas** covered by hour and minute hands
- ☂️ **Umbrella sections** - ribs divide the circle into sectors
- 🚗 **Windshield wipers** - area cleaned by wiper blades
- 💡 **Lighthouse coverage** - warning light covers a sector of the sea
- 🐴 **Grazing area of tethered animals** - sector of a circle

🔑 KEY DEFINITIONS (MUST MEMORIZE!)

1. Sector of a Circle:

A **sector** is the region enclosed by two radii of a circle and the arc between them.

Think of it as: A pizza slice - bounded by two radii and the curved edge (arc).

Key Points:

- **Minor Sector:** The smaller region (angle < 180°)
- **Major Sector:** The larger region (angle > 180°)
- **Angle of Sector:** The angle formed by two radii at the center

- Unless specified, "sector" means minor sector

2. Segment of a Circle:

A **segment** is the region enclosed by a chord and the arc between the chord's endpoints.

Think of it as: The area between a straight line (chord) and the curved edge (arc) - like a bow shape.

Key Points:

- **Minor Segment:** The smaller region
- **Major Segment:** The larger region
- The perpendicular from center to chord bisects both the chord and the segment
- Unless specified, "segment" means minor segment

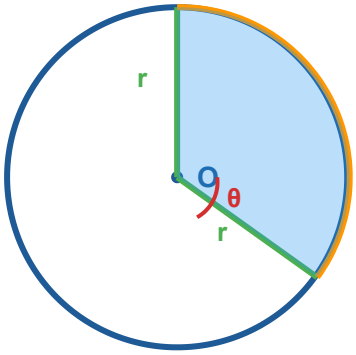
3. Arc of a Circle:

An **arc** is a continuous portion of the circumference of a circle.

Arc Length: The distance along the curved line forming the arc.

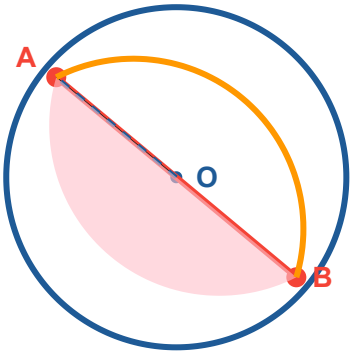
Visual Understanding: Sector and Segment

SECTOR



Shaded Region = SECTOR

SEGMENT



Shaded Region = SEGMEN

💯 ESSENTIAL FORMULAS (MEMORIZE THESE!)

FORMULA 1: Area of Sector

$$\text{Area of Sector} = (\theta/360) \times \pi r^2$$

where **r** = radius of circle, **θ** = angle of sector in degrees

FORMULA 2: Length of Arc

$$\text{Arc Length} = (\theta/360) \times 2\pi r$$

where **r** = radius of circle, **θ** = angle of sector in degrees

FORMULA 3: Area of Segment

$$\text{Area of Segment} = \text{Area of Sector} - \text{Area of Triangle}$$

$$= (\theta/360) \times \pi r^2 - \text{Area of } \triangle OAB$$

where △OAB is the triangle formed by two radii and the chord

📝 Important Relations:

For Major Sector:

Area of Major Sector = **πr² − Area of Minor Sector**

OR

Area of Major Sector = **[(360−θ)/360] × πr²**

For Major Segment:

Area of Major Segment = **πr² − Area of Minor Segment**

Special Cases:

- **Semicircle (θ = 180°):** Area = πr²/2
- **Quadrant (θ = 90°):** Area = πr²/4
- **One-third circle (θ = 120°):** Area = πr²/3

📊 DETAILED FORMULA DERIVATION

Understanding Area of Sector Using Unitary Method:

Step 1: We know that the area of a complete circle = **πr²**

This corresponds to angle = 360°

Step 2: Using Unitary Method:

When angle = 360°, Area = πr²

When angle = 1°, Area = πr²/360

When angle = θ°, Area = (πr²/360) × θ = **(θ/360) × πr²**

Therefore:

Area of Sector = (θ/360) × πr²

Understanding Arc Length Using Unitary Method:

Step 1: We know that the circumference of a circle = **2πr**

This corresponds to angle = 360°

Step 2: Using Unitary Method:

When angle = 360°, Arc Length = 2πr

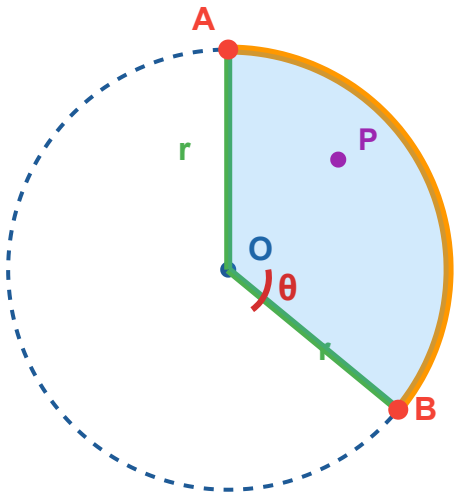
When angle = 1°, Arc Length = 2πr/360

When angle = θ°, Arc Length = (2πr/360) × θ = **(θ/360) × 2πr**

Therefore:

Arc Length = (θ/360) × 2πr

Sector: Area and Arc Length



Area of Sector OAPB = (θ/360) × πr²

Arc Length APB = (θ/360) × 2πr

SOLVED EXAMPLES (NCERT BASED)

Example 1: Finding Area of Sector and Major Sector

Question: Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the area of the corresponding major sector. (Use π = 3.14)

Given: r = 4 cm, θ = 30°, π = 3.14

To Find: (i) Area of minor sector, (ii) Area of major sector

Solution:

1. **Area of Minor Sector OAPB:**
   Using formula: Area = (θ/360) × πr²
   = (30/360) × 3.14 × 4 × 4
   = (1/12) × 3.14 × 16
   = (3.14 × 16)/12
   = 50.24/12
   = 4.19 cm² (approx.)

2. **Area of Major Sector:**
   Method 1: Area of Major Sector = πr² − Area of Minor Sector
   = (3.14 × 16) − 4.19
   = 50.24 − 4.19
   = 46.05 cm²
   = **46.1 cm²** (approx.)

3. **Alternatively (Method 2):**
   Angle of major sector = 360° − 30° = 330°
   Area = (330/360) × πr²
   = (330/360) × 3.14 × 16
   = (11/12) × 50.24

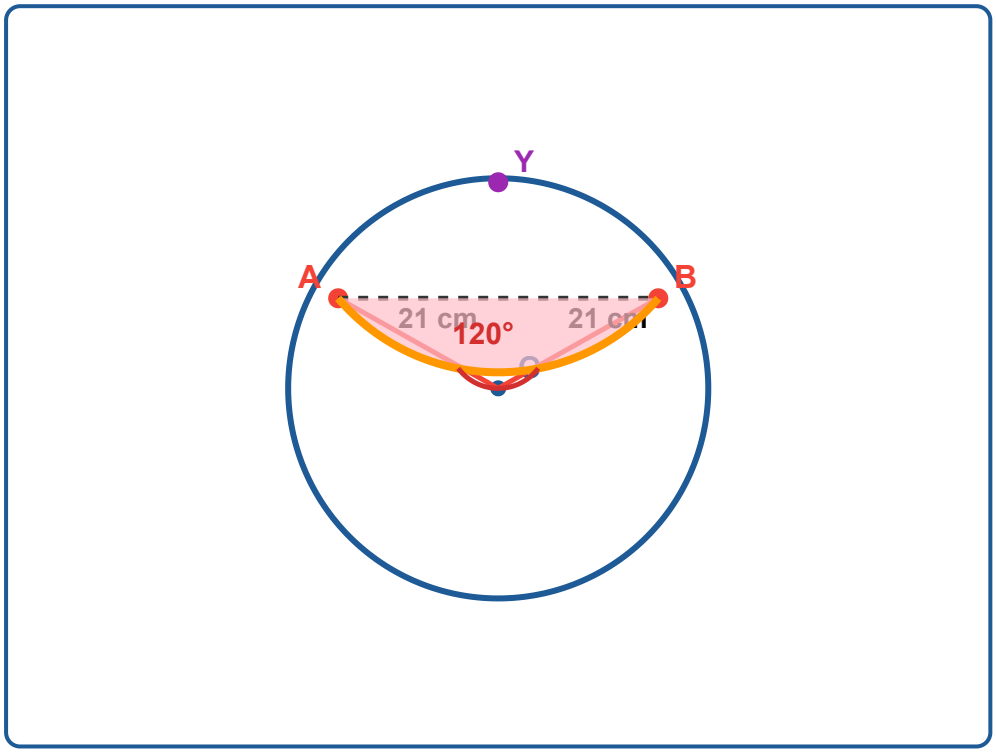
= 46.05 cm²

= **46.1 cm²** (approx.)

Answer: Minor Sector = 4.19 cm², Major Sector = 46.1 cm²

Example 2: Finding Area of Segment (VERY IMPORTANT!)

Question: Find the area of the segment AYB shown in the figure, if radius of the circle is 21 cm and ∠AOB = 120°. (Use π = 22/7)



Given: r = 21 cm, θ = 120°, π = 22/7

To Find: Area of segment AYB (the pink shaded region)

Solution:

Step 1: Find Area of Sector OAYB

1. Area of sector = (θ/360) × πr²
   = (120/360) × (22/7) × 21 × 21
   = (1/3) × (22/7) × 441
   = (22 × 441)/(7 × 3)
   = 9702/21
   = **462 cm²**

Step 2: Find Area of Triangle OAB

For this, we need to find the height of the triangle.

Draw OM ⊥ AB (M is the midpoint of AB)

2. Since OA = OB (radii), △AMO ≅ △BMO (by RHS congruence)
   Therefore, M is the midpoint of AB
   Also, ∠AOM = ∠BOM = 120°/2 = 60°
3. In right triangle △OMA:
   cos 60° = OM/OA
   1/2 = OM/21
   OM = 21/2 cm = 10.5 cm
4. Also, sin 60° = AM/OA
   √3/2 = AM/21
   AM = (21√3)/2 cm
5. Therefore, AB = 2 × AM = 2 × (21√3)/2 = **21√3 cm**
6. Area of △OAB = (1/2) × Base × Height
   = (1/2) × AB × OM
   = (1/2) × 21√3 × (21/2)
   = (21 × 21√3)/4
   = **(441√3)/4 cm²**

Step 3: Find Area of Segment AYB

7. Area of segment = Area of sector − Area of triangle
   = 462 − (441√3)/4
   = 462 − 110.25√3 [Taking √3 ≈ 1.732]
   = 462 − 190.97
   = **271.03 cm²**
   OR
   = **(21/4)(88 − 21√3) cm²** (exact form)

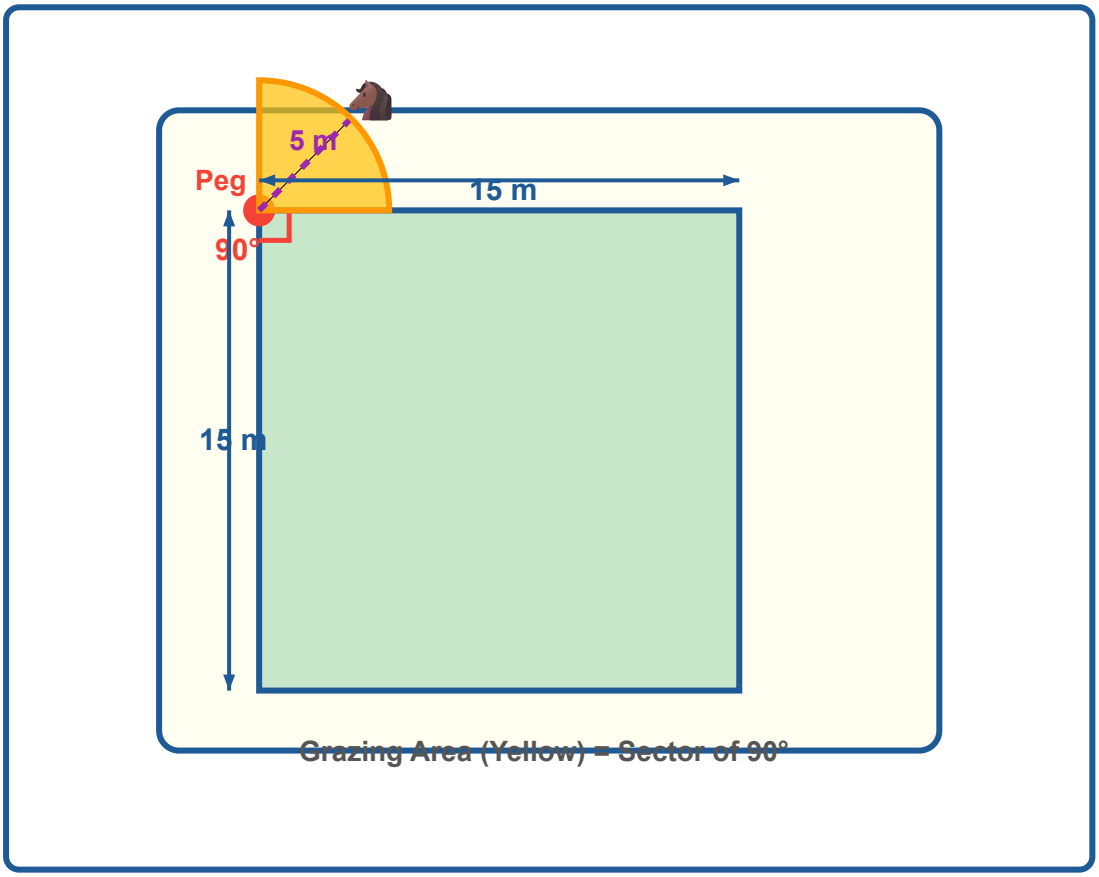
Answer: Area of Segment = 271.03 cm² (approx.) OR (21/4)(88 − 21√3) cm²

Example 3: Real-Life Application - Horse Grazing Area

Question: A horse is tied to a peg at one corner of a square-shaped grass field of side 15 m by means of a 5 m long rope. Find:

(i) the area of that part of the field in which the horse can graze.

(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)



Given: Side of square field = 15 m, Length of rope = 5 m, π = 3.14

To Find: (i) Grazing area with 5 m rope, (ii) Increase if rope is 10 m

Solution:

Part (i): Grazing area with 5 m rope

1. The horse can graze in a **sector of 90°** (quarter circle) with radius 5 m

2. Area of grazing = Area of sector of 90°
   = (θ/360) × πr²
   = (90/360) × 3.14 × 5 × 5
   = (1/4) × 3.14 × 25
   = (3.14 × 25)/4
   = 78.5/4
   = **19.625 m²**

Part (ii): Increase if rope is 10 m

3. With 10 m rope, grazing area = (90/360) × π × 10²
   = (1/4) × 3.14 × 100
   = 314/4
   = **78.5 m²**
4. Increase in grazing area = 78.5 − 19.625
   = **58.875 m²**

Answer: (i) 19.625 m², (ii) Increase = 58.875 m²

🚫 COMMON MISTAKES TO AVOID

❌ Mistake 1: Confusing Sector and Segment

Wrong Understanding: Thinking sector and segment are the same.

Correct Understanding:

- **Sector:** Bounded by TWO RADII and an ARC (like a pizza slice) - includes the triangular portion
- **Segment:** Bounded by a CHORD and an ARC ONLY (like a bow) - does NOT include radii

Solution: Always draw the figure first to identify correctly!

❌ Mistake 2: Forgetting to Subtract Triangle Area for Segment

Wrong Approach: Calculating segment area as just the sector area.

Correct Approach: Area of Segment = Area of Sector − Area of Triangle

Solution: Remember the formula! Segment is ALWAYS less than sector.

❌ Mistake 3: Using Wrong Angle for Major Sector/Segment

Wrong: Using the given angle θ for major sector.

Correct: Angle of major sector = **360° − θ**

Solution: Always verify: Minor angle < 180°, Major angle > 180°

❌ Mistake 4: Not Converting Units

Wrong: Mixing cm and m, or mm and cm without conversion.

Correct: Always convert to the same unit before calculation.

Solution: 1 m = 100 cm, 1 cm = 10 mm. Convert first, then calculate!

❌ Mistake 5: Incorrect Use of Trigonometry for Triangle Area

Wrong: Not using the perpendicular from center to find triangle height.

Correct: Draw perpendicular from center to chord, use trigonometry to find base and height.

Solution: For segment problems, always draw OM ⊥ AB to create right triangles!

📋 IMPORTANT FORMULAS SUMMARY

Complete Formula Chart - MUST MEMORIZE!

S.No.	Quantity	Formula
1	**Area of Sector**	$(\theta/360) \times \pi r^2$
2	**Arc Length**	$(\theta/360) \times 2\pi r$
3	**Area of Segment**	$(\theta/360) \times \pi r^2$ − Area of △
4	**Area of Major Sector**	πr^2 − Area of Minor Sector
5	**Area of Major Segment**	πr^2 − Area of Minor Segment
6	**Perimeter of Sector**	2r + Arc Length = 2r + $(\theta/360)\times 2\pi r$
7	**Area of Semicircle**	$\pi r^2/2$
8	**Area of Quadrant**	$\pi r^2/4$

🎯 BOARD EXAM PATTERN 2025-26

📊 Question Pattern Analysis (CBSE 2025-26):

Question Type	Marks	Expected Questions
Short Answer (SA-I)	2 marks	1-2 questions
Short Answer (SA-II)	3 marks	1 question
Long Answer	5 marks	0-1 question
Total from Areas Related to Circles		**8-10 marks**

🎯 Most Frequently Asked Topics:

1. **Area of sector (basic calculation)** - 90% probability
2. **Area of segment** - 85% probability
3. **Real-life application (horse grazing, wiper, etc.)** - 75% probability
4. **Combination with circles/triangles** - 70% probability
5. **Arc length calculation** - 60% probability
6. **Brooch/table cover/umbrella design problems** - 65% probability

📚 PRACTICE QUESTIONS (NCERT EXERCISE 11.1)

Quick Practice Problems:

Q1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

Answer: (60/360) × (22/7) × 36 = 132/7 cm² ≈ 18.86 cm²

Q2. Find the area of a quadrant of a circle whose circumference is 22 cm.

Hint: $2\pi r = 22$, find r first, then area of quadrant = $\pi r^2/4$

Answer: 9.625 cm²

Q3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Hint: In 60 minutes, hand covers 360°. In 5 minutes = (5/60)×360° = 30°

Answer: (30/360) × (22/7) × 196 = 154/3 cm² ≈ 51.33 cm²

Q4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) minor segment

(ii) major sector (Use π = 3.14)

Answer: (i) 28.5 cm², (ii) 235.5 cm²

🚀 LAST MINUTE REVISION CHECKLIST

☑ Must Complete Before Exam:

- ☑ **Memorize all 3 main formulas** (Sector area, Arc length, Segment area)
- ☑ **Practice NCERT Exercise 11.1** completely (all 14 questions)
- ☑ **Master Example 2** (Area of segment with trigonometry)
- ☑ **Remember special cases:** Semicircle, Quadrant, 120° sector
- ☑ **Know how to find triangle area** using perpendicular from center
- ☑ **Practice minimum 80 problems** on areas
- ☑ **Solve last 5 years' board papers** questions from this chapter
- ☑ **Revise common mistakes** to avoid silly errors

💡 EXPERT TIPS FROM TOPPERS

🏆 How to Score Full Marks in Areas Related to Circles:

Tip 1: Always Draw the Figure

- Even if figure is given, redraw it with all labels
- Mark the angle θ, radius r, and shade the required region

Tip 2: For Segment Problems

- Step 1: Find sector area first
- Step 2: Draw perpendicular from O to chord
- Step 3: Use trigonometry to find triangle base and height
- Step 4: Subtract triangle area from sector area

Tip 3: Unit Conversion

- Convert all measurements to same unit BEFORE starting
- Write the converted value in the solution

Tip 4: Use π = 22/7 Unless Specified

- If question says "Use π = 3.14", use it
- Otherwise, use π = 22/7 for exact answers

Tip 5: Exam Day Mantra

Draw → Identify (Sector/Segment) → Formula → Calculate → Verify Units!

📖 CHAPTER SUMMARY

Complete Chapter Summary:

1. Key Definitions:

- ✓ **Sector:** Region bounded by two radii and arc (includes triangular part)
- ✓ **Segment:** Region bounded by chord and arc ONLY (bow shape, no radii)
- ✓ **Arc:** Continuous portion of circumference

2. Essential Formulas:

- ✓ Area of Sector = $(\theta/360) \times \pi r^2$
- ✓ Arc Length = $(\theta/360) \times 2\pi r$
- ✓ Area of Segment = $(\theta/360) \times \pi r^2 - \text{Area of } \triangle OAB$

3. Important Points:

- ✓ Always check if asked for minor or major sector/segment
- ✓ For segment, draw perpendicular from center to chord
- ✓ Use trigonometry to find triangle dimensions
- ✓ Remember special cases: semicircle (180°), quadrant (90°)

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