CIRCLES

CHAPTER 10 - CLASS 10 MATHEMATICS

CBSE Board Exam 2025-26

Prepared by: **MATH LOVE INSTITUTE**

Education as a Service (EaaS) | Contact: +91-7869553517

📚 COMPREHENSIVE STUDY MATERIAL - TANGENTS TO A CIRCLE 📚

🎯 CHAPTER AT A GLANCE

Chapter Overview:

- **Chapter Name:** Circles
- **Main Topic:** Tangents to a Circle
- **Weightage in Board Exam:** 3 marks (MCQs only)
- **Difficulty Level:** Medium
- **Question Types:** Multiple Choice Questions (1 mark each)
- **Number of Theorems:** 2 Major Theorems (with proofs)
- **Total NCERT Exercises:** 2 (Exercise 10.1 & 10.2)
- **Time Required:** 10-12 days for complete mastery
- **Practice Problems Needed:** Minimum 50-60 problems

📖 INTRODUCTION

Revision from Class IX:

A **circle** is a collection of all points in a plane which are at a constant distance (radius) from a fixed point (centre).

Important Terms (Quick Recap):

- **Centre:** The fixed point from which all points on the circle are equidistant
- **Radius:** The constant distance from the centre to any point on the circle
- **Chord:** A line segment joining any two points on the circle
- **Diameter:** A chord passing through the centre (longest chord = 2 × radius)
- **Segment:** The region between a chord and the corresponding arc
- **Sector:** The region between two radii and the corresponding arc
- **Arc:** A continuous piece of the circle

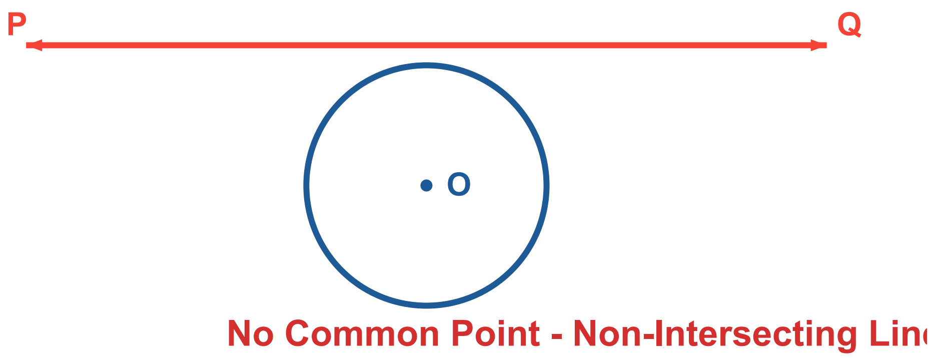
🔑 POSITION OF A LINE WITH RESPECT TO A CIRCLE

Three Possible Positions:

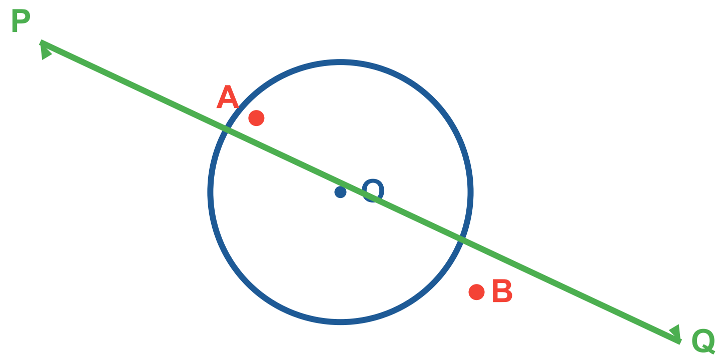
When we have a circle and a line in a plane, there can be **EXACTLY THREE possibilities:**

1. **Non-Intersecting Line:** The line and circle have NO common point
2. **Secant:** The line intersects the circle at TWO points
3. **Tangent:** The line touches the circle at EXACTLY ONE point

Position 1: Non-Intersecting Line (0 common points)

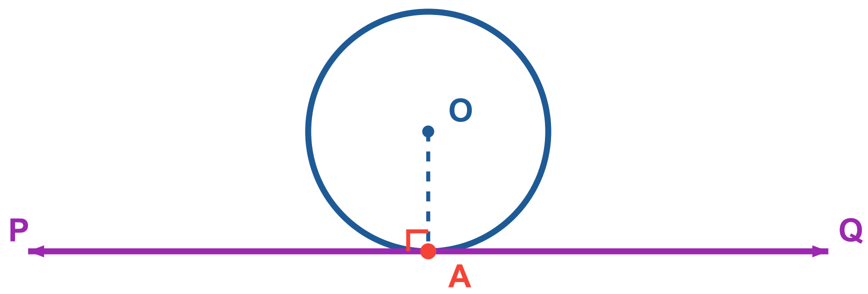


Position 2: Secant (2 common points)



Two Common Points (A, B) - SECANT

Position 3: Tangent (1 common point)



One Common Point (A) - TANGENT

📚 KEY DEFINITIONS (MUST MEMORIZE!)

1. Tangent to a Circle:

A tangent to a circle is a line that intersects (touches) the circle at **only one point**.

Key Point: Tangent TOUCHES the circle, it doesn't cut through it!

2. Point of Contact:

The common point of the tangent and the circle is called the **point of contact**.

The point where the tangent line touches the circle.

3. Secant:

A line that intersects the circle at **two points** is called a **secant**.

Key Point: Secant CUTS the circle at 2 points!

4. Normal:

The line containing the radius through the point of contact is called the **normal** to the circle at that point.

Normal is perpendicular to the tangent at point of contact.

📝 Important Note:

"Tangent comes from the Latin word 'tangere' meaning 'to touch'"

It was introduced by Danish mathematician Thomas Fineke in 1583.

Key Observation: A tangent to a circle is a special case of the secant, when the two end points of its corresponding chord coincide.

🚲 Real-Life Example - Bicycle Wheel:

When a bicycle wheel moves on the ground:

- The ground acts as a **tangent** to the circular wheel
- The point where wheel touches ground is the **point of contact**
- All spokes of the wheel are **radii** of the circle
- The radius through point of contact is **perpendicular** to the ground

This real-world observation leads us to our first theorem!

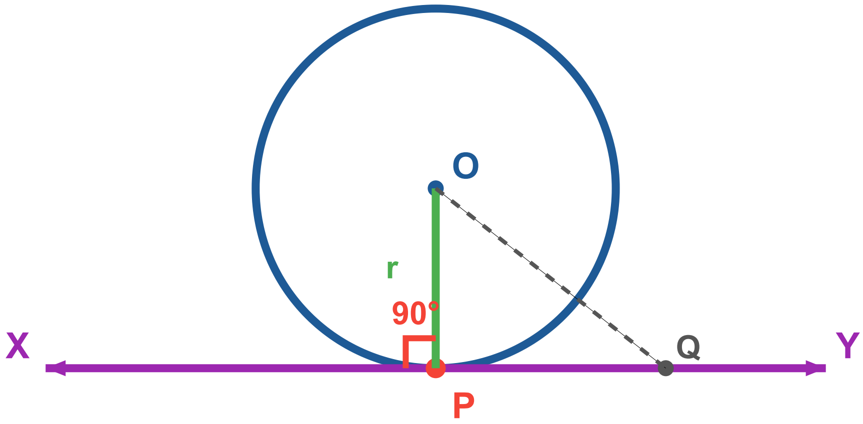
⚡ THEOREM 10.1 (MOST IMPORTANT!)

🔥 THEOREM 10.1

The tangent at any point of a circle is perpendicular to the radius through the point of contact.

In Simple Words: Tangent ⊥ Radius (90° angle)

Understanding Theorem 10.1



XY is tangent at point P
OP ⊥ XY (Theorem 10.1)

Given: XY is a tangent to the circle at point P, O is the centre

To Prove: OP ⊥ XY

📐 PROOF OF THEOREM 10.1:

Given: A circle with centre O, and XY is a tangent to the circle at point P.

To Prove: OP ⊥ XY

Proof:

1. Take any point Q on the tangent XY (other than P).
2. Join OQ.
3. Since Q is a point on the tangent (and Q ≠ P), **Q must lie outside the circle.**
   Why? If Q lies inside the circle, XY would become a secant (cutting at 2 points), not a tangent.
4. Since Q lies outside the circle, we have:
   OQ > OP
   (Because OP is the radius, and any point outside the circle is at a greater distance from O)
5. This is true for **every point on line XY except point P**.
6. Therefore, **OP is the shortest distance** from O to line XY.
7. By the theorem: *"The shortest distance from a point to a line is the perpendicular distance"*
8. **Hence, OP ⊥ XY**

Therefore, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

💡 Important Consequences of Theorem 10.1:

Consequence 1:

At any point on a circle, there can be **one and only one tangent**.

Consequence 2:

The line containing the radius through the point of contact is called the **NORMAL** to the circle at that point.

Consequence 3:

Tangent ⊥ Radius is the **most frequently used property** in circle problems!

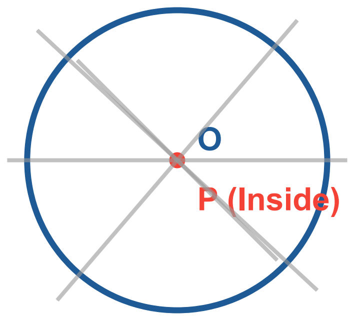
⚡ **Exam Tip: Always draw the radius to the point of contact when you see a tangent. It creates a 90° angle which helps in applying Pythagoras theorem!**

📊 NUMBER OF TANGENTS FROM A POINT

Three Cases Based on Point Position:

Case	Position of Point	Number of Tangents
Case 1	Point **INSIDE** the circle	**ZERO (0) tangents**
Case 2	Point **ON** the circle	**ONE (1) tangent**
Case 3	Point **OUTSIDE** the circle	**TWO (2) tangents**

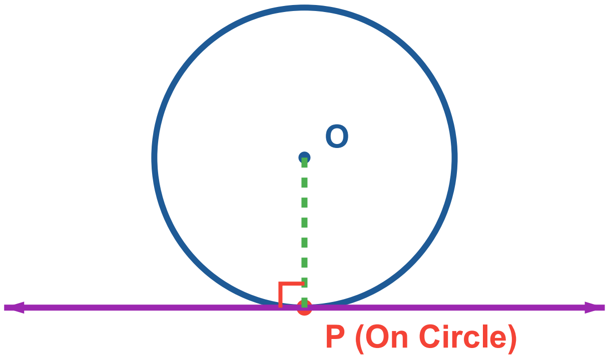
Case 1: Point P Inside Circle → NO TANGENT (0)



All lines through P intersect circle at 2 points
Number of tangents = 0

Why? Any line through P intersects the circle at two points, so it's a secant, not a tangent.

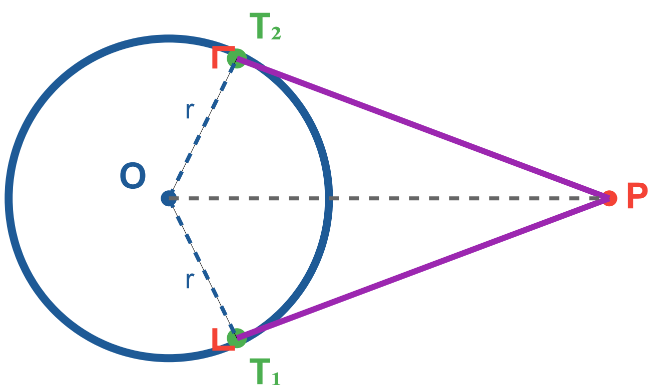
Case 2: Point P On Circle → ONE TANGENT (1)



Exactly ONE tangent at point P

The unique tangent at P is perpendicular to radius OP (by Theorem 10.1)

Case 3: Point P Outside Circle → TWO TANGENTS (2)



Exactly TWO tangents PT_1 and PT_2

$PT_1 = PT_2$ (Always Equal!)

From an external point P, exactly two tangents can be drawn to the circle.

📏 Length of a Tangent:

Definition: The length of the segment of the tangent from the external point P to the point of contact with the circle is called the length of the tangent from point P to the circle.

Example: In Case 3 above, PT_1 and PT_2 are the lengths of the tangents from P to the circle.

> **Important:** PT_1 and PT_2 have a special property - they are always equal! This is Theorem 10.2.

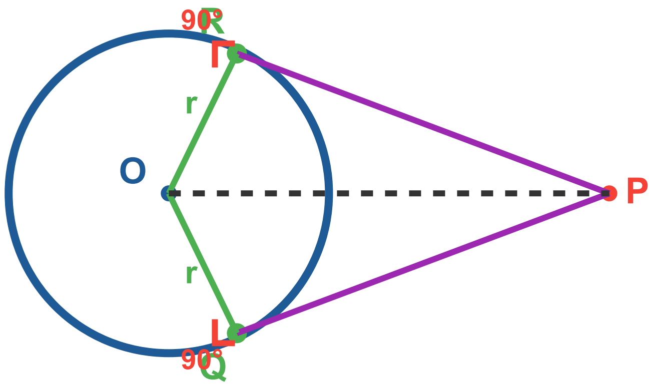
⚡ THEOREM 10.2 (VERY IMPORTANT!)

🔥 THEOREM 10.2

The lengths of tangents drawn from an external point to a circle are equal.

In Simple Words: If PT_1 and PT_2 are tangents from external point P, then $PT_1 = PT_2$

Diagram for Theorem 10.2



To Prove: PQ = PR

Given: Circle with centre O, external point P, tangents PQ and PR

To Prove: PQ = PR

PROOF OF THEOREM 10.2:

Given: A circle with centre O, an external point P, and two tangents PQ and PR touching the circle at Q and R respectively.

To Prove: PQ = PR

Proof:

1. Join OP, OQ, and OR.
2. Since PQ is a tangent at Q and OQ is the radius through Q,
   By Theorem 10.1: $\angle OQP = 90^\circ$
3. Since PR is a tangent at R and OR is the radius through R,
   By Theorem 10.1: $\angle ORP = 90^\circ$
4. Now consider **right triangles $\triangle OQP$ and $\triangle ORP$**
5. In these triangles:
   • **OQ = OR** (Both are radii of the same circle)
   • **OP = OP** (Common side)
   • $\angle OQP = \angle ORP = 90^\circ$ (From steps 2 and 3)
6. By **RHS (Right angle-Hypotenuse-Side) congruence**:
   $\triangle OQP \cong \triangle ORP$
7. By **CPCT (Corresponding Parts of Congruent Triangles)**:
   PQ = PR

Hence Proved: The lengths of tangents drawn from an external point to a circle are equal.

📝 Alternative Proof Using Pythagoras Theorem:

Theorem 10.2 can also be proved using Pythagoras Theorem:

In right triangle △OQP:

$$PQ^2 = OP^2 - OQ^2$$

In right triangle △ORP:

$$PR^2 = OP^2 - OR^2$$

Since OQ = OR (both radii):

$$PQ^2 = OP^2 - OQ^2 = OP^2 - OR^2 = PR^2$$

Therefore, PQ = PR

💡 Additional Results from Theorem 10.2:

Result 1: ∠OPQ = ∠OPR

(Can be proved using the congruence △OQP ≅ △ORP)

Result 2: OP is the angle bisector of ∠QPR

(Follows from Result 1)

Result 3: The centre O lies on the bisector of the angle between the two tangents

(Important for construction problems)

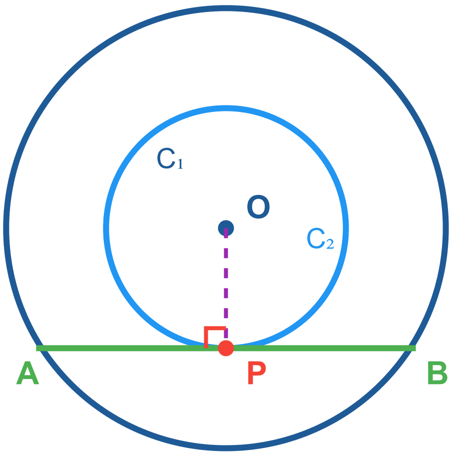
Result 4: If tangents are drawn from external point, ∠QPR + ∠QOR = 180°

(The angles are supplementary)

📝 SOLVED EXAMPLES (NCERT BASED)

Example 1: Concentric Circles

Question: Prove that in two concentric circles, the chord of the larger circle which touches the smaller circle is bisected at the point of contact.



To Prove: AP = BP

Given: Two concentric circles C_1 and C_2 with centre O. AB is a chord of larger circle C_1 which touches smaller circle C_2 at point P.

To Prove: AP = BP

Solution:

1. Join OP.
2. AB is a tangent to C_2 at P, and OP is the radius.
3. By Theorem 10.1: **OP ⊥ AB**

4. Now, AB is a chord of circle C_1 and OP $\perp$ AB.
5. We know the theorem: *"The perpendicular from the centre to a chord bisects the chord"*
6. Therefore, **AP = BP**

Hence Proved: The chord is bisected at the point of contact.

Example 2: Angle Relationship

Question: Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2∠OPQ.

Given: Circle with centre O, external point T, tangents TP and TQ with P and Q as points of contact.

To Prove: ∠PTQ = 2∠OPQ

Solution:

1. Let ∠PTQ = θ
2. By Theorem 10.2: **TP = TQ**
3. Therefore, △TPQ is isosceles (with TP = TQ)
4. In isosceles triangle TPQ:
   ∠TPQ = ∠TQP = ½(180° − θ) = 90° − θ/2
5. By Theorem 10.1: ∠**OPT = 90°**
6. Now, ∠OPQ = ∠OPT − ∠TPQ
7. ∠OPQ = 90° − (90° − θ/2)
8. ∠OPQ = θ/2
9. Therefore, θ = 2∠OPQ
10. Hence, ∠**PTQ = 2**∠**OPQ**

Hence Proved

Example 3: Finding Length of Tangent

Question: PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at point T. Find the length TP.

Given: PQ = 8 cm, radius = 5 cm

To Find: Length TP

Solution:

1. Join OT. Let it intersect PQ at point R.
2. Since $\triangle TPQ$ is isosceles (TP = TQ by Theorem 10.2), TO is the angle bisector of $\angle PTQ$.
3. Therefore, **OT** $\perp$ **PQ** and OT bisects PQ.
4. So, **PR = RQ = 4 cm**
5. In right $\triangle ORP$:
   $OR^2 = OP^2 - PR^2$
   $OR^2 = 5^2 - 4^2 = 25 - 16 = 9$
   OR = 3 cm
6. Now, $\angle TPR + \angle RPO = 90° = \angle TPR + \angle PTR$
   (Since $\angle OPT = 90°$ by Theorem 10.1)
7. So, $\angle RPO = \angle PTR$
8. Therefore, right $\triangle TRP \sim$ right $\triangle PRO$ (by AA similarity)
9. By proportionality:
   TP/PO = PR/RO
10. TP/5 = 4/3
11. **TP = 20/3 cm = 6.67 cm (approx.)**

Answer: TP = 20/3 cm ≈ 6.67 cm

🚫 COMMON MISTAKES TO AVOID

❌ Mistake 1: Forgetting Tangent ⊥ Radius

Wrong Approach: Assuming any random angle between tangent and radius.

Correct Approach: ALWAYS remember: **Tangent is PERPENDICULAR to radius at point of contact (90°)**

Solution: As first step in ANY tangent problem - draw radius to point of contact and mark 90°!

❌ Mistake 2: Not Using Equal Tangent Property

Wrong Approach: Calculating both tangent lengths separately.

Correct Approach: Tangents from external point are ALWAYS equal (Theorem 10.2).

Solution: If PT_1 and PT_2 are tangents from P, immediately write $\mathbf{PT_1 = PT_2}$

❌ Mistake 3: Confusing Secant and Tangent

Wrong Understanding: Thinking secant and tangent are the same.

Correct Understanding:

- **Secant:** Intersects circle at **2 points**
- **Tangent:** Touches circle at **exactly 1 point**

Solution: Always count intersection points before naming the line!

❌ Mistake 4: Wrong Number of Tangents

Wrong: Drawing 2 tangents from a point ON the circle.

Correct:

- Point INSIDE circle → **0** tangents
- Point ON circle → **1** tangent
- Point OUTSIDE circle → **2** tangents

Solution: Check point position FIRST before drawing tangents!

💯 IMPORTANT FORMULAS & PROPERTIES

Essential Formulas - MUST MEMORIZE!

S.No.	Property/Formula	Description
1	**Tangent ⊥ Radius**	∠(tangent, radius) = 90°
2	**$PT_1 = PT_2$**	Equal tangents from external point
3	**$TP^2 = OP^2 - r^2$**	Pythagoras in △OTP
4	**$OP^2 = OT^2 + TP^2$**	Pythagoras theorem
5	**∠QPR + ∠QOR = 180°**	Supplementary angles
6	**OP bisects ∠TPQ**	Centre lies on angle bisector
7	**AB + CD = AD + BC**	Quadrilateral circumscribing circle

🔑 Key Properties to Remember:

1. In Concentric Circles:

The chord of the larger circle which touches the smaller circle is bisected at the point of contact.

2. Tangents at Ends of Diameter:

Tangents drawn at the ends of a diameter of a circle are parallel.

3. Perpendicular at Point of Contact:

The perpendicular at the point of contact to the tangent passes through the centre.

4. Parallelogram Circumscribing Circle:

A parallelogram circumscribing a circle is a rhombus.

🎯 BOARD EXAM PATTERN

📊 Question Pattern Analysis (CBSE 2025-26):

Question Type	Marks	Expected Questions
MCQ (Multiple Choice)	1 mark each	3 questions
Total from Circles Chapter		**3 marks (MCQs only)**

🎯 Most Frequently Asked Topics:

1. **Tangent perpendicular to radius (Theorem 10.1)** - 85% probability
2. **MCQ on tangent properties** - 95% probability
3. **Number of tangents from a point** - 80% probability
4. **Equal tangent property (Theorem 10.2)** - 75% probability
5. **Finding tangent length using Pythagoras** - 70% probability

📚 PRACTICE QUESTIONS

Exercise 10.1 - Basic Questions

Q1. How many tangents can a circle have?

Answer: A circle can have infinite tangents.

Q2. Fill in the blanks:

(i) A tangent to a circle intersects it in **one** point(s).

(ii) A line intersecting a circle in two points is called a **secant**.

(iii) A circle can have **two** parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called **point of contact**.

Q3. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is:

(A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ cm

Answer: (D) $\sqrt{119}$ cm

Exercise 10.2 - Advanced Questions

Q1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is:

(A) 7 cm (B) 12 cm (C) 15 cm (D) 24.5 cm

Answer: (A) 7 cm

Q2. If TP and TQ are two tangents to a circle with centre O such that ∠POQ = 110°, then ∠PTQ is equal to:

(A) 60° (B) 70° (C) 80° (D) 90°

Answer: (B) 70°

Q3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then ∠POA is equal to:

(A) 50° (B) 60° (C) 70° (D) 80°

Answer: (A) 50°

🚀 LAST MINUTE REVISION CHECKLIST

☑ Must Complete Before Exam:

- ☑ **Memorize both theorem statements** word-by-word
- ☑ **Practice both theorem proofs** - write in 5 minutes each
- ☑ **Remember: Tangent ⊥ Radius** (most important property)
- ☑ **Remember: $PT_1 = PT_2$** (equal tangents from external point)
- ☑ **Know number of tangents:** Inside=0, On=1, Outside=2
- ☑ **Solve NCERT Exercise 10.1 & 10.2** completely
- ☑ **Practice minimum 50 problems** on circles
- ☑ **Solve last 5 years' MCQs** from board papers
- ☑ **Practice drawing neat diagrams** with compass

💡 EXPERT TIPS FROM TOPPERS

🏆 How to Score Full Marks in Circles:

Tip 1: Master the Diagrams

- Always draw radius to point of contact - it creates 90° angle!
- Use compass for perfect circles, ruler for straight lines

Tip 2: Identify Right Triangles

- Tangent problem → Look for right triangle → Apply Pythagoras

Tip 3: MCQ Strategy

- Read question twice, draw quick diagram
- Eliminate obviously wrong options first

Tip 4: Exam Day Mantra

See tangent → Draw radius → Mark 90° → Apply Pythagoras!

📖 CHAPTER SUMMARY

Complete Chapter Summary:

1. Main Theorems:

- ✓ **Theorem 10.1:** Tangent ⊥ Radius at point of contact
- ✓ **Theorem 10.2:** Tangents from external point are equal ($PT_1 = PT_2$)

2. Number of Tangents:

- ✓ Point INSIDE circle → **0 tangents**
- ✓ Point ON circle → **1 tangent**
- ✓ Point OUTSIDE circle → **2 tangents**

3. Key Formulas:

- ✓ Tangent ⊥ Radius: $\angle$**OTP = 90°**
- ✓ Pythagoras: **$TP^2 = OT^2 - OP^2$**
- ✓ Equal tangents: **$PT_1 = PT_2$**

📞 MATH LOVE INSTITUTE

Education as a Service (EaaS)

📱 **Phone:** +91-7869553517

🌐 **Website:** www.mathlove.in

📧 **Email:** info@mathlove.in

🏢 **Address:** H-1 Street 2, V V Vihar, Shankar Nagar, Raipur (C.G.)

🎯 Our Services:

Document ID: CIRCLES-CH10-V3.0 | Generated: December 2024