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COORDINATE GEOMETRY

Complete Study Material for CBSE Class 10 (2025-26)

Chapter 7

🌟 FASCINATING FACTS ABOUT COORDINATE

GEOMETRY

Did You Know?

🎭 **René Descartes - The Father:** Coordinate geometry was invented by French mathematician René Descartes (1596-1650). Legend says he invented it while lying in bed watching a fly crawl on the ceiling, wondering how to describe its position!

📍 **Real Name:** Coordinate geometry is also called "Cartesian geometry" or "Analytic geometry" - Cartesian comes from Descartes' Latin name "Cartesius".

🗺️ **GPS Technology:** Your smartphone GPS uses coordinate geometry! Every location on Earth is identified using coordinates (latitude and longitude).

🎮 **Video Games:** Every character, object, and movement in video games uses coordinate geometry. When Mario jumps, programmers use distance formulas and coordinates!

🚀 **Space Missions:** NASA uses coordinate geometry to calculate distances between planets, plot spacecraft trajectories, and land rovers on Mars!

🏗️ **Architecture & Engineering:** Buildings, bridges, and roads are designed using coordinate geometry. Engineers use it to ensure structures are level, aligned, and stable.

📱 **Computer Graphics:** Every pixel on your screen has coordinates! When you draw, edit photos, or watch animations - it's all coordinate geometry.

⚽ **Sports Analytics:** Player positions, ball trajectories, and game strategies in football, cricket, and basketball are analyzed using coordinate geometry!

IN **Indian Contribution:** Ancient Indian mathematicians like Brahmagupta and Bhaskara II laid foundations for algebraic concepts that contributed to coordinate geometry's development.

📋 CHAPTER OVERVIEW

Chapter	Coordinate Geometry (Chapter 7)
Weightage	6 marks (As per CBSE marking scheme 2025-26)
Difficulty Level	Medium to High
Expected Questions	1-2 questions (2 marks, 3 marks, or 4 marks)
Time Required	10-15 minutes in exam
Prerequisites	Class IX Coordinate Geometry, Pythagoras Theorem, Basic Algebra

Topics Covered:

1. **Coordinate System - Revision**
2. **Distance Formula**
3. **Section Formula (Internal Division)**
4. **Mid-Point Formula**
5. **Applications in Triangles, Quadrilaterals**
6. **Collinearity of Points**
7. **Area of Triangle (Optional - Not in CBSE)**

⚠️ IMPORTANT NOTE - CBSE 2025-26:

Area of Triangle using coordinates is NOT in the CBSE Class 10 syllabus for 2025-26. However, it's included at the end for competitive exam preparation.

Focus mainly on: **Distance Formula** and **Section Formula**

🎯 SECTION 1: COORDINATE SYSTEM (REVISION)

Understanding the Coordinate Plane

What is a Coordinate System?

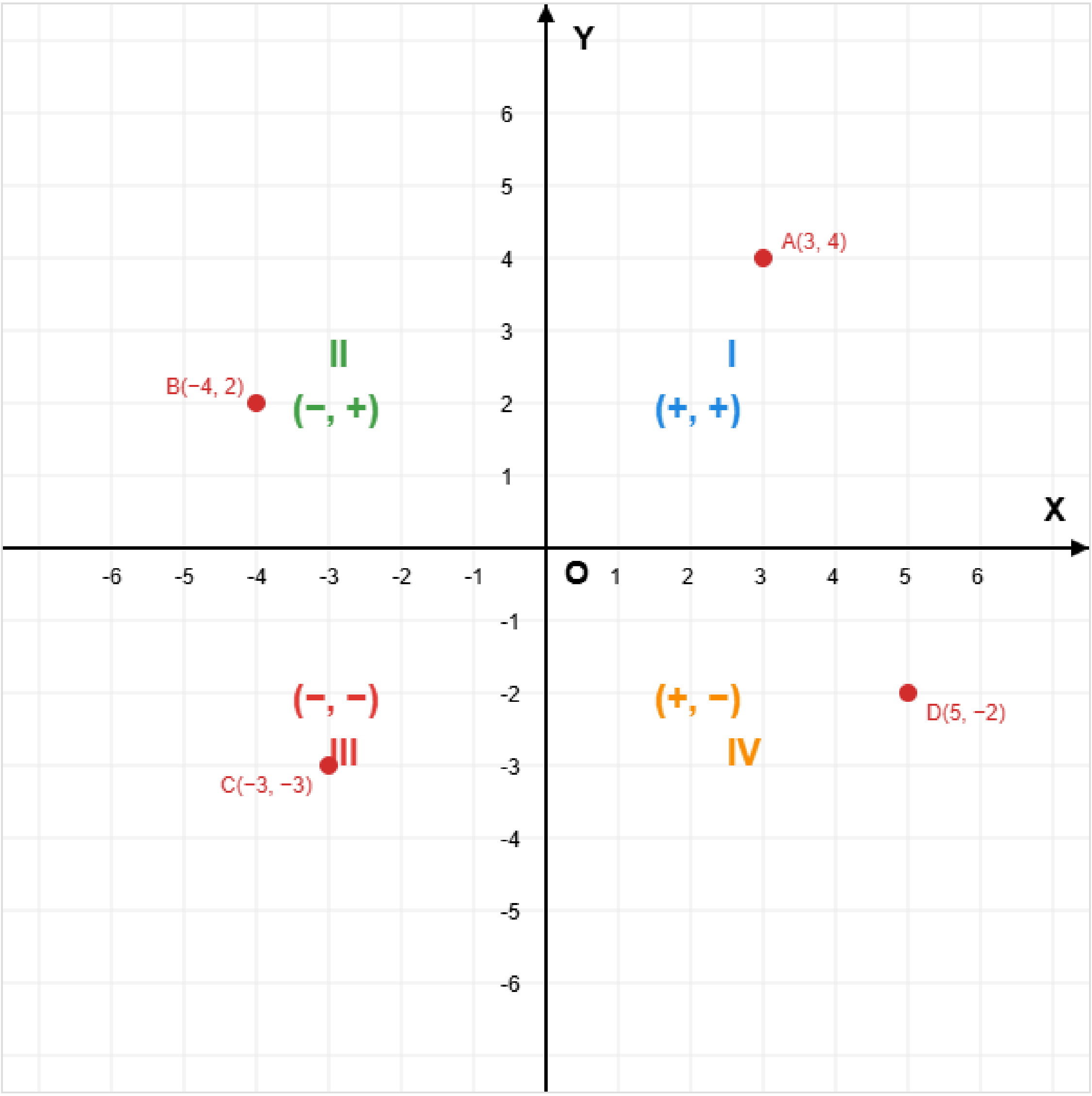
A coordinate system is formed by two perpendicular lines:

- **X-axis:** Horizontal line
- **Y-axis:** Vertical line
- **Origin (O):** Point where both axes meet, coordinates (0, 0)

Coordinates of a Point: P(x, y)

- **x-coordinate (Abscissa):** Distance from Y-axis (horizontal distance)
- **y-coordinate (Ordinate):** Distance from X-axis (vertical distance)

The Coordinate Plane with Four Quadrants



Sign Convention:

- **Quadrant I:** (+, +) - Both coordinates positive
- **Quadrant II:** (−, +) - x negative, y positive
- **Quadrant III:** (−, −) - Both coordinates negative
- **Quadrant IV:** (+, −) - x positive, y negative

Important Points to Remember

📌 Key Points About Coordinates:

- ✓ Points on **X-axis** have y-coordinate = 0, form: **(x, 0)**
- ✓ Points on **Y-axis** have x-coordinate = 0, form: **(0, y)**
- ✓ **Origin** has coordinates **(0, 0)**
- ✓ Distance is **always positive**
- ✓ Order matters: (3, 4) ≠ (4, 3)
- ✓ **Perpendicular distance** from Y-axis = |x-coordinate|
- ✓ **Perpendicular distance** from X-axis = |y-coordinate|

Example 1: In which quadrant do the following points lie?

(i) (3, 5)
Both coordinates positive → **Quadrant I**

(ii) (−2, 7)
x negative, y positive → **Quadrant II**

(iii) (−4, −6)
Both coordinates negative → **Quadrant III**

(iv) (8, −3)
x positive, y negative → **Quadrant IV**

(v) (0, 5)
x = 0 → **On positive Y-axis**

(vi) (−7, 0)
y = 0 → **On negative X-axis**

🎯 SECTION 2: DISTANCE FORMULA

DISTANCE BETWEEN TWO POINTS

$$PQ = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

where $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points

SPECIAL CASE: Distance from Origin

$$OP = \sqrt{(x^2 + y^2)}$$

where P(x, y) is any point and O(0, 0) is origin

Derivation of Distance Formula

Using Pythagoras Theorem:

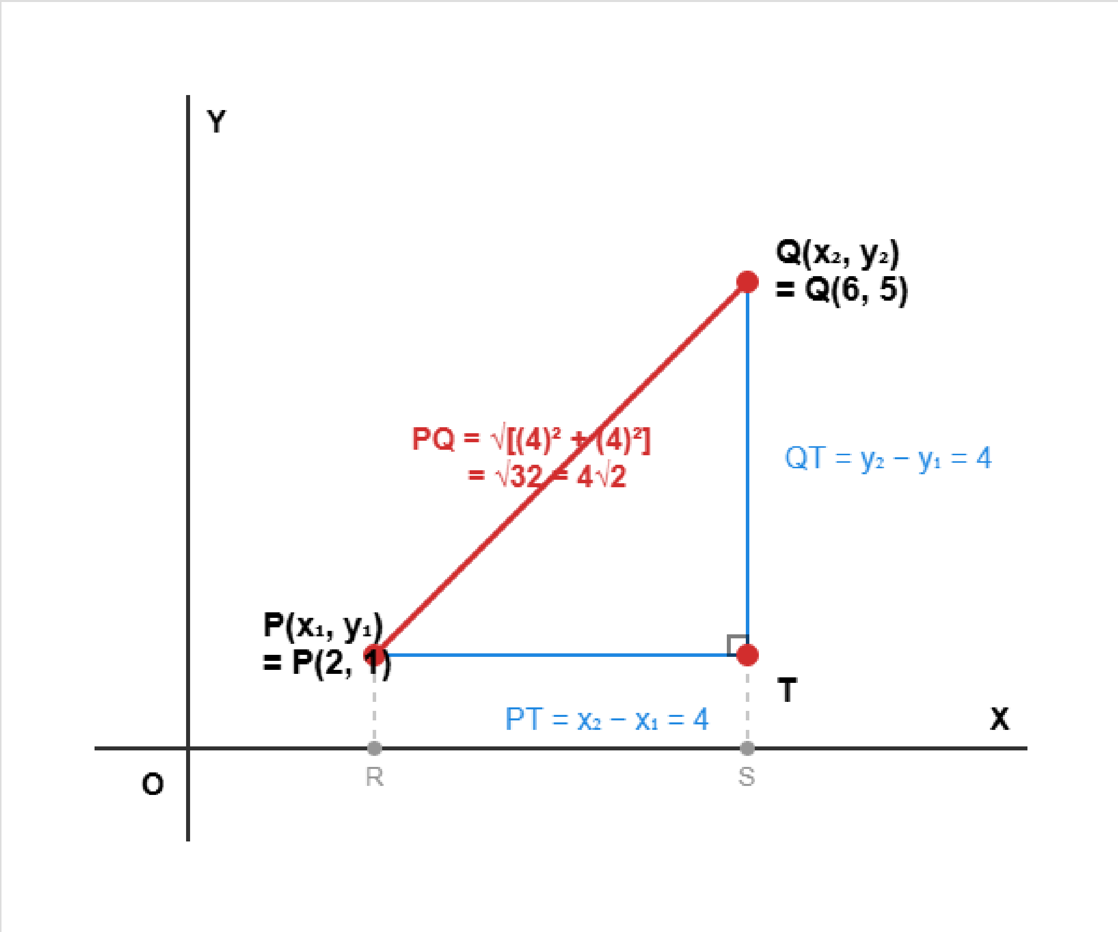
Consider two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in a coordinate plane.

Steps:

1. Draw perpendiculars from P and Q to X-axis meeting at R and S
2. Draw PT perpendicular to QS, meeting at T
3. Now PTQ forms a right triangle with right angle at T
4. PT = RS = OS − OR = $x_2 - x_1$ (horizontal distance)
5. QT = QS − TS = QS − PR = $y_2 - y_1$ (vertical distance)
6. By Pythagoras theorem in △PTQ:

$$PQ^2 = PT^2 + QT^2$$
$$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$\mathbf{PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Visualizing Distance Formula



The distance PQ is the hypotenuse of right triangle PTQ

Understanding the Formula

💡 Important Points About Distance Formula:

- ✓ Distance is **always positive** (we take positive square root)
- ✓ Order doesn't matter: Distance from P to Q = Distance from Q to P
- ✓ $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]} = \sqrt{[(x_1 - x_2)^2 + (y_1 - y_2)^2]}$
- ✓ For points on same horizontal line (same y): Distance = $|x_2 - x_1|$
- ✓ For points on same vertical line (same x): Distance = $|y_2 - y_1|$
- ✓ **Always simplify the square root** in your final answer

TYPE 1: Finding Distance Between Two Points

Example 2: Find the distance between the points (2, 3) and (4, 1).

Solution:

Let $P(2, 3) = P(x_1, y_1)$ and $Q(4, 1) = Q(x_2, y_2)$

Using distance formula:

$PQ = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

$PQ = \sqrt{[(4 - 2)^2 + (1 - 3)^2]}$

$PQ = \sqrt{[(2)^2 + (-2)^2]}$

$PQ = \sqrt{[4 + 4]}$

$PQ = \sqrt{8}$

$PQ = \sqrt{(4 \times 2)}$

$PQ = 2\sqrt{2}$ units

Answer: Distance = $2\sqrt{2}$ units ≈ 2.83 units

Example 3: Find the distance between (−5, 7) and (−1, 3).

Solution:

Let A(−5, 7) and B(−1, 3)

$$AB = \sqrt{[(-1 - (-5))^2 + (3 - 7)^2]}$$

$$AB = \sqrt{[(-1 + 5)^2 + (-4)^2]}$$

$$AB = \sqrt{[(4)^2 + (-4)^2]}$$

$$AB = \sqrt{[16 + 16]}$$

$$AB = \sqrt{32}$$

$$AB = \sqrt{(16 \times 2)}$$

$$AB = 4\sqrt{2} \text{ units}$$

Answer: Distance = $4\sqrt{2}$ units ≈ 5.66 units

Example 4: Find the distance of point (3, 4) from the origin.

Solution:

Origin O = (0, 0), Point P = (3, 4)

Using distance formula:

$OP = \sqrt{[(3 - 0)^2 + (4 - 0)^2]}$

$OP = \sqrt{[9 + 16]}$

$OP = \sqrt{25}$

OP = 5 units

Alternative: Using special formula

$OP = \sqrt{(x^2 + y^2)}$

$OP = \sqrt{(3^2 + 4^2)}$

$OP = \sqrt{(9 + 16)}$

$OP = \sqrt{25} = 5$ units

Answer: Distance from origin = 5 units

TYPE 2: Checking Triangle Type

Example 5: Do the points (3, 2), (−2, −3) and (2, 3) form a triangle? If so, name the type of triangle.

Solution:

Let P(3, 2), Q(−2, −3) and R(2, 3)

Step 1: Find all three distances

$PQ = \sqrt{[(-2 - 3)^2 + (-3 - 2)^2]}$

$PQ = \sqrt{[(-5)^2 + (-5)^2]}$

$PQ = \sqrt{[25 + 25]}$

$PQ = \sqrt{50} = 5\sqrt{2} \approx 7.07$ units

$QR = \sqrt{[(2 - (-2))^2 + (3 - (-3))^2]}$

$QR = \sqrt{[(4)^2 + (6)^2]}$

$QR = \sqrt{[16 + 36]}$

$QR = \sqrt{52} = 2\sqrt{13} \approx 7.21$ units

$PR = \sqrt{[(2 - 3)^2 + (3 - 2)^2]}$

$PR = \sqrt{[(-1)^2 + (1)^2]}$

$PR = \sqrt{[1 + 1]}$

$PR = \sqrt{2} \approx 1.41$ units

Step 2: Check triangle inequality

For a triangle: Sum of any two sides > Third side

$PQ + PR = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} \approx 8.48 > 7.21 = QR$ ✓

$QR + PR = 2\sqrt{13} + \sqrt{2} \approx 8.62 > 7.07 = PQ$ ✓

$PQ + QR = 5\sqrt{2} + 2\sqrt{13} \approx 14.28 > 1.41 = PR$ ✓

YES, these points form a triangle

Step 3: Check type of triangle

Check if $PQ^2 + PR^2 = QR^2$

$PQ^2 = 50$

$PR^2 = 2$

$QR^2 = 52$

$PQ^2 + PR^2 = 50 + 2 = 52 = QR^2$

By converse of Pythagoras theorem: **$\angle P = 90°$**

```
Answer: YES, points form a RIGHT-ANGLED TRIANGLE
```

TYPE 3: Identifying Shape of Quadrilateral

Example 6: Show that (1, 7), (4, 2), (−1, −1) and (−4, 4) are vertices of a square.

Solution:

Let A(1, 7), B(4, 2), C(−1, −1), D(−4, 4)

For a square: All four sides equal AND diagonals equal

Step 1: Find all four sides

$AB = \sqrt{(4 - 1)^2 + (2 - 7)^2}$

$AB = \sqrt{9 + 25} = \sqrt{34}$

$BC = \sqrt{(-1 - 4)^2 + (-1 - 2)^2}$

$BC = \sqrt{25 + 9} = \sqrt{34}$

$CD = \sqrt{(-4 - (-1))^2 + (4 - (-1))^2}$

$CD = \sqrt{9 + 25} = \sqrt{34}$

$DA = \sqrt{(1 - (-4))^2 + (7 - 4)^2}$

$DA = \sqrt{25 + 9} = \sqrt{34}$

All four sides equal = √34 ✓

Step 2: Find diagonals

$AC = \sqrt{(-1 - 1)^2 + (-1 - 7)^2}$

$AC = \sqrt{4 + 64} = \sqrt{68}$

$BD = \sqrt{(-4 - 4)^2 + (4 - 2)^2}$

$BD = \sqrt{64 + 4} = \sqrt{68}$

Both diagonals equal = √68 ✓

Since all sides equal AND diagonals equal:

ABCD is a SQUARE ✓

Answer: Points are vertices of a square

TYPE 4: Finding Unknown Coordinate

Example 7: Find a point on the y-axis which is equidistant from A(6, 5) and B(−4, 3).

Solution:

Point on y-axis has form P(0, y)

Given: PA = PB (equidistant)

Method: $PA^2 = PB^2$ (to avoid square roots)

$PA^2 = (6 - 0)^2 + (5 - y)^2$

$PA^2 = 36 + 25 - 10y + y^2$

$PA^2 = 61 - 10y + y^2$

$PB^2 = (-4 - 0)^2 + (3 - y)^2$

$PB^2 = 16 + 9 - 6y + y^2$

$PB^2 = 25 - 6y + y^2$

Since $PA^2 = PB^2$:

$61 - 10y + y^2 = 25 - 6y + y^2$

$61 - 10y = 25 - 6y$

$61 - 25 = 10y - 6y$

$36 = 4y$

$y = 9$

Verification:

$PA = \sqrt{[(6-0)^2 + (5-9)^2]} = \sqrt{[36+16]} = \sqrt{52}$

$PB = \sqrt{[(-4-0)^2 + (3-9)^2]} = \sqrt{[16+36]} = \sqrt{52}$ ✓

Answer: Point on y-axis is (0, 9)

Example 8: Find values of y for which distance between P(2, −3) and Q(10, y) is 10 units.

Solution:

Given: PQ = 10

Using distance formula:

$PQ^2 = (10 - 2)^2 + (y - (-3))^2$

$10^2 = 8^2 + (y + 3)^2$

$100 = 64 + (y + 3)^2$

$36 = (y + 3)^2$

$\pm 6 = y + 3$

Case 1: $y + 3 = 6$

$y = 3$

Case 2: $y + 3 = -6$

$y = -9$

Answer: y = 3 or y = −9

Both values are valid!

TYPE 5: Collinearity of Points

Example 9: Check whether A(3, 1), B(6, 4) and C(8, 6) are collinear.

Solution:

Three points are collinear if: AB + BC = AC

$AB = \sqrt{[(6 - 3)^2 + (4 - 1)^2]}$

$AB = \sqrt{[9 + 9]} = \sqrt{18} = 3\sqrt{2}$

$BC = \sqrt{[(8 - 6)^2 + (6 - 4)^2]}$

$BC = \sqrt{[4 + 4]} = \sqrt{8} = 2\sqrt{2}$

$AC = \sqrt{[(8 - 3)^2 + (6 - 1)^2]}$

$AC = \sqrt{[25 + 25]} = \sqrt{50} = 5\sqrt{2}$

Check: $AB + BC = 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2} = AC$ ✓

Answer: Points A, B, C are COLLINEAR (lie on same line)

⚠️ COMMON MISTAKES IN DISTANCE FORMULA:

❌ **Mistake 1:** Sign errors with negative coordinates

- Wrong: $(-2 - (-5))^2 = (-2 - 5)^2 = (-7)^2$
- Correct: $(-2 - (-5))^2 = (-2 + 5)^2 = (3)^2$
- **Remember:** Minus negative = Plus!

❌ **Mistake 2:** Not simplifying square roots

- Wrong: $\sqrt{32}$ (leaving as is)
- Correct: $\sqrt{32} = \sqrt{(16 \times 2)} = 4\sqrt{2}$
- Always simplify: $\sqrt{50} = 5\sqrt{2}$, $\sqrt{18} = 3\sqrt{2}$, $\sqrt{72} = 6\sqrt{2}$

❌ **Mistake 3:** Taking negative square root

- Distance is ALWAYS positive
- Always take positive square root

❌ **Mistake 4:** Wrong formula application

- Wrong: $\sqrt{[(x_2 + x_1)^2 + (y_2 + y_1)^2]}$
- Correct: $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$
- It's MINUS, not plus!

❌ **Mistake 5:** Calculation errors

- $(-5)^2 = 25$, NOT -25
- $3^2 + 4^2 = 9 + 16 = 25$, NOT 49
- Always double-check arithmetic

🎯 SECTION 3: SECTION FORMULA

SECTION FORMULA (INTERNAL DIVISION)

If point P(x, y) divides line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$

internally in ratio $m_1 : m_2$, then:

$$x = (m_1x_2 + m_2x_1)/(m_1 + m_2)$$

$$y = (m_1y_2 + m_2y_1)/(m_1 + m_2)$$

Or in coordinate form:

$$P(x, y) = ((m_1x_2 + m_2x_1)/(m_1 + m_2), (m_1y_2 + m_2y_1)/(m_1 + m_2))$$

Understanding Section Formula

Point P divides AB in ratio AP : PB = $m_1 : m_2$

Understanding the Formula

What does "divides in ratio $m_1 : m_2$" mean?

- $AP/PB = m_1/m_2$
- If $m_1 = 2$ and $m_2 = 3$, then $AP/PB = 2/3$
- This means: P is closer to A than to B
- The point P divides the line segment **internally** (lies between A and B)

💡 Key Points About Section Formula:

- ✓ The formula gives coordinates of the **dividing point**
- ✓ **m_1** is associated with **x_2, y_2** (second point)
- ✓ **m_2** is associated with **x_1, y_1** (first point)
- ✓ Cross multiplication: m_1 with far point, m_2 with near point
- ✓ If ratio is k : 1, formula becomes: $x = (kx_2 + x_1)/(k + 1)$, $y = (ky_2 + y_1)/(k + 1)$
- ✓ Always add both parts of ratio in denominator

TYPE 1: Finding Point Dividing in Given Ratio

Example 10: Find coordinates of point which divides line segment joining (−1, 7) and (4, −3) in ratio 2 : 3.

Solution:

Let $A(-1, 7) = (x_1, y_1)$ and $B(4, -3) = (x_2, y_2)$

Given ratio: $m_1 : m_2 = 2 : 3$

Using section formula:

$x = (m_1x_2 + m_2x_1)/(m_1 + m_2)$

$x = (2\times4 + 3\times(-1))/(2 + 3)$

$x = (8 - 3)/5$

$x = 5/5 = 1$

$y = (m_1y_2 + m_2y_1)/(m_1 + m_2)$

$y = (2\times(-3) + 3\times7)/(2 + 3)$

$y = (-6 + 21)/5$

$y = 15/5 = 3$

Answer: Required point is (1, 3)

Example 11: Find coordinates of point which divides join of (4, −3) and (8, 5) in ratio 3 : 1 internally.

```
Solution:
A(4, -3), B(8, 5), ratio = 3 : 1

x = (3×8 + 1×4)/(3 + 1)
x = (24 + 4)/4
x = 28/4 = 7

y = (3×5 + 1×(-3))/(3 + 1)
y = (15 - 3)/4
y = 12/4 = 3

Answer: Point is (7, 3)
```

TYPE 2: Finding Ratio of Division

Example 12: In what ratio does point (−4, 6) divide line segment joining A(−6, 10) and B(3, −8)?

Solution:

Let point P(−4, 6) divide AB in ratio $m_1 : m_2$

A(−6, 10), B(3, −8), P(−4, 6)

Using section formula for x-coordinate:

$-4 = (m_1 \times 3 + m_2 \times (-6))/(m_1 + m_2)$

$-4(m_1 + m_2) = 3m_1 - 6m_2$

$-4m_1 - 4m_2 = 3m_1 - 6m_2$

$-4m_1 - 3m_1 = -6m_2 + 4m_2$

$-7m_1 = -2m_2$

$7m_1 = 2m_2$

$m_1/m_2 = 2/7$

So, $m_1 : m_2 = 2 : 7$

Verification using y-coordinate:

$y = (m_1 y_2 + m_2 y_1)/(m_1 + m_2)$

$y = (2 \times (-8) + 7 \times 10)/(2 + 7)$

$y = (-16 + 70)/9$

$y = 54/9 = 6$ ✓

Answer: Ratio is 2 : 7

Example 13: Determine ratio in which line segment joining (1, −5) and (−4, 5) is divided by x-axis. Also find point of division.

```
Solution:
Point on x-axis has form P(x, 0)
Let A(1, −5), B(−4, 5)
Let ratio be k : 1

Since P lies on x-axis, y-coordinate = 0

Using y-coordinate in section formula:
0 = (k×5 + 1×(−5))/(k + 1)
0 = (5k − 5)/(k + 1)
5k − 5 = 0
5k = 5
k = 1

So ratio is 1 : 1 (midpoint!)

Now find x-coordinate:
x = (1×(−4) + 1×1)/(1 + 1)
x = (−4 + 1)/2
x = −3/2

Answer: Ratio = 1 : 1, Point = (−3/2, 0) or (−1.5, 0)
```

TYPE 3: Points of Trisection

Example 14: Find coordinates of points of trisection (dividing in three equal parts) of line segment joining A(2, −2) and B(−7, 4).

Solution:

Trisection means dividing into 3 equal parts

Let P and Q be points of trisection

Finding P:

P divides AB in ratio 1 : 2 (one part from A, two parts to B)

$m_1 = 1, m_2 = 2$

$x = (1×(-7) + 2×2)/(1 + 2)$

$x = (-7 + 4)/3 = -3/3 = -1$

$y = (1×4 + 2×(-2))/(1 + 2)$

$y = (4 - 4)/3 = 0/3 = 0$

P = (-1, 0)

Finding Q:

Q divides AB in ratio 2 : 1 (two parts from A, one part to B)

$m_1 = 2, m_2 = 1$

$x = (2×(-7) + 1×2)/(2 + 1)$

$x = (-14 + 2)/3 = -12/3 = -4$

$y = (2×4 + 1×(-2))/(2 + 1)$

$y = (8 - 2)/3 = 6/3 = 2$

Q = (-4, 2)

Answer: Points of trisection are (-1, 0) and (-4, 2)

🎯 SECTION 4: MID-POINT FORMULA

MID-POINT FORMULA

Mid-point of line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is:

$$M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$$

OR

$$x = (x_1 + x_2)/2 \qquad y = (y_1 + y_2)/2$$

Derivation from Section Formula:

Mid-point divides line segment in ratio 1 : 1

Using section formula with $m_1 = 1$, $m_2 = 1$:

$x = (1\times x_2 + 1\times x_1)/(1 + 1) = (x_1 + x_2)/2$

$y = (1\times y_2 + 1\times y_1)/(1 + 1) = (y_1 + y_2)/2$

Mid-point Formula: $M((x_1 + x_2)/2, (y_1 + y_2)/2)$

💡 Key Points About Mid-Point:

- ✓ Mid-point is **special case** of section formula (ratio 1:1)
- ✓ Simply **average** the x-coordinates and y-coordinates
- ✓ Much easier than section formula - use when finding midpoint
- ✓ **Faster method** for 50-50 division
- ✓ In parallelogram, diagonals bisect each other (same midpoint)

Applications of Mid-Point Formula

Example 15: Find mid-point of line segment joining (6, 1) and (8, 2).

Solution:

Let A(6, 1) and B(8, 2)

Mid-point $M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$

$M = ((6 + 8)/2, (1 + 2)/2)$

$M = (14/2, 3/2)$

$M = (7, 1.5)$

Answer: Mid-point is (7, 3/2) or (7, 1.5)

Example 16: Find coordinates of point A, where AB is diameter of circle whose centre is (2, −3) and B is (1, 4).

```
Solution:
Let A(x, y)
Centre C(2, -3), B(1, 4)

Since AB is diameter, centre is mid-point of AB:
C = Mid-point of AB

2 = (x + 1)/2
4 = x + 1
x = 3

-3 = (y + 4)/2
-6 = y + 4
y = -10

Answer: Coordinates of A are (3, -10)

Verification:
Mid-point of (3, -10) and (1, 4) = ((3+1)/2,
(-10+4)/2) = (2, -3) ✓
```

Example 17: If (1, 2), (4, y), (x, 6) and (3, 5) are vertices of parallelogram taken in order, find x and y.

Solution:

Let A(1, 2), B(4, y), C(x, 6), D(3, 5)

In parallelogram: Diagonals bisect each other

So, mid-point of AC = mid-point of BD

Mid-point of AC:

((1 + x)/2, (2 + 6)/2) = ((1 + x)/2, 4)

Mid-point of BD:

((4 + 3)/2, (y + 5)/2) = (7/2, (y + 5)/2)

Equating both:

(1 + x)/2 = 7/2

1 + x = 7

x = 6

4 = (y + 5)/2

8 = y + 5

y = 3

Answer: x = 6, y = 3

Example 18: If A and B are (−2, −2) and (2, −4), find coordinates of P such that AP = 3/7 AB and P lies on AB.

```
Solution:
Given: AP = (3/7)AB
This means AP/AB = 3/7

Since P lies on AB:
AP + PB = AB
AP + PB = (7/3)AP  [since AB = (7/3)AP]
PB = (7/3)AP - AP
PB = (7/3 - 1)AP
PB = (4/3)AP

So, AP/PB = 3/4
Therefore, P divides AB in ratio 3 : 4

Using section formula:
A(-2, -2), B(2, -4), ratio = 3 : 4

x = (3×2 + 4×(-2))/(3 + 4)
x = (6 - 8)/7 = -2/7

y = (3×(-4) + 4×(-2))/(3 + 4)
y = (-12 - 8)/7 = -20/7

Answer: P = (-2/7, -20/7)
```

TYPE 4: Dividing Into Equal Parts

Example 19: Find coordinates of points which divide line segment joining A(−2, 2) and B(2, 8) into four equal parts.

Solution:

Let P, Q, R be three points dividing AB into 4 equal parts

Point P: Divides AB in ratio 1 : 3

x = (1×2 + 3×(−2))/(1 + 3) = (2 − 6)/4 = −4/4 = −1

y = (1×8 + 3×2)/(1 + 3) = (8 + 6)/4 = 14/4 = 3.5

P = (−1, 3.5)

Point Q: Divides AB in ratio 2 : 2 = 1 : 1 (midpoint!)

Q = ((−2 + 2)/2, (2 + 8)/2) = (0, 5)

Q = (0, 5)

Point R: Divides AB in ratio 3 : 1

x = (3×2 + 1×(−2))/(3 + 1) = (6 − 2)/4 = 4/4 = 1

y = (3×8 + 1×2)/(3 + 1) = (24 + 2)/4 = 26/4 = 6.5

R = (1, 6.5)

Answer: Points are (−1, 3.5), (0, 5), and (1, 6.5)

⚠️ COMMON MISTAKES IN SECTION FORMULA:

❌ **Mistake 1:** Confusing which coordinate goes where

- Wrong: $x = (m_1x_1 + m_2x_2)/(m_1 + m_2)$
- Correct: $x = (m_1x_2 + m_2x_1)/(m_1 + m_2)$
- **Remember:** m_1 with x_2, m_2 with x_1 (cross multiplication!)

❌ **Mistake 2:** Forgetting to add denominators

- Wrong: $x = (m_1x_2 + m_2x_1)/m_1$ or $/m_2$
- Correct: $x = (m_1x_2 + m_2x_1)/(m_1 + m_2)$
- Always: $\mathbf{m_1 + m_2}$ in denominator

❌ **Mistake 3:** Wrong ratio interpretation

- If ratio is 2:3, then $m_1 = 2$, $m_2 = 3$
- Don't interchange the values
- First part of ratio = m_1, second part = m_2

❌ **Mistake 4:** Using section formula for midpoint

- For midpoint, use simpler formula: $((x_1+x_2)/2, (y_1+y_2)/2)$
- No need to use section formula with 1:1

❌ **Mistake 5:** Sign errors

- Be careful with negative coordinates
- Example: $(3\times(-4) + 2\times5)/(3+2) = (-12+10)/5 = -2/5$

🎯 SECTION 5: COMBINED APPLICATIONS

Real-Life Applications

Example 20: To conduct Sports Day, in rectangular school ground ABCD, lines drawn at 1m distance. 100 flower pots placed 1m apart along AD. Niharika runs 1/4th distance AD on 2nd line and posts green flag. Preet runs 1/5th distance AD on 8th line and posts red flag. What is distance between flags? If Rashmi posts blue flag halfway between the two flags, where should she post?

Solution:

Let A be at origin (0, 0)

AD = 100m (along y-axis)

Niharika's position:

- On 2nd line means $x = 2$
- 1/4th of AD means $y = 100/4 = 25$

Green flag: N(2, 25)

Preet's position:

- On 8th line means $x = 8$
- 1/5th of AD means $y = 100/5 = 20$

Red flag: P(8, 20)

Distance between flags:

$NP = \sqrt{[(8 - 2)^2 + (20 - 25)^2]}$

$NP = \sqrt{[6^2 + (-5)^2]}$

$NP = \sqrt{[36 + 25]}$

$NP = \sqrt{61} \approx 7.81$ m

Rashmi's position (midpoint):

$R = ((2 + 8)/2, (25 + 20)/2)$

$R = (5, 22.5)$

Answers:

Distance between flags = $\sqrt{61}$ m ≈ 7.81 m

Blue flag position = (5, 22.5) i.e., 5th line, 22.5m from A

Example 21: Find area of rhombus if vertices are (3, 0), (4, 5), (−1, 4) and (−2, −1) taken in order.

Solution:

Let A(3, 0), B(4, 5), C(−1, 4), D(−2, −1)

Area of rhombus = $(1/2) \times d_1 \times d_2$

where d_1 and d_2 are lengths of diagonals

Diagonal AC:

$AC = \sqrt{[(-1 - 3)^2 + (4 - 0)^2]}$

$AC = \sqrt{[16 + 16]}$

$AC = \sqrt{32} = 4\sqrt{2}$

Diagonal BD:

$BD = \sqrt{[(-2 - 4)^2 + (-1 - 5)^2]}$

$BD = \sqrt{[36 + 36]}$

$BD = \sqrt{72} = 6\sqrt{2}$

Area:

$\text{Area} = (1/2) \times 4\sqrt{2} \times 6\sqrt{2}$

$\text{Area} = (1/2) \times 24 \times 2$

Area = 24 square units

Answer: Area of rhombus = 24 square units

📝 PREVIOUS YEARS' BOARD QUESTIONS

2 MARK QUESTIONS

Q1. Find distance between points (0, 0) and (36, 15). (2020)

Solution:

A(0, 0), B(36, 15)

$AB = \sqrt{(36 - 0)^2 + (15 - 0)^2}$

$AB = \sqrt{1296 + 225}$

$AB = \sqrt{1521}$

AB = 39 units

Answer: 39 units

Q2. Find coordinates of point which divides join of (−1, 7) and (4, −3) in ratio 2:3. (2022)

Solution:

A(−1, 7), B(4, −3), ratio = 2:3

$x = (2\times4 + 3\times(-1))/(2 + 3) = (8 - 3)/5 = 1$

$y = (2\times(-3) + 3\times7)/(2 + 3) = (-6 + 21)/5 = 3$

Answer: (1, 3)

Q3. Find point on x-axis equidistant from (2, −5) and (−2, 9). (2023)

Solution:

Point on x-axis: P(x, 0)

Given: PA = PB

$PA^2 = (x - 2)^2 + (0 - (-5))^2$

$PA^2 = (x - 2)^2 + 25$

$PB^2 = (x - (-2))^2 + (0 - 9)^2$

$PB^2 = (x + 2)^2 + 81$

Since $PA^2 = PB^2$:

$(x - 2)^2 + 25 = (x + 2)^2 + 81$

$x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$

$-4x + 29 = 4x + 85$

$-8x = 56$

$x = -7$

Answer: (−7, 0)

3 MARK QUESTIONS

Q4. Determine if points (1, 5), (2, 3) and (−2, −11) are collinear. (2021)

Solution:

Let A(1, 5), B(2, 3), C(−2, −11)

$AB = \sqrt{(2 - 1)^2 + (3 - 5)^2}$

$AB = \sqrt{1 + 4} = \sqrt{5}$

$BC = \sqrt{(-2 - 2)^2 + (-11 - 3)^2}$

$BC = \sqrt{16 + 196} = \sqrt{212} = 2\sqrt{53}$

$AC = \sqrt{(-2 - 1)^2 + (-11 - 5)^2}$

$AC = \sqrt{9 + 256} = \sqrt{265}$

Check: $AB + BC = \sqrt{5} + 2\sqrt{53} \neq \sqrt{265}$

Answer: Points are NOT collinear

Q5. Check whether (5, −2), (6, 4) and (7, −2) are vertices of isosceles triangle. (2024)

Solution:

Let A(5, −2), B(6, 4), C(7, −2)

$AB = \sqrt{(6 - 5)^2 + (4 - (-2))^2}$

$AB = \sqrt{1 + 36} = \sqrt{37}$

$BC = \sqrt{(7 - 6)^2 + (-2 - 4)^2}$

$BC = \sqrt{1 + 36} = \sqrt{37}$

$AC = \sqrt{(7 - 5)^2 + (-2 - (-2))^2}$

$AC = \sqrt{4 + 0} = 2$

Since $AB = BC = \sqrt{37}$

Answer: YES, it's an ISOSCELES triangle

Q6. Find ratio in which line segment joining (−3, 10) and (6, −8) is divided by (−1, 6). (2023)

Solution:

Let A(-3, 10), B(6, -8), P(-1, 6)

Let ratio = k : 1

Using x-coordinate:

$-1 = (k\times6 + 1\times(-3))/(k + 1)$

$-1(k + 1) = 6k - 3$

$-k - 1 = 6k - 3$

$2 = 7k$

$k = 2/7$

Ratio = 2/7 : 1 = 2 : 7

Verification with y-coordinate:

$y = (2\times(-8) + 7\times10)/(2 + 7)$

$y = (-16 + 70)/9 = 54/9 = 6$ ✓

Answer: Ratio = 2 : 7

4 MARK QUESTIONS

Q7. Name type of quadrilateral formed by points (−1, −2), (1, 0), (−1, 2), (−3, 0). Give reasons. (2022)

Solution:

Let A(−1, −2), B(1, 0), C(−1, 2), D(−3, 0)

$AB = \sqrt{[(1 - (-1))^2 + (0 - (-2))^2]}$

$AB = \sqrt{[4 + 4]} = \sqrt{8} = 2\sqrt{2}$

$BC = \sqrt{[(-1 - 1)^2 + (2 - 0)^2]}$

$BC = \sqrt{[4 + 4]} = \sqrt{8} = 2\sqrt{2}$

$CD = \sqrt{[(-3 - (-1))^2 + (0 - 2)^2]}$

$CD = \sqrt{[4 + 4]} = \sqrt{8} = 2\sqrt{2}$

$DA = \sqrt{[(-1 - (-3))^2 + (-2 - 0)^2]}$

$DA = \sqrt{[4 + 4]} = \sqrt{8} = 2\sqrt{2}$

All sides equal = $2\sqrt{2}$

Check diagonals:

$AC = \sqrt{[(-1 - (-1))^2 + (2 - (-2))^2]}$

$AC = \sqrt{[0 + 16]} = 4$

$BD = \sqrt{[(-3 - 1)^2 + (0 - 0)^2]}$

$BD = \sqrt{[16 + 0]} = 4$

Both diagonals equal = 4

Since all sides equal AND diagonals equal:

Answer: SQUARE

Q8. If Q(0, 1) is equidistant from P(5, −3) and R(x, 6), find values of x. Also find distances QR and PR. (2024)

Solution:

Given: QP = QR

$QP^2 = (5 - 0)^2 + (-3 - 1)^2$

$QP^2 = 25 + 16 = 41$

$QR^2 = (x - 0)^2 + (6 - 1)^2$

$QR^2 = x^2 + 25$

Since $QP^2 = QR^2$:

$41 = x^2 + 25$

$x^2 = 16$

$x = \pm 4$

Distance QR:

$QR = \sqrt{41}$ units

Distance PR when x = 4:

$PR = \sqrt{[(4 - 5)^2 + (6 - (-3))^2]}$

$PR = \sqrt{[1 + 81]} = \sqrt{82}$ units

Distance PR when x = −4:

$PR = \sqrt{[(-4 - 5)^2 + (6 - (-3))^2]}$

$PR = \sqrt{[81 + 81]} = \sqrt{162} = 9\sqrt{2}$ units

Answer: x = 4 or x = −4

$QR = \sqrt{41}$ units

$PR = \sqrt{82}$ or $9\sqrt{2}$ units

💯 EXAM STRATEGY & TIME MANAGEMENT

Question Type	Marks	Time	Strategy
Distance between points	2	2-3 min	Apply formula directly. Simplify square root. Show all steps.
Section formula	2-3	3-4 min	Identify ratio. Apply formula for both coordinates. Simplify.
Finding ratio	3	4-5 min	Use one coordinate to find ratio. Verify with other coordinate.
Collinearity check	3	4-5 min	Find all three distances. Check if AB + BC = AC.
Triangle/Quadrilateral type	3-4	5-7 min	Find all sides. Check diagonals if needed. State type with reason.
Equidistant problems	3-4	5-7 min	Set $PA^2 = PB^2$. Solve equation. Verify answer.

⏰ Time Allocation Tips:

- **Read carefully:** Note what is given and what to find (30 sec)
- **Choose right formula:** Distance or Section formula (15 sec)
- **Show formula first:** Always write formula before substituting
- **Simplify radicals:** $\sqrt{32} = 4\sqrt{2}$, $\sqrt{50} = 5\sqrt{2}$
- **Box final answer:** Make it clearly visible
- **Verify if time permits:** Especially for 3-4 mark questions

⚠️ COMMON MISTAKES TO AVOID (CONSOLIDATED)

Top 20 Mistakes Students Make:

1. **Sign errors with negatives:** $(-2 - (-5)) = (-2 + 5) = 3$, NOT -7
2. **Wrong formula:** Distance uses MINUS ($x_2 - x_1$), not plus
3. **Not simplifying √:** $\sqrt{32}$ must be written as $4\sqrt{2}$
4. **Section formula confusion:** m_1 with x_2, m_2 with x_1 (cross!)
5. **Forgetting ($m_1 + m_2$):** Always in denominator
6. **Taking negative √:** Distance is always positive
7. **Calculation errors:** $(-5)^2 = 25$, NOT -25
8. **Order in coordinates:** $(3, 4) \neq (4, 3)$
9. **Not checking collinearity:** Must verify AB + BC = AC
10. **Wrong diagonal check:** For square, need sides AND diagonals equal
11. **Incomplete working:** Show all steps for full marks
12. **Not using midpoint formula:** For 1:1, use easier formula
13. **Ratio interpretation:** 2:3 means $m_1=2$, $m_2=3$
14. **Forgetting verification:** Always verify with both coordinates
15. **Not stating conclusion:** "Therefore, it's a square"
16. **Wrong Pythagoras check:** $a^2 + b^2 = c^2$ for right triangle
17. **Arithmetic mistakes:** $3^2 + 4^2 = 9 + 16 = 25$, NOT 49
18. **Not boxing answer:** Final answer should be highlighted
19. **Incomplete triangle check:** Check ALL three combinations
20. **Not reading "equidistant":** Set $PA^2 = PB^2$, don't calculate both

📚 IMPORTANT FORMULAS - QUICK REFERENCE

1. DISTANCE FORMULA

$PQ = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

From origin: $OP = \sqrt{(x^2 + y^2)}$

2. SECTION FORMULA (Internal Division)

Point dividing in ratio $m_1 : m_2$

$x = (m_1x_2 + m_2x_1)/(m_1 + m_2)$

$y = (m_1y_2 + m_2y_1)/(m_1 + m_2)$

3. MID-POINT FORMULA

$M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$

4. COLLINEARITY

Points A, B, C collinear if: AB + BC = AC

5. TRIANGLE TYPES

- Right-angled: $a^2 + b^2 = c^2$
- Isosceles: Two sides equal
- Equilateral: All three sides equal

- Scalene: All sides different

6. QUADRILATERAL TYPES

- Square: All sides equal, diagonals equal
- Rectangle: Opposite sides equal, diagonals equal
- Rhombus: All sides equal, diagonals NOT equal
- Parallelogram: Opposite sides equal

🚀 LAST MINUTE REVISION CHECKLIST

☑ Theory to Remember:

- ☑ Distance formula - write it 10 times!
- ☑ Section formula - m_1 with x_2, m_2 with x_1
- ☑ Mid-point = Section with ratio 1:1
- ☑ Collinearity: AB + BC = AC
- ☑ Right triangle: $a^2 + b^2 = c^2$
- ☑ Square: All sides equal AND diagonals equal

☑ Quick Checks:

- ☑ Did I simplify the square root?
- ☑ Are my signs correct with negatives?
- ☑ Did I verify my answer?
- ☑ Have I shown all steps?
- ☑ Is final answer boxed/highlighted?

☑ Common Question Types:

- ☑ Find distance between two points
- ☑ Find point dividing in given ratio
- ☑ Find ratio of division
- ☑ Check if triangle (collinearity)
- ☑ Identify type of triangle/quadrilateral
- ☑ Find point equidistant from two points
- ☑ Mid-point and trisection problems

☑ Before Exam:

- ☑ Practice 50+ questions
- ☑ Solve last 5 years' board questions
- ☑ Time yourself - 3-5 min per question

- ☑ Revise all formulas
- ☑ Go through common mistakes
- ☑ Practice simplifying square roots

💡 EXPERT TIPS FOR SCORING FULL MARKS

How to Score 100% in Coordinate Geometry:

- **1. Formula first:** Always write formula before substituting
- **2. Simplify radicals:** $\sqrt{32} = 4\sqrt{2}$, $\sqrt{50} = 5\sqrt{2}$, $\sqrt{72} = 6\sqrt{2}$
- **3. Show all working:** Every step matters for marks
- **4. Verify answers:** Check with both coordinates
- **5. Box final answer:** Make it stand out
- **6. Be careful with signs:** Double-check negative numbers
- **7. State conclusions:** "Therefore, it's a square"
- **8. Use shortcuts wisely:** Midpoint formula for 1:1
- **9. Check arithmetic:** Simple errors cost marks
- **10. Practice regularly:** Speed comes with practice

📖 PRACTICE QUESTIONS FOR SELF-ASSESSMENT

Section A: 2 Marks Questions

1. Find distance between (3, 4) and (−1, 1).
2. Find mid-point of segment joining (2, −5) and (4, 3).
3. Find coordinates dividing (1, 2) and (5, 6) in ratio 1:3.
4. In which ratio does (2, 3) divide segment joining (1, 2) and (4, 5)?
5. Find distance of point (5, −12) from origin.

Section B: 3 Marks Questions

6. Find point on y-axis equidistant from (−5, −2) and (3, 2).
7. Check if (−2, 2), (8, −2) and (−4, −3) are collinear.
8. Find ratio in which (3, 4) divides join of (1, 2) and (6, 7).
9. Find coordinates of points of trisection of segment joining (1, −2) and (−3, 4).
10. If (2, 3), (4, k), (6, 5) are collinear, find k.

Section C: 4 Marks Questions

11. Show that (0, 7), (3, 1), (−3, 1) form an isosceles triangle. Find area.
12. Prove that (4, 3), (6, 4), (5, 6), (3, 5) are vertices of square.
13. If Q(0, −1) is equidistant from P(5, −3) and R(x, 6), find x and distances.
14. Find coordinates of A if AB is diameter with center (2, −3) and B(1, 4).
15. Show that A(3, 0), B(6, 4), C(−1, 3) and D(−4, −1) form rhombus.

🎓 BONUS: AREA OF TRIANGLE (NOT IN CBSE SYLLABUS)

⚠️ IMPORTANT NOTE:

This topic is NOT in CBSE Class 10 syllabus 2025-26.

Included here for competitive exams (JEE, NTSE, Olympiad) preparation only.

Skip this if preparing only for CBSE Board Exams.

AREA OF TRIANGLE (For Competitive Exams)

Area of triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$:

$$\text{Area} = (1/2)|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Take absolute value to ensure positive area

Example 22 (Bonus): Find area of triangle with vertices (1, 2), (3, 5), (5, 1).

Solution:

Let A(1, 2), B(3, 5), C(5, 1)

$$\text{Area} = (1/2)|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$\text{Area} = (1/2)|1(5 - 1) + 3(1 - 2) + 5(2 - 5)|$$

$$\text{Area} = (1/2)|1(4) + 3(-1) + 5(-3)|$$

$$\text{Area} = (1/2)|4 - 3 - 15|$$

$$\text{Area} = (1/2)|-14|$$

Area = 7 square units

Answer: Area = 7 square units

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