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Complete NCERT Solutions — Compiled Volume Class IX Mathematics · Ganita Manjari Part I

All available chapters · Every exercise set & end-of-chapter exercise, solved step-wise

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A Note on This Document: Parts of this material have been organised and formatted with the help of AI-based tools, and every solution has thereafter been checked against the NCERT text and the CBSE marking scheme. Even so, an occasional slip may remain. Please treat the written working and final answers as the reference, and bring anything that looks doubtful to your teacher — that conversation is itself a useful part of learning. **All diagrams in this document are redrawn schematics of the textbook figures — correct in construction, but not to scale.**

Coverage note. NCERT has released **Ganita Manjari Part I** with 8 chapters for Session 2026–27. This compiled volume carries step-wise NCERT solutions for **all 8 chapters** — every exercise set together with the end-of-chapter exercises. Diagrams are redrawn schematics: correct in construction, but not drawn to scale.

⚠ IMPORTANT — this volume is NOT the whole examinable syllabus

NCERT has so far released only **Ganita Manjari Part I (8 chapters)**. The official CBSE Mathematics curriculum for Class IX, 2026–27 (Subject Code 041) is built on **6 units / 15 chapters**. The 8 chapters solved in this volume therefore cover **only about half of the 80-mark annual paper**. The units below are **fully examinable** but await *Ganita Manjari Part II*:

Unit	Topics still to come in Part II	Approx. marks
II — Algebra (remainder)	Linear Equations in Two Variables	part of 20
IV — Geometry	Euclid's axioms & postulates; Lines & Angles; Triangles (congruence); Quadrilaterals	25
V — Mensuration (remainder)	Surface Areas and Volumes	part of 14
VI — Statistics & Probability (remainder)	Statistics (data handling, graphs, measures of central tendency)	part of 10

What this means for you: do not treat this book as “the complete Class 9 Maths course”. MLI will issue a Part II companion volume as soon as NCERT releases the textbook. Until then, cover Geometry and Statistics from the topics listed in the official CBSE curriculum document, and ask your faculty for the MLI interim Geometry handouts.

Marks scheme note: the 2026–27 unit weightage has been revised — Number System 7, Algebra 20, Coordinate Geometry 4, Geometry 25, Mensuration 14, Statistics & Probability 10 (total 80). The older 10 / 20 / 4 / 27 / 13 / 6 split no longer applies.

✓ How to use this document — and what it does not replace

This volume is written so that you do **not** need a second guide or market notes for the chapters it covers. Four honest points, so nothing surprises you later:

- **This volume covers *Ganita Manjari Part I* only.** NCERT has not yet released Part II, so Linear Equations in Two Variables, Euclid's axioms, Lines & Angles, Triangles, Quadrilaterals, Surface Areas and Volumes, and Statistics are **not here** — yet they are fully examinable. See the red banner above. MLI will issue a Part II companion as soon as the book is out.
- **Keep your NCERT *Ganita Manjari* open beside this.** Every figure here is an **MLI-redrawn schematic** — correct in construction but **not drawn to scale**, and not a copy of any textbook figure. Each one carries a note saying so. Never measure a length or an angle off these diagrams; always work from the given data.
- **Constructions are meant to be drawn by hand.** Where a question asks for a construction or a proof, draw the figure yourself on plain paper — a proof submitted without a figure loses a mark, and copying a printed figure teaches you nothing about the construction.
- **If anything here disagrees with your printed NCERT book, the book wins.** Tell your faculty so this volume can be corrected. Parts of this material were organised with AI assistance and then checked; every numerical answer has been worked through, but an occasional slip may still remain.

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CHAPTER 1

Orienting Yourself: The Use of Coordinates

Exercise Sets 1.1 – 1.2 + End-of-Chapter (with textbook figures)

EXERCISE SETS 1.1 – 1.2 + END-OF-CHAPTER (WITH TEXTBOOK FIGURES)



Weightage (School Annual Exam, Per Official CBSE Curriculum)

Marks Distribution (Out of 80 — Annual Theory Paper)

✓ Verified directly against the official CBSE curriculum document (cbseacademic.nic.in, Mathematics Class IX, 2026–27) — not a secondary source. The document lists "Coordinate Geometry" as Unit III with exactly these topics: brief history of coordinate geometry, the 2-D Cartesian coordinate system, distance between two points, and the midpoint of a line-segment. It is allotted only **6 teaching periods** — the smallest of any unit in the whole syllabus.

Detail	Value
Unit	Coordinate Geometry (Unit III)
Chapters in this unit	Coordinate Geometry is the only chapter (this chapter, "Orienting Yourself")
Marks allotted	4 marks out of 80 — the lowest-weightage unit in the entire Class 9 syllabus
Teaching periods	6 periods (out of ~200 total for the year)
Internal Assessment (separate, 20 marks)	Pen-Paper Test & Multiple Assessment (10), Portfolio (5), Lab Practical (5)

✦ Because this chapter carries so few marks, it is one of the most efficient chapters to fully master — a small, well-defined set of skills (plotting points, distance formula, midpoint) covers the entire unit.

Predicted Question Types

Format	Marks	Likely Topic
MCQ / Very Short Answer	1	Identifying the quadrant of a point; reading coordinates off a grid; sign conventions
Short Answer	2	Finding the distance between two points; checking collinearity
Short/Long Answer	2–3	Finding a midpoint or an unknown endpoint given a midpoint; checking if a triangle is right-angled using coordinates
Application/Reasoning	2–3	Floor-plan style questions (locating objects, doors, furniture on a grid) similar to the chapter's own "Reiaan's room" example

All Key Formulas & Definitions

Coordinate plane / Cartesian plane / xy-plane: the plane formed by two perpendicular number lines — the horizontal x-axis and vertical y-axis.

Origin O: the point where the x-axis and y-axis intersect; its coordinates are (0, 0).

Coordinates of a point P(x, y): x is the perpendicular distance of P from the y-axis (measured along the x-axis); y is the perpendicular distance of P from the x-axis (measured along the y-axis).

Quadrant sign conventions: Quadrant I $\rightarrow (+, +)$; Quadrant II $\rightarrow (-, +)$; Quadrant III $\rightarrow (-, -)$; Quadrant IV $\rightarrow (+, -)$.

Points on the axes: a point on the x-axis has the form (x, 0); a point on the y-axis has the form (0, y).

(x, y) = (y, x) only when $x = y$; otherwise $(x, y) \neq (y, x)$ — order matters.

Distance formula (from the Baudhāyana–Pythagoras Theorem): the distance between (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Special cases: distance between (x_1, y) and (x_2, y) [same y] is simply $|x_2 - x_1|$; distance between (x, y_1) and (x, y_2) [same x] is simply $|y_2 - y_1|$.

Midpoint of a line segment: the midpoint M of the segment joining (x_1, y_1) and (x_2, y_2) is $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.

Collinearity test using distances: three points A, B, C (in that order along a line) are collinear if the sum of the two shorter distances equals the longest distance (e.g. $AB + BC = AC$).

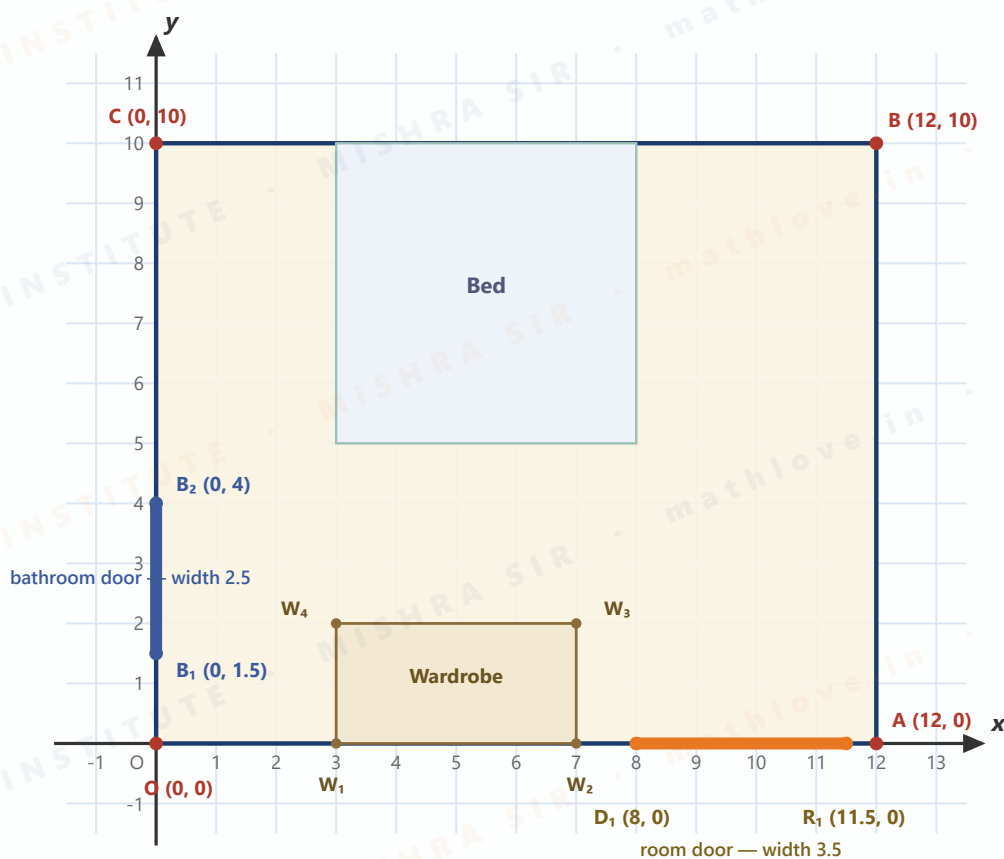
Shortcuts & Study Tips

1. **Reading a point's quadrant fast:** look only at the signs of x and y — you don't need the exact values to name the quadrant.
2. **Distance formula shortcut:** if two points share the same x -coordinate or the same y -coordinate, skip the full formula — just subtract the other coordinates directly (no square roots needed).
3. **Midpoint formula:** just average the x 's and average the y 's separately — a very common small error is averaging x with y by mistake.
4. **Finding a missing endpoint from a midpoint:** rearrange the midpoint formula as $x_2 = 2M_x - x_1$ and $y_2 = 2M_y - y_1$ — don't try to guess-and-check.
5. **Checking collinearity without plotting:** compute all three pairwise distances, then check if the two smaller ones add up exactly to the largest one.
6. **Checking a triangle is right-angled using coordinates:** compute all three side lengths, then check if the Pythagoras relation (sum of squares of two sides = square of the third) holds.
7. **Floor-plan style questions:** always write down what a positive/negative x or y physically means in the picture (e.g., "up," "right," "into the next room") before answering — this prevents sign mix-ups.

NCERT Solutions — Exercise Set 1.1

🔑 Approach for This Exercise

This exercise refers to Fig. 1.3 in the textbook (Reiaan's room, with O at the origin). Read the coordinates of the labelled points directly off the grid, then answer using simple subtraction — no formulas needed yet.



Reiaan's room (drawn to scale from Fig. 1.3): room $O(0,0)$ – $A(12,0)$ – $B(12,10)$ – $C(0,10)$; room door D_1R_1 on the x-axis; bathroom door B_1B_2 on the y-axis.

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari, Part I*.

Referring to Fig. 1.3: (i) door's distance from left wall & x-axis (ii) coordinates of D_1 (iii) width of the door & comfort check (iv) is the bathroom door narrower or wider?

4 questions

GIVEN: Room corners $O(0,0)$, $A(12,0)$, $B(12,10)$, $C(0,10)$; door points D_1 and R_1 on the x-axis, with $R_1 = (11.5, 0)$; bathroom door points $B_1(0, 1.5)$ and $B_2(0, 4)$.

LAYOUT DESCRIPTION: The room is a 12×10 rectangle with O at the bottom-left (origin), A at the bottom-right, B at the top-right, and C at the top-left. The room's main door sits on the bottom wall (the x-axis) between points D_1 and R_1 . The bathroom door sits on the left wall (the y-axis) between points B_1 and B_2 .

SOLUTION:

(i) Reading D_1 off the grid, $D_1 = (8, 0)$. Its distance from the left wall (the y-axis) is its x-coordinate: **8 units**. Since D_1 lies exactly on the x-axis, its distance from the x-axis is **0 units**. **1 mark**

(ii) $D_1 = (8, 0)$ **1 mark**

(iii) Since $D_1 = (8, 0)$ and $R_1 = (11.5, 0)$ lie on the same line (the x-axis), $\text{width} = |11.5 - 8| = 3.5$ units. Taking the unit as 1 foot, that's $3.5 \text{ ft} \approx 42$ inches. Standard residential doors are about 30–36 inches wide, so this door is comfortably wide — even slightly more generous than average. A wheelchair typically needs at least 32 inches of clear width; since $42 \text{ in} > 32 \text{ in}$, a wheelchair user should be able to enter easily. **1 mark**

(iv) Bathroom door width $= |4 - 1.5| = 2.5$ units, compared to the room door's 3.5 units. Since $2.5 < 3.5$, **the bathroom door is narrower than the room door**. **1 mark**

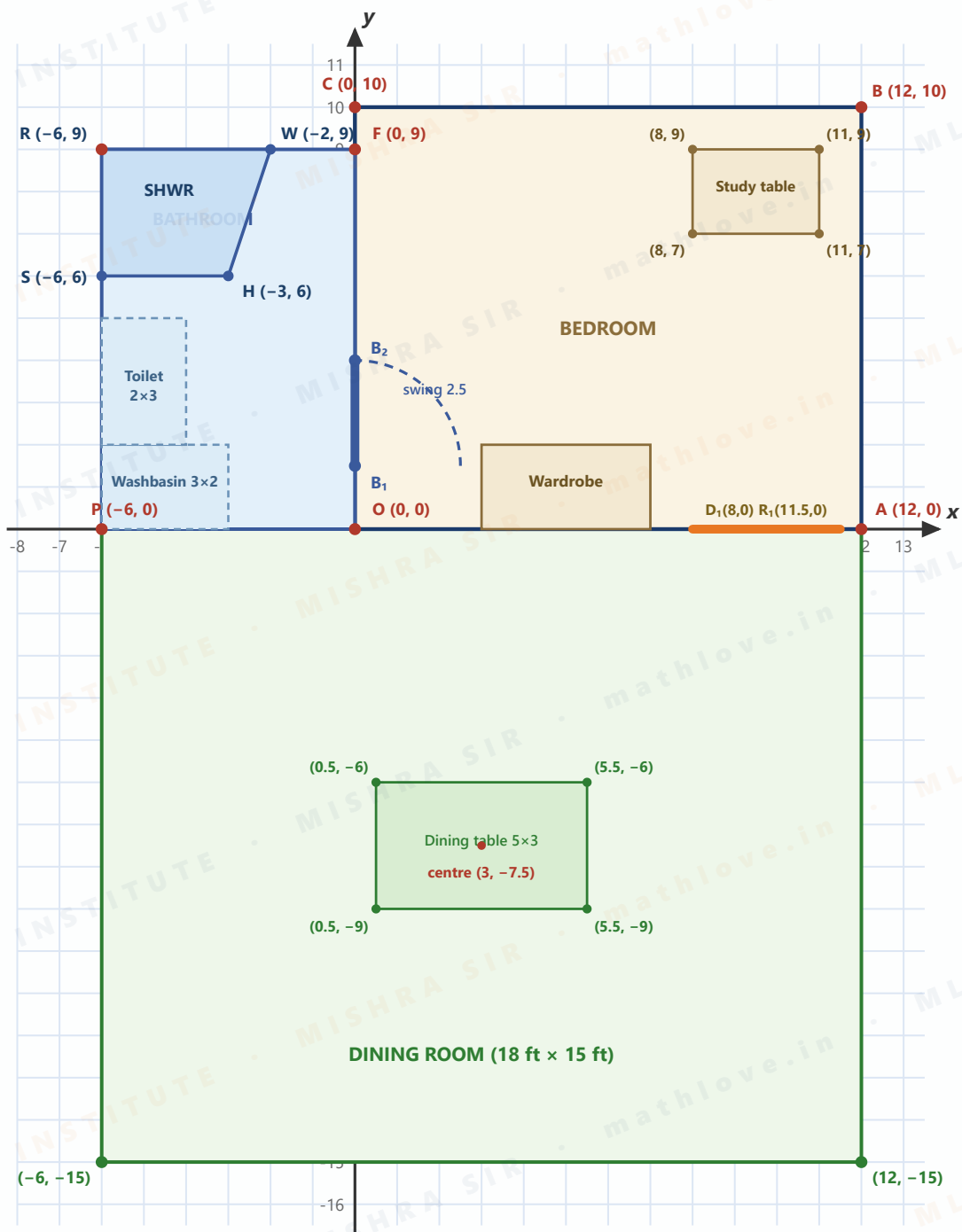
Think and Reflect (discussion prompts — sample responses)

- Standard door widths are typically 30–36 inches (about 2.5–3 ft) for interior residential doors; door widths vary by country and building code, so students should physically check doors at home and school.
- Accessible design guidelines (e.g., in India, the Harmonised Guidelines for accessibility) generally recommend a minimum clear width of about 32 inches (0.81 m) for wheelchair access — students should compare their own school's doors against this benchmark.

— End of Exercise Set 1.1 —

🔑 **Approach for This Exercise**

This exercise refers to the extended Fig. 1.5 (bathroom + bedroom + dining room, all sharing the same axes). The full diagram below is drawn to scale directly from the textbook figure, so every coordinate quoted in the solutions can be read straight off it.



Complete layout drawn to scale from Fig. 1.5: bathroom $O-F-R-P$ (left of the y -axis), showering trapezium $SHWR$ sharing corner $R(-6, 9)$, example washbasin/toilet placements, the door-swing arc from hinge B_1 , and the dining room from $P(-6, 0)$ to $A(12, 0)$ with the centred 5×3 table.

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari, Part I*.

Q1. Place a rectangular study table with three feet at (8,9), (11,9), (11,7). (i) fourth foot? (ii) good spot? (iii) width, length, height?

3 marks

GIVEN: Three corners of a rectangular table: (8,9), (11,9), (11,7).

SOLUTION:

(i) In a rectangle, opposite sides are parallel to the axes here. (8,9) and (11,9) share $y = 9$ (one side); (11,9) and (11,7) share $x = 11$ (an adjacent side). The fourth corner must match the "outer" x from the first point ($x = 8$) and the "outer" y from the third point ($y = 7$): **fourth foot = (8, 7)**

1 mark

(ii) Yes — this is a reasonable spot: the table sits fully inside the room boundary, away from the doorway (which is near the x -axis, $y = 0$) and doesn't overlap the bed or wardrobe area described in the chapter, so it doesn't block movement through the room.

1 mark

(iii) Length = $|11 - 8| = 3$ units (along the x -direction, from $x=8$ to $x=11$); Width = $|9 - 7| = 2$ units (along the y -direction, from $y=9$ to $y=7$). The height of the table **cannot be determined** from this figure, because the sketch is a top-view (floor plan) showing only two dimensions (x and y) — there is no third (vertical/height) axis represented.

1 mark

Q2. If the bathroom door has a hinge at B_1 and opens into the bedroom, will it hit the wardrobe?

2 marks

GIVEN: Bathroom door hinge at $B_1 = (0, 1.5)$; door width 2.5 units (from Exercise 1.1(iv)); wardrobe's near edge at $x = 3$ (from Fig. 1.5).

SOLUTION: When a door swings open from its hinge, it sweeps out a quarter-circle whose radius equals the door's width. Here, the door width is 2.5 units, so the swept path reaches at most 2.5 units from the hinge in any direction into the room.

Comparison: the wardrobe's nearest edge is 3 units from the y -axis (the hinge's axis). Since the swing radius (2.5 units) is less than this distance (3 units), **the door will not hit the wardrobe.**

Suggestion if the door is widened: if the door were widened beyond 3 ft, its swing radius would reach or pass the wardrobe's edge, so either the wardrobe should be moved further away, or the door should be set to open outward (into the bathroom/corridor) instead of into the bedroom.

2 marks

Q3. Look at Reiaan's bathroom: (i) coordinates of corners O,F,R,P (ii) shape of showering area SHWR, its corners (iii) washbasin & toilet space coordinates

3 marks

SOLUTION: (read directly from Fig. 1.5, redrawn to scale above)

(i) The bathroom is the rectangle to the left of the y-axis: $O = (0, 0)$, $F = (0, 9)$, $R = (-6, 9)$, $P = (-6, 0)$.

1 mark

(ii) Tracing $S(-6, 6) \rightarrow H(-3, 6) \rightarrow W(-2, 9) \rightarrow R(-6, 9)$: side SH is horizontal ($y = 6$) and side WR is horizontal ($y = 9$) — one pair of opposite sides is parallel, while the other pair (HW and RS) is not. So

SHWR is a trapezium, with corners $S(-6, 6)$, $H(-3, 6)$, $W(-2, 9)$, $R(-6, 9)$.

1 mark

(iii) Taking convenient empty corners inside the bathroom boundary (below the trapezium, avoiding overlap): washbasin ($3 \text{ ft} \times 2 \text{ ft}$) at corners $(-6, 0)$, $(-3, 0)$, $(-3, 2)$, $(-6, 2)$; toilet ($2 \text{ ft} \times 3 \text{ ft}$) at corners $(-6, 2)$, $(-4, 2)$, $(-4, 5)$, $(-6, 5)$. (Any non-overlapping placement inside the bathroom is equally acceptable — this is one valid example.)

1 mark

Q4. Other rooms: (i) sketch the dining room ($18 \text{ ft} \times 15 \text{ ft}$, from P to A) (ii) place a 5×3 dining table at its centre

2 marks

SOLUTION:

(i) $P = (-6, 0)$ (the bathroom's bottom-left corner, from Q3) and $A = (12, 0)$ — so PA runs along the x-axis with length $|12 - (-6)| = 18 \text{ ft}$ ✓, matching the given length. The dining room extends 15 ft downward (negative y-direction) from PA , giving corners $P(-6, 0)$, $A(12, 0)$, $(12, -15)$, $(-6, -15)$.

1 mark

(ii) Centring a $5 \text{ ft} \times 3 \text{ ft}$ table: margin along the length $= \frac{18 - 5}{2} = 6.5 \text{ ft}$ on each end; margin along the width $= \frac{15 - 3}{2} = 6 \text{ ft}$ on each side. Starting from corner $(-6, 0)$: the table's four feet are at $(-6 + 6.5, 0 - 6) = (0.5, -6)$, $(0.5 + 5, -6) = (5.5, -6)$, $(5.5, -6 - 3) = (5.5, -9)$, $(0.5, -9)$.

1 mark

— End of Exercise Set 1.2 —

Worked Example — Distance Formula (Triangle ADM)

Approach

This is how the textbook itself derives and illustrates the distance formula — shown here as a fully worked, verified example since it underpins nearly every question in the End-of-Chapter Exercises.

Find the lengths of the sides of triangle ADM, where A(3,4), D(7,1), M(9,6).

Textbook Example

SETUP: Picture point A(3,4) plotted 3 units right and 4 units up from the origin; point D(7,1) plotted 7 units right and 1 unit up; point M(9,6) plotted 9 units right and 6 units up. All three lie in Quadrant I. To find AD, introduce a helper point C(3,1) — it shares its x-coordinate (3) with A, and its y-coordinate (1) with D. This makes AC a vertical segment and CD a horizontal segment, meeting at a right angle at C — exactly the two legs of a right triangle whose hypotenuse is AD.

SOLUTION: The helper point C(3,1) shares its x-coordinate with A and its y-coordinate with D, forming a right angle at C.

$$CD = x\text{-coordinate of } D - x\text{-coordinate of } A = 7 - 3 = 4$$

$$AC = y\text{-coordinate of } A - y\text{-coordinate of } D = 4 - 1 = 3$$

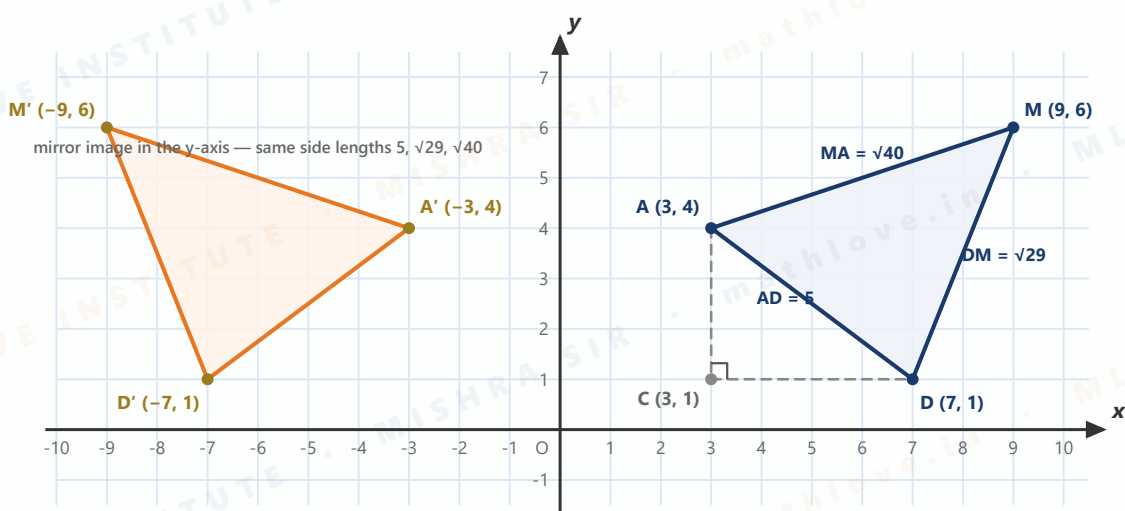
By the Baudhāyana–Pythagoras Theorem: $AD = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$ units

✓

Similarly:

$$DM = \sqrt{(9-7)^2 + (6-1)^2} = \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29} \text{ units}$$

$$MA = \sqrt{(9-3)^2 + (6-4)^2} = \sqrt{6^2 + 2^2} = \sqrt{36 + 4} = \sqrt{40} \text{ units}$$



Triangle ADM with helper point C(3, 1) forming the right-angled construction, and its reflection A'D'M' in the y-axis (lengths preserved).

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

Reflection Example (Fig. 1.9)

Triangle AMD is reflected in the y-axis. Verify that reflection preserves side lengths.

Textbook Example

SETUP: The original triangle has $A(3,4)$, $D(7,1)$, $M(9,6)$, all in Quadrant I (to the right of the y-axis). Reflecting each point in the y-axis flips the sign of the x-coordinate only, keeping the y-coordinate unchanged, and moves each point to the mirror-image position in Quadrant II (to the left of the y-axis): $A(3,4) \rightarrow A'(-3,4)$, $D(7,1) \rightarrow D'(-7,1)$, $M(9,6) \rightarrow M'(-9,6)$. Picture the reflected triangle $A'D'M'$ as the "mirror twin" of ADM , flipped left-right across the y-axis, the same distance from the axis on the other side.

SOLUTION:

$$A'D' = \sqrt{(-3 - (-7))^2 + (4 - 1)^2} = \sqrt{4^2 + 3^2} = 5$$

$$D'M' = \sqrt{(-7 - (-9))^2 + (1 - 6)^2} = \sqrt{2^2 + 5^2} = \sqrt{29}$$

$$M'A' = \sqrt{(-9 - (-3))^2 + (6 - 4)^2} = \sqrt{6^2 + 2^2} = \sqrt{40}$$

These are **identical to the original triangle's side lengths** (5, $\sqrt{29}$, $\sqrt{40}$). Reflection preserves distances (it is a "rigid" transformation); only the position, not the shape or size, changes.

Think and Reflect (sample responses)

1. What stays the same after reflection: the shape and all side lengths of the triangle. What changes: the position (every point's x-coordinate has its sign flipped, so the whole figure moves to the mirror-image side of the y-axis).
2. Yes — if $\triangle ADM$ were reflected in the x-axis instead, the same conclusion would hold (side lengths preserved), except now it's the y-coordinates whose signs would flip, not the x-coordinates.

NCERT Solutions — End-of-Chapter Exercises

Approach

Questions 1–5 and 14–16 are standard exam-style questions. Questions marked **Enrichment** (6–13) carry an asterisk in the textbook, meaning NCERT does not require them for the school exam — but full solutions are given below since they build essential skills (collinearity, midpoints, circles) that reinforce everything else in the chapter.

Q1. What are the x-coordinate and y-coordinate of the point of intersection of the two axes?

1 mark

SOLUTION: The two axes intersect at the origin, whose coordinates are **(0, 0)** — so both the x-coordinate and y-coordinate are **0**.

1 mark

Q2. Point W has x-coordinate -5. Predict the coordinates of point H, on the line through W parallel to the y-axis. Which quadrants can H lie in?

2 marks

GIVEN: W has x-coordinate -5; H lies on a line through W parallel to the y-axis.

SOLUTION: A line parallel to the y-axis has a constant x-coordinate for every point on it. So H must also have x-coordinate -5: **H = (-5, y)** for some value of y.

1 mark

Since $x = -5$ is negative: if $y > 0$, H lies in **Quadrant II**; if $y < 0$, H lies in **Quadrant III**; if $y = 0$, H lies on the negative x-axis itself (not inside any quadrant).

1 mark

Q3. Points R(3,0), A(0,-2), M(-5,-2), P(-5,2) joined as RAMP. Predict: (i) two perpendicular sides (ii) a side parallel to an axis (iii) two points that are mirror images in one axis.

3 marks

SOLUTION:

(i) Side AM runs from A(0,-2) to M(-5,-2): since both have $y = -2$, AM is horizontal. Side MP runs from M(-5,-2) to P(-5,2): since both have $x = -5$, MP is vertical. A horizontal and a vertical line are always perpendicular, so **AM $\perp$ MP**.

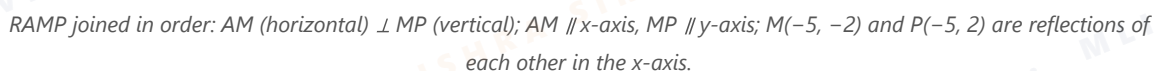
1 mark

(ii) AM (horizontal, $y = -2$ throughout) is **parallel to the x-axis**; MP (vertical, $x = -5$ throughout) is **parallel to the y-axis**.

1 mark

(iii) M(-5,-2) and P(-5,2) share the same x-coordinate, and their y-coordinates are exact opposites (-2 and 2). Reflecting a point (x,y) in the x-axis gives (x,-y) — applying this to M gives exactly P. **M and P are mirror images of each other in the x-axis.**

1 mark



Q4. Plot $Z(5, -6)$. Construct a right-angled triangle IZN and find the lengths of the three sides. (Comment: Answers may differ from person to person.)

SOLUTION: Since this question is explicitly open-ended (as the textbook itself notes), here is one valid construction:

Then IZ is vertical (from $(5,0)$ to $(5,-6)$): length $= |0 - (-6)| = 6$

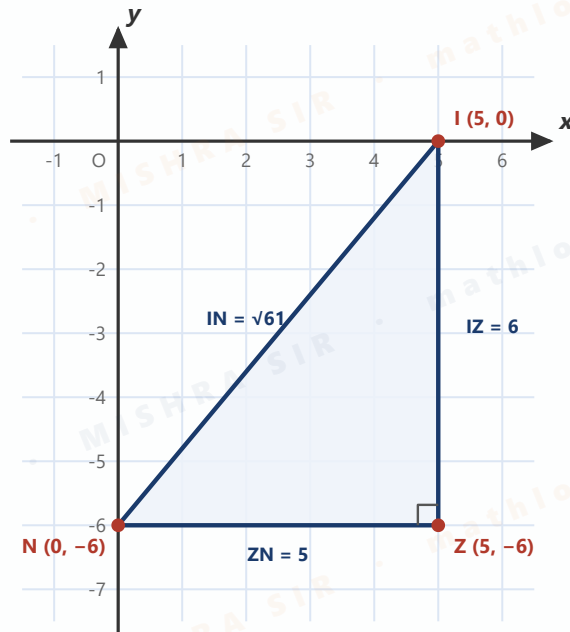
ZN is horizontal (from $(5, -6)$ to $(0, -6)$): length = $|5 - 0| = 5$

Since IZ is vertical and ZN is horizontal, the right angle is at Z .

$$IN = \sqrt{(5-0)^2 + (0-(-6))^2} = \sqrt{25+36} = \sqrt{61} \text{ (the hypotenuse)}$$

1½ marks

Sides: $IZ = 6$, $ZN = 5$, $IN = \sqrt{61}$ (any other valid right-angled construction through Z is equally acceptable)



One valid construction for Q4: right angle at $Z(5, -6)$ with $I(5, 0)$ and $N(0, -6)$; sides 6, 5 and hypotenuse $\sqrt{61}$.

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari*, Part I.

Q5. What would a coordinate system be like without negative numbers? Would it allow us to locate all points on a 2-D plane?

2 marks

SOLUTION: Without negative numbers, we could only represent points where both coordinates are non-negative — that is, only points in **Quadrant I** (plus the non-negative parts of the two axes).

1 mark

No, this would not allow us to locate all points on the plane — points in Quadrants II, III, and IV, which require at least one negative coordinate, would be impossible to describe. This is precisely why Brahmagupta's formalisation of negative numbers was essential for the four-quadrant Cartesian plane to work.

1 mark

***Q6. Are M(-3, -4), A(0,0), G(6,8) on the same straight line? Suggest a method to check this without plotting.**

Enrichment

GIVEN: M(-3, -4), A(0,0), G(6,8).

SUGGESTED METHOD: Compute the distance between each pair of points using the distance formula.

Three points are collinear if the sum of the two shorter distances exactly equals the longest distance (this only happens when the "in-between" point lies exactly on the segment joining the other two).

SOLUTION:

$$AM = \sqrt{(0 - (-3))^2 + (0 - (-4))^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$AG = \sqrt{(6 - 0)^2 + (8 - 0)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$

$$MG = \sqrt{(6 - (-3))^2 + (8 - (-4))^2} = \sqrt{81 + 144} = \sqrt{225} = 15$$

$$\text{Check: } AM + AG = 5 + 10 = 15 = MG \quad \checkmark$$

Since $AM + AG = MG$ exactly, **yes, M, A and G are collinear** (with A lying between M and G).

***Q7. Use your method from Q6 to check if R(-5, -1), B(-2, -5), C(4, -12) are collinear.**

Enrichment

GIVEN: R(-5, -1), B(-2, -5), C(4, -12).

SOLUTION:

$$RB = \sqrt{(-2 - (-5))^2 + (-5 - (-1))^2} = \sqrt{3^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$BC = \sqrt{(4 - (-2))^2 + (-12 - (-5))^2} = \sqrt{6^2 + (-7)^2} = \sqrt{36 + 49} = \sqrt{85} \approx 9.22$$

$$RC = \sqrt{(4 - (-5))^2 + (-12 - (-1))^2} = \sqrt{9^2 + (-11)^2} = \sqrt{81 + 121} = \sqrt{202} \approx 14.21$$

Check: $RB + BC = 5 + \sqrt{85} \approx 14.22$, but $RC \approx 14.21$. These are close but **not exactly equal**

$$(5 + \sqrt{85})^2 = 110 + 10\sqrt{85} \approx 202.2 \neq 202 = RC^2.$$

Since $RB + BC \neq RC$, **R, B and C are not collinear** — this is a good example of why exact calculation (not rounding to 1–2 decimal places) matters, since the numbers look deceptively close.

***Q8. Using the origin as one vertex, plot: (i) a right-angled isosceles triangle (ii) an isosceles triangle with one vertex in Quadrant III and one in Quadrant IV.**

Enrichment

SOLUTION: (open-ended — one valid example for each)

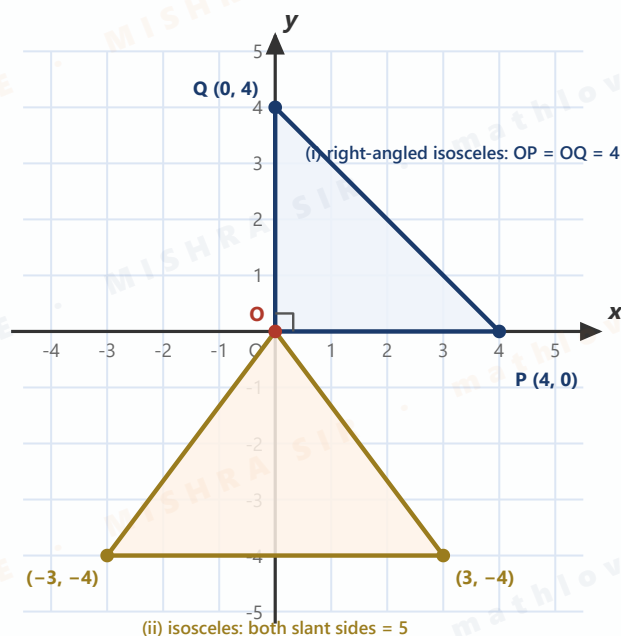
(i) Let $O = (0, 0)$, $P = (4, 0)$, $Q = (0, 4)$. Then $OP = 4$ and $OQ = 4$ (equal legs — isosceles), and since OP lies along the x -axis and OQ along the y -axis, the angle at O is 90° (right-angled).

$$PQ = \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}.$$

✓ right-angled isosceles

(ii) Let $O = (0, 0)$, one vertex = $(-3, -4)$ in Quadrant III, another vertex = $(3, -4)$ in Quadrant IV.

Distance from O to each: $\sqrt{9 + 16} = 5$ for both — equal sides, so the triangle is isosceles with O as the apex.



Q8 sample answers: (i) right-angled isosceles triangle OPQ (teal); (ii) isosceles triangle with vertices in Quadrants III and IV (gold) — both using the origin as one vertex.

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari*, Part I.

***Q9. Table of S, M, T coordinates: state whether M is the midpoint of ST in each row, with reasons. What connection exists between M, S, T when M is the midpoint?**

Enrichment

APPROACH: M is the midpoint of ST exactly when $M = \left(\frac{S_x + T_x}{2}, \frac{S_y + T_y}{2} \right)$.

SOLUTION:

Row 1: $S(-3,0)$, $M(0,0)$, $T(3,0) \rightarrow$ midpoint of $ST = \left(\frac{-3+3}{2}, \frac{0+0}{2} \right) = (0,0) = M$. **Yes**, M is the midpoint. ✓

Row 2: $S(2,3)$, $M(3,4)$, $T(4,5) \rightarrow$ midpoint of $ST = \left(\frac{2+4}{2}, \frac{3+5}{2} \right) = (3,4) = M$. **Yes**, M is the midpoint. ✓

Row 3: $S(0,0)$, $M(0,5)$, $T(0,-10) \rightarrow$ midpoint of $ST = \left(\frac{0+0}{2}, \frac{0-10}{2} \right) = (0,-5) \neq (0,5) = M$. **No**, M is not the midpoint. ✗

Row 4: $S(-8,7)$, $M(0,-2)$, $T(6,-3) \rightarrow$ midpoint of $ST = \left(\frac{-8+6}{2}, \frac{7-3}{2} \right) = (-1,2) \neq (0,-2) = M$. **No**, M is not the midpoint. ✗

Connection: When M is the midpoint of ST , $M_x = \frac{S_x + T_x}{2}$ and $M_y = \frac{S_y + T_y}{2}$ — this is the midpoint formula.

***Q10. Use the midpoint connection to find B, given $M(-7,1)$ is the midpoint of $A(3,-4)$ and $B(x,y)$.**

Enrichment

GIVEN: $M(-7,1)$ is the midpoint of $A(3,-4)$ and $B(x,y)$.

SOLUTION: Using the midpoint formula: $-7 = \frac{3+x}{2}$ and $1 = \frac{-4+y}{2}$

From the first equation: $-14 = 3+x \Rightarrow x = -17$

From the second equation: $2 = -4+y \Rightarrow y = 6$

B = (-17, 6)

***Q11. P, Q are trisection points of AB (P closer to A, Q closer to B). Find P and Q for $A(4,7)$, $B(16,-2)$.**

Enrichment

GIVEN: $A(4,7)$, $B(16,-2)$; P, Q divide AB into three equal parts.

SOLUTION: Moving from A to B, the total change is $x : 16 - 4 = 12$, $y : -2 - 7 = -9$. Dividing into 3 equal steps: each step changes x by $\frac{12}{3} = 4$ and y by $\frac{-9}{3} = -3$.

P is one step ($\frac{1}{3}$ of the way) from A: $P = (4+4, 7-3) = (8,4)$

Q is two steps ($\frac{2}{3}$ of the way) from A: $Q = (4+8, 7-6) = (12,1)$

***Q12. (i) Show A(1, -8), B(-4, 7), C(-7, -4) lie on a circle K centred at O(0, 0); find its radius. (ii) Check whether D(-5, 6) and E(0, 9) lie inside, on, or outside circle K.**

Enrichment

SOLUTION (I): A point lies on a circle centred at O exactly when its distance from O equals the radius.

Computing all three:

$$OA = \sqrt{1^2 + (-8)^2} = \sqrt{1 + 64} = \sqrt{65}$$

$$OB = \sqrt{(-4)^2 + 7^2} = \sqrt{16 + 49} = \sqrt{65}$$

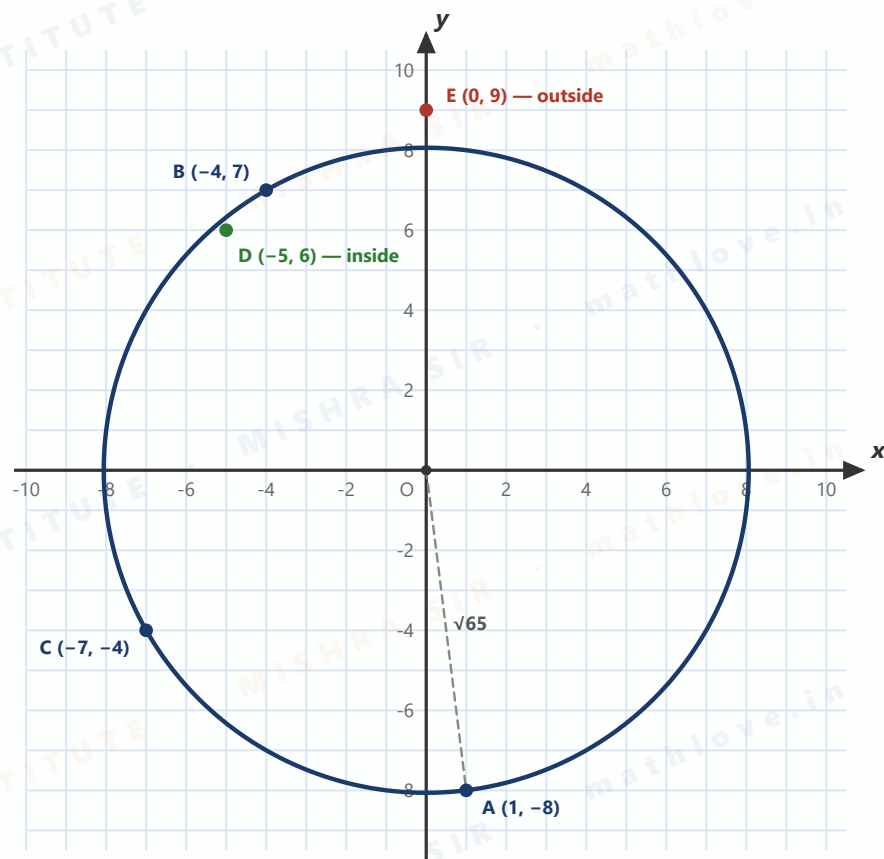
$$OC = \sqrt{(-7)^2 + (-4)^2} = \sqrt{49 + 16} = \sqrt{65}$$

All three distances equal $\sqrt{65}$, so A, B, C all lie on the same circle centred at O. **Radius = $\sqrt{65}$ units** (≈ 8.06 units). 

SOLUTION (II): Compare each point's distance from O to the radius $\sqrt{65} \approx 8.06$:

$$OD = \sqrt{(-5)^2 + 6^2} = \sqrt{25 + 36} = \sqrt{61} \approx 7.81. \text{ Since } 7.81 < 8.06, \text{ **D lies inside the circle.**}$$

$$OE = \sqrt{0^2 + 9^2} = \sqrt{81} = 9. \text{ Since } 9 > 8.06, \text{ **E lies outside the circle.**}$$



Circle K centred at O with radius $\sqrt{65} \approx 8.06$: A, B, C all lie on it; D (distance $\sqrt{61} \approx 7.81$) falls inside; E (distance 9) falls outside.

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari, Part I*.

***Q13. The midpoints of the sides of triangle ABC are D(5,1), E(6,5), F(0,3). Find A, B, C.**

Enrichment

GIVEN: D, E, F are the midpoints of BC, CA, AB respectively.

KEY RELATIONSHIP: for a triangle's midpoint triangle, each original vertex equals the sum of the two nearby midpoints minus the opposite midpoint: $A = E + F - D$, $B = D + F - E$, $C = D + E - F$.

1 mark

SOLUTION:

$$A = E + F - D = (6, 5) + (0, 3) - (5, 1) = (6 + 0 - 5, 5 + 3 - 1) = (1, 7)$$

$$B = D + F - E = (5, 1) + (0, 3) - (6, 5) = (5 + 0 - 6, 1 + 3 - 5) = (-1, -1)$$

$$C = D + E - F = (5, 1) + (6, 5) - (0, 3) = (5 + 6 - 0, 1 + 5 - 3) = (11, 3)$$

1 mark

CHECK: midpoint of BC = $\left(\frac{-1 + 11}{2}, \frac{-1 + 3}{2}\right) = (5, 1) = D$ ✓; midpoint of CA = $(6, 5) = E$ ✓;

midpoint of AB = $(0, 3) = F$ ✓

1 mark

Q14. A city's streets: N-S and E-W roads 200m apart, 10 in each direction.

Intersections named (street#, street#). Find: (a) how many intersections can be called (4,3)? (b) how many can be called (3,4)?

2 marks

SOLUTION: Each intersection is uniquely identified by exactly one ordered pair (N-S street number, E-W street number) — one specific N-S street crossing one specific E-W street gives exactly one point.

(a) The label (4,3) refers to exactly **one** intersection: where the 4th N-S street crosses the 3rd E-W street.

1 mark

(b) The label (3,4) also refers to exactly **one** intersection — but a different one (3rd N-S street crossing 4th E-W street). This illustrates exactly why order matters in coordinates: $(4,3) \neq (3,4)$ as they name two distinct locations.

1 mark

Q15. Screen 800×600 px, origin bottom-left. Circle A: centre(100,150), radius 80. Circle B: centre(250,230), radius 100. (i) does any part lie outside the screen? (ii) do the circles intersect?

3 marks

SOLUTION (I):

Circle A's extreme points: x from $100 - 80 = 20$ to $100 + 80 = 180$; y from $150 - 80 = 70$ to $150 + 80 = 230$. Since $0 \leq 20$ and $180 \leq 800$, and $0 \leq 70$ and $230 \leq 600$, **Circle A lies entirely within the screen.**

1 mark

Circle B's extreme points: x from $250 - 100 = 150$ to $250 + 100 = 350$; y from $230 - 100 = 130$ to $230 + 100 = 330$. All within $[0,800]$ and $[0,600]$, so **Circle B also lies entirely within the screen.**

1 mark

SOLUTION (II): Distance between centres

$$= \sqrt{(250 - 100)^2 + (230 - 150)^2} = \sqrt{150^2 + 80^2} = \sqrt{22500 + 6400} = \sqrt{28900} = 170.$$

Sum of radii $= 80 + 100 = 180$; difference of radii $= 100 - 80 = 20$. Since $20 < 170 < 180$ (the distance lies strictly between the difference and the sum of the radii), **the two circles intersect** (they overlap, crossing at two points).

1 mark

Q16. Plot A(2,1), B(-1,2), C(-2,-1), D(1,-2). Is ABCD a square? Explain. What is its area?

3 marks

SOLUTION: Computing all four side lengths:

$$AB = \sqrt{(2 - (-1))^2 + (1 - 2)^2} = \sqrt{9 + 1} = \sqrt{10}$$

$$BC = \sqrt{(-1 - (-2))^2 + (2 - (-1))^2} = \sqrt{1 + 9} = \sqrt{10}$$

$$CD = \sqrt{(-2 - 1)^2 + (-1 - (-2))^2} = \sqrt{9 + 1} = \sqrt{10}$$

$$DA = \sqrt{(1 - 2)^2 + (-2 - 1)^2} = \sqrt{1 + 9} = \sqrt{10}$$

1 mark

All four sides are equal ($\sqrt{10}$ each). Now checking the diagonals:

$$AC = \sqrt{(2 - (-2))^2 + (1 - (-1))^2} = \sqrt{16 + 4} = \sqrt{20}$$

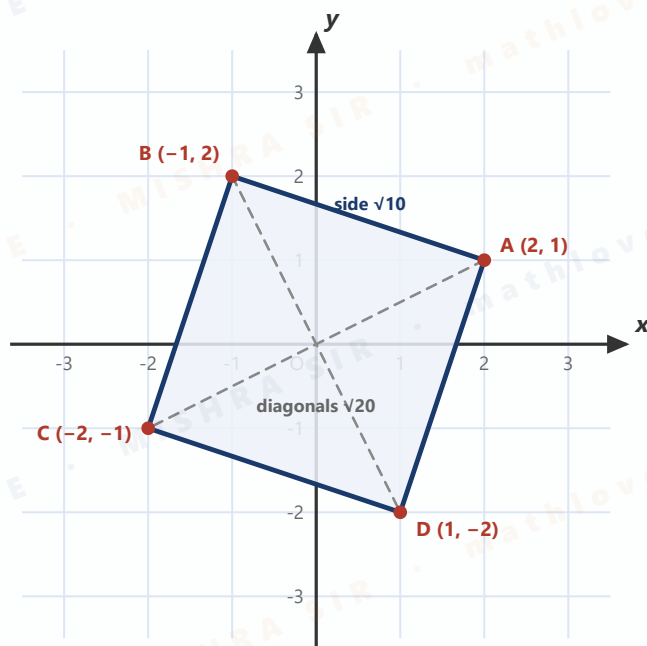
$$BD = \sqrt{(-1 - 1)^2 + (2 - (-2))^2} = \sqrt{4 + 16} = \sqrt{20}$$

1 mark

Since all four sides are equal **and** both diagonals are equal, ABCD is a **square** (a rhombus — equal sides — with equal diagonals is always a square).

$$\text{Area} = \text{side}^2 = (\sqrt{10})^2 = 10 \text{ square units}$$

1 mark



ABCD: all four sides $\sqrt{10}$, both diagonals $\sqrt{20}$ — a square of area 10 sq. units, tilted 45° relative to the axes.

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

— End of Chapter 1 NCERT Solutions —

Additional Practice Questions

A Plotting, Quadrants & Basic Reading (6 Questions)

A1. In which quadrant or on which axis does each point lie: (i) $(-4, 7)$ (ii) $(0, -6)$ (iii) $(5, -3)$ (iv) $(2, 0)$?

2 marks

SOLUTION: (i) $(-4, 7)$: x negative, y positive $\rightarrow$ **Quadrant II**. (ii) $(0, -6)$: $x = 0 \rightarrow$ **on the y-axis**. (iii) $(5, -3)$: x positive, y negative $\rightarrow$ **Quadrant IV**. (iv) $(2, 0)$: $y = 0 \rightarrow$ **on the x-axis**.

2 marks

A2. Write the coordinates of a point that lies 6 units to the left of the y-axis and 3 units below the x-axis.

1 mark

SOLUTION: 6 units left of the y-axis means $x = -6$; 3 units below the x-axis means $y = -3$.

Point = $(-6, -3)$

1 mark

A3. If a point $P(a, b)$ lies in Quadrant III, what can you say about the signs of a and b ?

1 mark

SOLUTION: In Quadrant III, both coordinates are negative: $a < 0$ and $b < 0$

1 mark

A4. For what value of k does the point $(k, 5)$ lie on the y -axis?

1 mark

SOLUTION: A point on the y -axis has x -coordinate 0. So $k = 0$

1 mark

A5. Points $P(4, y)$ and $Q(4, -y)$ are plotted. If $y \neq 0$, in which two quadrants (or on which axes) could P and Q lie?

2 marks

SOLUTION: Both P and Q have $x = 4$ (positive), so both lie on the right half of the plane. If $y > 0$: $P(4, y)$ is in **Quadrant I** and $Q(4, -y)$ is in **Quadrant IV**. If $y < 0$: P is in **Quadrant IV** and Q is in **Quadrant I**. Either way, P and Q always lie in Quadrants I and IV (one each).

2 marks

A6. Is it possible for a point (x, y) to satisfy $(x, y) = (y, x)$ while lying in Quadrant II? Justify.

2 marks

SOLUTION: $(x, y) = (y, x)$ only when $x = y$. In Quadrant II, x must be negative and y must be positive — so $x \neq y$ always (one is negative, the other positive). **No, this is not possible** — a point satisfying $(x, y) = (y, x)$ can only lie in Quadrant I, Quadrant III, or on the line through the origin where $x = y$ (never in Quadrant II or IV).

2 marks

B Distance Formula (6 Questions)

B1. Find the distance between the points $(2, 3)$ and $(5, 7)$.

2 marks

SOLUTION: $d = \sqrt{(5-2)^2 + (7-3)^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$ units

2 marks

B2. Find the distance between $(-2, -3)$ and $(4, 5)$.

2 marks

SOLUTION: $d = \sqrt{(4-(-2))^2 + (5-(-3))^2} = \sqrt{6^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = 10$ units

2 marks

B3. A point lies on the x-axis at (7, 0) and another on the y-axis at (0, 24). Find the distance between them.

2 marks

SOLUTION: $d = \sqrt{(7-0)^2 + (0-24)^2} = \sqrt{49 + 576} = \sqrt{625} = 25$ units

2 marks

B4. Show that the points A(1,1), B(4,5), C(5,-2) form a right-angled triangle.

3 marks

SOLUTION: $AB = \sqrt{(4-1)^2 + (5-1)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$AC = \sqrt{(5-1)^2 + (-2-1)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$BC = \sqrt{(5-4)^2 + (-2-5)^2} = \sqrt{1+49} = \sqrt{50}$

1½ marks

Check: $AB^2 + AC^2 = 25 + 25 = 50 = BC^2$ ✓ — the Pythagoras relation holds, so $\triangle ABC$ is right-angled, with the right angle at A (between the two equal sides AB and AC).

1½ marks

(Note: always test all three combinations of sides before concluding whether or where a right angle exists — here $AB^2 + AC^2 = BC^2$ is the one that matches.)

B5. Find the value of y if the distance between (2, y) and (10, 8) is 10 units.

3 marks

SOLUTION: $\sqrt{(10-2)^2 + (8-y)^2} = 10$

$64 + (8-y)^2 = 100$

$(8-y)^2 = 36$

$8-y = \pm 6$

$y = 2$ or $y = 14$

3 marks

B6. Which of the points P(-1, 1) or Q(3, -4) is closer to the origin?

2 marks

SOLUTION: $OP = \sqrt{(-1)^2 + 1^2} = \sqrt{2} \approx 1.41$

$OQ = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

Since $\sqrt{2} < 5$, **P is closer to the origin.**

2 marks

C Midpoint & Collinearity (5 Questions)

C1. Find the midpoint of the segment joining (-3, 6) and (7, -2).

2 marks

SOLUTION: Midpoint = $\left(\frac{-3+7}{2}, \frac{6+(-2)}{2}\right) = \left(\frac{4}{2}, \frac{4}{2}\right) = (2, 2)$

2 marks

C2. The midpoint of segment AB is (4, -1). If A = (9, 2), find B.

2 marks

SOLUTION: $4 = \frac{9 + B_x}{2} \Rightarrow B_x = 8 - 9 = -1$

$-1 = \frac{2 + B_y}{2} \Rightarrow B_y = -2 - 2 = -4$

B = (-1, -4)

2 marks

C3. Check whether the points (1, 2), (3, 6), (5, 10) are collinear using the distance method.

3 marks

SOLUTION: Let $P = (1, 2), Q = (3, 6), R = (5, 10)$.

$PQ = \sqrt{2^2 + 4^2} = \sqrt{20}; QR = \sqrt{2^2 + 4^2} = \sqrt{20}; PR = \sqrt{4^2 + 8^2} = \sqrt{80}$

Check: $PQ + QR = \sqrt{20} + \sqrt{20} = 2\sqrt{20} = \sqrt{4 \times 20} = \sqrt{80} = PR \checkmark$

Yes, the three points are collinear (Q lies exactly midway between P and R).

3 marks

C4. Find the coordinates of the point which divides the segment from (0,0) to (10, 15) exactly in half.

1 mark

SOLUTION: Midpoint = $\left(\frac{0+10}{2}, \frac{0+15}{2}\right) = (5, 7.5)$

1 mark

C5. If the midpoints of the sides of a triangle are (2,3), (4,5), (6,1), is it possible to find the original triangle's vertices? Explain your approach (no need to compute).

2 marks

SOLUTION: Yes — using the relation that each vertex equals the sum of the two nearby midpoints minus the opposite one ($A = E + F - D$, and cyclically for B and C), the three original vertices can always be reconstructed uniquely from the three given midpoints.

2 marks

D Application / Floor-Plan Style (3 Questions)

D1. A rectangular garden has corners at (0,0), (20,0), (20,15), (0,15) (in metres). A tree is planted at the centre of the garden. Find the tree's coordinates.

2 marks

SOLUTION: The centre of a rectangle is the midpoint of its diagonal. Using corners (0,0) and

(20,15): centre = $\left(\frac{0+20}{2}, \frac{0+15}{2}\right) = (10, 7.5)$

2 marks

D2. Two friends' houses are located at A(3, 4) and B(15, 9) on a city map where 1 unit = 1 km. Find the straight-line distance between their houses.

2 marks

SOLUTION: $AB = \sqrt{(15 - 3)^2 + (9 - 4)^2} = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13 \text{ km}$

2 marks

D3. On a computer screen with origin at the bottom-left, an icon needs to be placed exactly halfway between two markers at (50, 400) and (350, 100). Find the icon's coordinates.

2 marks

SOLUTION: $\text{Midpoint} = \left(\frac{50 + 350}{2}, \frac{400 + 100}{2} \right) = \left(\frac{400}{2}, \frac{500}{2} \right) = (200, 250)$

2 marks

— End of Additional Practice Questions —

CHAPTER 2

Introduction to Linear Polynomials

Exercise Sets 2.1 – 2.6 + End-of-Chapter

EXERCISE SETS 2.1 – 2.6 + END-OF-CHAPTER

 **Weightage (School Annual Exam, Per Official CBSE Curriculum)**

Marks Distribution (Out of 80 — Annual Theory Paper)

✓ Verified directly against the official CBSE curriculum document (cbseacademic.nic.in, Mathematics Class IX, 2026–27) — not a secondary source. Unlike Chapter 1 (Coordinate Geometry), which is the *only* chapter in its unit, this chapter sits inside **Unit II: Algebra**, which is shared across **four** chapters — Introduction to Polynomials (this chapter), Sequences and Progressions, Exploring Algebraic Identities, and Linear Equations in Two Variables. The 20 marks of the unit are distributed across all four; there is no separate official mark-split for this chapter alone.

Detail	Value
Unit	Algebra (Unit II)
Chapters sharing this unit	Introduction to Polynomials (this chapter) · Sequences and Progressions · Exploring Algebraic Identities · Linear Equations in Two Variables
Marks allotted to the whole unit	20 marks out of 80 — the second-highest-weightage unit after Geometry (25 marks)
Teaching periods for the whole unit	66 periods (out of ~200 total for the year) — the largest number of periods of any unit
Internal Assessment (separate, 20 marks)	Pen-Paper Test & Multiple Assessment (10), Portfolio (5), Lab Practical (5)

✦ Because Algebra is a large, high-weightage unit spread across four chapters, this chapter's specific ideas — degree, terms, coefficients, evaluating a polynomial, and linear growth/decay/relationships — are foundational skills examiners lean on heavily when setting questions for the *later* Algebra chapters (Algebraic Identities, Linear Equations) too. Mastering

this chapter will pay off across the whole unit, not just in questions that look like Chapter 2 questions.

Predicted Question Types

Format	Marks	Likely Topic
MCQ / Very Short Answer	1	Identifying the degree of a polynomial; naming linear/quadratic/cubic/constant polynomials; reading off a coefficient or the constant term
Short Answer	2	Evaluating a polynomial at a given value of the variable; writing the n th-term expression for a simple linear pattern
Short/Long Answer	2–3	Word problems modelled by a linear equation (ages, coins, fencing, rectangles) solved by forming and solving $ax + b = c$; identifying linear growth vs. linear decay from a table or a real-life description
Application/Reasoning	2–3	Finding the linear relationship $y = ax + b$ from two given data points (bills, fares, temperature scales); reading slope and y -intercept off an equation; explaining why two lines are or are not parallel

All Key Formulas & Definitions

Algebraic expression: a combination of numbers, variables, and operation symbols, e.g. $4x + 5y + 3$.

Terms, coefficients, constant: in $4x + 5y + 3$, the terms are $4x$, $5y$, 3 ; the coefficients of x and y are 4 and 5; the constant term is 3.

Polynomial (univariate): an algebraic expression in one variable and its non-negative integer powers, e.g. $5y^3 + y^2 + 2y - 1$.

Degree of a polynomial: the highest power of the variable appearing in it.

Classification by degree: degree 0 → **constant polynomial** (e.g. 8); degree 1 → **linear polynomial** (e.g. $3z + 7$); degree 2 → **quadratic polynomial** (e.g. $x^2 + 5x + 1$); degree 3 → **cubic polynomial** (e.g. $5y^3 + y^2 + 2y - 1$).

Linear pattern: a sequence of numbers in which the difference between consecutive terms is constant.

Linear growth: a quantity increases by a fixed amount over equal intervals (positive slope).

Linear decay: a quantity decreases by a fixed amount over equal intervals (negative slope).

Linear equation (in one variable): obtained by equating a linear polynomial to a constant, e.g. $2x + 10 = 64$.

Function / input-output view: a polynomial such as $2x + 3$ can be treated as a rule that turns an input x into an output value.

Linear relationship: a relationship between two variables x and y expressed as $y = ax + b$, which graphs as a straight line.

Slope and y-intercept: in $y = ax + b$, a is the **slope** of the line (the constant amount y changes for every unit increase in x); b is the **y-intercept**, the point $(0, b)$ where the line cuts the y -axis.

Parallel lines: two lines $y = ax + b$ with the **same slope** a but **different y-intercepts** b are parallel — they never meet, only shift up or down.

Finding a and b from two data points: substitute both (x_1, y_1) and (x_2, y_2) into $y = ax + b$ to get two equations in a and b , then solve them simultaneously (usually by substitution or elimination).

Shortcuts & Study Tips

- Finding the degree fast:** scan for the largest exponent on the variable — ignore the number of terms or their order; a polynomial doesn't need to be written "neatly" from highest to lowest power.
- Constant polynomials are still degree 0, not "no degree":** a lone number like -9 can be thought of as $-9x^0$, so its degree is 0, not undefined.
- Coefficient of a "missing" power is 0:** in $4z^3 + 5z^2 = 11$, there's no z^1 term at all — so the coefficient of z is 0, not no answer.

4. **Evaluating a polynomial:** always substitute the given value in brackets first, then simplify — e.g. write $5(2)^2 - 3(2) + 7$ before multiplying, to avoid sign errors with negative inputs.
5. **Spotting linear growth vs. decay instantly:** just check the sign of the constant change — increasing by a fixed amount → growth (positive slope); decreasing by a fixed amount → decay (negative slope).
6. **Word problems that become linear equations:** name the unknown with a single letter, translate every "more than," "less than," "times," and "sum" phrase one at a time into symbols, then solve — don't try to set up the whole equation in your head at once.
7. **Finding a and b from two points quickly:** subtracting one equation from the other eliminates b immediately, leaving a one-step equation for a — always eliminate before you try to solve for both unknowns at once.
8. **Checking whether two lines are parallel:** rewrite both equations in $y = ax + b$ form first (watch for equations given as $2y = 4x + 7$ — divide through before comparing) — only then compare the slopes.

■ NCERT Solutions — Exercise Set 2.1

🔑 Approach for This Exercise

Everything in this set is about **reading a polynomial correctly** rather than calculating with it. Three ideas cover all five questions. The **degree** is the highest power of the variable that actually appears. A **coefficient** is the number multiplying a particular power — and it carries the sign in front of it. The **constant term** is the term with no variable, which is really the coefficient of x^0 (since $x^0 = 1$). The single biggest trap is a *missing* power: if a term is absent, its coefficient is 0, not "nothing".

Q1. Find the degrees of the following polynomials: (i) $2x^2 - 5x + 3$ (ii) $y^3 + 2y - 1$ (iii) -9

(iv) $4z - 3$

4 parts

SOLUTION:

Rule used: the degree of a polynomial in one variable is the highest power of that variable among the terms with a non-zero coefficient. Look at every term, note its power, then take the largest.

(i) $2x^2 - 5x + 3$

Term by term: $2x^2$ has power 2; $-5x$ has power 1; 3 is constant, power 0.

The highest of 2, 1, 0 is 2.

Therefore the **degree is 2** — this is a quadratic polynomial.

1 mark

(ii) $y^3 + 2y - 1$

Powers present: $y^3 \rightarrow 3$; $2y \rightarrow 1$; $-1 \rightarrow 0$.

Note the y^2 term is missing, but that does not matter — we take the highest power that is present.

Therefore the **degree is 3** — a cubic polynomial.

1 mark

(iii) -9

There is no variable visible. But any non-zero constant can be written as $-9 \times 1 = -9x^0$, because $x^0 = 1$.

So the only power present is 0, and the **degree is 0** — a constant polynomial.

1 mark

(iv) $4z - 3$

Powers present: $4z \rightarrow 1$; $-3 \rightarrow 0$. The highest is 1.

Therefore the **degree is 1** — a linear polynomial.

1 mark

📌 **Worth remembering.** Degree 0 = constant, 1 = linear, 2 = quadratic, 3 = cubic. One special case: the polynomial 0 on its own has no defined degree, which is why part (iii) uses -9 rather than 0.

Q2. Write polynomials of degrees 1, 2 and 3.

Open-ended

SOLUTION:

Any correct example is acceptable here; what the examiner checks is that the highest power matches the degree asked for, and that the leading coefficient is not zero. One valid set:

Degree 1 (linear): $2x + 3$

Highest power of x is 1, and its coefficient $2 \neq 0$. ✓

1 mark

Degree 2 (quadratic): $x^2 + 2x + 1$

Highest power is 2, coefficient $1 \neq 0$. ✓

1 mark

Degree 3 (cubic): $x^3 - 2x + 5$

Highest power is 3, coefficient $1 \neq 0$. Note that leaving out the x^2 term is perfectly allowed — it does not change the degree. ✓

1 mark

✳ **Careful.** Writing something like $0x^2 + 3x + 1$ for degree 2 is wrong: once the leading coefficient is 0, that term vanishes and the polynomial is really of degree 1.

Q3. What are the coefficients of x^2 and x^3 in the polynomial $x^4 - 3x^3 + 6x^2 - 2x + 7$?

2 marks

SOLUTION:

Rule used: the coefficient of a power is the number multiplying it, including the sign written in front of the term.

Write the polynomial out term by term with each power labelled:

$x^4 \rightarrow$ coefficient 1

$-3x^3 \rightarrow$ coefficient -3

$6x^2 \rightarrow$ coefficient 6

$-2x \rightarrow$ coefficient -2

$7 \rightarrow$ constant term

Reading off the two asked for:

Coefficient of $x^3 = -3$ 1 mark

Coefficient of $x^2 = 6$ 1 mark

✦ **The usual mistake** is answering 3 instead of -3 . The minus sign belongs to the coefficient, not to the operation.

Q4. What is the coefficient of z in the polynomial $4z^3 + 5z^2 - 11$?

1 mark

SOLUTION:

List the powers actually present: $4z^3 \rightarrow$ power 3; $5z^2 \rightarrow$ power 2; $-11 \rightarrow$ power 0.

There is no term containing z^1 . A missing power does not mean the coefficient is undefined — it means the term is present with coefficient zero. We can make this explicit by writing the polynomial as $4z^3 + 5z^2 + 0 \cdot z - 11$

Therefore the **coefficient of z is 0.** 1 mark

✦ **Why this is asked.** The examiner wants to see that you treat an absent term as having coefficient 0. Answers such as "no coefficient" or "does not exist" do not earn the mark.

Q5. What is the constant term of the polynomial $9x^3 + 5x^2 - 8x - 10$?

1 mark

SOLUTION:

The constant term is the one with no variable attached — equivalently, the coefficient of x^0 .

Scanning the terms: $9x^3$ has x ; $5x^2$ has x ; $-8x$ has x ; -10 has none.

Therefore the **constant term is -10** , carrying its minus sign.

1 mark

 **Useful check.** The constant term is always the value of the polynomial at $x = 0$. Here

$9(0)^3 + 5(0)^2 - 8(0) - 10 = -10$ ✓ — a quick way to confirm your answer.

— End of Exercise Set 2.1 —

NCERT Solutions — Exercise Set 2.2

Approach for This Exercise

Q1–Q2 are pure **substitution**: replace the variable with the given number, always wrapping a negative value in brackets, then simplify one step at a time. Q3–Q7 are **word problems**, and each follows the same four-step routine: (1) name the unknown with a letter, (2) express every other quantity in terms of that letter, (3) form one equation from the condition given, (4) solve, then state the answer in words *with units*. Always finish with a check — it is quick and it catches sign and arithmetic slips.

Q1. Find the value of the linear polynomial $5x - 3$ if: (i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$

3 parts

SOLUTION:

Substitute each value into $5x - 3$, keeping brackets around negative numbers.

(i) $x = 0$

$5(0) - 3 = 0 - 3 = -3$ (note this is the constant term — the value at $x = 0$ always is)

1 mark

(ii) $x = -1$

$5(-1) - 3 = -5 - 3$

Both terms are negative, so they add in size: $= -8$

1 mark

(iii) $x = 2$

$5(2) - 3 = 10 - 3 = 7$

1 mark

 **Habit worth building.** Writing $5(-1)$ rather than 5×-1 prevents the commonest error in part (ii), where students compute $-5 + 3 = -2$.

Q2. Find the value of the quadratic polynomial $7s^2 - 4s + 6$ if: (i) $s = 0$ (ii) $s = -3$ (iii)

$s = 4$

3 parts

SOLUTION:

Substitute, square before multiplying, and mind the sign of the middle term.

(i) $s = 0$

$$7(0)^2 - 4(0) + 6 = 0 - 0 + 6 = 6$$

1 mark

(ii) $s = -3$

First the square: $(-3)^2 = 9$ — a negative squared is positive.

Then the middle term: $-4(-3) = +12$ — minus times minus is plus.

$$7(9) + 12 + 6 = 63 + 12 + 6 = 81$$

1 mark

(iii) $s = 4$

$$(4)^2 = 16, \text{ so } 7(16) = 112; \text{ and } -4(4) = -16.$$

$$112 - 16 + 6 = 102$$

1 mark

📌 **Two sign traps in (ii).** Students often write $(-3)^2 = -9$, or $-4(-3) = -12$. Both are wrong, and together they turn 81 into 39. Do the square and the middle term on separate lines.

Q3. The present age of Salil's mother is three times Salil's present age. After 5 years, their ages will add up to 70 years. Find their present ages.

3 marks

SOLUTION:

Step 1 — name the unknown. Let Salil's present age be x years.

Step 2 — express the other quantity. His mother's present age is three times his, so it is $3x$ years.

Step 3 — move to "after 5 years". Everyone ages by the same 5 years:

Salil $\rightarrow x + 5$; mother $\rightarrow 3x + 5$.

1 mark

Step 4 — form the equation from "add up to 70":

$$(x + 5) + (3x + 5) = 70$$

$$4x + 10 = 70$$

$$4x = 60$$

$$x = 15$$

1 mark

Step 5 — answer in words. Salil's present age = 15 years; his mother's present age = $3 \times 15 = 45$ years.

1 mark

Check: after 5 years the ages are 20 and 50, and $20 + 50 = 70$ ✓

✂ **The classic error** is adding 5 only once, giving $4x + 5 = 70$. Each person ages 5 years, so the total gains 10.

Q4. The difference between two positive integers is 63. The ratio of the two integers is 2 : 5. Find the two integers.

3 marks

SOLUTION:

Step 1 — use the ratio to name both numbers with one letter. Since they are in the ratio 2 : 5, write them as $2k$ and $5k$ for some positive number k . This is better than two separate letters, because it builds the ratio in from the start. **1 mark**

Step 2 — form the equation from the difference (larger minus smaller):

$$5k - 2k = 63$$

$$3k = 63$$

$$k = 21$$
 1 mark

Step 3 — find the actual integers.

$$\text{Smaller} = 2(21) = 42; \text{ larger} = 5(21) = 105. \quad \text{1 mark}$$

Check: $105 - 42 = 63$ ✓ and $42 : 105 = 2 : 5$ (divide both by 21) ✓

📌 **Why $2k$ and $5k$ works.** Any pair in the ratio 2 : 5 is a common multiple of 2 and 5 parts. Naming the "one part" as k turns two unknowns into one.

Q5. Ruby has 3 times as many two-rupee coins as she has five-rupee coins. If she has a total of ₹88, how many coins does she have of each type?

3 marks

SOLUTION:

Step 1 — name the unknown. Let the number of five-rupee coins be x .

Step 2 — the other quantity. Two-rupee coins are 3 times as many, so there are $3x$ of them.

1 mark

Step 3 — form the equation using value, not count. This is the key step: ₹88 is the total value, so multiply each count by the coin's worth.

Value of five-rupee coins = $5x$; value of two-rupee coins = $2(3x) = 6x$.

$$5x + 6x = 88$$

$$11x = 88$$

$$x = 8$$

1 mark

Step 4 — answer. Five-rupee coins = 8; two-rupee coins = $3 \times 8 = 24$. (Total = 32 coins.)

1 mark

Check: $5(8) + 2(24) = 40 + 48 = 88$ ✓

✂ **Count versus value.** Writing $x + 3x = 88$ treats 88 as a number of coins and gives a non-integer answer — a signal that the wrong quantity was equated.

Q6. A farmer cuts a 300 feet fence into two pieces of different sizes. The longer piece is four times as long as the shorter piece. How long are the two pieces?

2 marks

SOLUTION:

Step 1 — name the unknown. Let the shorter piece be x feet. Then the longer piece is $4x$ feet.

Step 2 — form the equation. The two pieces together make the whole fence:

$$x + 4x = 300$$

$$5x = 300$$

$$x = 60$$

1 mark

Step 3 — answer with units. Shorter piece = 60 feet; longer piece = $4 \times 60 = 240$ feet.

1 mark

Check: $60 + 240 = 300$ ✓ and $240 = 4 \times 60$ ✓

✦ **Always restate both pieces.** Stopping at $x = 60$ answers only half the question.

Q7. If the length of a rectangle is three more than twice its width and its perimeter is 24 cm, what are the dimensions of the rectangle?

3 marks

SOLUTION:

Step 1 — name the unknown. Let the width be w cm.

Step 2 — express the length. "Three more than twice the width" means $l = 2w + 3$ cm.

1 mark

Step 3 — use the perimeter formula. $P = 2(l + w)$, so

$$2(l + w) = 24$$

Dividing both sides by 2 first makes the algebra much lighter: $l + w = 12$.

Substituting $l = 2w + 3$:

$$(2w + 3) + w = 12$$

$$3w + 3 = 12$$

$$3w = 9$$

$$w = 3$$

1 mark

Step 4 — find the length and answer. $l = 2(3) + 3 = 9$.

Width = 3 cm; Length = 9 cm.

1 mark

Check: perimeter = $2(9 + 3) = 2(12) = 24$ cm ✓ and the length 9 is indeed three more than twice 3 ✓

✦ **Divide by 2 early.** Expanding to $2(2w + 3) + 2w = 24$ also works, but creates larger numbers and more chances to slip.

— End of Exercise Set 2.2 —

■ NCERT Solutions — Exercise Set 2.3

🔑 Approach for This Exercise

Every question here describes a real quantity that changes by a fixed amount at each step — the same structure as the chess-club and auto-rickshaw examples in the chapter. Identify the starting value and the constant step, then write the n th-term expression as (starting value) $\pm$ (step) $\times n$.

Q1. A student has ₹500 in her savings bank account. She gets ₹150 every month as pocket money. How much money will she have at the end of every month from the second month onwards? Find a linear expression to represent the amount she will have in the n th month.

2 marks

SOLUTION: Starting amount = ₹500; each month adds a fixed ₹150.

Amount at the end of month n : $500 + 150n$

End of month 1: $500 + 150 = 650$; end of month 2: $500 + 300 = 800$; end of month 3: $500 + 450 = 950$; and so on.

Linear expression: $500 + 150n$ (linear growth — the amount grows by a fixed ₹150 every month)

2 marks

Q2. A rally starts with 120 members. Each hour, 9 members drop out of the group. How many members will remain after 1, 2, 3, ... hours? Find a linear expression to represent the number of members at the end of the n th hour.

2 marks

SOLUTION: Starting members = 120; each hour removes a fixed 9 members.

Members at the end of hour n : $120 - 9n$

After 1 hour: $120 - 9 = 111$; after 2 hours: $120 - 18 = 102$; after 3 hours: $120 - 27 = 93$; and so on.

Linear expression: $120 - 9n$

2 marks

Q3. Suppose the length of a rectangle is 13 cm. Find the area if the breadth is (i) 12 cm, (ii) 10 cm, (iii) 8 cm. Find the linear pattern representing the area of the rectangle.

2 marks

SOLUTION: Area = length $\times$ breadth = $13 \times b$

(i) $b = 12$: Area = $13 \times 12 = 156 \text{ cm}^2$

(ii) $b = 10$: Area = $13 \times 10 = 130 \text{ cm}^2$

(iii) $b = 8$: Area = $13 \times 8 = 104 \text{ cm}^2$

Linear pattern: Area = $13b$ — a linear polynomial in the breadth b

2 marks

Q4. Suppose the length of a rectangular box is 7 cm and breadth is 11 cm. Find the volume if the height is (i) 5 cm, (ii) 9 cm, (iii) 13 cm. Find the linear pattern representing the volume of the rectangular box.

2 marks

SOLUTION: $\text{Volume} = \text{length} \times \text{breadth} \times \text{height} = 7 \times 11 \times h = 77h$

(i) $h = 5$: $\text{Volume} = 77 \times 5 = 385 \text{ cm}^3$

(ii) $h = 9$: $\text{Volume} = 77 \times 9 = 693 \text{ cm}^3$

(iii) $h = 13$: $\text{Volume} = 77 \times 13 = 1001 \text{ cm}^3$

Linear pattern: $\text{Volume} = 77h$ — a linear polynomial in the height h

2 marks

Q5. Sarita is reading a book of 500 pages. She reads 20 pages every day. How many pages will be left after 15 days? Express this as a linear pattern.

2 marks

SOLUTION: $\text{Pages left after } d \text{ days} = 500 - 20d$

After 15 days: $500 - 20(15) = 500 - 300 = 200$ pages

Linear pattern: $500 - 20d$ (linear decay)

2 marks

— End of Exercise Set 2.3 —

■ NCERT Solutions — Exercise Set 2.4

Approach for This Exercise

Each question models linear growth or linear decay. Write the function as (starting value) $\pm$ (constant rate) $\times$ (time variable), then explain growth/decay by pointing to the sign of the rate — positive means growth, negative means decay.

Q1. Suppose a plant has height 1.75 feet and it grows by 0.5 feet each month. (i) Find the height after 7 months. (ii) Make a table for $t = 0$ to 10 months. (iii) Find an expression relating h and t , and explain why it represents linear growth.

3 parts

SOLUTION: $h(t) = 1.75 + 0.5t$

(i) $h(7) = 1.75 + 0.5(7) = 1.75 + 3.5 = 5.25$ feet

(ii) Table:

t (months)	h (feet)
0	1.75
1	2.25
2	2.75
3	3.25
4	3.75
5	4.25
6	4.75
7	5.25
8	5.75
9	6.25
10	6.75

(iii) $h = 1.75 + 0.5t$ is a linear polynomial in t with a **positive slope (0.5)** — the height increases by a fixed 0.5 feet for every extra month, so it represents **linear growth**.

3 marks

Q2. A mobile phone is bought for ₹10,000. Its value decreases by ₹800 every year. (i) Find the value after 3 years. (ii) Make a table for $t = 0$ to 8 years. (iii) Find an expression relating v and t , and explain why it represents linear decay.

3 parts

SOLUTION: $v(t) = 10000 - 800t$

(i) $v(3) = 10000 - 800(3) = 10000 - 2400 = 7600 \rightarrow \text{₹7600}$

(ii) Table:

t (years)	v (₹)
0	10000
1	9200
2	8400
3	7600
4	6800
5	6000
6	5200
7	4400
8	3600

(iii) $v = 10000 - 800t$ is a linear polynomial in t with a **negative slope (-800)** — the value decreases by a fixed ₹800 for every extra year, so it represents **linear decay**.

3 marks

Q3. The initial population of a village is 750. Every year, 50 people move from a nearby city to the village. (i) Find the population after 6 years. (ii) Make a table for $t = 0$ to 10 years. (iii) Find an expression relating P and t , and explain why it represents linear growth.

3 parts

SOLUTION: $P(t) = 750 + 50t$

(i) $P(6) = 750 + 50(6) = 750 + 300 = 1050$

(ii) Table:

t (years)	P
0	750
1	800
2	850
3	900
4	950
5	1000
6	1050
7	1100
8	1150
9	1200
10	1250

(iii) $P = 750 + 50t$ is a linear polynomial in t with a **positive slope (50)** — the population increases by a fixed 50 people every year, so it represents **linear growth**.

3 marks

Q4. A telecom company charges ₹600 for a certain recharge scheme. This prepaid balance is reduced by ₹15 each day after the recharge. (i) Write an equation for $b(x)$ after x days and explain why it represents linear decay. (ii) After how many days will the balance run out? (iii) Make a table for $x = 1$ to 10 days.

3 parts

SOLUTION:

(i) $b(x) = 600 - 15x$. Since the balance decreases by a fixed ₹15 every day (negative slope -15), this represents **linear decay**.

(ii) Balance runs out when $b(x) = 0$: $600 - 15x = 0 \Rightarrow 15x = 600 \Rightarrow x = 40$ days

(iii) Table:

x (days)	$b(x)$ (₹)
1	585
2	570
3	555
4	540
5	525
6	510
7	495
8	480
9	465
10	450

3 marks

— End of Exercise Set 2.4 —

 **NCERT Solutions — Exercise Set 2.5**

Approach for This Exercise

Each question describes a **fixed charge plus a rate**, which is exactly the linear model $y = ax + b$: a is the rate (cost per unit) and b is the fixed part. Two data pairs give two equations. The efficient route is always the same — **subtract one equation from the other**, which cancels b instantly and leaves a alone. Find a first, then put it back into either equation to get b . Finish by saying what a and b mean in the situation, since the question is about a real context.

Q1. A learning platform charges a fixed monthly fee and an additional cost per module. 10 modules cost ₹400; 14 modules cost ₹500. If $y = ax + b$, find a and b .

3 marks

SOLUTION:

Here x is the number of modules and y the total monthly cost, so a is the cost per module and b the fixed fee.

Step 1 — write both equations.

$$400 = 10a + b \dots (1)$$

$$500 = 14a + b \dots (2)$$

1 mark

Step 2 — eliminate b by subtracting (1) from (2). Since b appears identically in both, it cancels:

$$500 - 400 = (14a - 10a) + (b - b)$$

$$100 = 4a$$

$$a = 25$$

1 mark

Step 3 — substitute back into (1) to find b .

$$400 = 10(25) + b = 250 + b$$

$$b = 400 - 250 = 150$$

1 mark

Answer: $a = 25$ — the cost per module is ₹25; $b = 150$ — the fixed monthly fee is ₹150.

Check with the second pair: $y = 25(14) + 150 = 350 + 150 = 500$ ✓

 **Interpret, don't just solve.** Marks are given for saying that ₹25 is per module and ₹150 is fixed. A bare " $a = 25$, $b = 150$ " is an incomplete answer to a word problem.

Q2. A gym charges a fixed monthly fee and an additional cost per hour for the badminton court. 10 hours cost ₹800; 15 hours cost ₹1100. If $y = ax + b$, find a and b .

3 marks

SOLUTION:

Here x is hours of court use and y the monthly total.

Step 1 — the two equations.

$$800 = 10a + b \dots (1)$$

$$1100 = 15a + b \dots (2)$$

1 mark

Step 2 — subtract (1) from (2) to remove b .

$$1100 - 800 = 15a - 10a$$

$$300 = 5a$$

$$a = 60$$

1 mark

Step 3 — back-substitute into (1).

$$800 = 10(60) + b = 600 + b$$

$$b = 800 - 600 = 200$$

1 mark

Answer: $a = 60$ — the court costs ₹60 per hour; $b = 200$ — the fixed monthly fee is ₹200.

Check: $y = 60(15) + 200 = 900 + 200 = 1100$ ✓

✦ **Sanity check the rate.** Between the two data points, cost rose ₹300 for 5 extra hours — ₹60 per hour, matching a . Doing this mentally first tells you straight away whether your answer is plausible.

Q3. Consider the relationship $^{\circ}\text{C} = a \times ^{\circ}\text{F} + b$. Ice melts at $0^{\circ}\text{C} / 32^{\circ}\text{F}$; water boils at $100^{\circ}\text{C} / 212^{\circ}\text{F}$. Find a and b .

3 marks

SOLUTION:

Here $^{\circ}\text{F}$ plays the role of x and $^{\circ}\text{C}$ the role of y . Substitute each pair of temperatures.

Step 1 — the two equations.

Ice melting: $0 = 32a + b \dots (1)$

Water boiling: $100 = 212a + b \dots (2)$

1 mark

Step 2 — subtract (1) from (2).

$$100 - 0 = (212a - 32a)$$

$$100 = 180a$$

$$a = \frac{100}{180}. \text{ Reduce by HCF 20: } a = \frac{5}{9}$$

1 mark

Step 3 — substitute into (1).

$$0 = 32 \times \frac{5}{9} + b, \text{ so } b = -\frac{160}{9}$$

1 mark

Answer: $a = \frac{5}{9}$ and $b = -\frac{160}{9}$.

Putting these back gives $^{\circ}\text{C} = \frac{5}{9} ^{\circ}\text{F} - \frac{160}{9} = \frac{5}{9} (^{\circ}\text{F} - 32)$ — the familiar Fahrenheit-to-Celsius conversion formula, now derived rather than memorised.

Check at $^{\circ}\text{F} = 212$: $\frac{5}{9}(212) - \frac{160}{9} = \frac{1060 - 160}{9} = \frac{900}{9} = 100 \checkmark$

✳ **Note what a and b mean here.** $a = \frac{5}{9}$ is the scale factor — a Celsius degree is $\frac{9}{5}$ times bigger than a Fahrenheit degree — and b shifts the zero point, because the two scales do not start together.

— End of Exercise Set 2.5 —

 **NCERT Solutions — Exercise Set 2.6**

Approach for This Exercise

To plot any line $y = ax + b$ by hand, work out just two points (usually $x = 0$ and one more convenient value), mark them on graph paper, and draw a straight line through them extending in both directions.

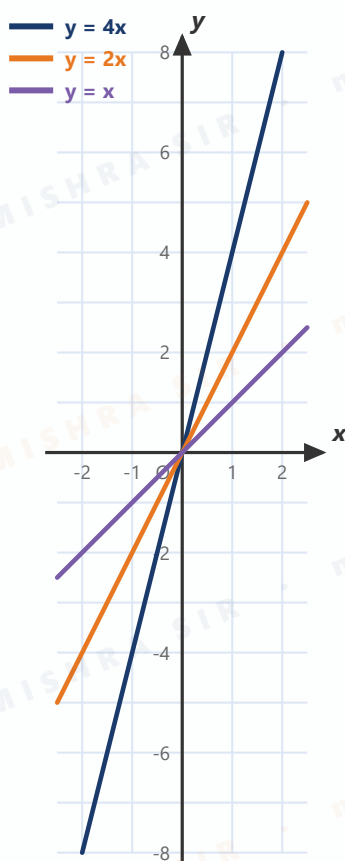
Below, each line's two easy plotting points are given, along with what the comparison reveals about the roles of a and b — draw these yourself on graph paper using the points given.

Q1(i). $y = 4x$, $y = 2x$, $y = x$

Draw + Reflect

POINTS TO PLOT: $y = 4x$: $(0,0)$ and $(1,4)$. $y = 2x$: $(0,0)$ and $(1,2)$. $y = x$: $(0,0)$ and $(1,1)$.

REFLECTION: All three lines pass through the origin (since $b = 0$ in every case). Among lines with positive slope, the **larger the value of a , the steeper the line** — $y = 4x$ is steepest, then $y = 2x$, then $y = x$. ✓



all pass through O; bigger $a \Rightarrow$ steeper

Ex 2.6 Q1(i): $y = 4x$, $y = 2x$ and $y = x$ — all through the origin; the larger the slope a , the steeper the line.

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

Q1(ii). $y = -6x$, $y = -3x$, $y = -x$

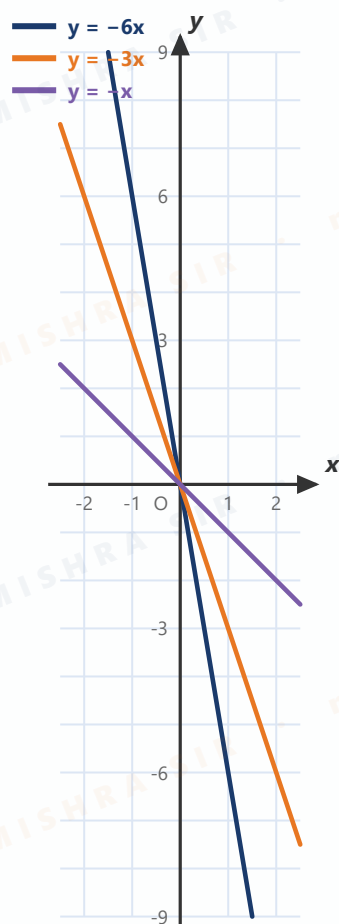
Draw + Reflect

POINTS TO PLOT: $y = -6x$: $(0,0)$ and $(1,-6)$. $y = -3x$: $(0,0)$ and $(1,-3)$. $y = -x$: $(0,0)$ and $(1,-1)$.

REFLECTION: All three lines pass through the origin and slope downward left-to-right (negative slope).

The larger $|a|$ is (ignoring sign), the steeper the downward line — $y = -6x$ falls fastest.

✓



negative slope $\Rightarrow$ falls left-to-right

Ex 2.6 Q1(ii): $y = -6x$, $y = -3x$ and $y = -x$ — through the origin, sloping downward; larger $|a|$ falls faster.

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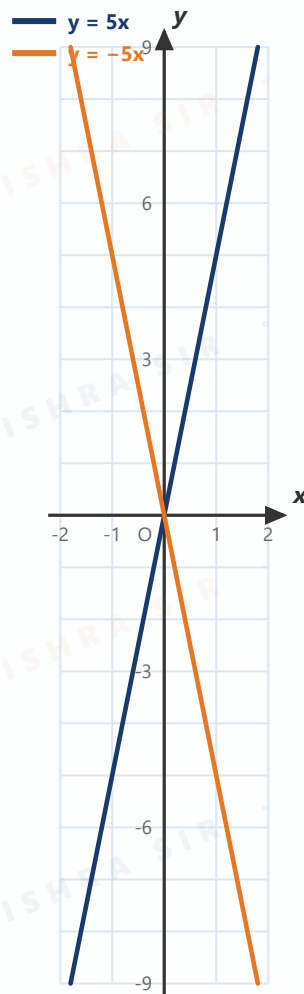
Q1(iii). $y = 5x$, $y = -5x$

Draw + Reflect

POINTS TO PLOT: $y = 5x$: $(0,0)$ and $(1,5)$. $y = -5x$: $(0,0)$ and $(1,-5)$.

REFLECTION: Both lines pass through the origin and are equally steep (same $|a| = 5$) but slope in opposite directions — they are mirror images of each other across the x -axis.

✓



mirror images across the x-axis

Ex 2.6 Q1(iii): $y = 5x$ and $y = -5x$ — equally steep, opposite directions; mirror images of each other.

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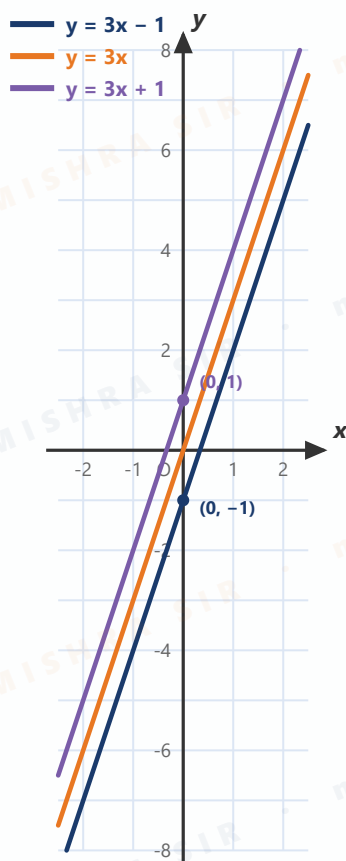
Q1(iv). $y = 3x - 1$, $y = 3x$, $y = 3x + 1$

Draw + Reflect

POINTS TO PLOT: $y = 3x - 1$: $(0, -1)$ and $(1, 2)$. $y = 3x$: $(0, 0)$ and $(1, 3)$. $y = 3x + 1$: $(0, 1)$ and $(1, 4)$.

REFLECTION: All three lines have the **same slope** $a = 3$ — they are equally steep and **parallel** to each other. Only b changes $(-1, 0, 1)$, which shifts each line up or down without changing its direction.





same slope 3 $\Rightarrow$ parallel; b shifts up/down

Ex 2.6 Q1(iv): $y = 3x - 1$, $y = 3x$, $y = 3x + 1$ — same slope, so all three are parallel; only the y-intercept b changes.

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

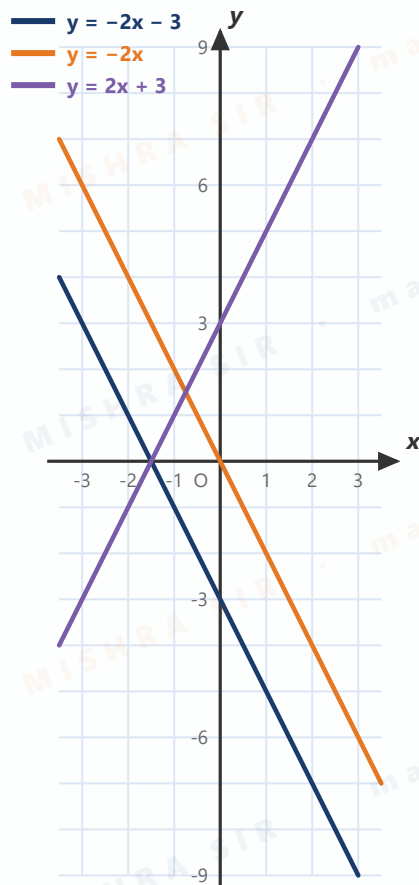
Q1(v). $y = -2x - 3$, $y = -2x$, $y = 2x + 3$

Draw + Reflect

POINTS TO PLOT: $y = -2x - 3$: $(0, -3)$ and $(1, -5)$. $y = -2x$: $(0, 0)$ and $(1, -2)$. $y = 2x + 3$: $(0, 3)$ and $(1, 5)$.

REFLECTION: The first two lines ($y = -2x - 3$ and $y = -2x$) share the **same slope** $a = -2$, so they are **parallel** to each other. The third line, $y = 2x + 3$, has a **different slope** ($a = 2$) and slopes the opposite way, so it is **not parallel** to the other two — it will cross both of them.





first two parallel; third crosses both

Ex 2.6 Q1(v): $y = -2x - 3$ and $y = -2x$ share slope -2 (parallel); $y = 2x + 3$ slopes the opposite way and crosses both.

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— End of Exercise Set 2.6 —

End-of-Chapter Exercises

Q1. Write a polynomial of degree 3 in the variable x , in which the coefficient of the x^2 term is -7 .

Open-ended

SOLUTION: (Any correct example is acceptable.) $x^3 - 7x^2 + 2x + 5$ — the highest power is 3 (so degree 3), and the coefficient of x^2 is -7 as required.

2 marks

Q2. Find the values of the following polynomials at the indicated values of the variables. (i) $5x^2 - 3x + 7$ if $x = 1$ (ii) $4t^3 - t^2 + 6$ if $t = a$

2 parts

SOLUTION:

(i) $5(1)^2 - 3(1) + 7 = 5 - 3 + 7 = 9$

(ii) $4t^3 - t^2 + 6$ at $t = a$: substitute directly $\rightarrow 4a^3 - a^2 + 6$

2 marks

Q3. If we multiply a number by $\frac{5}{2}$ and add $\frac{2}{3}$ to the product, we get $-\frac{7}{12}$. Find the number.

3 marks

SOLUTION: Let the number be x . Then $\frac{5}{2}x + \frac{2}{3} = -\frac{7}{12}$

$$\frac{5}{2}x = -\frac{7}{12} - \frac{2}{3} = -\frac{7}{12} - \frac{8}{12} = -\frac{15}{12} = -\frac{5}{4}$$

$$x = -\frac{5}{4} \times \frac{2}{5} = -\frac{10}{20} = -\frac{1}{2}$$

CHECK: $\frac{5}{2} \left(-\frac{1}{2}\right) + \frac{2}{3} = -\frac{5}{4} + \frac{2}{3} = -\frac{15}{12} + \frac{8}{12} = -\frac{7}{12} \checkmark$

3 marks

Q4. A positive number is 5 times another number. If 21 is added to both the numbers, then one of the new numbers becomes twice the other new number. What are the numbers?

3 marks

SOLUTION: Let the smaller number be y , so the larger (positive) number is $5y$.

After adding 21: new numbers are $y + 21$ and $5y + 21$. Since $5y + 21$ is already larger, it must be the one that becomes twice the other:

$$5y + 21 = 2(y + 21)$$

$$5y + 21 = 2y + 42 \Rightarrow 3y = 21 \Rightarrow y = 7$$

The numbers are $y = 7$ and $5y = 35$.

CHECK: $7 + 21 = 28$; $35 + 21 = 56$; and $56 = 2 \times 28 \checkmark$

3 marks

Q5. If you have ₹800 and you save ₹250 every month, find the amount you have after (i) 6 months (ii) 2 years. Express this as a linear pattern.

2 marks

SOLUTION: Amount after n months $= 800 + 250n$

(i) After 6 months: $800 + 250(6) = 800 + 1500 = 2300 \rightarrow \text{₹2300}$

(ii) After 2 years ($n = 24$): $800 + 250(24) = 800 + 6000 = 6800 \rightarrow \text{₹6800}$

Linear pattern: $800 + 250n$

2 marks

***Q6. The digits of a two-digit number differ by 3. If the digits are interchanged, and the resulting number is added to the original number, we get 143. Find both the numbers.**

Enrichment

GIVEN: Let the tens digit be x and the units digit be y , so the number is $10x + y$. The interchanged number is $10y + x$.

SOLUTION: Sum: $(10x + y) + (10y + x) = 11x + 11y = 11(x + y) = 143 \Rightarrow x + y = 13$

Also given $|x - y| = 3$. Taking $x - y = 3$ together with $x + y = 13$: adding gives $2x = 16 \Rightarrow x = 8, y = 5$.

The number is 85 (digits 8 and 5, differing by 3); interchanging gives 58.

CHECK: $85 + 58 = 143$ ✓, and $8 - 5 = 3$ ✓. (Equivalently, if we'd started with $x = 5, y = 8$, the number would be 58 — the same pair of digits, just read the other way round.)

3 marks

***Q7. Draw the graph of the following equations, and identify their slopes and y-intercepts. Also find the coordinates where these lines cut the y-axis. (i) $y = -3x + 4$ (ii) $2y = 4x + 7$ (iii) $5y = 6x - 10$ (iv) $3y = 6x - 11$. Are any of the lines parallel?**

Enrichment

SOLUTION: Rewrite each equation in the form $y = ax + b$ first:

(i) $y = -3x + 4 \rightarrow$ slope $a = -3$, y-intercept $b = 4$, cuts y-axis at **(0, 4)**

(ii) $2y = 4x + 7 \Rightarrow y = 2x + 3.5 \rightarrow$ slope $a = 2$, y-intercept $b = 3.5$, cuts y-axis at **(0, 3.5)**

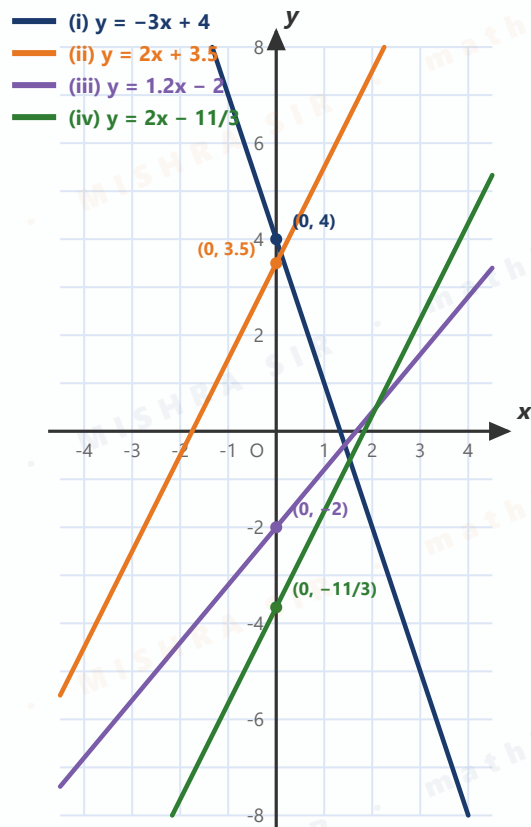
(iii) $5y = 6x - 10 \Rightarrow y = 1.2x - 2 \rightarrow$ slope $a = 1.2$, y-intercept $b = -2$, cuts y-axis at **(0, -2)**

(iv) $3y = 6x - 11 \Rightarrow y = 2x - \frac{11}{3} \rightarrow$ slope $a = 2$, y-intercept $b = -\frac{11}{3} \approx -3.67$, cuts y-axis at **$(0, -\frac{11}{3})$**

PARALLEL CHECK: Lines (ii) and (iv) both have slope $a = 2$, so **(ii) and (iv) are parallel** to each other.

None of the other pairs share a slope.

4 marks



(ii) and (iv): same slope 2 $\Rightarrow$ parallel

End-of-Chapter Q7: the four lines with their y -intercepts marked. Lines (ii) and (iv) have the same slope ($a = 2$), so they are parallel and never meet.

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***Q8. Temperature in Kelvin (x K) and Fahrenheit (y °F) is given by $y = \frac{9}{5}(x - 273) + 32$. (i) Find y in Fahrenheit if $x = 313$ K. (ii) If the temperature is 158°F , find it in Kelvin.**

Enrichment

SOLUTION:

$$\text{(i)} \quad y = \frac{9}{5}(313 - 273) + 32 = \frac{9}{5}(40) + 32 = 72 + 32 = 104^\circ\text{F}$$

$$\text{(ii)} \quad 158 = \frac{9}{5}(x - 273) + 32$$

$$126 = \frac{9}{5}(x - 273) \Rightarrow x - 273 = 126 \times \frac{5}{9} = 70 \Rightarrow x = 343 \text{ K}$$

3 marks

***Q9.** The work done by a body on application of a constant force is the product of the constant force and the distance travelled. Express this as a linear equation in two variables (work w , distance d), and draw its graph taking constant force = 3 units. What is the work done when the distance travelled is 2 units? Verify by plotting.

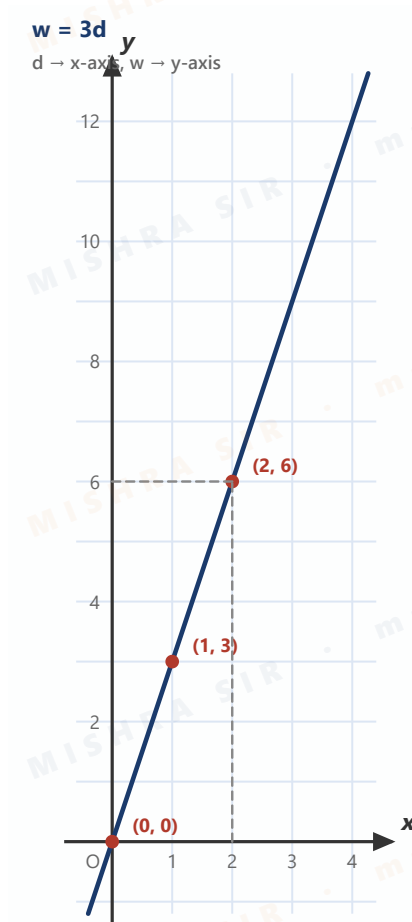
Enrichment

SOLUTION: Work = Force $\times$ distance, so with a constant force of 3 units: $w = 3d$, i.e. $w - 3d = 0$ — a linear equation in the two variables w and d , which graphs as a straight line through the origin with slope 3.

POINTS TO PLOT: $(0, 0)$, $(1, 3)$, $(2, 6)$.

Work done when $d = 2$: $w = 3(2) = 6$ units. This matches the point $(2, 6)$ that lies on the graphed line.

3 marks



End-of-Chapter Q9: $w = 3d$, a straight line through the origin with slope 3. Reading the graph at $d = 2$ gives $w = 6$ — matching the calculation.

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***Q10.** The graph of a linear polynomial $p(x)$ passes through the points $(1, 5)$ and $(3, 11)$. (i) Find $p(x)$. (ii) Find the coordinates where the graph cuts the axes. (iii) Draw the graph and verify.

Enrichment

SOLUTION: Let $p(x) = ax + b$.

$$5 = a + b \dots (1) \text{ and } 11 = 3a + b \dots (2)$$

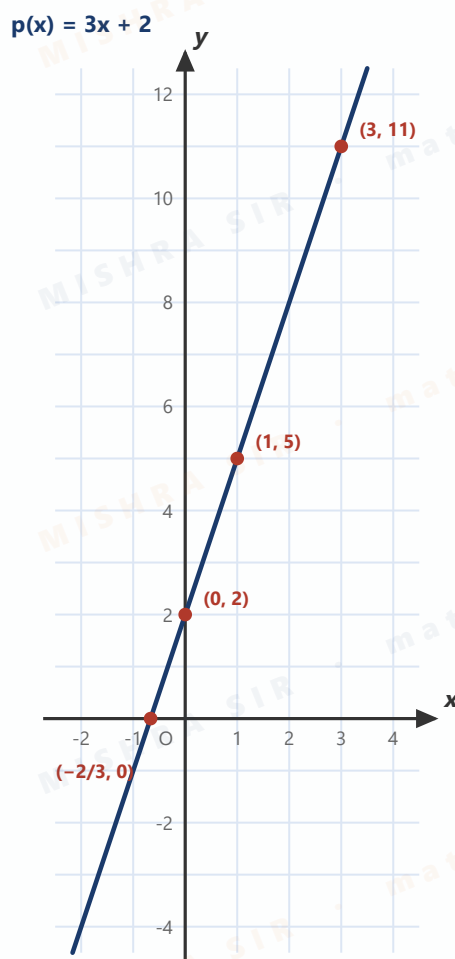
Subtracting (1) from (2): $6 = 2a \Rightarrow a = 3$; from (1): $b = 5 - 3 = 2$

(i) $p(x) = 3x + 2$

(ii) Cuts y -axis at $x = 0$: $p(0) = 2 \rightarrow (0, 2)$. Cuts x -axis where $p(x) = 0$: $3x + 2 = 0 \Rightarrow x = -\frac{2}{3} \rightarrow (-\frac{2}{3}, 0)$

(iii) Plot $(1, 5)$, $(3, 11)$, $(0, 2)$, $(-\frac{2}{3}, 0)$ — check: $p(1) = 3 + 2 = 5 \checkmark$; $p(3) = 9 + 2 = 11 \checkmark$

4 marks



End-of-Chapter Q10: the graph of $p(x) = 3x + 2$ passes through $(1, 5)$ and $(3, 11)$, cuts the y -axis at $(0, 2)$ and the x -axis at $(-\frac{2}{3}, 0)$.

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***Q11.** Let $p(x)=ax+b$ and $q(x)=cx+d$ be two linear polynomials such that: (i) $p(0)=5$
(ii) $p(x)-q(x)$ cuts the x -axis at $(3,0)$ (iii) $p(x)+q(x) = 6x+4$ for all real x . Find $p(x)$
and $q(x)$.

Enrichment

SOLUTION:

From (i): $p(0) = b = 5$

From (iii): $p(x) + q(x) = (a + c)x + (b + d) = 6x + 4 \Rightarrow a + c = 6$ and $b + d = 4 \Rightarrow d = 4 - 5 = -1$

From (ii): $p(x) - q(x) = (a - c)x + (b - d)$ has a zero at $x = 3$: $(a - c)(3) + (b - d) = 0$

$b - d = 5 - (-1) = 6$, so $3(a - c) + 6 = 0 \Rightarrow a - c = -2$

Now solve $a + c = 6$ and $a - c = -2$ together: adding gives $2a = 4 \Rightarrow a = 2$, so $c = 4$

$p(x) = 2x + 5$; $q(x) = 4x - 1$

CHECK: $p(x) - q(x) = (2x + 5) - (4x - 1) = -2x + 6$, zero at $x = 3$ ✓; $p(x) + q(x) = 6x + 4$ ✓

4 marks

***Q12.** First three stages of a growing pattern of hexagons made using matchsticks — a new hexagon shares a side with the last one at every stage. (i) Draw next two stages; how many matchsticks required? (ii) Complete the table for $n=1$ to n . (iii) Rule for the n th stage. (iv) Matchsticks at the 15th stage? (v) Can 200 matchsticks form a stage?

Enrichment

SOLUTION: A single hexagon needs 6 matchsticks. Each new hexagon shares exactly one side with the previous hexagon, so it only needs 5 new matchsticks (6 sides minus the 1 shared side).

Stage 1: 6. Stage 2: $6 + 5 = 11$. Stage 3: $11 + 5 = 16$.

(i) Stage 4: $16 + 5 = 21$ matchsticks; Stage 5: $21 + 5 = 26$ matchsticks.

(ii) Table:

Stage (n)	Matchsticks
1	6
2	11
3	16
4	21
5	26
...	...
n	$5n + 1$

(iii) Rule: **matchsticks** = $5n + 1$ (check: $n = 3$: $5(3) + 1 = 16$ ✓)

(iv) 15th stage: $5(15) + 1 = 75 + 1 = 76$ matchsticks

(v) Set $5n + 1 = 200 \Rightarrow 5n = 199 \Rightarrow n = 39.8$ — not a whole number, so **200 matchsticks cannot form any stage** of this pattern.

5 marks



6 matchsticks



11 matchsticks



16 matchsticks



21 matchsticks

each new hexagon shares one side $\Rightarrow$ adds only 5 sticks: 6, 11, 16, 21, 26, ... (rule $5n + 1$)

End-of-Chapter Q12: stages 1–4 of the hexagon matchstick pattern. Each new hexagon shares one side with the previous one, so each stage adds exactly 5 matchsticks.

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***Q13.** Let $p(x) = ax + b$ and $q(x) = cx + d$ be two linear polynomials such that: (i) graph of $p(x)$ passes through (2,3) and (6,11) (ii) graph of $q(x)$ passes through (4,-1) (iii) graph of $q(x)$ is parallel to the graph of $p(x)$. Find $p(x)$ and $q(x)$; also find where these lines meet the x -axis.

Enrichment

SOLUTION: From (i): $3 = 2a + b \dots(1)$ and $11 = 6a + b \dots(2)$

Subtracting (1) from (2): $8 = 4a \Rightarrow a = 2$; from (1): $b = 3 - 4 = -1$

$$p(x) = 2x - 1$$

Since $q(x)$ is parallel to $p(x)$, it has the same slope: $c = 2$. Using (ii), $q(4) = -1$:

$$-1 = 2(4) + d = 8 + d \Rightarrow d = -9$$

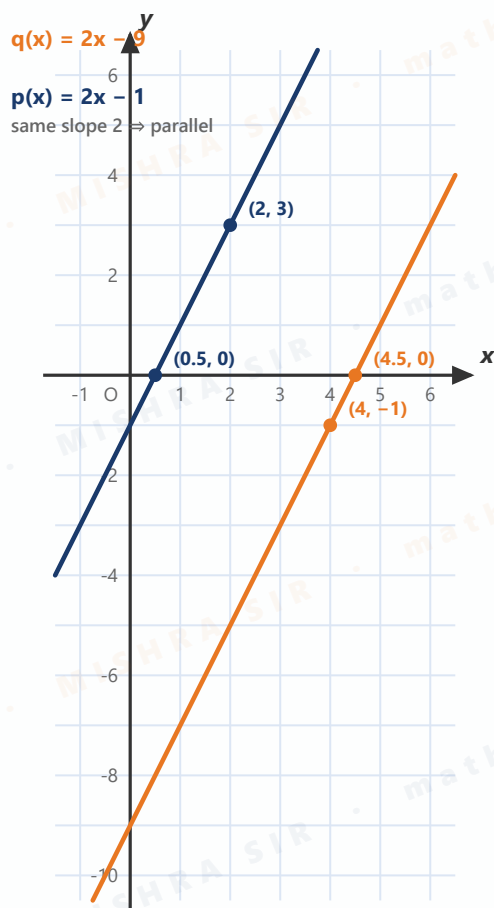
$$q(x) = 2x - 9$$

Where each line meets the x -axis ($y = 0$):

$$p(x) = 0: 2x - 1 = 0 \Rightarrow x = 0.5 \rightarrow (0.5, 0)$$

$$q(x) = 0: 2x - 9 = 0 \Rightarrow x = 4.5 \rightarrow (4.5, 0)$$

4 marks



End-of-Chapter Q13: $p(x) = 2x - 1$ and the parallel $q(x) = 2x - 9$, meeting the x -axis at $(0.5, 0)$ and $(4.5, 0)$ respectively.

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***Q14. What do all linear functions of the form $f(x) = ax + a$, $a > 0$, have in common?**

Enrichment

SOLUTION: Factor out a : $f(x) = ax + a = a(x + 1)$

For every value of a , $f(-1) = a(-1 + 1) = a(0) = 0$.

Every such line passes through the same point, $(-1, 0)$, regardless of the value of a — only the steepness of the line changes as a varies; the x -intercept stays fixed at $x = -1$.

3 marks

— End of Chapter 2 NCERT Solutions —

 Additional Practice Questions

A Degree, Terms & Coefficients (5 Questions)

A1. Find the degree of the polynomial $7x^5 - 2x^3 + x - 9$.

1 mark

SOLUTION: The highest power of x present is 5. **Degree = 5**

1 mark

A2. Is $3x^2 - 2x + \sqrt{5}$ a polynomial? If so, state its degree.

1 mark

SOLUTION: Yes — every power of x present (2 and 1) is a non-negative integer; $\sqrt{5}$ is just a constant coefficient/term, which is allowed. **It is a polynomial of degree 2.**

1 mark

A3. What is the coefficient of x in the polynomial $8 - 5x + 2x^2$?

1 mark

SOLUTION: The term containing x^1 is $-5x$. **Coefficient of $x = -5$**

1 mark

A4. Write a constant polynomial and state its degree.

1 mark

SOLUTION: Any single number works, e.g. **7**, which can be written as $7x^0$. **Degree = 0**

1 mark

A5. State whether $2\sqrt{x} + 3$ is a polynomial. Justify your answer.

2 marks

SOLUTION: $2\sqrt{x} + 3 = 2x^{1/2} + 3$. Since the exponent $\frac{1}{2}$ is **not** a non-negative integer, this expression **is not a polynomial** in x — it fails the defining condition for polynomials even though it looks similar to one.

2 marks

B Evaluating Polynomials (5 Questions)

B1. Find the value of $3x^2 - 5x + 2$ at $x = 2$.

2 marks

SOLUTION: $3(2)^2 - 5(2) + 2 = 3(4) - 10 + 2 = 12 - 10 + 2 = 4$

2 marks

B2. Find the value of $-2x^3 + 4x - 7$ at $x = -1$.

2 marks

SOLUTION: $-2(-1)^3 + 4(-1) - 7 = -2(-1) - 4 - 7 = 2 - 4 - 7 = -9$

2 marks

B3. If $p(x) = 5x - 3$, find $p(0)$, $p(2)$ and $p(-3)$.

2 marks

SOLUTION: $p(0) = 5(0) - 3 = -3$

$p(2) = 5(2) - 3 = 10 - 3 = 7$

$p(-3) = 5(-3) - 3 = -15 - 3 = -18$

2 marks

B4. Find the value of x for which the polynomial $4x - 8$ becomes zero.

1 mark

SOLUTION: $4x - 8 = 0 \Rightarrow 4x = 8 \Rightarrow x = 2$

1 mark

B5. For the polynomial $p(y) = 7 - 2y$, find $p(1) + p(-1)$.

2 marks

SOLUTION: $p(1) = 7 - 2(1) = 5$; $p(-1) = 7 - 2(-1) = 7 + 2 = 9$

$p(1) + p(-1) = 5 + 9 = 14$

2 marks

C Linear Patterns, Growth & Decay (5 Questions)

C1. A tank holds 500 L and drains at a constant 25 L/min. Write an expression for the water left after t minutes. Is this growth or decay?

2 marks

SOLUTION: $V(t) = 500 - 25t$. Since the amount decreases by a fixed 25 L every minute (negative slope), this is **linear decay**.

2 marks

C2. A company's profit is modelled by $P(n) = 2000 + 150n$, where n is the number of units sold. Find the profit when $n = 12$.

2 marks

SOLUTION: $P(12) = 2000 + 150(12) = 2000 + 1800 = 3800 \rightarrow \text{₹}3800$

2 marks

C3. Find the constant difference for the linear pattern 7, 12, 17, 22, ... and write its n th-term rule.

2 marks

SOLUTION: Difference between consecutive terms $= 12 - 7 = 5$ (constant). n th term

$= 7 + 5(n - 1) = 5n + 2$

CHECK: $n = 1$: $5 + 2 = 7$ ✓; $n = 4$: $20 + 2 = 22$ ✓ **n th term $= 5n + 2$**

2 marks

C4. A candle 20 cm tall burns down at 1.5 cm per hour. Write $h(t)$ and find its height after 6 hours.

2 marks

SOLUTION: $h(t) = 20 - 1.5t$. At $t = 6$: $h(6) = 20 - 1.5(6) = 20 - 9 = 11$ cm

2 marks

C5. Is the sequence 3, 9, 15, 21, ... a linear pattern? Find the common difference and the 10th term.

2 marks

SOLUTION: Difference = $9 - 3 = 6$, and this stays constant throughout, so **yes, it is a linear pattern.**

n th term = $3 + 6(n - 1) = 6n - 3$. 10th term: $6(10) - 3 = 60 - 3 = 57$

2 marks

D Linear Relationships & Applications (5 Questions)

D1. A taxi charges a base fare plus a per-km rate. 5 km costs ₹150; 12 km costs ₹290. Find $y = ax + b$.

3 marks

SOLUTION: $150 = 5a + b \dots(1)$; $290 = 12a + b \dots(2)$

Subtracting: $140 = 7a \Rightarrow a = 20$; from (1): $b = 150 - 100 = 50$

$y = 20x + 50$ (₹20 per km, ₹50 base fare)

3 marks

D2. Find the slope and y-intercept of the line $y = -4x + 9$.

1 mark

SOLUTION: Comparing with $y = ax + b$: **slope = -4; y-intercept = 9**

1 mark

D3. A line has slope 3 and passes through (2, 7). Find its equation in the form $y = ax + b$.

2 marks

SOLUTION: $a = 3$. Substituting (2,7): $7 = 3(2) + b = 6 + b \Rightarrow b = 1$

$y = 3x + 1$

2 marks

D4. Are the lines $y = 2x + 5$ and $y = 2x - 3$ parallel? Explain.

1 mark

SOLUTION: Both lines have the **same slope ($a = 2$)** but different y-intercepts (5 and -3). **Yes, they are parallel** — they run in the same direction but never meet.

1 mark

D5. If a straight line $y = ax + b$ passes through the origin, what can you say about b ? Justify with an example.

2 marks

SOLUTION: Substituting the origin $(0, 0)$: $0 = a(0) + b \Rightarrow b = 0$. **So b must be 0.**

Example: $y = 4x$ passes through the origin, and here $b = 0$, confirming the rule.

2 marks

— End of Additional Practice Questions —

CHAPTER 3

The World of Numbers

Exercise Sets 3.1 – 3.5 + End-of-Chapter

EXERCISE SETS 3.1 – 3.5 + END-OF-CHAPTER

 **Weightage (School Annual Exam, Per Official CBSE Curriculum)**

Marks Distribution (Out of 80 — Annual Theory Paper)

✓ Verified directly against the official CBSE curriculum document (cbseacademic.nic.in, Mathematics Class IX, 2026–27) — not a secondary source. Like Chapter 1 (Coordinate Geometry), this chapter is the **only** chapter in its unit — **Unit I: Number System** — so its marks are not shared with any other chapter, unlike Chapter 2 (which shares Unit II: Algebra with three other chapters).

Detail	Value
Unit	Number System (Unit I)
Chapters in this unit	Number System is the only chapter (this chapter, "The World of Numbers")
Marks allotted	7 marks out of 80
Teaching periods	12 periods (out of ~200 total for the year)
Internal Assessment (separate, 20 marks)	Pen-Paper Test & Multiple Assessment (10), Portfolio (5), Lab Practical (5)

✦ Official CBSE learning outcomes for this unit: understand rational numbers and represent them on the number line, understand the properties of rational numbers, explain density of rational numbers, compute decimal representations of rational numbers, understand irrational numbers, prove irrationality, and construct the square root spiral. Nearly every one of these outcomes maps directly onto a section of this chapter.

Predicted Question Types

Format	Marks	Likely Topic
MCQ / Very Short Answer	1	Classifying a number as rational/irrational; identifying whether a fraction's decimal terminates or repeats just from the denominator's prime factors
Short Answer	2	Rational number arithmetic (add/subtract/multiply/divide); converting a terminating or repeating decimal to p/q form; finding a rational number between two given numbers
Short/Long Answer	2–3	Proving $\sqrt{2}$, $\sqrt{3}$, or $\sqrt{5}$ is irrational using proof by contradiction; converting a general repeating decimal (non-repeating part + repeating part) to p/q form
Application/Reasoning	2–3	Word problems using Brahmagupta's integer rules (debt/fortune contexts); density-of-rationals reasoning; constructing/interpreting the square root spiral

All Key Formulas & Definitions

Natural numbers ($\mathbb{N}$): $\{1, 2, 3, 4, \dots\}$ — the basic counting numbers.

Integers ($\mathbb{Z}$): $\{\dots, -2, -1, 0, 1, 2, \dots\}$ — natural numbers, their negatives, and zero.

Brahmagupta's rules for zero: $a + 0 = a$; $a - 0 = a$; $a \times 0 = 0$.

Brahmagupta's rules for signed numbers: fortune + fortune = fortune; debt + debt = debt; debt $\times$ fortune = debt (negative); debt $\times$ debt = fortune (positive).

Rational number ($\mathbb{Q}$): any number expressible as $\frac{p}{q}$, where p, q are integers and $q \neq 0$.

Equality of rationals: $\frac{a}{b} = \frac{c}{d}$ exactly when $ad = bc$.

Rational arithmetic: $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$; $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$; $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$;
 $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$ (with all denominators $\neq 0$).

Density of rational numbers: between any two rational numbers there is always another rational number — e.g. their average $\frac{a+b}{2}$. This means there are infinitely many rationals between any two rationals.

Absolute value: $|x|$ is the distance of x from 0 on the number line; $|x| \geq 0$ always. The distance between two rationals a and b is $|a - b|$.

Irrational number: a number that cannot be written as $\frac{p}{q}$ for any integers p, q — e.g. $\sqrt{2}$, $\sqrt{3}$, π .

Real numbers ($\mathbb{R}$): the union of all rational and irrational numbers — the complete, unbroken number line.

Terminating vs. repeating decimals: for $\frac{p}{q}$ in lowest terms, the decimal **terminates** exactly when the only prime factors of q are 2, 5, or both; otherwise it **repeats**.

Converting a terminating decimal to $\frac{p}{q}$: write the digits over the matching power of 10, then simplify — e.g. $0.35 = \frac{35}{100} = \frac{7}{20}$.

Converting a pure repeating decimal: let x = the decimal; multiply by 10^n (n = number of repeating digits); subtract the original x ; solve for x .

Converting a general repeating decimal: multiply first by 10^m (m = number of non-repeating digits after the point) to clear the non-repeating part, then by 10^n more (n = number of repeating digits) to shift one full cycle; subtract and solve.

Cyclic numbers: the repeating block of certain fractions (like $\frac{1}{7} = 0.\overline{142857}$) shifts its digits cyclically when multiplied by 2, 3, 4,

Constructing $\sqrt{n}$: using a right triangle with legs $\sqrt{n-1}$ and 1 (Baudhāyana–Pythagoras Theorem), the hypotenuse has length $\sqrt{n}$ — repeating this builds the **square root spiral**.

Shortcuts & Study Tips

1. **Sign rule shortcut:** same signs multiply/divide to a positive result; different signs give a negative result — this is exactly Brahmagupta's "debt $\times$ debt = fortune" rule in modern notation.

2. **Predicting terminating vs. repeating without dividing:** factorise only the denominator (after simplifying to lowest terms) — if you see only 2s and/or 5s, it terminates; any other prime factor (3, 7, 11, 13, ...) means it repeats.
3. **Counting decimal places in a terminating decimal:** the number of decimal places equals $\max(\text{power of 2, power of 5})$ in the denominator, since that's how many extra factors you need to reach a power of 10.
4. **Converting repeating decimals fast:** always count digits carefully — non-repeating digits determine the first multiplier's power of 10; repeating digits determine the second.
5. **Finding a rational number between two others:** averaging always works, but for finding *several* at once, convert both numbers to a common denominator large enough that whole numbers exist strictly between the two numerators.
6. **Proving irrationality by contradiction:** always follow the same 8-step template — assume rational in lowest terms, square, show the numerator's square is divisible by the prime, deduce the numerator itself is divisible by that prime, substitute back, repeat the argument for the denominator, then spot the contradiction with "lowest terms."
7. **Square root spiral shortcut:** you never need to compute a fresh Pythagorean sum for each triangle — the hypotenuse of the n -th triangle is always exactly $\sqrt{n+1}$, since each step just adds 1 to the previous squared length.

NCERT Solutions — Exercise Set 3.1

Approach for This Exercise

This exercise checks basic understanding of natural numbers, their historical origins (the Ishango Bone), and the closure property. No heavy computation is needed — careful reasoning is the key skill here.

Q1. A merchant in the port city of Lothal is exchanging bags of spices for copper ingots. He receives 15 ingots for every 2 bags of spices. If he brings 12 bags of spices to the market, how many copper ingots will he leave with?

2 marks

SOLUTION: Rate: 2 bags $\rightarrow$ 15 ingots, so 1 bag $\rightarrow \frac{15}{2}$ ingots.

For 12 bags: $12 \times \frac{15}{2} = 6 \times 15 = 90$ copper ingots

2 marks

Q2. Look at the sequence of numbers on one column of the Ishango bone: 11, 13, 17, 19. What do these numbers have in common? List the next three numbers that fit this pattern.

2 marks

SOLUTION: All four numbers are **prime numbers** (numbers with exactly two factors: 1 and themselves) — in fact, they are exactly the prime numbers between 10 and 20.

Next three primes after 19: 23, 29, 31

2 marks

Q3. We know that Natural Numbers are closed under addition. Are they closed under subtraction? Provide a couple of examples to justify your answer.

2 marks

SOLUTION: No, natural numbers are not closed under subtraction.

Example 1: $3 - 7 = -4$, which is **not** a natural number.

Example 2: $5 - 5 = 0$, which is also **not** a natural number (since $\mathbb{N} = \{1, 2, 3, \dots\}$ does not include 0).

So subtracting two natural numbers can give a result outside the set of natural numbers — closure fails.

2 marks

***Q4. Ancient Indians used the joints of their fingers to count. Each finger has 3 joints, and the thumb is used to count them. How many can you count on one hand? How does this relate to the ancient base-12 counting systems?**

Enrichment

SOLUTION: One hand has 4 fingers (excluding the thumb, which is used as the "pointer" to touch the joints), and each finger has 3 joints.

Total countable joints = $4 \times 3 = 12$ on one hand.

This directly explains the origin of **base-12 (duodecimal) counting systems** — since a person can naturally count up to 12 using just one hand's finger joints (touched by the thumb), many ancient number systems (and even modern conventions like 12 inches in a foot, 12 hours on a clock face) are built around groups of 12 rather than groups of 10.

3 marks

— End of Exercise Set 3.1 —

 **NCERT Solutions — Exercise Set 3.2**

Approach for This Exercise

Every question here is a direct application of Brahmagupta's rules — think in terms of "fortune" (positive) and "debt" (negative), and apply the sign rules for addition, subtraction, and multiplication carefully.

Q1. The temperature in the high-altitude desert of Ladakh is recorded as 4°C at noon. By midnight, it drops by 15°C . What is the midnight temperature?

1 mark

SOLUTION: Midnight temperature $= 4 - 15 = -11^{\circ}\text{C}$

1 mark

Q2. A spice trader takes a loan (debt) of ₹850. The next day, he makes a profit (fortune) of ₹1,200. The following week, he incurs a loss of ₹450. Write this sequence as an equation using integers and calculate his final financial standing.

2 marks

SOLUTION: Loan (debt) $= -850$; profit (fortune) $= +1200$; loss $= -450$.

Equation: $-850 + 1200 - 450$

$= 350 - 450 = -100$

Final standing: $-\text{₹}100$, i.e. the trader ends up with a net debt of ₹100.

2 marks

Q3. Calculate the following using Brahmagupta's laws: (i) $(-12) \times 5$ (ii) $(-8) \times (-7)$ (iii) $0 - (-14)$ (iv) $(-20) \div 4$

4 parts

SOLUTION:

(i) Debt $\times$ fortune $=$ debt: $(-12) \times 5 = -60$

(ii) Debt $\times$ debt $=$ fortune: $(-8) \times (-7) = 56$

(iii) $0 - (-14) = 0 + 14 = 14$

(iv) $(-20) \div 4 = -5$

4 marks

Q4. Explain, using a real-world example of debt, why subtracting a negative number is the same as adding a positive number (e.g., $10 - (-5) = 15$).

2 marks

SOLUTION: Suppose you have ₹10, and separately you owe someone ₹5 (a debt of -5). If that debt is removed — say, the person forgives it — you are effectively ₹5 richer than before, because you no longer have to pay it back.

So removing (subtracting) a debt of ₹5 has the same effect as gaining ₹5:

$10 - (-5) = 10 + 5 = 15$. This is exactly why subtracting a negative number is equivalent to adding its positive value.

2 marks

 NCERT Solutions — Exercise Set 3.3

 Approach for This Exercise

This set applies **Brahmagupta's rules for rational numbers**, set out in the *Brāhmasphuṭasiddhānta* (628 CE) — the first systematic treatment of arithmetic with negative numbers and fractions. Four rules cover everything here: equality by cross-multiplication, addition and subtraction through a common denominator, multiplication straight across, and division by multiplying with the reciprocal. Two habits earn marks in every part: **always reduce the final answer to lowest terms**, and when a negative sign appears, attach it to the *numerator* and carry it through the working rather than dropping it and restoring it at the end.

Q1. Prove that the following rational numbers are equal: (i) $\frac{2}{3}$ and $\frac{4}{6}$ (ii) $\frac{5}{4}$ and $\frac{10}{8}$ (iii)

$-\frac{3}{5}$ and $-\frac{6}{10}$ (iv) $\frac{9}{3}$ and 3

4 parts

SOLUTION:

Rule used: $\frac{a}{b} = \frac{c}{d}$ exactly when $ad = bc$. This is called cross-multiplication: multiply the numerator of each fraction by the denominator of the other. If the two products agree, the fractions name the same number. Note that we never need to convert to decimals.

(i) $\frac{2}{3}$ and $\frac{4}{6}$ — here $a = 2$, $b = 3$, $c = 4$, $d = 6$.

First cross-product: $ad = 2 \times 6 = 12$

Second cross-product: $bc = 3 \times 4 = 12$

Since $12 = 12$, the two cross-products agree, so $\frac{2}{3} = \frac{4}{6}$ ✓ **1 mark**

(ii) $\frac{5}{4}$ and $\frac{10}{8}$ — here $a = 5$, $b = 4$, $c = 10$, $d = 8$.

$ad = 5 \times 8 = 40$

$bc = 4 \times 10 = 40$

Since $40 = 40$, $\frac{5}{4} = \frac{10}{8}$ ✓ (check: $\frac{10}{8}$ reduces to $\frac{5}{4}$ on dividing both parts by 2) **1 mark**

(iii) $-\frac{3}{5}$ and $-\frac{6}{10}$ — write these as $\frac{-3}{5}$ and $\frac{-6}{10}$, keeping the sign with the numerator.

$ad = (-3) \times 10 = -30$

$bc = 5 \times (-6) = -30$

Since $-30 = -30$, $-\frac{3}{5} = -\frac{6}{10}$ ✓ **1 mark**

(iv) $\frac{9}{3}$ and 3 — every integer is a rational number with denominator 1, so write $3 = \frac{3}{1}$.

$ad = 9 \times 1 = 9$

$bc = 3 \times 3 = 9$

Since $9 = 9$, $\frac{9}{3} = 3$ ✓ **1 mark**

✳ **Examiner's tip.** In part (iv) the step that earns the mark is writing $3 = \frac{3}{1}$. Many students jump straight to " $9 \div 3 = 3$ ", which gives the right answer but does not prove equality by the rule the question asks for.

Q2. Find the sum: (i) $\frac{2}{5} + \frac{3}{10}$ (ii) $\frac{7}{12} + \frac{5}{8}$ (iii) $-\frac{4}{7} + \frac{3}{14}$

3 parts

SOLUTION:

Rule used: fractions can only be added once they share a denominator. Take the LCM of the two denominators, rewrite each fraction with that denominator, then add the numerators and keep the denominator unchanged.

(i) $\frac{2}{5} + \frac{3}{10}$

LCM of 5 and 10 is 10.

Rewrite the first fraction: $\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$; the second already has denominator 10.

Add the numerators: $\frac{4}{10} + \frac{3}{10} = \frac{4+3}{10} = \frac{7}{10}$

7 and 10 share no common factor, so this is already in lowest terms: $\frac{7}{10}$

1 mark

(ii) $\frac{7}{12} + \frac{5}{8}$

LCM of 12 and 8: $12 = 2^2 \times 3$, $8 = 2^3$, so $LCM = 2^3 \times 3 = 24$.

$$\frac{7}{12} = \frac{7 \times 2}{12 \times 2} = \frac{14}{24} \quad \text{and} \quad \frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$
$$\frac{14}{24} + \frac{15}{24} = \frac{29}{24}$$

29 is prime and does not divide 24, so the answer stays as $\frac{29}{24}$ (an improper fraction; it may also be written $1\frac{5}{24}$)

1 mark

(iii) $-\frac{4}{7} + \frac{3}{14}$

LCM of 7 and 14 is 14.

$$-\frac{4}{7} = \frac{-4 \times 2}{7 \times 2} = -\frac{8}{14}$$
$$-\frac{8}{14} + \frac{3}{14} = \frac{-8+3}{14} = -\frac{5}{14}$$

Therefore the sum is $-\frac{5}{14}$

1 mark

 **Common slip.** In (iii) the numerators are added as directed numbers: $-8 + 3 = -5$, not -11 .

Adding a positive to a negative moves towards zero.

Q3. Find the difference: (i) $\frac{5}{6} - \frac{1}{4}$ (ii) $\frac{11}{8} - \frac{3}{4}$ (iii) $-\frac{7}{9} - \left(-\frac{2}{3}\right)$

3 parts

SOLUTION:

Rule used: the same common-denominator method as for addition. For part (iii) also recall that subtracting a negative is the same as adding the positive: $a - (-b) = a + b$.

(i) $\frac{5}{6} - \frac{1}{4}$

LCM of 6 and 4: $6 = 2 \times 3$, $4 = 2^2$, so $LCM = 2^2 \times 3 = 12$.

$$\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12} \text{ and } \frac{1}{4} = \frac{1 \times 3}{4 \times 3} = \frac{3}{12}$$
$$\frac{10}{12} - \frac{3}{12} = \frac{10-3}{12} = \frac{7}{12}$$

1 mark

(ii) $\frac{11}{8} - \frac{3}{4}$

LCM of 8 and 4 is 8.

$$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$
$$\frac{11}{8} - \frac{6}{8} = \frac{11-6}{8} = \frac{5}{8}$$

1 mark

(iii) $-\frac{7}{9} - \left(-\frac{2}{3}\right)$

First deal with the double sign: subtracting $-\frac{2}{3}$ means adding $+\frac{2}{3}$.

$$-\frac{7}{9} - \left(-\frac{2}{3}\right) = -\frac{7}{9} + \frac{2}{3}$$
$$LCM \text{ of } 9 \text{ and } 3 \text{ is } 9, \text{ and } \frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$$
$$-\frac{7}{9} + \frac{6}{9} = \frac{-7+6}{9} = -\frac{1}{9}$$

1 mark

 **Examiner's tip.** In (iii), write the "minus a minus becomes plus" step on its own line. It carries the mark, and skipping straight to the common denominator is where sign errors creep in.

Q4. Find the product: (i) $\frac{2}{3} \times \frac{3}{10}$ (ii) $\frac{7}{11} \times \frac{5}{8}$ (iii) $-\frac{4}{7} \times \frac{5}{14}$

3 parts

SOLUTION:

Rule used: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$ — multiply numerators together and denominators together. No common denominator is needed. Then reduce.

$$\text{(i)} \quad \frac{2}{3} \times \frac{3}{10} \\ = \frac{2 \times 3}{3 \times 10} = \frac{6}{30}$$

Reduce: the HCF of 6 and 30 is 6, so $\frac{6 \div 6}{30 \div 6} = \frac{1}{5}$

(Quicker: the 3 in the numerator cancels the 3 in the denominator before multiplying, leaving $\frac{2}{10} = \frac{1}{5}$)

1 mark

$$\text{(ii)} \quad \frac{7}{11} \times \frac{5}{8} \\ = \frac{7 \times 5}{11 \times 8} = \frac{35}{88}$$

$35 = 5 \times 7$ and $88 = 2^3 \times 11$ share no common factor, so the answer is already in lowest terms: $\frac{35}{88}$

1 mark

$$\text{(iii)} \quad -\frac{4}{7} \times \frac{5}{14}$$

A negative times a positive is negative.

$$= \frac{-4 \times 5}{7 \times 14} = \frac{-20}{98}$$

Reduce: HCF of 20 and 98 is 2, so $\frac{-20 \div 2}{98 \div 2} = -\frac{10}{49}$

1 mark

 **Marks are lost here** more than anywhere else in the set — by stopping at $\frac{6}{30}$ or $\frac{-20}{98}$. The final reduction is part of the answer, not an optional tidy-up.

Q5. Find the quotient: (i) $\frac{2}{3} \div \frac{3}{10}$ (ii) $\frac{7}{11} \div \frac{5}{8}$ (iii) $-\frac{4}{7} \div \frac{5}{14}$

3 parts

SOLUTION:

Rule used: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ — dividing by a fraction is multiplying by its reciprocal (the fraction turned upside down). Write the reciprocal step explicitly; it carries the mark.

(i) $\frac{2}{3} \div \frac{3}{10}$

The reciprocal of $\frac{3}{10}$ is $\frac{10}{3}$.

$$\frac{2}{3} \div \frac{3}{10} = \frac{2}{3} \times \frac{10}{3} = \frac{2 \times 10}{3 \times 3} = \frac{20}{9}$$

20 and 9 share no factor, so this is in lowest terms.

1 mark

(ii) $\frac{7}{11} \div \frac{5}{8}$

The reciprocal of $\frac{5}{8}$ is $\frac{8}{5}$.

$$\frac{7}{11} \times \frac{8}{5} = \frac{7 \times 8}{11 \times 5} = \frac{56}{55}$$

1 mark

(iii) $-\frac{4}{7} \div \frac{5}{14}$

The reciprocal of $\frac{5}{14}$ is $\frac{14}{5}$.

$$-\frac{4}{7} \times \frac{14}{5} = \frac{-4 \times 14}{7 \times 5} = \frac{-56}{35}$$

Reduce: HCF of 56 and 35 is 7, so $\frac{-56 \div 7}{35 \div 7} = -\frac{8}{5}$

1 mark

✎ **Compare Q4 and Q5.** Part (iii) of each uses the same two fractions. Multiplying gave $-\frac{10}{49}$ (a number less than 1 in size); dividing gave $-\frac{8}{5}$ (greater than 1). Dividing by a fraction smaller than 1 always makes the result larger in magnitude — a useful check.

Q6. Show that: $\left(\frac{1}{2} + \frac{3}{4}\right) \times \frac{8}{3} = \frac{1}{2} \times \frac{8}{3} + \frac{3}{4} \times \frac{8}{3}$

2 marks

SOLUTION:

This question asks us to **verify the distributive law** $a(b + c) = ab + ac$ for a particular set of rational numbers. The method is to evaluate each side separately and show the two results agree.

Left-hand side:

First work inside the bracket. LCM of 2 and 4 is 4: $\frac{1}{2} = \frac{2}{4}$, so $\frac{2}{4} + \frac{3}{4} = \frac{5}{4}$.

Now multiply: $\frac{5}{4} \times \frac{8}{3} = \frac{5 \times 8}{4 \times 3} = \frac{40}{12}$.

Reduce by HCF 4: $\frac{40 \div 4}{12 \div 4} = \frac{10}{3}$.

So LHS = $\frac{10}{3}$.

1 mark

Right-hand side:

First product: $\frac{1}{2} \times \frac{8}{3} = \frac{8}{6} = \frac{4}{3}$

Second product: $\frac{3}{4} \times \frac{8}{3} = \frac{24}{12} = 2$

Add them, writing 2 = $\frac{6}{3}$: $\frac{4}{3} + \frac{6}{3} = \frac{10}{3}$.

So RHS = $\frac{10}{3}$.

1 mark

Conclusion: LHS = RHS = $\frac{10}{3}$, so the **distributive law is verified** for these numbers.

✳ **Write the conclusion sentence.** A "show that" question is only complete when you state that the two sides are equal. Stopping after computing both values typically loses the final half mark.

Q7. Simplify the following using the distributive property: $\frac{7}{9} \left(\frac{6}{7} - \frac{3}{4} \right)$

2 marks

SOLUTION:

The question specifies the method, so we must expand using $a(b - c) = ab - ac$ rather than simplifying the bracket first.

$$\frac{7}{9} \left(\frac{6}{7} - \frac{3}{4} \right) = \frac{7}{9} \times \frac{6}{7} - \frac{7}{9} \times \frac{3}{4}$$

First product: $\frac{7}{9} \times \frac{6}{7} = \frac{42}{63}$. Reduce by HCF 21: $\frac{42 \div 21}{63 \div 21} = \frac{2}{3}$.

Second product: $\frac{7}{9} \times \frac{3}{4} = \frac{21}{36}$. Reduce by HCF 3: $\frac{21 \div 3}{36 \div 3} = \frac{7}{12}$.

Now subtract. LCM of 3 and 12 is 12, and $\frac{2}{3} = \frac{8}{12}$:

$$\frac{8}{12} - \frac{7}{12} = \frac{1}{12}$$

2 marks

 **Check by the other route.** Simplifying the bracket first gives $\frac{6}{7} - \frac{3}{4} = \frac{24 - 21}{28} = \frac{3}{28}$, and $\frac{7}{9} \times \frac{3}{28} = \frac{21}{252} = \frac{1}{12}$ — the same answer, which confirms the working. In the exam, still present the distributive route, since that is what was asked.

Q8. Find the rational number x such that: $\frac{5}{6}\left(x + \frac{3}{5}\right) = \frac{5}{6}x + \frac{1}{2}$

2 marks

SOLUTION:

Begin by expanding the left-hand side with the distributive property, then compare the two sides term by term.

$$\frac{5}{6}\left(x + \frac{3}{5}\right) = \frac{5}{6} \times x + \frac{5}{6} \times \frac{3}{5}$$

Evaluate the constant term: $\frac{5}{6} \times \frac{3}{5} = \frac{5 \times 3}{6 \times 5} = \frac{15}{30} = \frac{1}{2}$.

So the left-hand side becomes $\frac{5}{6}x + \frac{1}{2}$. 1 mark

But this is exactly the right-hand side, $\frac{5}{6}x + \frac{1}{2}$.

The two sides are therefore identical, whatever value x takes. Subtracting $\frac{5}{6}x + \frac{1}{2}$ from both sides gives $0 = 0$, which is true for every x and places no restriction on it.

Answer: the equation holds for every rational number x — it is an identity, not an equation with a single solution. 1 mark

✦ **Why this question is set.** It looks like a "solve for x " problem, and students often manufacture an answer such as $x = 0$. The real point is that $\frac{5}{6} \times \frac{3}{5} = \frac{1}{2}$ matches the constant on the right exactly, so the statement is just the distributive property written out — true for all x . Recognising an identity, rather than solving, is the skill being tested.

— End of Exercise Set 3.3 —

■ **NCERT Solutions — Exercise Set 3.4**

Approach for This Exercise

Number-line placement questions ask you to describe the exact position of a fraction between two integers — divide the unit interval into equal parts as shown in the chapter's Fig. 3.5–3.7. "Find rationals between" questions are solved fastest by converting to a common denominator large enough to leave room, or by simple averaging.

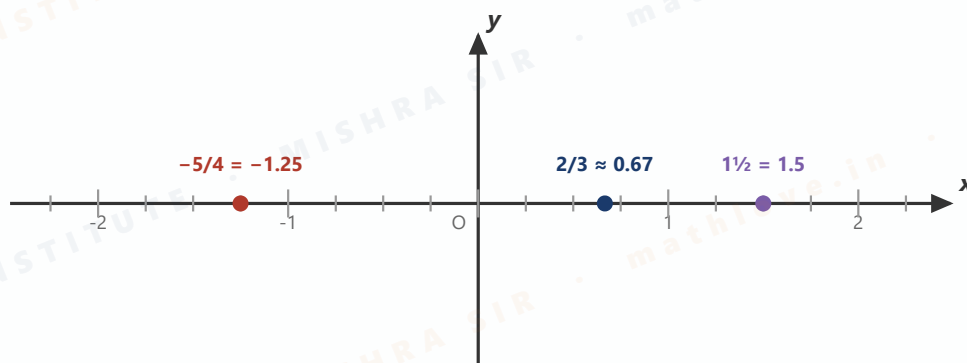
Q1. Represent the rational numbers $\frac{2}{3}$, $-\frac{5}{4}$ and $1\frac{1}{2}$ on a single number line.

2 marks

SOLUTION: Convert to decimals to place them: $\frac{2}{3} \approx 0.67$ (between 0 and 1, two-thirds of the way from 0 to 1); $-\frac{5}{4} = -1.25$ (between -2 and -1, one-quarter of the way past -1 towards -2); $1\frac{1}{2} = 1.5$ (exactly halfway between 1 and 2).

On a single number line marked from -2 to 2: place $-\frac{5}{4}$ between -2 and -1 (closer to -1), place $\frac{2}{3}$ between 0 and 1 (closer to 1), and place $1\frac{1}{2}$ exactly at the midpoint of 1 and 2.

2 marks



Ex 3.4 Q1: $-\frac{5}{4}$ sits a quarter of the way from -1 towards -2; $\frac{2}{3}$ sits two-thirds of the way from 0 to 1; $1\frac{1}{2}$ sits exactly midway between 1 and 2. (Minor ticks = quarters.)

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

Q2. Find three distinct rational numbers that lie strictly between $-\frac{1}{2}$ and $\frac{1}{4}$.

2 marks

SOLUTION: $-\frac{1}{2} = -0.5$ and $\frac{1}{4} = 0.25$.

Three valid choices: $-\frac{1}{4}$, 0, $\frac{1}{8}$

CHECK: $-0.5 < -0.25 < 0 < 0.125 < 0.25$ ✓ — all three lie strictly between.

2 marks

Q3. Simplify the expression: $(-1/4) + (5/12)$

1 mark

SOLUTION: $LCM(4, 12) = 12$: $-\frac{3}{12} + \frac{5}{12} = \frac{2}{12} = \frac{1}{6}$

1 mark

Q4. A tailor has $15\frac{3}{4}$ metres of fine silk. If making one kurta requires $2\frac{1}{4}$ metres of silk, exactly how many kurtas can he make?

2 marks

SOLUTION: $15\frac{3}{4} = \frac{63}{4}$ m; $2\frac{1}{4} = \frac{9}{4}$ m per kurta.

Number of kurtas = $\frac{63}{4} \div \frac{9}{4} = \frac{63}{4} \times \frac{4}{9} = \frac{63}{9} = 7$ kurtas

2 marks

Q5. Find three rational numbers between 3.1415 and 3.1416.

2 marks

SOLUTION: Appending an extra decimal digit makes the gap easier to see: 3.14150 to 3.14160.

Three valid choices: **3.14151, 3.14152, 3.14153** (any three distinct values strictly between 3.14150 and 3.14160 work).

2 marks

***Q6. Can you think of other way(s) to find a rational number between any two rational numbers?**

Enrichment

SOLUTION: Besides taking the average $\frac{a+b}{2}$, two other reliable methods:

Method 1 — Common denominator with room to spare: convert both numbers to the same denominator (choosing it large enough), then pick any whole number strictly between the two numerators as the new numerator. For example, between $\frac{1}{2}$ and $\frac{2}{3}$: using denominator 12, these are $\frac{6}{12}$ and $\frac{8}{12}$, so $\frac{7}{12}$ lies between them.

Method 2 — Decimal insertion: write both numbers as decimals to enough places, then insert any digit strictly between the corresponding decimal digits — this is exactly the method used in Q5 above.

2 marks

— End of Exercise Set 3.4 —

■ NCERT Solutions — Exercise Set 3.5

🔑 Approach for This Exercise

To predict terminating vs. repeating without doing long division, simplify the fraction to lowest terms and factorise only the denominator — only 2s and 5s means terminating

Q1. Without performing long division, determine which of the following rational numbers will have terminating decimals and which will be repeating: $\frac{7}{20}$, $\frac{4}{15}$ and $\frac{13}{250}$. Then check by performing the long divisions.

3 marks

SOLUTION: Predictions from prime factors of the denominator:

$$\frac{7}{20}: 20 = 2^2 \times 5 \text{ — only 2s and 5s} \rightarrow \text{terminating}$$

$$\frac{4}{15}: 15 = 3 \times 5 \text{ — has a factor of 3} \rightarrow \text{repeating}$$

$$\frac{13}{250}: 250 = 2 \times 5^3 \text{ — only 2s and 5s} \rightarrow \text{terminating}$$

VERIFICATION BY LONG DIVISION:

$$\frac{7}{20} = 0.35 \text{ (terminates)} \checkmark$$

$$\frac{4}{15} = 0.2\overline{6} = 0.2666 \dots \text{ (repeats)} \checkmark$$

$$\frac{13}{250} = 0.052 \text{ (terminates)} \checkmark$$

3 marks

Q2. Perform the long division for $\frac{1}{13}$. Identify the repeating block of digits. Does it show cyclic properties if you evaluate $\frac{2}{13}$? Now compute $\frac{3}{13}$, $\frac{4}{13}$, etc. What do you notice?

3 marks

SOLUTION: $\frac{1}{13} = 0.\overline{076923}$ — repeating block: **076923** (6 digits).

$\frac{2}{13} = 0.\overline{153846}$. Check: $076923 \times 2 = 153846$ — the block **changes** here — 153846 is not a rotation of 076923. The rotations of 076923 are 769230, 692307, 923076, 230769 and 307692 only. So 13 has

two separate cycles: $\frac{k}{13}$ for $k = 1, 3, 4, 9, 10, 12$ all use the 076923 cycle, while $k = 2, 5, 6, 7, 8, 11$ use a second cycle starting 153846.

$$\frac{3}{13} = 0.\overline{230769}: 076923 \times 3 = 230769 \checkmark$$

$$\frac{4}{13} = 0.\overline{307692}: 076923 \times 4 = 307692 \checkmark$$

Observation: just like $\frac{1}{7}$'s repeating block 142857, the block 076923 behaves cyclically, but only **partly**: its six rotations cover $k = 1, 3, 4, 9, 10, 12$. Unlike $\frac{1}{7}$, whose block 142857 is a true cyclic number covering all of $k = 1 \dots 6$, 13 splits into two cycles because its repeating block is 6 digits long, not 12.

3 marks

Q3. Classify the following numbers as rational or irrational. Find the explicit fractions in case they are rational. (i) $\sqrt{81}$ (ii) $\sqrt{12}$ (iii) $0.33333\dots$ (iv) $0.123451234512345\dots$ (v) $1.01001000100001\dots$ (vi) 23.560185612239874790120

6 parts

SOLUTION:

(i) $\sqrt{81} = 9$ — a whole number, so it's **rational**: $\frac{9}{1}$

(ii) $\sqrt{12} = 2\sqrt{3} \approx 3.464$ — 12 is not a perfect square, so this is **irrational**

(iii) $0.33333\dots = 0.\bar{3}$ — a pure repeating decimal, so **rational**:

$$x = 0.\bar{3} \Rightarrow 10x = 3.\bar{3} \Rightarrow 9x = 3 \Rightarrow x = \frac{3}{9} = \frac{1}{3}$$

(iv) $0.\overline{12345}$ — the block "12345" (5 digits) repeats, so **rational**:

$$100000x - x = 12345 \Rightarrow 99999x = 12345 \Rightarrow x = \frac{12345}{99999} = \frac{4115}{33333}$$

(v) $1.01001000100001\dots$ — the gaps of zeros between the 1s keep **growing** (1 zero, then 2, then 3, ...), so there is no fixed repeating block. This is **irrational** — it's a classic example of a number that looks patterned but never actually repeats.

(vi) 23.560185612239874790120 is written with a definite, finite number of digits (it terminates), so it is **rational** — it can be written as an integer over 10^{21} (a terminating decimal is always rational, since it terminates).

6 marks

Q4. The number $0.\bar{9}$ (which means $0.99999\dots$) is a rational number. Using algebra (let $x = 0.\bar{9}$, multiply by 10, and subtract), explain why $0.\bar{9}$ is exactly equal to 1.

2 marks

SOLUTION: Let $x = 0.\bar{9} = 0.9999\dots$

Multiply both sides by 10: $10x = 9.9999\dots$

Subtract the first equation from the second: $10x - x = 9.9999\dots - 0.9999\dots = 9$

$$9x = 9 \Rightarrow x = 1$$

Therefore $0.\bar{9} = 1$ **exactly** — not "almost 1" or "very close to 1," but precisely equal, since the algebra leaves no room for any tiny gap between them.

2 marks

***Q5. We have seen that the repeating block of $1/7$ is a cyclic number. Try to find more numbers (n) whose reciprocals ($1/n$) produce decimals with repeating blocks that are cyclic.**

Enrichment

SOLUTION: This cyclic behaviour happens for certain prime denominators where the repeating block has the maximum possible length (one less than the prime itself). The primes that do this are exactly those whose block length is $n - 1$, called **full reptend primes**. $n = 7$ qualifies (block length $6 = 7 - 1$); $n = 13$ does **not**, since its block is only 6 digits, not 12. Genuine examples include $n = 17$ (a 16-digit cyclic block) and $n = 19$ (an 18-digit cyclic block). For example, $\frac{1}{17} = 0.0588235294117647$ — multiplying this 16-digit block by 2, 3, 4, ... up to 16 simply cycles the same digits, exactly like $\frac{1}{7}$ and $\frac{1}{13}$.

3 marks

— End of Exercise Set 3.5 —

■ End-of-Chapter Exercises

Q1. Convert the following rational numbers into the form of a terminating decimal or non-terminating repeating decimal, by the process of long division: (i) $3/50$ (ii) $2/9$

2 parts

SOLUTION:

- (i) $\frac{3}{50}$: $50 = 2 \times 5^2$ — terminating. Long division gives **0.06**
- (ii) $\frac{2}{9}$: $9 = 3^2$ — repeating. Long division gives **$0.\overline{2} = 0.2222 \dots$**

2 marks

Q2. Prove that $\sqrt{5}$ is an irrational number.**4 marks****SOLUTION (PROOF BY CONTRADICTION):**

Step 1: Assume $\sqrt{5}$ is rational, so $\sqrt{5} = \frac{p}{q}$ in lowest terms (p, q co-prime, $q \neq 0$).

Step 2: Squaring: $5 = \frac{p^2}{q^2}$

Step 3: $5q^2 = p^2$

Step 4: Since p^2 is a multiple of 5, p itself must be a multiple of 5 (5 is prime). Let $p = 5k$.

Step 5: Substituting: $5q^2 = (5k)^2 = 25k^2 \Rightarrow q^2 = 5k^2$

Step 6: This means q^2 is a multiple of 5, so q must also be a multiple of 5.

Step 7 (Contradiction): Both p and q share a common factor of 5 — but we assumed $\frac{p}{q}$ was in lowest terms, sharing no common factors!

Conclusion: Our assumption was wrong. **Therefore, $\sqrt{5}$ cannot be expressed as a fraction — it is irrational.**

4 marks**Q3. Convert the following decimal numbers into the form of p/q: (i) 12.6 (ii) 0.0120 (iii) $3.\overline{052}$ (iv) $1.\overline{235}$ (v) $0.\overline{23}$ (vi) $2.\overline{05}$ (vii) $2.\overline{125}$ (viii) $3.\overline{125}$ (ix) $2.\overline{1625}$** **9 parts****SOLUTION:**

(i) $12.6 = \frac{126}{10} = \frac{63}{5}$

(ii) $0.0120 = \frac{120}{10000} = \frac{3}{250}$

(iii) $3.\overline{052} = 3.052525252 \dots$: non-repeating "0" (1 digit), repeating "52" (2 digits). $10x = 30.5252 \dots$
 $1000x = 3052.5252 \dots$; subtracting: $990x = 3022 \Rightarrow x = \frac{3022}{990} = \frac{1511}{495}$

(iv) $1.\overline{235} = 1.235353535 \dots$: non-repeating "2" (1 digit), repeating "35" (2 digits). $10x = 12.3535 \dots$
 $1000x = 1235.3535 \dots$; subtracting: $990x = 1223 \Rightarrow x = \frac{1223}{990}$

(v) $0.\overline{23} = 0.2323 \dots$; $100x = 23.2323 \dots$; $100x - x = 23 \Rightarrow 99x = 23 \Rightarrow x = \frac{23}{99}$

(vi) $2.\overline{05} = 2.05555 \dots$: non-repeating "0" (1 digit), repeating "5" (1 digit). $10x = 20.5555 \dots$
 $100x = 205.5555 \dots$; $90x = 185 \Rightarrow x = \frac{37}{18}$

(vii) $2.\overline{125} = 2.125555 \dots$: non-repeating "12" (2 digits), repeating "5" (1 digit). $100x = 212.5555 \dots$
 $1000x = 2125.5555 \dots$; $900x = 1913 \Rightarrow x = \frac{1913}{900}$

(viii) $3.\overline{125} = 3.125555 \dots$: non-repeating "12" (2 digits), repeating "5" (1 digit). $100x = 312.5555 \dots$
 $1000x = 3125.5555 \dots$; subtracting: $900x = 2813 \Rightarrow x = \frac{2813}{900}$

(ix) $2.\overline{1625} = 2.16251625 \dots$: pure repeating, 4-digit block. $10000x = 21625.1625 \dots$
 $9999x = 21623 \Rightarrow x = \frac{21623}{9999}$

9 marks

Q4. Locate the following rational numbers on the number line: (i) 0.532 (ii) $1.\overline{15}$

2 marks

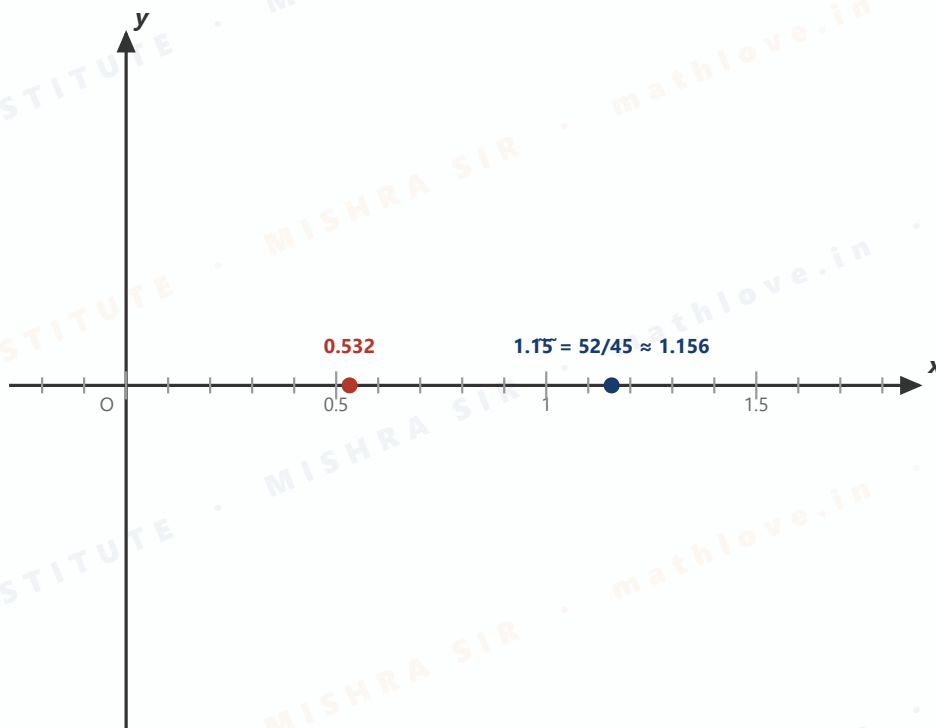
SOLUTION:

(i) $0.532 = \frac{532}{1000} = \frac{133}{250}$ — this lies between 0 and 1, just past the midpoint (roughly 53% of the way from 0 to 1).

(ii) $1.\overline{15} = 1.15555\dots$: converting, $10x = 11.555\dots$, $100x = 115.555\dots$,

$90x = 104 \Rightarrow x = \frac{104}{90} = \frac{52}{45} \approx 1.156$ — this lies between 1 and 2, close to 1.16.

2 marks



End-of-Chapter Q4: 0.532 lies just past the midpoint of 0 and 1 (between 0.5 and 0.6); $1.\overline{15} = 52/45 \approx 1.156$ lies between 1.1 and 1.2, close to 1.16. (Minor ticks = tenths.)

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

Q5. Find 6 rational numbers between 3 and 4.

2 marks

SOLUTION: Writing 3 and 4 with denominator 7: $3 = \frac{21}{7}$, $4 = \frac{28}{7}$.

Six rationals strictly between: $\frac{22}{7}$, $\frac{23}{7}$, $\frac{24}{7}$, $\frac{25}{7}$, $\frac{26}{7}$, $\frac{27}{7}$

2 marks

Q6. Find 5 rational numbers between $\frac{2}{5}$ and $\frac{3}{5}$.

2 marks

SOLUTION: Using denominator 30: $\frac{2}{5} = \frac{12}{30}$, $\frac{3}{5} = \frac{18}{30}$.

Five rationals strictly between: $\frac{13}{30}$, $\frac{14}{30}$, $\frac{15}{30}$, $\frac{16}{30}$, $\frac{17}{30}$

2 marks

Q7. Find 5 rational numbers between $\frac{1}{6}$ and $\frac{2}{5}$.

2 marks

SOLUTION: $LCM(6,5)=30$: $\frac{1}{6} = \frac{5}{30}$, $\frac{2}{5} = \frac{12}{30}$.

Five rationals strictly between: $\frac{6}{30}$, $\frac{7}{30}$, $\frac{8}{30}$, $\frac{9}{30}$, $\frac{10}{30}$ (i.e. $\frac{1}{5}$, $\frac{7}{30}$, $\frac{4}{15}$, $\frac{3}{10}$, $\frac{1}{3}$)

2 marks

Q8. If $\frac{x}{3} + \frac{x}{5} = \frac{16}{15}$, find the rational number x .

2 marks

SOLUTION: Common denominator 15: $\frac{5x}{15} + \frac{3x}{15} = \frac{16}{15}$

$$\frac{8x}{15} = \frac{16}{15} \Rightarrow 8x = 16 \Rightarrow x = 2$$

2 marks

Q9. Let a and b be two non-zero rational numbers such that $a + \frac{1}{b} = 0$. Without assigning any numerical values, determine whether ab is positive or negative. Justify your answer.

2 marks

SOLUTION: From $a + \frac{1}{b} = 0$, we get $a = -\frac{1}{b}$.

$$\text{So } ab = \left(-\frac{1}{b}\right) \times b = -1$$

Since -1 is negative, ab is always negative, regardless of the specific values of a and b .

2 marks

Q10. A rational number has a terminating decimal expansion whose last non-zero digit occurs in the 4th decimal place. Show that such a number can be written in the form $p/10^4$, where p is an integer not divisible by 10. Is it necessary that the denominator, in lowest form, is divisible by 2^4 or 5^4 ? Give reasons.

3 marks

SOLUTION: If the decimal terminates exactly at the 4th place (with a non-zero digit there), then the whole number can be written as an integer p (the digits with the decimal point removed) divided by 10^4 : $x = \frac{p}{10^4}$.

p cannot be divisible by 10 — if it were, $\frac{p}{10^4}$ would simplify to $\frac{p/10}{10^3}$, meaning the decimal would actually terminate by the 3rd place, contradicting that the last non-zero digit is at the 4th place.

Is the lowest-form denominator necessarily divisible by 2^4 or 5^4 ? No, not necessarily. Since $10^4 = 2^4 \times 5^4$, the lowest-form denominator will only ever contain the primes 2 and/or 5 — but if p itself contains factors of 2 or 5, the fraction reduces further and the denominator may end up with a smaller power of 2 or 5 than 4 (or even none of one of them). For example, $\frac{625}{10^4} = \frac{1}{16}$ — here the denominator in lowest form is 2^4 only, with no factor of 5 at all.

3 marks

Q11. Without performing division, determine whether the decimal expansion of $18/125$ is terminating or non-terminating. If it terminates, state the number of decimal places.

2 marks

SOLUTION: $125 = 5^3$ — only the prime 5 is present, so the decimal **terminates**.

To reach a power of 10, multiply numerator and denominator by $2^3 = 8$:

$$\frac{18}{125} = \frac{18 \times 8}{1000} = \frac{144}{1000} = 0.144$$

It terminates in 3 decimal places.

2 marks

Q12. A rational number in its lowest form has denominator $2^3 \times 5$. How many decimal places will its decimal expansion have? Explain your answer.

2 marks

SOLUTION: Denominator $= 2^3 \times 5^1$. To convert to a power of 10, we need $\max(3, 1) = 3$ — multiply by 5^2 to balance the powers: $2^3 \times 5^1 \times 5^2 = 2^3 \times 5^3 = 10^3$.

The decimal expansion terminates in 3 decimal places.

2 marks

***Q13.** Let $a = 7/12$ and $b = 5/6$. Express both a and b in the form k_1/m and k_2/m where k_1, k_2, m are integers and $k_2 - k_1 > 6$. Using the same denominator m , write exactly five distinct rational numbers lying between a and b keeping an integer numerator. Explain why the condition $k_2 - k_1 > n + 1$ is necessary to find n such rational numbers.

Enrichment

SOLUTION: Using $m = 120$ (a multiple of the LCM 12): $a = \frac{7}{12} = \frac{70}{120}$, $b = \frac{5}{6} = \frac{100}{120}$.

So $k_1 = 70$, $k_2 = 100$, $m = 120$, and $k_2 - k_1 = 30 > 6$ ✓

Five distinct rationals between a and b (with denominator 120): $\frac{75}{120}, \frac{80}{120}, \frac{85}{120}, \frac{90}{120}, \frac{95}{120}$

Why $k_2 - k_1 > n + 1$ is necessary: to find n distinct integer numerators strictly between k_1 and k_2 , we need the count of whole numbers in the open interval (k_1, k_2) — which is exactly $k_2 - k_1 - 1$ — to be at least n . This requires $k_2 - k_1 - 1 \geq n$, i.e. $k_2 - k_1 \geq n + 1$. Without a large enough gap between k_1 and k_2 , there simply wouldn't be enough integers available to pick from.

4 marks

***Q14.** Three rational numbers x, y, z satisfy $x + y + z = 0$ and $xy + yz + zx = 0$. Show that all the rational numbers x, y, z must be simultaneously zero.

Enrichment

SOLUTION: Square the first equation: $(x + y + z)^2 = 0^2 = 0$

Expanding: $x^2 + y^2 + z^2 + 2(xy + yz + zx) = 0$

Using the second given equation, $xy + yz + zx = 0$:

$$x^2 + y^2 + z^2 + 2(0) = 0 \Rightarrow x^2 + y^2 + z^2 = 0$$

Since x, y, z are rational (hence real) numbers, each of x^2, y^2, z^2 is **non-negative**. A sum of non-negative terms can only equal 0 if **every** term equals 0.

$$\text{So } x^2 = 0, y^2 = 0, z^2 = 0 \Rightarrow x = y = z = 0$$

4 marks

***Q15.** Show that the rational number $(a + b)/2$ lies between the rational numbers a and b .

Enrichment

SOLUTION: Without loss of generality, assume $a < b$.

$$\frac{a+b}{2} - a = \frac{a+b-2a}{2} = \frac{b-a}{2}. \text{ Since } b > a, \text{ this is positive, so } \frac{a+b}{2} > a.$$

$$b - \frac{a+b}{2} = \frac{2b-a-b}{2} = \frac{b-a}{2}. \text{ This is also positive, so } b > \frac{a+b}{2}.$$

$$\text{Combining both: } a < \frac{a+b}{2} < b$$

Hence the average $\frac{a+b}{2}$ always lies strictly between a and b (when $a \neq b$).

3 marks

Q16. Find the lengths of the hypotenuses of all the right triangles in Fig. 3.14, which is referred to as the square root spiral.

3 marks

SOLUTION: Each triangle in the spiral has one leg of length 1, and its other leg is the hypotenuse of the previous triangle. By the Baudhāyana–Pythagoras Theorem, each new hypotenuse is 1 unit longer (in squared terms) than the last:

1st triangle: legs 1 and 1 $\rightarrow$ hypotenuse $= \sqrt{1^2 + 1^2} = \sqrt{2}$

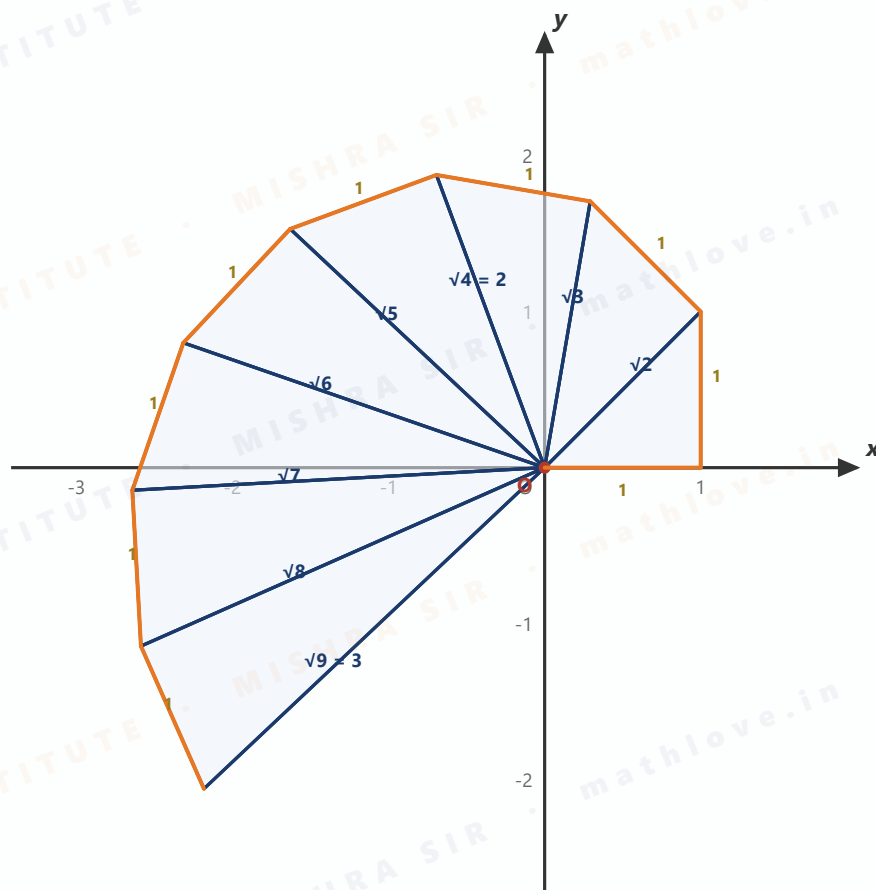
2nd triangle: legs $\sqrt{2}$ and 1 $\rightarrow$ hypotenuse $= \sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3}$

3rd triangle: legs $\sqrt{3}$ and 1 $\rightarrow$ hypotenuse $= \sqrt{4} = 2$

4th triangle: hypotenuse $= \sqrt{5}$; 5th: $\sqrt{6}$; 6th: $\sqrt{7}$; 7th: $\sqrt{8} = 2\sqrt{2}$; 8th: $\sqrt{9} = 3$; 9th: $\sqrt{10}$

General pattern: the hypotenuse of the n -th triangle is always $\sqrt{n+1}$. So the sequence of hypotenuses is $\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{9}, \sqrt{10}, \dots$ — continuing for as many triangles as are drawn in your own copy of Fig. 3.14.

3 marks



Q16 — the square root spiral (drawn to scale): every outer edge (gold) has length 1, and each successive hypotenuse from O measures $\sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}, \sqrt{7}, 2\sqrt{2}, 3, \sqrt{10}, \dots$ — the n -th is always $\sqrt{n+1}$.

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

— End of Chapter 3 NCERT Solutions —

Additional Practice Questions

A Integers & Brahmagupta's Rules (5 Questions)

A1. Calculate $(-9) + (-16)$ using Brahmagupta's "debt plus debt" rule.

1 mark

SOLUTION: Debt plus debt is a debt: $(-9) + (-16) = -25$

1 mark

A2. Calculate $(-15) \times 6$ using Brahmagupta's "debt times fortune" rule.

1 mark

SOLUTION: Debt $\times$ fortune = debt: $(-15) \times 6 = -90$

1 mark

A3. Calculate $(-7) \times (-9)$ using Brahmagupta's "debt times debt is a fortune" rule.

1 mark

SOLUTION: Debt $\times$ debt = fortune: $(-7) \times (-9) = 63$

1 mark

A4. A trader owes ₹340 (debt) and separately earns ₹550 (fortune). What is his net position?

2 marks

SOLUTION: $-340 + 550 = +210$ — a net fortune of ₹210

2 marks

A5. Simplify $8 - (-11)$ and explain using the "removing a debt" logic.

2 marks

SOLUTION: $8 - (-11) = 8 + 11 = 19$. Removing an ₹11 debt has the same effect as gaining ₹11 — that's why subtracting a negative equals adding its positive value.

2 marks

B Rational Number Operations (5 Questions)

B1. Add: $3/8 + 5/12$

2 marks

SOLUTION: $LCM(8,12)=24$: $\frac{9}{24} + \frac{10}{24} = \frac{19}{24}$

2 marks

B2. Subtract: $7/10 - 2/15$

2 marks

SOLUTION: $LCM(10,15)=30$: $\frac{21}{30} - \frac{4}{30} = \frac{17}{30}$

2 marks

B3. Multiply: $-5/6 \times 9/10$

2 marks

SOLUTION: $\frac{-5 \times 9}{6 \times 10} = \frac{-45}{60} = -\frac{3}{4}$

2 marks

B4. Divide: $4/9 \div (-8/27)$

2 marks

SOLUTION: $\frac{4}{9} \times \left(-\frac{27}{8}\right) = -\frac{108}{72} = -\frac{3}{2}$

2 marks

B5. Simplify using the distributive property: $3/5 \times (7/9 + 2/3)$

2 marks

SOLUTION: $\frac{7}{9} + \frac{2}{3} = \frac{7}{9} + \frac{6}{9} = \frac{13}{9}$; then $\frac{3}{5} \times \frac{13}{9} = \frac{39}{45} = \frac{13}{15}$

2 marks

C Irrationality & Number Classification (5 Questions)

C1. Classify $\sqrt{49}$ as rational or irrational.

1 mark

SOLUTION: $\sqrt{49} = 7$, a whole number — **rational**

1 mark

C2. Classify $\sqrt{50}$ as rational or irrational.

1 mark

SOLUTION: $\sqrt{50} = 5\sqrt{2} \approx 7.07$; 50 is not a perfect square — **irrational**

1 mark

C3. Is 0.1010010001... (with an increasing number of zeros each time) rational or irrational? Justify.

2 marks

SOLUTION: Irrational — there is no fixed block of digits that repeats; the gaps of zeros keep growing (1, then 2, then 3, ...), so the decimal never settles into a repeating cycle.

2 marks

C4. Prove $\sqrt{3}$ is irrational using proof by contradiction (brief outline).

3 marks

SOLUTION: Assume $\sqrt{3} = \frac{p}{q}$ in lowest terms. Squaring and rearranging: $p^2 = 3q^2$, so p^2 (and hence p) is divisible by 3. Let $p = 3k$; substituting gives $q^2 = 3k^2$, so q is also divisible by 3. This contradicts p, q being co-prime. **Hence $\sqrt{3}$ is irrational.**

3 marks

C5. Is the sum of a rational number and an irrational number always irrational?

2 marks

Give an example.

SOLUTION: Yes. Example: $2 + \sqrt{3}$ is irrational — if it were rational, say r , then $\sqrt{3} = r - 2$ would also be rational (difference of two rationals), contradicting the known irrationality of $\sqrt{3}$.

2 marks

D Decimal Conversions & Density (5 Questions)

D1. Without long division, state whether $7/16$ terminates.

1 mark

SOLUTION: $16 = 2^4$ — only the prime 2 → **terminates**

1 mark

D2. Without long division, state whether $5/21$ terminates.

1 mark

SOLUTION: $21 = 3 \times 7$ — has primes other than 2 and 5 → **repeats (non-terminating)**

1 mark

D3. Convert $0.\overline{7}$ to p/q form.

2 marks

SOLUTION: $x = 0.\overline{7} \Rightarrow 10x = 7.\overline{7} \Rightarrow 9x = 7 \Rightarrow x = \frac{7}{9}$

2 marks

D4. Find a rational number between $1/3$ and $1/2$.

2 marks

SOLUTION: Average: $\frac{\frac{1}{3} + \frac{1}{2}}{2} = \frac{\frac{2}{6} + \frac{3}{6}}{2} = \frac{\frac{5}{6}}{2} = \frac{5}{12}$

2 marks

D5. Convert 0.45 (terminating) into p/q form.

1 mark

SOLUTION: $0.45 = \frac{45}{100} = \frac{9}{20}$

1 mark

— End of Additional Practice Questions —

CHAPTER 4

Exploring Algebraic Identities

Exercise Sets 4.1 – 4.5 + End-of-Chapter

EXERCISE SETS 4.1 – 4.5 + END-OF-CHAPTER

 **Weightage (School Annual Exam, Per Official CBSE Curriculum)**

Marks Distribution (Out of 80 — Annual Theory Paper)

✓ Verified directly against the official CBSE curriculum document (cbseacademic.nic.in, Mathematics Class IX, 2026–27) — not a secondary source. Like Chapter 2 (Introduction to Linear Polynomials), this chapter sits inside **Unit II: Algebra**, which is shared across **four** chapters — Introduction to Polynomials, Sequences and Progressions, Exploring Algebraic Identities (this chapter), and Linear Equations in Two Variables. The 20 marks of the unit are distributed across all four; there is no separate official mark-split for this chapter alone.

Detail	Value
Unit	Algebra (Unit II)
Chapters sharing this unit	Introduction to Polynomials · Sequences and Progressions · Exploring Algebraic Identities (this chapter) · Linear Equations in Two Variables
Marks allotted to the whole unit	20 marks out of 80 — the second-highest-weightage unit after Geometry (25 marks)
Teaching periods for the whole unit	66 periods (out of ~200 total for the year) — the largest number of periods of any unit
Internal Assessment (separate, 20 marks)	Pen-Paper Test & Multiple Assessment (10), Portfolio (5), Lab Practical (5)

✦ This is one of the most calculation-dense chapters in the entire Class 9 syllabus — mastering the identities here directly speeds up numerical work in every other chapter, and the "splitting the middle term" and rational-expression simplification skills reappear constantly in Linear Equations and beyond.

Predicted Question Types

Format	Marks	Likely Topic
MCQ / Very Short Answer	1	Identifying which identity applies; stating the correct expansion of a simple binomial or trinomial square
Short Answer	2	Expanding a binomial/trinomial square; using an identity to compute a square or product quickly (e.g. 998^2 , 104×96)
Short/Long Answer	2–3	Factorising a quadratic trinomial by splitting the middle term; factorising using sum/difference of cubes; simplifying a rational algebraic expression
Application/Reasoning	2–4	Cube/three-variable identities ($x^3 + y^3 + z^3 - 3xyz$); geometric area/volume word problems solved by factorisation; "show that" proof-style questions using identities

All Key Formulas & Definitions

Algebraic identity: an equation that is true for *all* values of the variables in it (unlike an ordinary equation, which is only true for specific values).

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$

$$(x + y)(x - y) = x^2 - y^2$$

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

$$(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$ — and if $x + y + z = 0$, this simplifies to $x^3 + y^3 + z^3 = 3xyz$.

Splitting the middle term (to factor $x^2 + (a + b)x + ab$ back into $(x + a)(x + b)$): find two numbers whose sum is the middle coefficient and whose product is the constant term.

Simplifying rational expressions: factorise both numerator and denominator completely using identities, then cancel any common factor — provided that factor is not equal to zero.

💡 Shortcuts & Study Tips

- Squaring numbers near a round number fast:** write the number as (round number $\pm$ small difference), e.g. $998 = 1000 - 2$, then apply $(a - b)^2 = a^2 - 2ab + b^2$ — far faster than direct multiplication.
- Products of two numbers equidistant from a midpoint:** use $(a + b)(a - b) = a^2 - b^2$ — e.g. $23 \times 17 = (20 + 3)(20 - 3) = 400 - 9 = 391$.
- Splitting the middle term — always multiply first and last coefficients:** for $px^2 + qx + r$, find two numbers that multiply to $p \times r$ and add to q — this works even when $p \neq 1$.
- Common factor first, always:** before trying to match a trinomial to $a^2 \pm 2ab + b^2$, check for a common numerical factor across all terms (as in Example 7 of the chapter) — it often turns an awkward expression into a clean perfect square.
- Recognising a three-term perfect square with negative signs:** match $A^2 + B^2 + C^2$ parts first, then use the cross terms' signs to figure out whether each of A, B, C should be positive or negative in $(A + B + C)^2$.
- Cube identities — check the pattern of signs first:** $(x - y)^3$ alternates $+ - + -$; if a four-term cubic expression doesn't alternate signs like this, it isn't a simple binomial cube.
- The $x^3 + y^3 + z^3 - 3xyz$ shortcut:** whenever you see $x + y + z = 0$ given in a problem, immediately recall that $x^3 + y^3 + z^3 = 3xyz$ — this shortcut appears often in "find the value" problems.
- Simplifying rational expressions — factor completely before cancelling:** a very common error is cancelling individual terms instead of whole factors; only cancel entire bracketed factors that appear in both the numerator and denominator.

 Approach for This Exercise

Every part of this exercise runs on a single identity: $(a + b)^2 = a^2 + 2ab + b^2$. The whole skill lies in deciding *what plays the role of a* and *what plays the role of b* , then substituting carefully. Two traps: (1) when a term is a fraction or decimal, the middle term $2ab$ is where most marks are lost — always write it out in full *before* simplifying; (2) in Q2 the identity is used as a mental-maths shortcut, so the number must first be split into a convenient sum such as $60 + 4$.

Q1. Using the identity $(a + b)^2 = a^2 + 2ab + b^2$, expand the following: (i) $(7x + 4y)^2$ (ii)

(iii) $(2.5p + 1.5q)^2$ (iv) $\left(\frac{3}{4}s + 8t\right)^2$ (v) $\left(x + \frac{1}{2y}\right)^2$ (vi) $\left(\frac{1}{x} + \frac{1}{y}\right)^2$

6 parts

SOLUTION:

(i) $(7x + 4y)^2$ — here $a = 7x$ and $b = 4y$.

Square of the first: $a^2 = (7x)^2 = 49x^2$

Twice the product: $2ab = 2 \times 7x \times 4y = 56xy$

Square of the second: $b^2 = (4y)^2 = 16y^2$

Therefore $(7x + 4y)^2 = 49x^2 + 56xy + 16y^2$

1 mark

(ii) $\left(\frac{7}{5}x + \frac{3}{2}y\right)^2$ — here $a = \frac{7}{5}x$ and $b = \frac{3}{2}y$.

$a^2 = \left(\frac{7}{5}\right)^2 x^2 = \frac{49}{25}x^2$

$2ab = 2 \times \frac{7}{5} \times \frac{3}{2} \times xy = \frac{42}{10}xy = \frac{21}{5}xy$ (the 2 cancels with the denominator 2)

$b^2 = \left(\frac{3}{2}\right)^2 y^2 = \frac{9}{4}y^2$

Therefore the expansion is $\frac{49}{25}x^2 + \frac{21}{5}xy + \frac{9}{4}y^2$

1 mark

(iii) $(2.5p + 1.5q)^2$ — here $a = 2.5p$ and $b = 1.5q$.

$a^2 = (2.5)^2 p^2 = 6.25p^2$

$2ab = 2 \times 2.5 \times 1.5 \times pq = 7.5pq$

$b^2 = (1.5)^2 q^2 = 2.25q^2$

Therefore the expansion is $6.25p^2 + 7.5pq + 2.25q^2$

1 mark

(iv) $\left(\frac{3}{4}s + 8t\right)^2$ — here $a = \frac{3}{4}s$ and $b = 8t$.

$a^2 = \frac{9}{16}s^2$

$2ab = 2 \times \frac{3}{4} \times 8 \times st = \frac{48}{4}st = 12st$

$b^2 = (8t)^2 = 64t^2$

Therefore the expansion is $\frac{9}{16}s^2 + 12st + 64t^2$

1 mark

(v) $\left(x + \frac{1}{2y}\right)^2$ — here $a = x$ and $b = \frac{1}{2y}$.

$a^2 = x^2$

$2ab = 2 \times x \times \frac{1}{2y} = \frac{2x}{2y} = \frac{x}{y}$

$b^2 = \left(\frac{1}{2y}\right)^2 = \frac{1}{4y^2}$

Therefore the expansion is $x^2 + \frac{x}{y} + \frac{1}{4y^2}$

1 mark

(vi) $\left(\frac{1}{x} + \frac{1}{y}\right)^2$ — here $a = \frac{1}{x}$ and $b = \frac{1}{y}$.

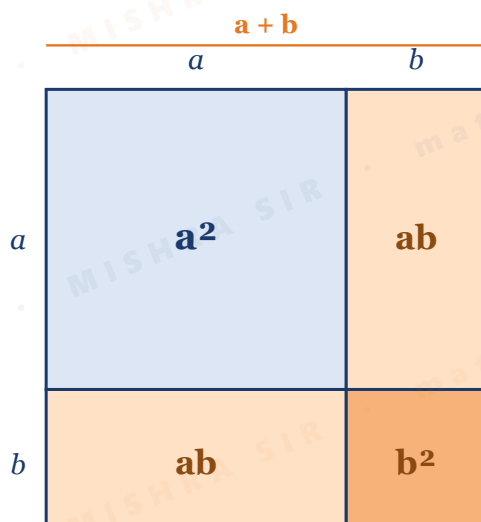
$$a^2 = \frac{1}{x^2}$$

$$2ab = 2 \times \frac{1}{x} \times \frac{1}{y} = \frac{2}{xy}$$

$$b^2 = \frac{1}{y^2}$$

Therefore the expansion is $\frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2}$ **1 mark**

✳ Examiner's tip. In parts (ii), (iv) and (v) the middle term is the only place where a fraction must be simplified. Write $2ab$ out in full before cancelling — a written middle term usually earns the mark even if the final simplification slips.



Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

Why $(a+b)^2 = a^2 + 2ab + b^2$: a square of side $(a+b)$ splits into one a^2 square, one b^2 square, and two ab rectangles — the identity is literally a picture of areas.

Q2. Using the same identity, find the values of the following: (i) $(64)^2$ (ii) $(105)^2$ (iii) $(205)^2$

3 parts

SOLUTION:

The idea here is the one shown in Example 4 of the textbook: split the number into a round number plus a small number, so that all three pieces of the identity become easy to compute mentally.

(i) $(64)^2$ — write $64 = 60 + 4$, so $a = 60$, $b = 4$.

$$\begin{aligned}(60 + 4)^2 &= 60^2 + 2 \times 60 \times 4 + 4^2 \\ &= 3600 + 480 + 16\end{aligned}$$

Therefore $(64)^2 = 4096$

1 mark

(ii) $(105)^2$ — write $105 = 100 + 5$, so $a = 100$, $b = 5$.

$$\begin{aligned}(100 + 5)^2 &= 100^2 + 2 \times 100 \times 5 + 5^2 \\ &= 10000 + 1000 + 25\end{aligned}$$

Therefore $(105)^2 = 11025$

1 mark

(iii) $(205)^2$ — write $205 = 200 + 5$, so $a = 200$, $b = 5$.

$$\begin{aligned}(200 + 5)^2 &= 200^2 + 2 \times 200 \times 5 + 5^2 \\ &= 40000 + 2000 + 25\end{aligned}$$

Therefore $(205)^2 = 42025$

1 mark

✦ **Why the split matters.** For 64 you could also write $64 = 70 - 6$, but that needs the $(a - b)^2$ identity. Always split so that a is a multiple of 10 or 100 — then a^2 and $2ab$ are both one-step calculations.

— End of Exercise Set 4.1 —

NCERT Solutions — Exercise Set 4.2

Approach for This Exercise

This set runs the identity **backwards**. In Set 4.1 we expanded; here we are given the expanded form and must recognise the pattern $a^2 \pm 2ab + b^2$ and write it back as a square. The method is always the same: take the square root of the first term to get a , the square root of the last term to get b , then **check that the middle term really equals** $2ab$ — that check is what turns a guess into a proof. For the starred (enrichment) parts, the expression is not a perfect square as it stands: take out a common factor first, as in Example 7 of the chapter, and the pattern appears.

Q1. Factor completely: (i) $9x^2 + 24xy + 16y^2$ (ii) $4s^2 + 20st + 25t^2$ (iii) $49x^2 + 28xy + 4y^2$ (iv)

$64p^2 + \frac{32}{3}pq + \frac{4}{9}q^2$ *(v) $3a^2 + 4ab + \frac{4}{3}b^2$ *(vi) $\frac{9}{5}s^2 + 6sv + 5v^2$

6 parts

SOLUTION:

(i) $9x^2 + 24xy + 16y^2$

First term: $9x^2 = (3x)^2$, so $a = 3x$. Last term: $16y^2 = (4y)^2$, so $b = 4y$.

Check the middle: $2ab = 2(3x)(4y) = 24xy$ ✓ — it matches.

$9x^2 + 24xy + 16y^2 = (3x)^2 + 2(3x)(4y) + (4y)^2 = (3x + 4y)^2$

1 mark

(ii) $4s^2 + 20st + 25t^2$

$4s^2 = (2s)^2 \Rightarrow a = 2s$; $25t^2 = (5t)^2 \Rightarrow b = 5t$.

Check: $2ab = 2(2s)(5t) = 20st$ ✓

$= (2s + 5t)^2$

1 mark

(iii) $49x^2 + 28xy + 4y^2$

$49x^2 = (7x)^2 \Rightarrow a = 7x$; $4y^2 = (2y)^2 \Rightarrow b = 2y$.

Check: $2ab = 2(7x)(2y) = 28xy$ ✓

$= (7x + 2y)^2$

1 mark

(iv) $64p^2 + \frac{32}{3}pq + \frac{4}{9}q^2$

$64p^2 = (8p)^2 \Rightarrow a = 8p$; $\frac{4}{9}q^2 = \left(\frac{2}{3}q\right)^2 \Rightarrow b = \frac{2}{3}q$.

Check: $2ab = 2(8p)\left(\frac{2}{3}q\right) = \frac{32}{3}pq$ ✓

$= \left(8p + \frac{2}{3}q\right)^2$

1 mark

*(v) $3a^2 + 4ab + \frac{4}{3}b^2$ (Enrichment — not exam-required)

Here $3a^2$ is not a perfect square, so the pattern does not fit directly. Notice that every coefficient is a

multiple of $\frac{1}{3}$: $3 = \frac{1}{3} \times 9$, $4 = \frac{1}{3} \times 12$, $\frac{4}{3} = \frac{1}{3} \times 4$.

Take out $\frac{1}{3}$: $3a^2 + 4ab + \frac{4}{3}b^2 = \frac{1}{3}(9a^2 + 12ab + 4b^2)$

Now the bracket is a perfect square: $9a^2 = (3a)^2$, $4b^2 = (2b)^2$, and $2(3a)(2b) = 12ab$ ✓

$= \frac{1}{3}[(3a)^2 + 2(3a)(2b) + (2b)^2] = \frac{1}{3}(3a + 2b)^2$

1 mark

*(vi) $\frac{9}{5}s^2 + 6sv + 5v^2$ (Enrichment — not exam-required)

Again take out the common factor $\frac{1}{5}$: $\frac{9}{5} = \frac{1}{5} \times 9$, $6 = \frac{1}{5} \times 30$, $5 = \frac{1}{5} \times 25$.

$= \frac{1}{5}(9s^2 + 30sv + 25v^2)$

Check the bracket: $9s^2 = (3s)^2$, $25v^2 = (5v)^2$, $2(3s)(5v) = 30sv$ ✓

$$= \frac{1}{5} [(3s)^2 + 2(3s)(5v) + (5v)^2] = \frac{1}{5} (3s + 5v)^2$$

1 mark

✳ **Never skip the middle-term check.** $9x^2 + 25y^2 + 16xy$ also has two perfect squares at the ends, but $2(3x)(5y) = 30xy \neq 16xy$, so it is not a perfect square. The check is what distinguishes the two cases.

Q2. Find the values using $(a - b)^2 = a^2 - 2ab + b^2$: (i) 79^2 (ii) 193^2 (iii) 299^2

3 parts

SOLUTION:

Each number sits just below a round number, so we write it as (round number – small number) and use the subtraction form of the identity. Watch the sign of the middle term: it is **negative**.

(i) 79^2 — write $79 = 80 - 1$, so $a = 80$, $b = 1$.

$$(80 - 1)^2 = 80^2 - 2 \times 80 \times 1 + 1^2$$

$$= 6400 - 160 + 1$$

$$= \mathbf{6241}$$

1 mark

(ii) 193^2 — write $193 = 200 - 7$, so $a = 200$, $b = 7$.

$$(200 - 7)^2 = 200^2 - 2 \times 200 \times 7 + 7^2$$

$$= 40000 - 2800 + 49$$

$$= \mathbf{37249}$$

1 mark

(iii) 299^2 — write $299 = 300 - 1$, so $a = 300$, $b = 1$.

$$(300 - 1)^2 = 300^2 - 2 \times 300 \times 1 + 1^2$$

$$= 90000 - 600 + 1$$

$$= \mathbf{89401}$$

1 mark

✳ **Choosing the split.** Compare with Set 4.1 Q2, where numbers sat just above a round figure and $(a + b)^2$ was used. The rule: pick whichever round number is nearest, then let the sign of the difference decide which identity to apply.

— End of Exercise Set 4.2 —

🔑 **Approach for This Exercise**

Q1 is about **choosing** the more convenient identity for each square — the nearest round number decides whether you use $(a + b)^2$ or $(a - b)^2$. Q2 and Q3 work the three-term identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ in both directions: factoring in Q2, expanding in Q3. Q4 is different in kind — it asks you to **test** whether a statement is an identity. Do not assume the answer is yes; expand fully and compare, and if the two sides differ, a single numerical counter-example settles it.

Q1. Find the following squares using one of the above identities. Determine which identity makes the calculation easier: (i) 117^2 (ii) 78^2 (iii) 198^2 (iv) 214^2 (v) 1104^2 (vi) 1120^2

6 parts

SOLUTION:

For each number, pick the nearest round value. If the number is above it, use $(a + b)^2$; if below, use $(a - b)^2$.

(i) 117^2 — nearest round value 100, and 117 is above it, so use $(a + b)^2$ with $a = 100$, $b = 17$.

$$= 100^2 + 2(100)(17) + 17^2 = 10000 + 3400 + 289 = \mathbf{13689}$$

1 mark

(ii) 78^2 — nearest round value 80, and 78 is below, so use $(a - b)^2$ with $a = 80$, $b = 2$.

$$= 80^2 - 2(80)(2) + 2^2 = 6400 - 320 + 4 = \mathbf{6084}$$

1 mark

(iii) 198^2 — below 200, so $(a - b)^2$ with $a = 200$, $b = 2$.

$$= 40000 - 800 + 4 = \mathbf{39204}$$

1 mark

(iv) 214^2 — above 200, so $(a + b)^2$ with $a = 200$, $b = 14$.

$$= 40000 + 5600 + 196 = \mathbf{45796}$$

1 mark

(v) 1104^2 — above 1100, so $(a + b)^2$ with $a = 1100$, $b = 4$.

$$= 1210000 + 8800 + 16 = \mathbf{1218816}$$

1 mark

(vi) 1120^2 — above 1100, so $(a + b)^2$ with $a = 1100$, $b = 20$.

$$= 1210000 + 44000 + 400 = \mathbf{1254400}$$

1 mark

 **Which identity is "easier"?** The one whose b is smallest, because b^2 and $2ab$ then stay simple.

For (vi) you could also take $a = 1000$, $b = 120$, but $120^2 = 14400$ is far more work than $20^2 = 400$.

Q2. Factor using suitable identities: (i) $16y^2 - 24y + 9$ (ii) $\frac{9}{4}s^2 + 6st + 4t^2$ (iii)

$\frac{m^2}{9} + \frac{mk}{3} + \frac{k^2}{4} + 3nk + 2mn + 9n^2$ (iv) $\frac{p^2}{16} - 2 + \frac{16}{p^2}$ (v) $9a^2 + 4b^2 + c^2 - 12ab + 6ac - 4bc$

5 parts

SOLUTION:

(i) $16y^2 - 24y + 9$

$16y^2 = (4y)^2 \Rightarrow a = 4y$; $9 = 3^2 \Rightarrow b = 3$. The middle term is negative, so we expect $(a - b)^2$.

Check: $2ab = 2(4y)(3) = 24y$ ✓

$= (4y)^2 - 2(4y)(3) + 3^2 = (4y - 3)^2$

1 mark

(ii) $\frac{9}{4}s^2 + 6st + 4t^2$

$\frac{9}{4}s^2 = \left(\frac{3}{2}s\right)^2 \Rightarrow a = \frac{3}{2}s$; $4t^2 = (2t)^2 \Rightarrow b = 2t$.

Check: $2ab = 2\left(\frac{3}{2}s\right)(2t) = 6st$ ✓

$= \left(\frac{3}{2}s + 2t\right)^2$

1 mark

(iii) $\frac{m^2}{9} + \frac{mk}{3} + \frac{k^2}{4} + 3nk + 2mn + 9n^2$

Six terms suggests the three-variable identity $(A + B + C)^2 = A^2 + B^2 + C^2 + 2AB + 2BC + 2CA$.

Read off the three squares: $\frac{m^2}{9} = \left(\frac{m}{3}\right)^2$, $\frac{k^2}{4} = \left(\frac{k}{2}\right)^2$, $9n^2 = (3n)^2$.

So try $A = \frac{m}{3}$, $B = \frac{k}{2}$, $C = 3n$, and verify all three cross terms:

$2AB = 2\left(\frac{m}{3}\right)\left(\frac{k}{2}\right) = \frac{mk}{3}$ ✓

$2BC = 2\left(\frac{k}{2}\right)(3n) = 3kn$ ✓

$2CA = 2(3n)\left(\frac{m}{3}\right) = 2mn$ ✓

All six terms are accounted for, so the expression is $\left(\frac{m}{3} + \frac{k}{2} + 3n\right)^2$

1 mark

(iv) $\frac{p^2}{16} - 2 + \frac{16}{p^2}$

$\frac{p^2}{16} = \left(\frac{p}{4}\right)^2 \Rightarrow a = \frac{p}{4}$; $\frac{16}{p^2} = \left(\frac{4}{p}\right)^2 \Rightarrow b = \frac{4}{p}$.

Check the middle: $2ab = 2\left(\frac{p}{4}\right)\left(\frac{4}{p}\right) = 2$ — and the term present is -2 , so the sign is negative.

$= \left(\frac{p}{4} - \frac{4}{p}\right)^2$

1 mark

(v) $9a^2 + 4b^2 + c^2 - 12ab + 6ac - 4bc$

Squares: $9a^2 = (3a)^2$, $4b^2 = (\pm 2b)^2$, $c^2 = (\pm c)^2$. The signs of the cross terms tell us which to take negative.

Try $A = 3a$, $B = -2b$, $C = c$:

$2AB = 2(3a)(-2b) = -12ab$ ✓

$$2BC = 2(-2b)(c) = -4bc \checkmark$$

$$2CA = 2(c)(3a) = 6ac \checkmark$$

All three match, so the expression is $(3a - 2b + c)^2$ **1 mark**

✦ **Method for three-term squares.** Match the three squares first to find A, B, C up to sign, then use the signs of the cross terms to decide which one carries the minus. In (v), two cross terms are negative and one positive — that pattern means exactly one of the three is negative.

Q3. Expand using $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$: **(i)** $(p + 3q + 7r)^2$ **(ii)**

$$(3x - 2y + 4z)^2$$

2 parts

SOLUTION:

(i) $(p + 3q + 7r)^2$ — take $a = p, b = 3q, c = 7r$.

$$\text{Squares: } a^2 = p^2; b^2 = (3q)^2 = 9q^2; c^2 = (7r)^2 = 49r^2$$

$$\text{Cross terms: } 2ab = 2(p)(3q) = 6pq; 2bc = 2(3q)(7r) = 42qr; 2ca = 2(7r)(p) = 14pr$$

$$= p^2 + 9q^2 + 49r^2 + 6pq + 42qr + 14pr$$
 1 mark

(ii) $(3x - 2y + 4z)^2$ — take $a = 3x, b = -2y, c = 4z$, keeping the minus with b .

$$\text{Squares: } a^2 = 9x^2; b^2 = (-2y)^2 = 4y^2 \text{ (positive); } c^2 = 16z^2$$

$$\text{Cross terms: } 2ab = 2(3x)(-2y) = -12xy; 2bc = 2(-2y)(4z) = -16yz; 2ca = 2(4z)(3x) = 24xz$$

$$= 9x^2 + 4y^2 + 16z^2 - 12xy - 16yz + 24xz$$
 1 mark

✦ **The squared term is always positive**, even when the original term is negative: $(-2y)^2 = +4y^2$.
Only the cross terms can carry a minus sign.

Q4. Is this an identity? $(a + b - c)^2 + (a - b + c)^2 + (a - b - c)^2 = 2a^2 + 2b^2 + 2c^2$

3 marks

SOLUTION:

An **identity** must hold for every value of the variables. To test the claim, expand the left-hand side fully and compare with the right.

Expand each square with $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$, keeping the signs with the terms:

$$(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$$

$$(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$$

$$(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$$

1 mark

Now add all three, collecting like terms:

Square terms: $3a^2 + 3b^2 + 3c^2$

ab terms: $2ab - 2ab - 2ab = -2ab$

bc terms: $-2bc - 2bc + 2bc = -2bc$

ca terms: $-2ca + 2ca - 2ca = -2ca$

So LHS = $3a^2 + 3b^2 + 3c^2 - 2ab - 2bc - 2ca$

1 mark

The right-hand side is $2a^2 + 2b^2 + 2c^2$. The two expressions are clearly different — the coefficients of the squares are 3 against 2, and the LHS carries cross terms that the RHS does not.

Counter-example. One numerical case is enough to disprove an identity. Take $a = b = c = 1$:

$$\text{LHS} = (1 + 1 - 1)^2 + (1 - 1 + 1)^2 + (1 - 1 - 1)^2 = 1^2 + 1^2 + (-1)^2 = 3$$

$$\text{RHS} = 2 + 2 + 2 = 6$$

Since $3 \neq 6$, the statement fails.

1 mark

Answer: this is NOT an identity. It holds only for special values — comparing the two sides, it is true exactly when $a^2 + b^2 + c^2 = 2(ab + bc + ca)$, not for all a, b, c .

✦ **How to answer "is it an identity?"** If yes, you must expand and show both sides agree in general — numbers alone are not proof. If no, one counter-example is a complete disproof. Here we did both: the general expansion shows why it fails, and the numbers confirm it.

— End of Exercise Set 4.3 —

NCERT Solutions — Exercise Set 4.4

Approach for This Exercise

Q1 requires splitting the middle term (multiply the first and last coefficients, find two numbers with that product and the middle coefficient as their sum). Q2 asks you to pick the fastest identity for each numerical product — usually $(a \pm b)^2$ or $(a + b)(a - b)$. Q3 combines factoring skills from the whole chapter.

Q1. Fill in the blanks to complete the following identities: (i) $s^2 - 11s + 24 = (\quad)(\quad)$ (ii) $(\quad)(x + 1) = 3x^2 - 4x - 7$ (iii) $10x^2 - 11x - 6 = (2x - \quad)(\quad + 2)$ (iv) $6x^2 + 7x + 2 = (\quad)(\quad)$

4 parts

SOLUTION:

(i) Need $a + b = -11$, $ab = 24$: $a = -3$, $b = -8$. $(s - 3)(s - 8)$

(ii) $3x^2 - 4x - 7$: split $-4x$ as $3x - 7x$ (since $3 \times -7 = -21$, and $3 \times (-7)/3 \dots$ using product

$3 \times (-7) = -21$, sum -4 : factors $3, -7$): $3x^2 + 3x - 7x - 7 = 3x(x + 1) - 7(x + 1) = (3x - 7)(x + 1)$.

Blank: $(3x - 7)$

(iii) $10x^2 - 11x - 6$: product $10 \times (-6) = -60$, sum -11 : factors $-15, 4$.

$10x^2 - 15x + 4x - 6 = 5x(2x - 3) + 2(2x - 3) = (5x + 2)(2x - 3)$. So $(2x - 3)(5x + 2)$

(iv) $6x^2 + 7x + 2$: product $6 \times 2 = 12$, sum 7 : factors $3, 4$.

$6x^2 + 3x + 4x + 2 = 3x(2x + 1) + 2(2x + 1) = (3x + 2)(2x + 1)$

4 marks

Q2. Select and use the identity that will help you find the following products without multiplying directly: (i) 41^2 (ii) 27^2 (iii) 23×17 (iv) 135^2 (v) 97^2 (vi) 18×29 (vii) 34×43 (viii) 205^2

8 parts

SOLUTION:

(i) $41^2 = (40 + 1)^2 = 1600 + 80 + 1 = 1681$

(ii) $27^2 = (30 - 3)^2 = 900 - 180 + 9 = 729$

(iii) $23 \times 17 = (20 + 3)(20 - 3) = 400 - 9 = 391$

(iv) $135^2 = (130 + 5)^2 = 16900 + 1300 + 25 = 18225$

(v) $97^2 = (100 - 3)^2 = 10000 - 600 + 9 = 9409$

(vi) $18 \times 29 = 18 \times (30 - 1) = 540 - 18 = 522$

(vii) $34 \times 43 = (38.5 + 4.5)(38.5 - 4.5) = 38.5^2 - 4.5^2 = 1482.25 - 20.25 = 1462$

(viii) $205^2 = (200 + 5)^2 = 40000 + 2000 + 25 = 42025$

8 marks

Q3. Factor the following: (i) $9a^2 + b^2 + 4c^2 - 6ab + 12ac - 4bc$ (ii) $16s^2 + 25t^2 - 40st$ (iii) $r^2 - r - 42$
(iv) $49g^2 + 14gh + h^2$ (v) $64u^2 + 121v^2 + 4w^2 - 176uv - 32uw + 44vw$

5 parts

SOLUTION:

(i) Matching $A = 3a, B = -b, C = 2c$: $2AB = -6ab, 2BC = -4bc, 2CA = 12ac$ — all match!

$$(3a - b + 2c)^2$$

$$(ii) 16s^2 + 25t^2 - 40st = (4s)^2 - 2(4s)(5t) + (5t)^2 = (4s - 5t)^2$$

$$(iii) r^2 - r - 42: \text{ need } a + b = -1, ab = -42: -7, 6. (r - 7)(r + 6)$$

$$(iv) 49g^2 + 14gh + h^2 = (7g)^2 + 2(7g)(h) + h^2 = (7g + h)^2$$

(v) Matching $A = 8u, B = -11v, C = -2w$: $2AB = -176uv, 2BC = 44vw, 2CA = -32uw$ — all match!

$$(8u - 11v - 2w)^2$$

5 marks

— End of Exercise Set 4.4 —

NCERT Solutions — Exercise Set 4.5

Approach for This Exercise

Factorise the numerator and denominator completely, using whichever identity fits each — perfect square, difference of squares, sum/difference of cubes, or splitting the middle term — then cancel any common bracketed factor.

Q1. Simplify the following rational expressions (denominators $\neq 0$): (i) $(3p^2 - 3pq - 18q^2)/(p^2 + 3pq - 10q^2)$ (ii) $(n^3 - 3n^2m + 3nm^2 - m^3)/(5m^2 - 10mn + 5n^2)$ (iii) $(w^3 - v^3 + x^3 + 3wvx)/(w^2 + v^2 + x^2 - 2wv - 2vx + 2wx)$ (iv) $(4y^2 - 20yz + 25z^2)/(25z^2 - 4y^2)$ (v) $[(x^2 + x - 6)(x^2 - 7x + 12)]/[(x^2 - 6x + 8)(x^2 - 9)]$ (vi) $(p^4 - 16)/(p^2 - 4p + 4)$

6 parts

SOLUTION:

(i) Numerator: $3p^2 - 3pq - 18q^2 = 3(p^2 - pq - 6q^2) = 3(p - 3q)(p + 2q)$

Denominator: $p^2 + 3pq - 10q^2 = (p + 5q)(p - 2q)$

No common factor — expression is already simplest form: $\frac{3(p - 3q)(p + 2q)}{(p + 5q)(p - 2q)}$

(ii) Numerator: $n^3 - 3n^2m + 3nm^2 - m^3 = (n - m)^3$

Denominator: $5m^2 - 10mn + 5n^2 = 5(n - m)^2$

Cancelling $(n - m)^2$: $\frac{n - m}{5}$

(iii) Using $A^3 + B^3 + C^3 - 3ABC = (A + B + C)(A^2 + B^2 + C^2 - AB - BC - CA)$ with

$A = w, B = -v, C = x$:

Numerator $= (w - v + x)(w^2 + v^2 + x^2 + wv + vx - wx)$

Denominator matches $(w - v + x)^2$

Cancelling one $(w - v + x)$: $\frac{w^2 + v^2 + x^2 + wv + vx - wx}{w - v + x}$

(iv) Numerator: $4y^2 - 20yz + 25z^2 = (2y - 5z)^2$

Denominator: $25z^2 - 4y^2 = (5z - 2y)(5z + 2y) = -(2y - 5z)(5z + 2y)$

Cancelling one $(2y - 5z)$: $\frac{5z - 2y}{5z + 2y}$

(v) Factor each: $x^2 + x - 6 = (x + 3)(x - 2)$; $x^2 - 7x + 12 = (x - 3)(x - 4)$;

$x^2 - 6x + 8 = (x - 2)(x - 4)$; $x^2 - 9 = (x - 3)(x + 3)$

All four factors cancel exactly: 1

(vi) Numerator: $p^4 - 16 = (p^2 - 4)(p^2 + 4) = (p - 2)(p + 2)(p^2 + 4)$

Denominator: $p^2 - 4p + 4 = (p - 2)^2$

Cancelling one $(p - 2)$: $\frac{(p + 2)(p^2 + 4)}{p - 2}$

6 marks

— End of Exercise Set 4.5 —



End-of-Chapter Exercises

Q1. Use suitable identities to find the following products: (i) $(-3x+4)^2$ (ii) $(2s+7)(2s-7)$ (iii) $(p^2+1/2)(p^2-1/2)$ (iv) $(2n+7)(2n-7)$ (v) $(s-2t)(s^2+2st+4t^2)$ (vi) $(1/2r-4r)^2$ (vii) $(-3m+4k-l)^2$ (viii) $(x-1/3y)^3$ (ix) $(7/2k-2/3m)^3$

9 parts

SOLUTION:

(i) $(-3x+4)^2 = 9x^2 - 24x + 16$

(ii) $(2s+7)(2s-7) = 4s^2 - 49$

(iii) $\left(p^2 + \frac{1}{2}\right)\left(p^2 - \frac{1}{2}\right) = p^4 - \frac{1}{4}$

(iv) $(2n+7)(2n-7) = 4n^2 - 49$

(v) $(s-2t)(s^2+2st+4t^2) = s^3 - (2t)^3 = s^3 - 8t^3$

(vi) $\left(\frac{1}{2r} - 4r\right)^2 = \frac{1}{4r^2} - 4 + 16r^2$

(vii) Matching $A = -3m, B = 4k, C = -l$: $(-3m+4k-l)^2 = 9m^2 + 16k^2 + l^2 - 24mk - 8kl + 6lm$

(viii) $\left(x - \frac{y}{3}\right)^3 = x^3 - x^2y + \frac{xy^2}{3} - \frac{y^3}{27}$

(ix) $\left(\frac{7k}{2} - \frac{2m}{3}\right)^3 = \frac{343k^3}{8} - \frac{49k^2m}{2} + \frac{14km^2}{3} - \frac{8m^3}{27}$

9 marks

Q2. Find the values using suitable identities: (i) 17×21 (ii) 104×96 (iii) 24×16 (iv) 147^3 (v) 199^3 (vi) 127^3 (vii) $(-107)^3$ (viii) $(-299)^3$

8 parts

SOLUTION:

(i) $17 \times 21 = (19-2)(19+2) = 361 - 4 = 357$

(ii) $104 \times 96 = (100+4)(100-4) = 10000 - 16 = 9984$

(iii) $24 \times 16 = (20+4)(20-4) = 400 - 16 = 384$

(iv)

$147^3 = (150-3)^3 = 150^3 - 3(150)^2(3) + 3(150)(3)^2 - 3^3 = 3375000 - 202500 + 4050 - 27 = 3176523$

(v) $199^3 = (200-1)^3 = 8000000 - 120000 + 600 - 1 = 7880599$

(vi) $127^3 = (130-3)^3 = 2197000 - 152100 + 3510 - 27 = 2048383$

(vii) $(-107)^3 = -(107)^3 = -(100+7)^3 = -(1000000 + 210000 + 14700 + 343) = -1225043$

(viii) $(-299)^3 = -(299)^3 = -(300-1)^3 = -(27000000 - 270000 + 900 - 1) = -26730899$

8 marks

Q3. Factor the following algebraic expressions: (i) $4y^2+1+1/16y^2$ (ii) $9m^2-1/25n^2$ (iii) $27b^3-1/64b^3$ (iv) $x^2+5x/6+1/6$ (v) $27u^3-1/125-27u^2/5+9u/25$ (vi) $64y^3+z^3/125$ (vii) $p^3+27q^3+r^3-9pqr$ (viii) $9m^2-12m+4$ (ix) $9x^3-8y^3/3+z^3/3+6xyz$ (x) $4x^2+9y^2+36z^2+12xy+36yz+24xz$ (xi) $27u^3-1/216-9u^2/2+u/4$

11 parts

SOLUTION:

(i) $4y^2 + 1 + \frac{1}{16y^2} = (2y)^2 + 2(2y)\left(\frac{1}{4y}\right) + \left(\frac{1}{4y}\right)^2 = \left(2y + \frac{1}{4y}\right)^2$

(ii) $9m^2 - \frac{1}{25n^2} = (3m)^2 - \left(\frac{1}{5n}\right)^2 = \left(3m - \frac{1}{5n}\right)\left(3m + \frac{1}{5n}\right)$

(iii) $27b^3 - \frac{1}{64b^3} = (3b)^3 - \left(\frac{1}{4b}\right)^3 = \left(3b - \frac{1}{4b}\right)\left(9b^2 + \frac{3}{4} + \frac{1}{16b^2}\right)$

(iv) $x^2 + \frac{5x}{6} + \frac{1}{6}$: need $a + b = \frac{5}{6}$, $ab = \frac{1}{6}$: $a = \frac{1}{2}$, $b = \frac{1}{3}$. $\left(x + \frac{1}{2}\right)\left(x + \frac{1}{3}\right)$

(v) Matches $\left(3u - \frac{1}{5}\right)^3 = 27u^3 - \frac{27u^2}{5} + \frac{9u}{25} - \frac{1}{125}$ exactly: $\left(3u - \frac{1}{5}\right)^3$

(vi) $64y^3 + \frac{z^3}{125} = (4y)^3 + \left(\frac{z}{5}\right)^3 = \left(4y + \frac{z}{5}\right)\left(16y^2 - \frac{4yz}{5} + \frac{z^2}{25}\right)$

(vii) Using $A^3 + B^3 + C^3 - 3ABC$ with $A = p$, $B = 3q$, $C = r$ (since $3ABC = 9pqr$):

$(p + 3q + r)(p^2 + 9q^2 + r^2 - 3pq - 3qr - pr)$

(viii) $9m^2 - 12m + 4 = (3m)^2 - 2(3m)(2) + 2^2 = (3m - 2)^2$

(ix) Factoring out $\frac{1}{3}$: $9x^3 - \frac{8y^3}{3} + \frac{z^3}{3} + 6xyz = \frac{1}{3}(27x^3 - 8y^3 + z^3 + 18xyz)$. Using

$A = 3x$, $B = -2y$, $C = z$ (since $-3ABC = 18xyz$):

$\frac{1}{3}(3x - 2y + z)(9x^2 + 4y^2 + z^2 + 6xy + 2yz - 3xz)$

(x) Matching $(A + B + C)^2$ with $A = 2x$, $B = 3y$, $C = 6z$: $2AB = 12xy$, $2BC = 36yz$, $2CA = 24xz$ — all match! $(2x + 3y + 6z)^2$

(xi) Matches $\left(3u - \frac{1}{6}\right)^3 = 27u^3 - \frac{9u^2}{2} + \frac{u}{4} - \frac{1}{216}$ exactly: $\left(3u - \frac{1}{6}\right)^3$

11 marks

Q4. Simplify the following (denominators $\neq 0$): (i) $(4x^2+4x+1)/(4x^2-1)$ (ii) $9(3a^3-24b^3)/(9a^2-36b^2)$ (iii) $(s^3+125t^3)/(s^2-2st-35t^2)$

3 parts

SOLUTION:

(i) Numerator = $(2x + 1)^2$; Denominator = $(2x - 1)(2x + 1)$. Cancel one $(2x + 1)$: $\frac{2x + 1}{2x - 1}$

(ii) Numerator = $9 \times 3(a - 2b)(a^2 + 2ab + 4b^2) = 27(a - 2b)(a^2 + 2ab + 4b^2)$; Denominator = $9(a - 2b)(a + 2b)$. Cancel 9 and $(a - 2b)$: $\frac{3(a^2 + 2ab + 4b^2)}{a + 2b}$

(iii) Numerator = $(s + 5t)(s^2 - 5st + 25t^2)$; Denominator = $(s - 7t)(s + 5t)$. Cancel $(s + 5t)$: $\frac{s^2 - 5st + 25t^2}{s - 7t}$

3 marks

Q5. Find possible expressions for the length and breadth of each rectangle whose area is given: (i) $25a^2 - 30ab + 9b^2$ (ii) $36s^2 - 49t^2$

2 marks

SOLUTION:

(i) $25a^2 - 30ab + 9b^2 = (5a - 3b)^2$ — **length = breadth = $(5a - 3b)$ units**

(ii) $36s^2 - 49t^2 = (6s - 7t)(6s + 7t)$ — **length = $(6s + 7t)$ units, breadth = $(6s - 7t)$ units**

2 marks

Q6. Find possible expressions for the length, breadth, and height of each cuboid whose volume is given: (i) $6a^2 - 24b^2$ (ii) $3ps^2 - 15ps + 12p$

2 marks

SOLUTION:

(i) $6a^2 - 24b^2 = 6(a^2 - 4b^2) = 6(a - 2b)(a + 2b)$ — **dimensions: 6, $(a - 2b)$, $(a + 2b)$ units**

(ii) $3ps^2 - 15ps + 12p = 3p(s^2 - 5s + 4) = 3p(s - 1)(s - 4)$ — **dimensions: 3p, $(s - 1)$, $(s - 4)$ units**

2 marks

Q7. The village playground is shaped as a square of side 40 metres. A path of width s metres is created around the playground. Find an expression for the area of the path in terms of s .

2 marks

SOLUTION: Outer boundary (playground + path on both sides) has side $(40 + 2s)$ m.

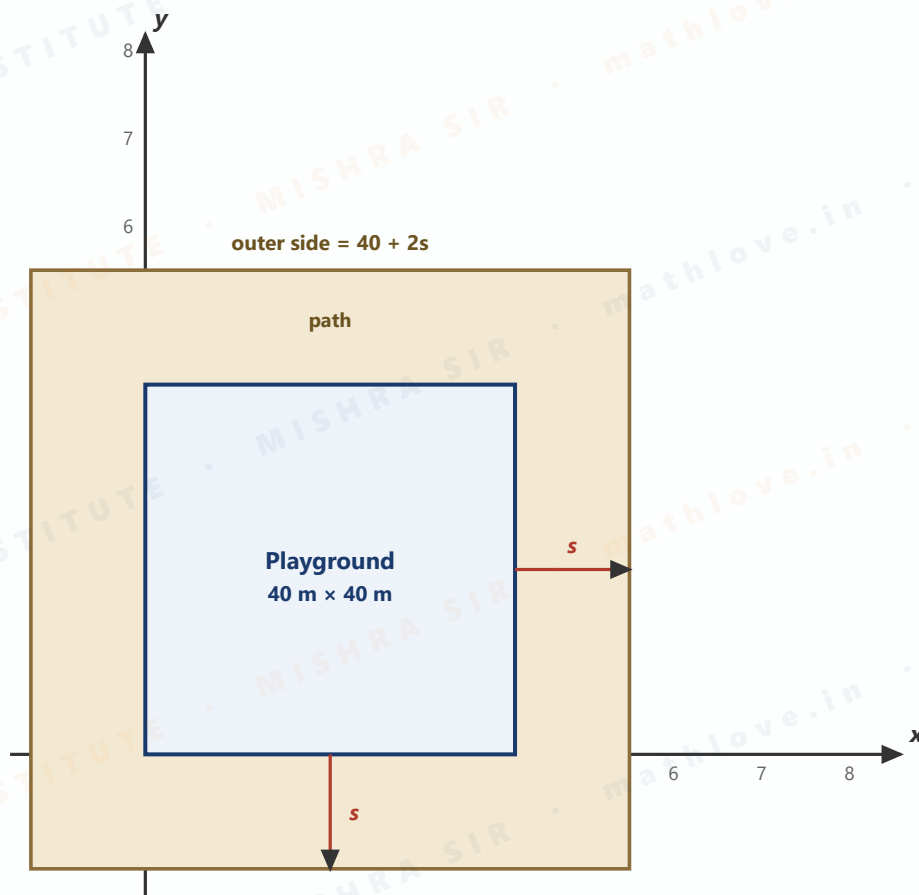
Area of path = $(40 + 2s)^2 - 40^2$

Using $a^2 - b^2 = (a + b)(a - b)$ with $a = 40 + 2s$, $b = 40$:

$= [(40 + 2s) + 40][(40 + 2s) - 40] = (80 + 2s)(2s) = 160s + 4s^2$

Area of path = $4s^2 + 160s$ sq. metres, i.e. $4s(s + 40)$ sq. metres

2 marks



Q7: the path of width s runs all the way around, so the outer square has side $(40 + 2s)$. Path area $= (40 + 2s)^2 - 40^2 = 4s^2 + 160s$ sq. m.

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari, Part I*.

Q8. If a number plus its reciprocal equals $\frac{10}{3}$, find the number.

3 marks

SOLUTION: Let the number be x : $x + \frac{1}{x} = \frac{10}{3}$

Multiplying by $3x$: $3x^2 + 3 = 10x \Rightarrow 3x^2 - 10x + 3 = 0$

Splitting: product $= 3 \times 3 = 9$, sum $= -10$: factors $-1, -9$.

$3x^2 - x - 9x + 3 = 0 \Rightarrow x(3x - 1) - 3(3x - 1) = 0 \Rightarrow (3x - 1)(x - 3) = 0$

$x = 3$ or $x = \frac{1}{3}$ (these are reciprocals of each other, as expected)

CHECK: $3 + \frac{1}{3} = \frac{10}{3}$ ✓

3 marks

Q9. A rectangular pool has area $2x^2 + 7x + 3$ square hastas. If its width is $2x + 1$ hastas, find its length.

2 marks

SOLUTION: Factor the area: $2x^2 + 7x + 3$: product = $2 \times 3 = 6$, sum = 7: factors 6, 1.

$$2x^2 + 6x + x + 3 = 2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3)$$

Since width = $2x + 1$, **length** = $(x + 3)$ hastas

2 marks

***Q10. If both $x - 2$ and $x - \frac{1}{2}$ are factors of $px^2 + 5x + r$, show that $p = r$.**

Enrichment

SOLUTION: If $x - 2$ is a factor, then $x = 2$ makes the expression zero:

$$p(2)^2 + 5(2) + r = 0 \Rightarrow 4p + 10 + r = 0 \Rightarrow 4p + r = -10 \dots (1)$$

If $x - \frac{1}{2}$ is a factor, then $x = \frac{1}{2}$ makes the expression zero:

$$p\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + r = 0 \Rightarrow \frac{p}{4} + \frac{5}{2} + r = 0. \text{ Multiplying by 4: } p + 10 + 4r = 0 \Rightarrow p + 4r = -10 \dots (2)$$

$$\text{Subtracting (2) from (1): } (4p + r) - (p + 4r) = 0 \Rightarrow 3p - 3r = 0 \Rightarrow p = r$$

3 marks

***Q11. If $a + b + c = 5$ and $ab + bc + ca = 10$, prove that $a^3 + b^3 + c^3 - 3abc = -25$.**

Enrichment

$$\text{SOLUTION: } (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$25 = a^2 + b^2 + c^2 + 2(10) \Rightarrow a^2 + b^2 + c^2 = 25 - 20 = 5$$

$$\text{Using the identity } a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca):$$

$$a^2 + b^2 + c^2 - ab - bc - ca = 5 - 10 = -5$$

$$a^3 + b^3 + c^3 - 3abc = (5)(-5) = -25 \text{ — proven.}$$

4 marks

***Q12. By factoring the expression, check that $n^3 - n$ is always divisible by 6 for all natural numbers n . Give reasons.**

Enrichment

$$\text{SOLUTION: } n^3 - n = n(n^2 - 1) = n(n - 1)(n + 1) = (n - 1)(n)(n + 1)$$

This is the product of **three consecutive integers**.

Among any three consecutive integers, **at least one is a multiple of 3** (since every third integer is), and **at least one is even** (since every second integer is). So the product is always divisible by both 2 and 3.

Since $\text{gcd}(2, 3) = 1$, the product is divisible by $2 \times 3 = 6$. Hence $n^3 - n$ is always divisible by 6 for every natural number n .

4 marks

***Q13. Find the value of: (i) $x^3 + y^3 - 12xy + 64$, when $x + y = -4$ (ii) $x^3 - 8y^3 - 36xy - 216$, when $x = 2y + 6$**

Enrichment

SOLUTION:

(i) Recognise this as $A^3 + B^3 + C^3 - 3ABC$ with $A = x, B = y, C = 4$ (since $4^3 = 64$ and $-3xy(4) = -12xy$):

$$x^3 + y^3 + 4^3 - 3xy(4) = (x + y + 4)(x^2 + y^2 + 16 - xy - 4y - 4x)$$

Given $x + y = -4$, so $x + y + 4 = 0$. Therefore the whole product is 0.

(ii) Recognise this as $A^3 + B^3 + C^3 - 3ABC$ with $A = x, B = -2y, C = -6$ (since $(-2y)^3 = -8y^3$, $(-6)^3 = -216$, and $-3x(-2y)(-6) = -36xy$):

$$x^3 + (-2y)^3 + (-6)^3 - 3x(-2y)(-6) = (x - 2y - 6)(\dots)$$

Given $x = 2y + 6$, so $x - 2y - 6 = 0$. Therefore the whole product is 0.

4 marks

— End of Chapter 4 NCERT Solutions —

Additional Practice Questions

A Basic Identity Expansion (5 Questions)

A1. Expand $(5x + 3y)^2$ using $(a + b)^2 = a^2 + 2ab + b^2$.

2 marks

SOLUTION: $(5x + 3y)^2 = 25x^2 + 30xy + 9y^2$

2 marks

A2. Expand $(4p - 5q)^2$ using $(a - b)^2 = a^2 - 2ab + b^2$.

2 marks

SOLUTION: $(4p - 5q)^2 = 16p^2 - 40pq + 25q^2$

2 marks

A3. Expand $(2x + 3y + z)^2$ using the three-term square identity.

2 marks

SOLUTION: $(2x + 3y + z)^2 = 4x^2 + 9y^2 + z^2 + 12xy + 6yz + 4xz$

2 marks

A4. Find 52^2 using a suitable identity.

1 mark

SOLUTION: $52^2 = (50 + 2)^2 = 2500 + 200 + 4 = 2704$

1 mark

A5. Find 48^2 using a suitable identity.

1 mark

SOLUTION: $48^2 = (50 - 2)^2 = 2500 - 200 + 4 = 2304$

1 mark

B Factorisation Using Identities (5 Questions)

B1. Factor $25x^2 + 30xy + 9y^2$.

1 mark

SOLUTION: $= (5x + 3y)^2$

1 mark

B2. Factor $49p^2 - 14pq + q^2$.

1 mark

SOLUTION: $= (7p - q)^2$

1 mark

B3. Factor $121a^2 - 64b^2$.

1 mark

SOLUTION: $= (11a - 8b)(11a + 8b)$

1 mark

B4. Factor $x^2 - 3x - 10$.

2 marks

SOLUTION: Need $a + b = -3$, $ab = -10$; $-5, 2$. $= (x - 5)(x + 2)$

2 marks

B5. Factor $8x^3 + 27y^3$ using the sum of cubes identity.

2 marks

SOLUTION: $= (2x)^3 + (3y)^3 = (2x + 3y)(4x^2 - 6xy + 9y^2)$

2 marks

C Numerical Calculation Shortcuts (5 Questions)

C1. Compute 102^3 using a suitable identity.

3 marks

SOLUTION: $102^3 = (100 + 2)^3 = 1000000 + 60000 + 1200 + 8 = 1061208$

3 marks

C2. Compute 19×21 using the difference of squares.

1 mark

SOLUTION: $(20 - 1)(20 + 1) = 400 - 1 = 399$

1 mark

C3. Compute 998^2 using a suitable identity.

2 marks

SOLUTION: $(1000 - 2)^2 = 1000000 - 4000 + 4 = 996004$

2 marks

C4. Compute 63^2 using a suitable identity.

1 mark

SOLUTION: $(60 + 3)^2 = 3600 + 360 + 9 = 3969$

1 mark

C5. Compute 34×26 using the difference of squares (hint: average is 30).

2 marks

SOLUTION: $(30 + 4)(30 - 4) = 900 - 16 = 884$

2 marks

D Simplifying Rational Expressions & Applications (5 Questions)

D1. Simplify $(x^2 - 9)/(x^2 - x - 6)$.

2 marks

SOLUTION: $\frac{(x - 3)(x + 3)}{(x - 3)(x + 2)} = \frac{x + 3}{x + 2}$

2 marks

D2. Simplify $(a^2 - b^2)/(a - b)$, for $a \neq b$.

1 mark

SOLUTION: $\frac{(a - b)(a + b)}{a - b} = a + b$

1 mark

D3. A rectangular field has area $x^2 + 9x + 20$ sq. units. If width = $(x + 4)$, find the length.

2 marks

SOLUTION: $x^2 + 9x + 20 = (x + 4)(x + 5)$. Since width = $(x + 4)$, **length = $(x + 5)$ units**

2 marks

D4. Find the value of x^3+y^3 , if $x+y=6$ and $xy=8$, using $x^3+y^3 = (x+y)^3-3xy(x+y)$.

2 marks

SOLUTION: $(x+y)^3 - 3xy(x+y) = 6^3 - 3(8)(6) = 216 - 144 = 72$

2 marks

D5. If $a-b=4$ and $ab=21$, find a^2+b^2 using $(a-b)^2 = a^2-2ab+b^2$.

2 marks

SOLUTION: $16 = a^2 + b^2 - 2(21) \Rightarrow a^2 + b^2 = 16 + 42 = 58$

2 marks

— End of Additional Practice Questions —

CHAPTER 5

I'm Up and Down, and Round and Round (Circles)

Exercise Set 5.1 · Exercise Set 5.2 · Exercise Set 5.3 · Exercise Set 5.4 · Exercise Set 5.5 · Exercise Set 5.6 · End-of-Chapter Exercises

EXERCISE SET 5.1

What Exercise Set 5.1 Tests

Theorem 1 — there is exactly one circle through three non-collinear points

Idea	Statement	Used in
Circumcentre	The point where the perpendicular bisectors of the sides meet; it is equidistant from all three vertices	Q1–Q3
Position of the circumcentre	All angles acute → inside ; one angle obtuse → outside ; one angle 90° → at the midpoint of the hypotenuse	Q1, Q2
Circles through two points	Infinitely many; all centres lie on the perpendicular bisector of the segment	Q4
Collinear points	No circle passes through three collinear points — the perpendicular bisectors are parallel	Think, Draw and Infer 1

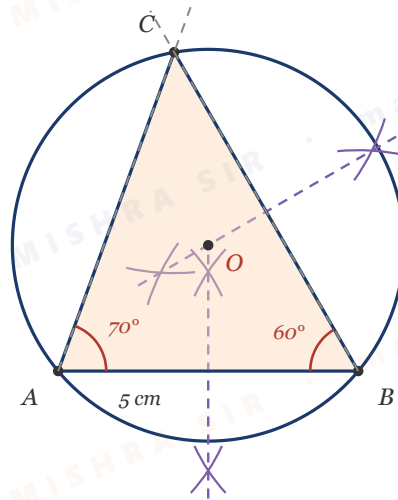
Reading the answer before you draw. You can state whether the centre is inside or outside just by finding the angles. The construction then confirms what you already know — that is the right order to work in.

Construction convention used in every figure below: **red arcs** are the arcs used to fix a side or an angle; **purple arcs and dashed lines** are the perpendicular-bisector construction; the teal circle is the finished circumcircle.

A Exercise Set 5.1 — Constructions and the circumcentre

Q1. Draw $\triangle ABC$ with $AB = 5\text{ cm}$, $\angle A = 70^\circ$ and $\angle B = 60^\circ$. Draw the circumcircle of $\triangle ABC$. Is the centre inside or outside the triangle?

2 marks



Construction for Q1 — red arcs mark the 70° and 60° angles at A and B; purple arcs give the perpendicular bisectors, which meet at the circumcentre O

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

CONSTRUCTION — STEP BY STEP:

Step 1. Draw a line segment $AB = 5\text{ cm}$.

Step 2. At A, use the protractor (or compasses) to construct $\angle BAX = 70^\circ$, and draw the ray AX. The small red arc at A records this angle.

Step 3. At B, construct $\angle ABY = 60^\circ$ on the same side of AB, and draw the ray BY. The rays AX and BY meet at C — this completes $\triangle ABC$.

Step 4. With centre A and a radius more than half of AB, draw arcs above and below AB. Repeat with the same radius from B. Join the two crossing points — this is the perpendicular bisector of AB.

Step 5. Repeat step 4 on side BC to get its perpendicular bisector. The two bisectors meet at O — the circumcentre.

Step 6. With centre O and radius OA, draw the circle. It passes through B and C as well, since $OA = OB = OC$.

ANSWER WITH REASONING:

Step 1 — Find the third angle. $\angle C = 180^\circ - 70^\circ - 60^\circ = 50^\circ$.

1

Step 2 — Classify the triangle. The angles are $70^\circ, 60^\circ, 50^\circ$ — all less than 90° , so $\triangle ABC$ is

acute-angled.

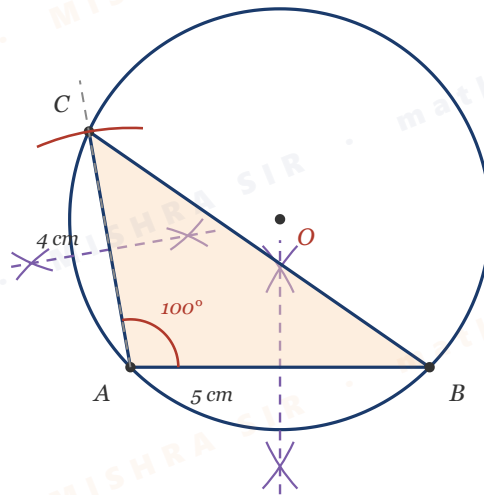
Step 3 — Apply the rule. For an acute-angled triangle the circumcentre lies inside the triangle.

The centre O lies inside the triangle.

1

Q2. Draw $\triangle ABC$ with $AB = 5$ cm, $\angle A = 100^\circ$, $AC = 4$ cm. Draw the circumcircle of $\triangle ABC$. Is the centre inside or outside the triangle?

2 marks



Construction for Q2 — the red arc at A marks the 100° angle and the second red arc cuts $AC = 4$ cm; the purple perpendicular-bisector arcs locate O, which falls outside $\triangle ABC$

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

CONSTRUCTION — STEP BY STEP:

Step 1. Draw $AB = 5$ cm.

Step 2. At A construct $\angle BAX = 100^\circ$ and draw the ray AX (red arc at A).

Step 3. With centre A and radius 4 cm, draw an arc cutting AX at C (the second red arc). Join BC — $\triangle ABC$ is complete.

Step 4. Construct the perpendicular bisector of AB using arcs from A and from B (purple).

Step 5. Construct the perpendicular bisector of AC in the same way. The two meet at O.

Step 6. With centre O and radius OA, draw the circumcircle.

ANSWER WITH REASONING:

Step 1 — Look at the given angle. $\angle A = 100^\circ > 90^\circ$, so $\triangle ABC$ is **obtuse-angled**.

1

Step 2 — Apply the rule. For an obtuse-angled triangle the circumcentre lies outside the triangle — on the far side of BC, the side opposite the obtuse angle.

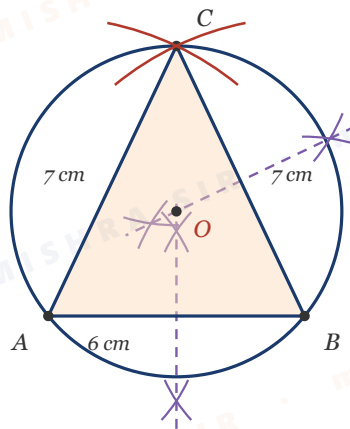
Step 3 — Why that must happen. $\angle A$ is the inscribed angle standing on BC. Since it exceeds 90° , the central angle for that arc exceeds 180° , which forces O to sit on the opposite side of BC from A.

The centre O lies outside the triangle.

1

Q3. Draw $\triangle ABC$ with $AB = 6$ cm, $BC = 7$ cm and $CA = 7$ cm. Draw the circumcircle of $\triangle ABC$. Let the circumcentre be O . Measure OA , OB , OC .

3 marks



Construction for Q3 — the two red arcs of radius 7 cm from A and from B cross at C; the purple arcs give the perpendicular bisectors of AB and BC, meeting at O

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari*, Part I.

CONSTRUCTION — STEP BY STEP:

Step 1. Draw $AB = 6$ cm.

Step 2. With centre A and radius 7 cm, draw an arc above AB (first red arc).

Step 3. With centre B and radius 7 cm, draw a second arc cutting the first at C (second red arc).

Join AC and BC — $\triangle ABC$ is complete.

Step 4. Construct the perpendicular bisector of AB: arcs of equal radius (more than 3 cm) from A and from B, above and below, then join the crossing points.

Step 5. Construct the perpendicular bisector of BC in the same way. The two bisectors meet at O.

Step 6. With centre O and radius OA, draw the circumcircle through A, B and C.

Step 7. Measure OA, OB and OC with the ruler.

WHAT YOU SHOULD FIND:

Step 1 — The three measurements agree. $OA = OB = OC$.

1

Step 2 — Why they must agree. O lies on the perpendicular bisector of AB, so $OA = OB$. It also lies on the perpendicular bisector of BC, so $OB = OC$. Hence all three are equal — that common length is the **circumradius**.

1

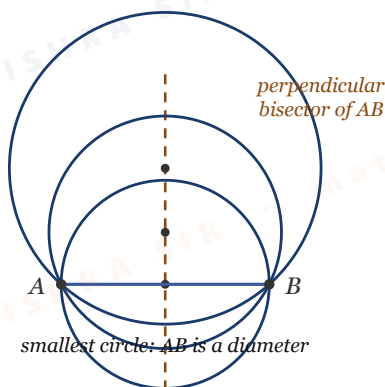
CHECK BY CALCULATION (ENRICHMENT): $\triangle ABC$ is isosceles with base $AB = 6$ and equal sides 7,

$$R = \frac{abc}{4 \text{ Area}} = \frac{7 \times 7 \times 6}{4 \times 6\sqrt{10}} = \frac{49}{4\sqrt{10}} \approx \mathbf{3.87 \text{ cm}}. \text{ Your measured value should be close to this.}$$

1

Q4. What is the least possible radius of a circle through two points A and B?

1 mark



Infinitely many circles pass through A and B — all their centres lie on the perpendicular bisector of AB. The smallest is the one whose centre is the midpoint of AB.

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SOLUTION:

Step 1 — Where can the centre be? The centre of a circle through A and B must satisfy $OA = OB$, so it lies on the perpendicular bisector of AB. 1

Step 2 — Write the radius in terms of one variable. If the centre is at distance t from the midpoint M of AB, then $\triangle OMA$ is right-angled at M and $r = \sqrt{t^2 + \left(\frac{AB}{2}\right)^2}$.

Step 3 — Minimise. r is smallest when $t = 0$, i.e. when the centre is at M itself.

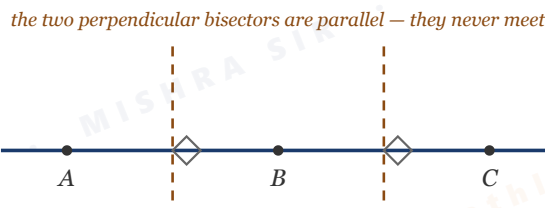
Least radius = $\frac{AB}{2}$ — obtained when AB is a diameter of the circle. 1

B

Exercise Set 5.1 — Think, Draw and Infer

TDI-1. A, B and C are three collinear points. Can you find a point P with $PA = PB = PC$? What can you say about the perpendicular bisectors of AB and BC? Is it possible for a circle to pass through collinear points? Can a line cut a circle in three distinct points?

3 marks



A, B, C collinear — the perpendicular bisectors of AB and of BC are both perpendicular to the same line, so they are parallel and never meet

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Step 1 — Is there such a point P?. No. $PA = PB$ would put P on the perpendicular bisector of AB; $PB = PC$ would put P on the perpendicular bisector of BC. P would have to lie on both.

1

Step 2 — The two perpendicular bisectors are parallel. A, B and C lie on one line ℓ . The perpendicular bisector of AB is perpendicular to ℓ , and so is the perpendicular bisector of BC. Two lines perpendicular to the same line are parallel. They are also distinct, because the midpoint of AB differs from the midpoint of BC. Parallel distinct lines never meet — confirming Step 1.

1

Step 3 — Can a circle pass through three collinear points?. No. Such a circle would need a centre equidistant from all three, and Step 1 shows no such point exists.

Step 4 — Can a line cut a circle at three points?. No. Three such points would be collinear and concyclic, contradicting Step 3. A line meets a circle in at most two points.

1

TDI-2. The circumcircle of a given $\triangle ABC$ is drawn. Can there be other triangles congruent to $\triangle ABC$ that share the same circumcircle?

2 marks

Step 1 — The answer. Yes — infinitely many.

1

Step 2 — The construction. Rotate $\triangle ABC$ about the centre O through any angle θ . A rotation is a rigid motion, so the image $\triangle A'B'C'$ is congruent to $\triangle ABC$.

Step 3 — Why the image is still inscribed. A circle is unchanged by rotation about its own centre, so all three image vertices still lie on the same circle. Since θ may take any value, there are infinitely many such triangles.

1

Step 4 — Another family. Reflecting $\triangle ABC$ in any diameter produces further congruent inscribed triangles.

The converse is **not** true: two triangles inscribed in the same circle need not be congruent — only their circumradii must agree.

⚠ Common Mistakes in Exercise Set 5.1

Not showing the construction arcs. In a construction question the arcs *are* the working. Faint pencil arcs must be left on the page — erasing them costs marks.

Using only one perpendicular bisector. One bisector gives a line of possible centres; you need two to fix the point O .

Guessing the position of the centre. Find the third angle first (Q1), or look at the given angle (Q2). The classification is what earns the mark, not the drawing.

Saying “no circle exists” without a reason (TDI-1). The reason is that the perpendicular bisectors are parallel, so there is no circumcentre.

Answering “AB” instead of “AB/2” in Q4. The least *radius* is half of AB; AB itself is the least *diameter*.

◆ END OF EXERCISE SET 5.1 ◆

EXERCISE SET 5.2



What Exercise Set 5.2 Tests

Theorems 2 and 3 — equal chords and equal angles at the centre

Result	Statement	Proved by
Theorem 2	Equal chords of a circle subtend equal angles at the centre	SSS congruence
Theorem 3 (converse)	Chords that subtend equal angles at the centre are equal	SAS congruence

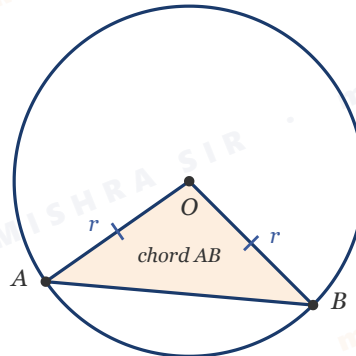
The one observation behind both questions: join the two ends of any chord to the centre. Both joining segments are *radii*, so they are equal — every chord automatically forms an isosceles triangle with the centre.

A

Exercise Set 5.2 — Chords and the angles they subtend

Q1. Show that the triangle formed by a chord and the centre of the circle is isosceles.

2 marks



Chord AB with the centre O joined to A and B . The tick marks show $OA = OB$, both being radii — so $\triangle OAB$ is isosceles with AB as base

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari*, Part I.

GIVEN: A circle with centre O and a chord AB . OA and OB are joined.

TO SHOW: $\triangle OAB$ is isosceles.

PROOF:

Step 1 — Identify the two sides. A lies on the circle, so OA is a radius. B lies on the circle, so OB is a radius. **1**

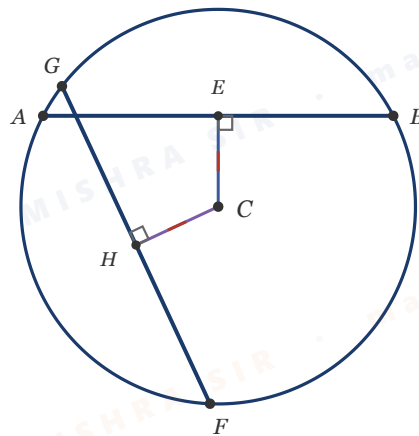
Step 2 — Equate them. All radii of the same circle are equal, so $OA = OB = r$.

Step 3 — Conclude. A triangle with two equal sides is isosceles. **1**

Hence $\triangle OAB$ is isosceles, with AB as its base and $\angle OAB = \angle OBA$.

Q2. Show that if two such isosceles triangles have equal base length, they are congruent to each other.

2 marks



$AB = FG \leftrightarrow CE = CH$ (tick marks show the equal distances)

Two equal chords AB and FG of the same circle, centre C . The triangles CAB and CFG have three pairs of equal sides

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari*, Part I.

GIVEN: A circle with centre C ; chords AB and FG with $AB = FG$. The triangles CAB and CFG are formed by joining each chord's ends to C .

TO SHOW: $\triangle CAB \cong \triangle CFG$.

PROOF:

Step 1 — First pair of sides. $CA = CF$ (both are radii of the same circle).

1

Step 2 — Second pair of sides. $CB = CG$ (both are radii of the same circle).

Step 3 — Third pair of sides. $AB = FG$ (given — the bases are equal).

Step 4 — Apply the criterion. By **SSS congruence**, $\triangle CAB \cong \triangle CFG$.

1

A USEFUL CONSEQUENCE: corresponding parts of congruent triangles are equal, so $\angle ACB = \angle FCG$. That is exactly **Theorem 2** — equal chords subtend equal angles at the centre.

Theorem 3 reverses this: given $\angle ACB = \angle FCG$, the two triangles are congruent by **SAS** (two radii and the included angle), which gives $AB = FG$.

⚠ Common Mistakes in Exercise Set 5.2

Not stating why $OA = OB$. Write "both are radii of the same circle". Without that phrase the step is unjustified.

Naming the wrong criterion. Q2 is SSS (three sides). Theorem 3 is SAS (two radii and the included angle). Marks are given for the correct name.

Assuming what is to be proved. Do not begin with "since the triangles are congruent..." — congruence is the conclusion, not the starting point.

◆ END OF EXERCISE SET 5.2 ◆

EXERCISE SET 5.3



What Exercise Set 5.3 Tests

Theorems 4 and 5 — the perpendicular and the midpoint of a chord

Result	Statement	Proved by
Theorem 4	The line joining the centre to the midpoint of a chord is perpendicular to it	SAS congruence
Theorem 5 (converse, Q1)	The perpendicular from the centre to a chord bisects it	RHS congruence

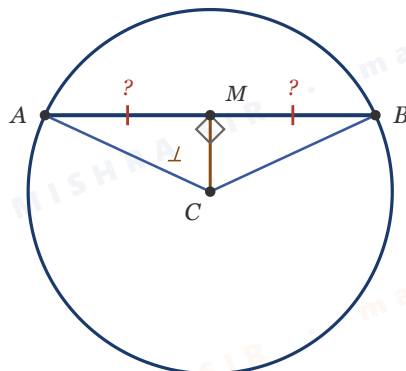
The master right triangle. Drop a perpendicular from the centre O to a chord AB , meeting it at M . Then $OM = d$, $AM = \frac{\ell}{2}$ and $OA = r$, and $\triangle OMA$ is right-angled at M . Almost every numerical question in this chapter is this one triangle: $r^2 = d^2 + \left(\frac{\ell}{2}\right)^2$.

A

Exercise Set 5.3 — Midpoints and perpendicular bisectors of chords

Q1. Explain why the converse of Theorem 4 is true — why does the perpendicular from the centre of a circle to a chord bisect the chord?

3 marks



given: $CM \perp AB \rightarrow$ to prove: $AM = BM$

We are given $CM \perp AB$ (the right-angle marker at M). We must prove that the two parts AM and MB, marked "?", are equal

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

GIVEN: A circle with centre C; chord AB; $CM \perp AB$, so $\angle CMA = \angle CMB = 90^\circ$.

TO SHOW: $AM = BM$.

PROOF — COMPARE $\triangle CMA$ AND $\triangle CMB$:

Step 1 — The right angle (R). $\angle CMA = \angle CMB = 90^\circ$ (given). 1

Step 2 — The hypotenuse (H). $CA = CB$, since both are radii of the same circle. 1

Step 3 — The common side (S). $CM = CM$.

Step 4 — Apply the criterion. By **RHS congruence**, $\triangle CMA \cong \triangle CMB$.

Step 5 — Read off the result. $AM = BM$ (corresponding parts of congruent triangles). 1

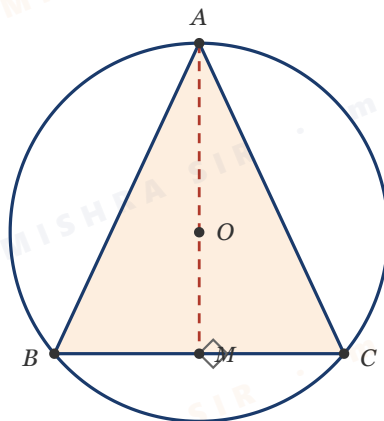
Hence the perpendicular from the centre bisects the chord — this is Theorem 5.

Note which criterion applies. Here we know a right angle, a hypotenuse and a side — that is **RHS**.

Theorem 4 goes the other way (two sides and the included angle) and uses **SAS**. Naming the wrong one loses the mark even if the rest is right.

Q2. An isosceles triangle ABC is inscribed in a circle, with $AB = AC$. Show that the altitude from A to BC passes through the centre of the circle.

3 marks



Isosceles $\triangle ABC$ inscribed in a circle with centre O; AM is the altitude from A to BC. Both A and O lie on the perpendicular bisector of BC

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari, Part I*.

GIVEN: $\triangle ABC$ inscribed in a circle with centre O, $AB = AC$. AM is the altitude from A to BC.

TO SHOW: AM passes through O.

STRATEGY: show that both A and O lie on the perpendicular bisector of BC, and that AM is that line.

PROOF:

Step 1 — A lies on the perpendicular bisector of BC. $AB = AC$ (given), so A is equidistant from B and C. Every point equidistant from B and C lies on the perpendicular bisector of BC.

1

Step 2 — O also lies on it. $OB = OC$ (both radii), so O is equidistant from B and C, and therefore lies on the same perpendicular bisector.

1

Step 3 — The two points determine the line. Two distinct points determine exactly one line, so the perpendicular bisector of BC is the line AO.

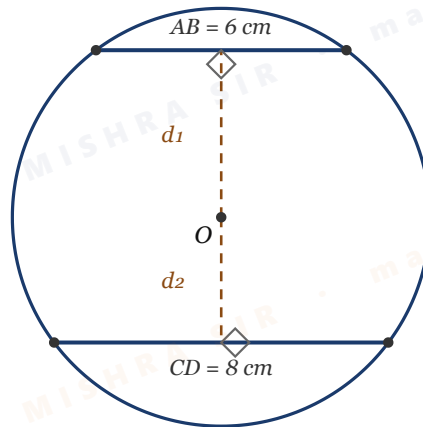
Step 4 — Identify the altitude. In an isosceles triangle the altitude from the apex to the base is the perpendicular bisector of the base. So AM lies along AO.

Therefore the altitude from A to BC passes through the centre O.

1

Q3. Two parallel chords of lengths 6 cm and 8 cm are on opposite sides of the centre of a circle. If the radius is 5 cm, find the distance between the midpoints of the chords.

3 marks



Chords $AB = 6\text{ cm}$ and $CD = 8\text{ cm}$ on **opposite** sides of the centre O of a circle of radius 5 cm — the required distance is $d_1 + d_2$ (not to scale)

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SOLUTION:

Step 1 — Set up. Let O be the centre, $r = 5\text{ cm}$. Drop a perpendicular from O to each chord. By Theorem 5, each foot is the **midpoint** of its chord.

Step 2 — Distance of the 6 cm chord. Half-chord $= 3$. $d_1 = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$ cm. 1

Step 3 — Distance of the 8 cm chord. Half-chord $= 4$. $d_2 = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3$ cm. 1

Step 4 — Combine — read the wording carefully. The chords are on **opposite sides**, so O lies between the two midpoints and the distances **add**: $4 + 3 = 7$.

Distance between the midpoints = 7 cm. 1

Watch the phrase. Had the chords been on the **same side**, the answer would be $d_1 - d_2 = 4 - 3 = 1$ cm. The words "opposite sides" / "same side" decide whether you add or subtract — the single commonest slip in this chapter.

⚠ Common Mistakes in Exercise Set 5.3

Confusing RHS with SAS (Q1). Given a right angle, hypotenuse and one side → RHS. Given two sides and the included angle → SAS.

Assuming the altitude passes through O (Q2). That is exactly what must be proved. Build the argument from $AB = AC$ and $OB = OC$.

Adding when you should subtract (Q3). Opposite sides → add. Same side → subtract. Read the sentence twice before writing the last line.

Forgetting to halve the chord. The leg of the right triangle is *half* the chord — 3 and 4, not 6 and 8.

◆ END OF EXERCISE SET 5.3 ◆

EXERCISE SET 5.4



What Exercise Set 5.4 Tests

Theorems 6 and 7 — equal chords are equidistant from the centre, and conversely

Result	Statement	Route 1	Route 2
Theorem 6	Chords of equal length are equidistant from the centre	SSS congruence of $\triangle CAB$ and $\triangle CFG$	Baudhāyana–Pythagoras
Theorem 7	Chords equidistant from the centre are equal in length	RHS congruence of $\triangle CEA$ and $\triangle CHF$	Baudhāyana–Pythagoras

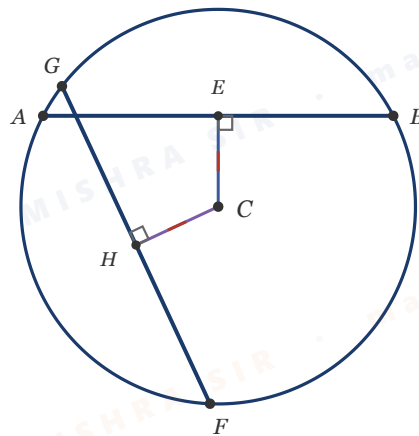
Why the exercise asks for both routes. The congruence proof trains you to name criteria correctly; the Pythagoras proof shows *why* the result is inevitable — the distance $d = \sqrt{r^2 - (\ell/2)^2}$ depends on nothing but r and ℓ .

A

Exercise Set 5.4 — Distance of chords from the centre

Q1. Use the Baudhāyana–Pythagoras theorem to show why Theorem 6 must be true.

3 marks



$AB = FG \leftrightarrow CE = CH$ (tick marks show the equal distances)

Equal chords AB and FG of a circle with centre C . $CE \perp AB$ and $CH \perp FG$ are the perpendicular distances; the tick marks record that they come out equal

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari*, Part I.

GIVEN: Circle with centre C and radius r ; chords AB and FG with $AB = FG$; $CE \perp AB$ and $CH \perp FG$.

TO SHOW: $CE = CH$.

PROOF:

Step 1 — Locate the feet of the perpendiculars. By Theorem 5, E and H are the midpoints of AB and FG , so $AE = \frac{AB}{2}$ and $FH = \frac{FG}{2}$.

Step 2 — Use the given equality. $AB = FG$, therefore $AE = FH$. 1

Step 3 — Apply Baudhāyana–Pythagoras in $\triangle CEA$. $CE^2 = CA^2 - AE^2 = r^2 - AE^2$. 1

Step 4 — Apply it again in $\triangle CHF$. $CH^2 = CF^2 - FH^2 = r^2 - FH^2$.

Step 5 — Compare. Since $AE = FH$, the two right-hand sides are equal, so $CE^2 = CH^2$, and as lengths are positive, $CE = CH$. 1

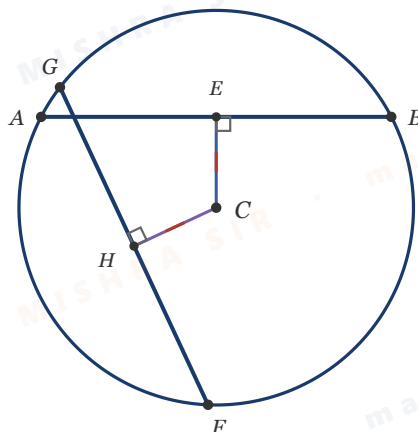
Hence chords of equal length are equidistant from the centre.

The formula worth memorising: $d = \sqrt{r^2 - \left(\frac{\ell}{2}\right)^2}$. It depends only on r and ℓ — equal chords in one circle must therefore give equal distances. That is the whole theorem in a

line.

Q2. Consider Fig. 5.15. If CE is perpendicular to AB , CH is perpendicular to GF , and $CE = CH$, show that $AB = GF$.

3 marks



$AB = GF \leftrightarrow CE = CH$ (tick marks show the equal distances)

Fig. 5.15 schematic — here the equal perpendicular distances $CE = CH$ are given, and the chord lengths are to be proved equal

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari*, Part I.

GIVEN: Circle with centre C ; $CE \perp AB$, $CH \perp GF$, and $CE = CH$.

TO SHOW: $AB = GF$.

PROOF — COMPARE $\triangle CEA$ AND $\triangle CHF$:

Step 1 — The right angle (R). $\angle CEA = \angle CHF = 90^\circ$ (given perpendiculars). 1

Step 2 — The hypotenuse (H). $CA = CF$, both being radii of the same circle.

Step 3 — The side (S). $CE = CH$ (given). 1

Step 4 — Apply the criterion. By **RHS congruence**, $\triangle CEA \cong \triangle CHF$, so $AE = FH$.

Step 5 — Double both. By Theorem 5, E and H are midpoints, so $AB = 2AE$ and $GF = 2FH$.

Hence $AB = 2AE = 2FH = GF$. 1

Hence $AB = GF$ — this establishes Theorem 7.

Q3. Solve the previous question using the Baudhāyana–Pythagoras theorem.

2 marks

PROOF:

Step 1 — Right triangle on the first chord. In $\triangle CEA$, right-angled at E:

$$AE^2 = CA^2 - CE^2 = r^2 - CE^2. \quad 1$$

Step 2 — Right triangle on the second chord. In $\triangle CHF$, right-angled at H:

$$FH^2 = CF^2 - CH^2 = r^2 - CH^2.$$

Step 3 — Use the given equality. $CE = CH$, so the two right-hand sides are equal:

$$AE^2 = FH^2, \text{ hence } AE = FH.$$

Step 4 — Double. $AB = 2AE = 2FH = GF. \quad 1$

Hence $AB = GF$.

Compare the two routes. The congruence proof needs the correct criterion (RHS); the Pythagoras proof needs no criterion — but both still need Theorem 5 to say that E and H are midpoints. Neither proof is complete without that step.

Common Mistakes in Exercise Set 5.4

Skipping Theorem 5. You cannot write $AE = AB/2$ without first saying that the perpendicular from the centre bisects the chord.

Using SSS where RHS is needed. In Q2 the given data are a right angle, a hypotenuse and a side — that is RHS.

Writing $CE^2 = CH^2$ and stopping. Add the line “since lengths are positive, $CE = CH$ ”.

Forgetting to double at the end of Q2/Q3. You prove $AE = FH$; the chords are twice these.

◆ END OF EXERCISE SET 5.4 ◆

EXERCISE SET 5.5



What Exercise Set 5.5 Tests

One formula, three rearrangements

You want	Formula	Question
Chord length ℓ	$\ell = 2\sqrt{r^2 - d^2}$	Q1, Q2
Distance d	$d = \sqrt{r^2 - \left(\frac{\ell}{2}\right)^2}$	Q3
Radius r	$r = \sqrt{d^2 + \left(\frac{\ell}{2}\right)^2}$	—

The master right triangle. Drop a perpendicular from the centre O to a chord AB, meeting it at M. Then $OM = d$, $AM = \frac{\ell}{2}$ and $OA = r$, and $\triangle OMA$ is right-angled at M. Almost every numerical question in this chapter is this one triangle: $r^2 = d^2 + \left(\frac{\ell}{2}\right)^2$.

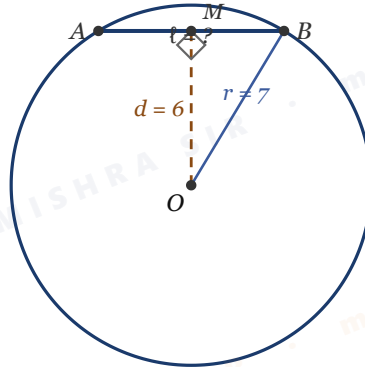
Theorem 8: of two unequal chords, the **longer** one is **nearer** the centre. The diameter (distance 0) is therefore the longest chord of all.

A

Exercise Set 5.5 — Chord length, radius and distance

Q1. Find the length of the chord of a circle where the radius is 7 cm and the perpendicular distance is 6 cm.

2 marks



Radius 7 cm; the chord lies 6 cm from the centre. $OM = 6$, $OA = 7$ and $\angle OMA = 90^\circ$ (schematic, not to scale)

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari, Part I*.

SOLUTION:

Step 1 — Name the parts. Let AB be the chord and M its midpoint, so $OM = 6$ cm, $OA = 7$ cm and $\angle OMA = 90^\circ$.

Step 2 — Find the half-chord. $AM^2 = OA^2 - OM^2 = 7^2 - 6^2 = 49 - 36 = 13$, so $AM = \sqrt{13}$ cm. 1

Step 3 — Double it. $AB = 2AM = 2\sqrt{13} \approx 2 \times 3.606$.

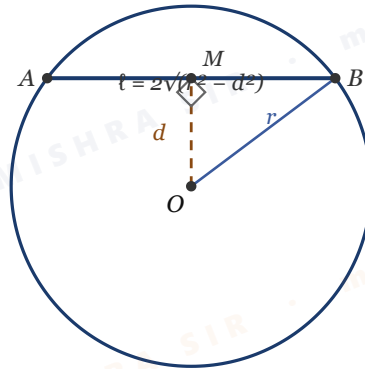
Chord $AB = 2\sqrt{13} \approx 7.21$ cm. 1

Do not stop at $\sqrt{13}$ — that is only half the chord. Forgetting to double is the commonest error here.

Q2. Explain why the following statement is true: if the perpendicular distance of a chord from the centre is d and the radius is r , then the chord length is

3 marks

$$2\sqrt{r^2 - d^2}.$$



The general figure — O the centre, M the foot of the perpendicular, $OM = d$, $OA = r$, $AM =$ half the chord

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari*, Part I.

GIVEN: A circle with centre O and radius r ; a chord AB at perpendicular distance d from O . M is the foot of the perpendicular from O to AB .

TO SHOW: $AB = 2\sqrt{r^2 - d^2}$.

PROOF:

Step 1 — M is the midpoint. By Theorem 5, the perpendicular from the centre bisects the chord, so $AB = 2 AM$. 1

Step 2 — Identify the right triangle. $\triangle OMA$ is right-angled at M , with hypotenuse $OA = r$ and $OM = d$. 1

Step 3 — Apply Baudhāyana–Pythagoras. $OA^2 = OM^2 + AM^2$, i.e. $r^2 = d^2 + AM^2$.

Step 4 — Solve for the half-chord. $AM^2 = r^2 - d^2$, so $AM = \sqrt{r^2 - d^2}$.

Step 5 — Double. $AB = 2 AM = 2\sqrt{r^2 - d^2}$. 1

Two sanity checks. Put $d = 0$: the chord passes through the centre and $\ell = 2r$ — the diameter, the longest chord. Put $d = r$: $\ell = 0$ — the chord has shrunk to a point on the circle. Both agree with the Comment following Theorem 8.

Q3.* In a circle, if the distance of chord AB from the centre is twice the distance of another chord CD from the centre, can we conclude that $CD = 2 AB$? Give reasons.

3 marks

ANSWER: NO — THE CONCLUSION DOES NOT FOLLOW.

1

REASONING:

Step 1 — Set up. Let the distance of CD be d ; then the distance of AB is $2d$. Using

$$\ell = 2\sqrt{r^2 - d^2}:$$

$$AB = 2\sqrt{r^2 - 4d^2}, \quad CD = 2\sqrt{r^2 - d^2}.$$

Step 2 — Test the claim. $CD = 2AB$ would require $\sqrt{r^2 - d^2} = 2\sqrt{r^2 - 4d^2}$.

Step 3 — Square and simplify. $r^2 - d^2 = 4r^2 - 16d^2 \Rightarrow 15d^2 = 3r^2 \Rightarrow d = \frac{r}{\sqrt{5}}$.

1

Step 4 — Interpret. The relation holds only for that single special distance — not in general.

COUNTER-EXAMPLE: take $r = 5$ cm and $d = 1$ cm (so AB is 2 cm from the centre).

$$AB = 2\sqrt{25 - 4} = 2\sqrt{21} \approx 9.17 \text{ cm}, \quad CD = 2\sqrt{25 - 1} = 2\sqrt{24} \approx 9.80 \text{ cm}.$$

Here CD is only slightly larger than AB — nowhere near double.

1

The idea to carry away: chord length is **not** proportional to distance from the centre — the relation involves squares. All that can safely be said (Theorem 8) is that the chord nearer the centre is the longer one, so $CD > AB$.

⚠ Common Mistakes in Exercise Set 5.5

Leaving the answer as the half-chord. $\sqrt{r^2 - d^2}$ is half. Double it.

Quoting Theorem 5 only in your head. The step “the perpendicular from the centre bisects the chord” must be written down in Q2 — it carries a mark.

Assuming proportionality (Q3). Double the distance does not halve the chord. Squares are involved.

Not checking that $2d < r$. In Q3 the chord AB only exists if its distance $2d$ is less than the radius.

◆ END OF EXERCISE SET 5.5 ◆

EXERCISE SET 5.6



What Exercise Set 5.6 Tests

Theorem 9 and its consequences

Result	Statement
Theorem 9	The angle an arc subtends at the centre is twice the angle it subtends at any point of the circle outside the arc
Same segment	Angles subtended by the same arc at different points on the same side are equal
Corollary	The angle in a semicircle is 90° (a diameter subtends 180° at the centre)
Theorem 10	If AB subtends equal angles at two points on the same side of AB, all four points are concyclic
Theorems 11 & 12	Opposite angles of a cyclic quadrilateral add to 180° , and conversely

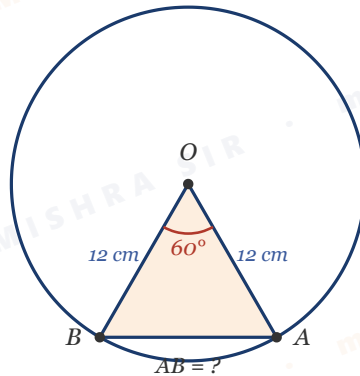
Halve when you step inward to outward. Moving from the centre to a point on the circle halves the angle. Moving the other way doubles it. Getting this direction wrong is the single biggest source of lost marks in this section.

A

Exercise Set 5.6 — Angles subtended by arcs

Q1. In a circle with centre O , the central angle AOB is 60° . If the radius of the circle is 12 cm, what is the length of the chord AB ?

2 marks



$OA = OB = 12\text{ cm}$ and the angle at O is drawn at its true size of 60° — which forces $\triangle OAB$ to be equilateral

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari*, Part I.

SOLUTION:

Step 1 — The triangle is isosceles. $OA = OB = 12\text{ cm}$ (radii), so $\angle OAB = \angle OBA$.

Step 2 — Find the base angles. $\angle OAB = \angle OBA = \frac{180^\circ - 60^\circ}{2} = 60^\circ$. 1

Step 3 — Classify. All three angles are 60° , so $\triangle OAB$ is **equilateral**.

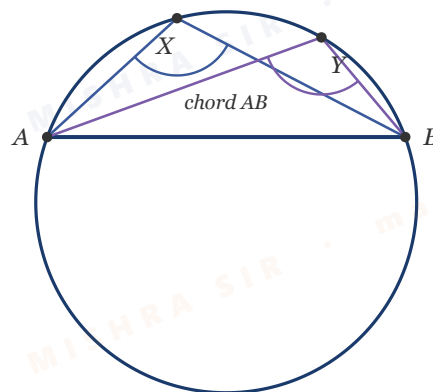
Step 4 — Read off the chord. $AB = OA = 12\text{ cm}$.

Chord $AB = 12\text{ cm}$. 1

Worth remembering: a chord subtending 60° at the centre always equals the radius.
That is precisely why a regular hexagon inscribed in a circle has side equal to the radius.

Q2. Let A and B be two points on a circle with centre O. (i) Are there points X, Y on the circle, on the same side of AB, such that $\angle AXB \neq \angle AYB$? (ii) If $\angle AXB = \angle AYB$, must X and Y lie on the same side? (iii) If $\angle AXB = \angle AYB$ and X, Y are not on the circle, does the circle through A, B, X also pass through Y?

3 marks



X and Y on the same side of $AB \rightarrow \angle AXB = \angle AYB$

X and Y both lie on the same side of chord AB . The marked angles $\angle AXB$ and $\angle AYB$ are equal, wherever X and Y are taken on that arc

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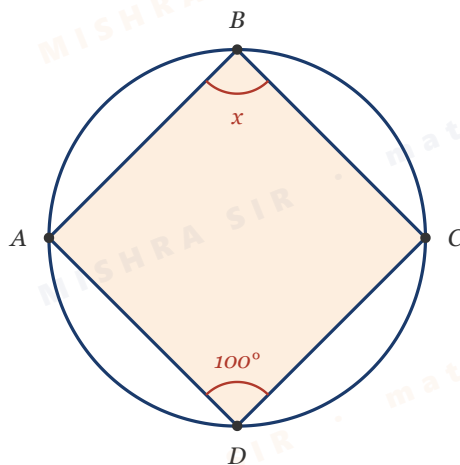
(I) NO SUCH POINTS EXIST. If X and Y are on the circle on the **same** side of AB , they lie on the same arc. By Theorem 9 each of $\angle AXB$ and $\angle AYB$ equals half the same central angle, so $\angle AXB = \angle AYB$ always. This is the “angles in the same segment” result. **1**

(II) NOT NECESSARILY. Points on **opposite** sides of AB give angles that add to 180° (Theorem 11). Two such angles are equal exactly when each is 90° — which happens when AB is a **diameter**. So if $\angle AXB = \angle AYB = 90^\circ$, X and Y may lie on opposite sides. For any other value, equality does force them onto the same side. **1**

(III) YES — PROVIDED X AND Y ARE ON THE SAME SIDE OF AB. This is exactly **Theorem 10**: if a segment AB subtends equal angles at two points on the same side of AB , the four points are concyclic. Hence the circle through A , B and X must also pass through Y . **1**

Q3. Find x in Fig. 5.26.

1 mark



Cyclic quadrilateral of Fig. 5.26 — the angle at D is 100° and x is the angle at B; these are opposite angles

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SOLUTION:

Step 1 — Identify the pair. In the cyclic quadrilateral, the angle marked 100° at D and the angle x at B are **opposite** angles.

Step 2 — Apply Theorem 11. The sum of a pair of opposite angles of a cyclic quadrilateral is 180° : $x + 100^\circ = 180^\circ$.

Step 3 — Solve. $x = 80^\circ$.

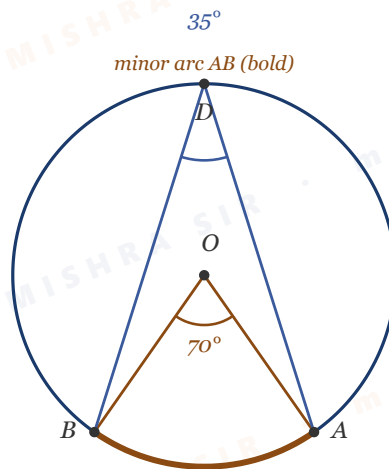
$x = 80^\circ$

1

B In-text exercises from Sections 5.7 and 5.8

In-text Exercise (page 107, Fig. 5.19). State whether arcs AKB and CLD are minor arcs or major arcs.

1 mark



How to classify an arc — measure the angle it subtends at the centre, sweeping **along that arc**. Here the bold arc subtends 70° , so it is a minor arc

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METHOD:

Step 1 — Sweep along the arc. Measure the angle at the centre as you move from one end of the arc to the other along that arc.

Step 2 — Apply the test. Angle at the centre less than $180^\circ \rightarrow$ **minor arc**. Greater than $180^\circ \rightarrow$ **major arc**.

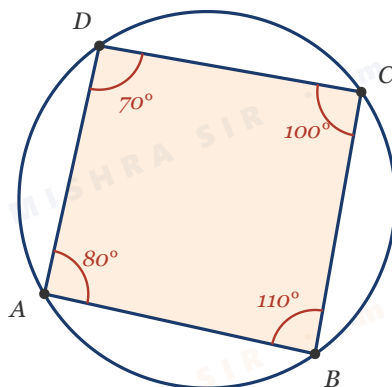
Step 3 — Read the figure. In Fig. 5.19, K lies on the shorter route from A to B, so **arc AKB is a minor arc**. L lies on the longer route from C to D, so **arc CLD is a major arc**.

1

The middle letter matters. "Arc AKB" means the route from A to B **through K** — it tells you which of the two possible arcs is meant.

In-text Exercise (page 113). A cyclic quadrilateral has $\angle A = 80^\circ$, $\angle B = 110^\circ$, $\angle C = 100^\circ$, $\angle D = 70^\circ$. Can such a quadrilateral be drawn? Explain.

2 marks



All four angles marked — both pairs of opposite angles add to 180° , so the quadrilateral is genuinely cyclic

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SOLUTION — TWO CHECKS ARE NEEDED:

Step 1 — Angle sum of a quadrilateral. $80^\circ + 110^\circ + 100^\circ + 70^\circ = 360^\circ$ ✓ 1

Step 2 — Opposite angles supplementary (Theorem 11). $\angle A + \angle C = 80^\circ + 100^\circ = 180^\circ$ ✓
and $\angle B + \angle D = 110^\circ + 70^\circ = 180^\circ$ ✓

Step 3 — Conclude. Both conditions hold, so by Theorem 12 (the converse) the four vertices are concyclic.

Yes — such a cyclic quadrilateral can be drawn. 1

Test yourself: $\angle A = 80^\circ$, $\angle B = 100^\circ$, $\angle C = 110^\circ$, $\angle D = 70^\circ$ also sums to 360° , but $\angle A + \angle C = 190^\circ$.
That quadrilateral exists — but it cannot be cyclic.

⚠ Common Mistakes in Exercise Set 5.6

Doubling when you should halve. Centre $\rightarrow$ circle: *halve*. Circle $\rightarrow$ centre: *double*.

Using “angles in the same segment” for points on opposite sides. Same side $\rightarrow$ equal.
Opposite sides $\rightarrow$ supplementary.

Missing the 90° exception in Q2(ii). Equal angles usually force the same side — but not when both are 90° and AB is a diameter.

Pairing adjacent angles in a cyclic quadrilateral. Only *opposite* angles are supplementary.

Ignoring the middle letter of an arc. Arc AKB and arc AXB are different arcs of the same chord.

◆ END OF EXERCISE SET 5.6 ◆

END-OF-CHAPTER EXERCISES

How the 26 Questions Split Up

Sort the question first — each family has one standard opening move

Family	Opening move	Questions
Chord – radius – distance	Draw the right triangle and use $r^2 = d^2 + \left(\frac{\ell}{2}\right)^2$	Q1, Q3, Q4, Q9, Q13, Q18, Q22, Q23
Angles in a circle	Angle at centre = $2 \times$ angle on the circle; angle in a semicircle = 90°	Q2, Q6, Q20, Q24
Cyclic quadrilaterals	Opposite angles add to 180°	Q7, Q8, Q10, Q11, Q21, Q26
Perpendicular / bisector proofs	Centre is equidistant from the ends of every chord	Q5, Q12, Q16, Q17, Q25
Inscribed figures	Every side is a chord; use the angle results on each	Q14, Q15, Q19

Before you start any numerical question: write down which of r , d , ℓ you are given and which you want. Two of the three are always supplied. This single habit prevents most errors in this exercise.

A Chord, radius and distance — Q1, Q3, Q4, Q9, Q13, Q18, Q22, Q23

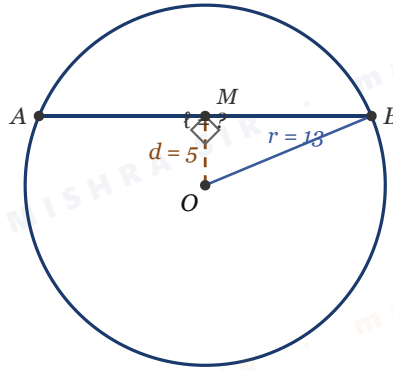
One formula for all of these

$r^2 = d^2 + \left(\frac{\ell}{2}\right)^2$ — radius, perpendicular distance from the centre, half the chord. The

Pythagorean triples (3, 4, 5), (5, 12, 13), (8, 15, 17) and (7, 24, 25) appear again and again here; spotting them saves time.

Q1. A chord is 5 cm from the centre. The radius is 13 cm. Find the length of the chord.

2 marks



Radius 13 cm, chord at perpendicular distance 5 cm from the centre (schematic, not to scale)

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

SOLUTION:

Step 1 — Name the parts. Let AB be the chord and M its midpoint. Then $OM = 5$, $OA = 13$ and $\angle OMA = 90^\circ$.

Step 2 — Find the half-chord. $AM^2 = 13^2 - 5^2 = 169 - 25 = 144$, so $AM = 12$ cm.

1

Step 3 — Double it. $AB = 2 \times 12 = 24$ cm.

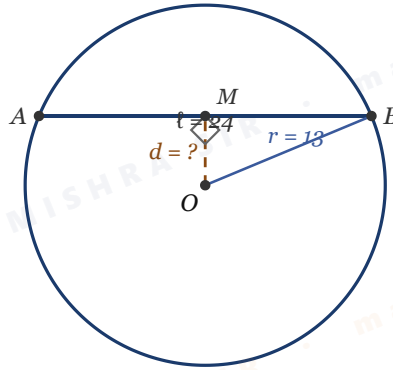
Chord $AB = 24$ cm.

1

The triple $(5, 12, 13)$ — recognise it and no square roots are needed.

Q3. The diameter of a circle is 26 cm. A chord of length 24 cm is drawn. Find the distance from the centre to the chord.

2 marks



Diameter 26 cm (so radius 13 cm) with a chord of 24 cm — the required distance is d

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SOLUTION:

Step 1 — Convert diameter to radius. Diameter = 26 cm, so $r = 13$ cm.

1

Step 2 — Halve the chord. Half-chord = $\frac{24}{2} = 12$ cm.

Step 3 — Apply the formula. $d^2 = r^2 - \left(\frac{\ell}{2}\right)^2 = 13^2 - 12^2 = 169 - 144 = 25$, so $d = 5$ cm.

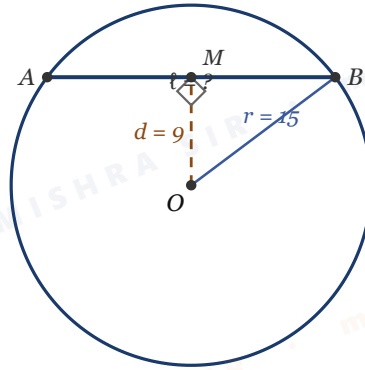
Distance from the centre = 5 cm.

1

Q1 and Q3 are the same triangle read in opposite directions. Halve the diameter before you begin — a very common slip.

**Q4. A circle has radius 15 cm. The distance from the centre to a chord is 9 cm.
What is the length of the chord?**

2 marks



Radius 15 cm, chord at distance 9 cm from the centre

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SOLUTION:

Step 1 — Find the half-chord. $\sqrt{15^2 - 9^2} = \sqrt{225 - 81} = \sqrt{144} = 12$ cm.

1

Step 2 — Double it. Chord = $2 \times 12 = 24$ cm.

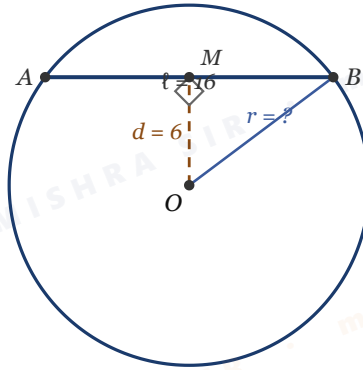
Chord = 24 cm.

1

This is (9, 12, 15) — the triple (3, 4, 5) multiplied by 3.

Q9. The distance of a chord of length 16 cm from the centre is 6 cm. Find the radius.

2 marks



Chord 16 cm at distance 6 cm from the centre — the radius is the hypotenuse

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SOLUTION:

Step 1 — List what is given. Half-chord = $\frac{16}{2} = 8$ cm; distance $d = 6$ cm. 1

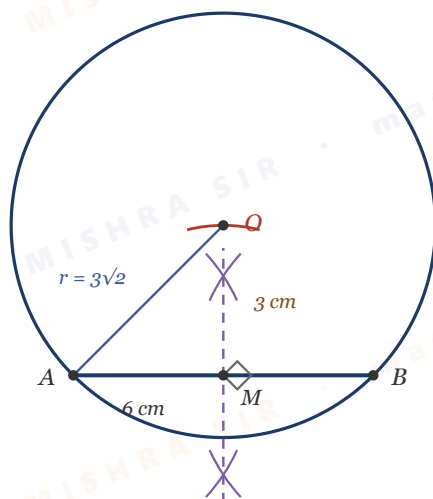
Step 2 — Apply the formula. $r^2 = d^2 + \left(\frac{\ell}{2}\right)^2 = 6^2 + 8^2 = 36 + 64 = 100$.

Step 3 — Take the square root. $r = 10$ cm.

Radius = 10 cm. 1

Q13.* Draw a circle in which a chord of 6 cm stands at a distance of 3 cm from the centre.

3 marks



Construction for Q13 — purple arcs from A and B give the perpendicular bisector of AB; the red arc marks off $OM = 3$ cm to fix the centre O

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FIRST FIND THE RADIUS:

$$r^2 = 3^2 + 3^2 = 9 + 9 = 18 \Rightarrow r = 3\sqrt{2} \approx 4.24 \text{ cm}$$

1

CONSTRUCTION — STEP BY STEP:

Step 1. Draw a line segment $AB = 6$ cm.

Step 2. With centre A and a radius more than 3 cm, draw arcs above and below AB. Repeat with the same radius from B. The purple arcs cross at two points.

Step 3. Join those two crossing points — this dashed line is the perpendicular bisector of AB, and it meets AB at its midpoint M.

Step 4. Along that perpendicular, use the compasses to mark off $OM = 3$ cm (the red arc). This fixes the centre O.

1

Step 5. With centre O and radius OA (≈ 4.2 cm) draw the circle. It passes through B as well, since $OA = OB$.

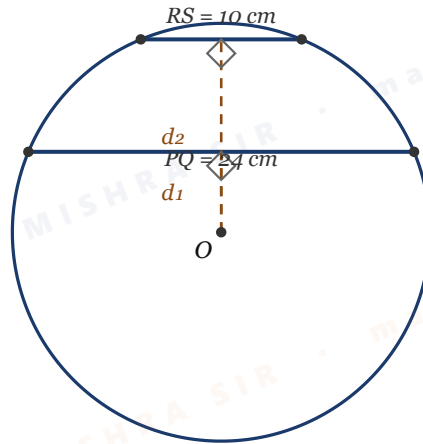
1

THE HINT — IS IT A CIRCUMCIRCLE? Yes. O is equidistant from A and B and from every point of the circle, so the circle drawn is the circumcircle of any triangle with AB as one side and its third

vertex on the circle — for instance the isosceles triangle OAB reflected, or $\triangle ABC$ for any C taken on the circle.

Q18. Two parallel chords of lengths 10 cm and 24 cm lie on the same side of the centre. The distance between them is 7 cm. Find the radius.

4 marks



Chords of 24 cm and 10 cm on the same side of the centre, 7 cm apart — here $d_2 - d_1 = 7$ (schematic, not to scale)

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SOLUTION: Let the distances of the 24 cm and 10 cm chords from the centre be d_1 and d_2 .

The longer chord lies nearer the centre (Theorem 8), so $d_1 < d_2$, and as they are on the **same side**:

$$d_2 - d_1 = 7 \dots (i)$$

1

From the master relation, using the same radius r for both:

$$r^2 = d_1^2 + 12^2 = d_1^2 + 144$$

$$r^2 = d_2^2 + 5^2 = d_2^2 + 25$$

1

$$\text{Equating: } d_1^2 + 144 = d_2^2 + 25 \Rightarrow d_2^2 - d_1^2 = 119$$

$$(d_2 - d_1)(d_2 + d_1) = 119 \Rightarrow 7(d_2 + d_1) = 119 \Rightarrow d_2 + d_1 = 17 \dots (ii)$$

1

Solving (i) and (ii): $d_2 = 12$, $d_1 = 5$.

$$r^2 = 5^2 + 144 = 169 \Rightarrow r = 13 \text{ cm}$$

1

The trick worth learning: do not solve for d_1 and d_2 separately by brute force. Use the difference of squares $d_2^2 - d_1^2 = (d_2 - d_1)(d_2 + d_1)$ — you already know the first bracket, so the second falls out at once.

Q22.* “There is no chord of a circle longer than its diameter.” Justify.

2 marks

PROOF 1 (USING THE CHORD FORMULA): A chord at perpendicular distance d from the centre has length $\ell = 2\sqrt{r^2 - d^2}$. Since $d \geq 0$, we get $r^2 - d^2 \leq r^2$, so

$\ell \leq 2\sqrt{r^2} = 2r = \text{the diameter.}$

1

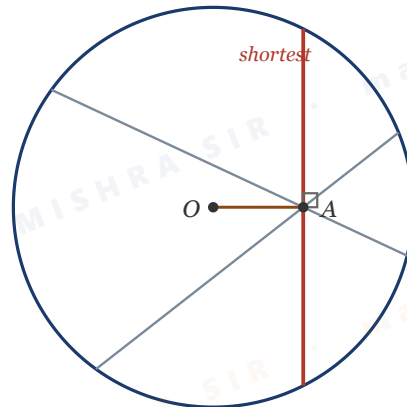
Equality holds only when $d = 0$, i.e. when the chord passes through the centre — so the diameter is the unique longest chord.

1

PROOF 2 (USING THE TRIANGLE INEQUALITY): For any chord AB , join OA and OB . In $\triangle OAB$, $AB \leq OA + OB = r + r = 2r$, with equality only when O lies on AB — that is, when AB is a diameter.

Q23.* A is any point inside a circle with centre O. Show that the shortest chord through A is the one perpendicular to OA.

3 marks



Several chords through the interior point A. The one perpendicular to OA (bold) is the shortest — it is the farthest of them from the centre.

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GIVEN: A point A inside a circle with centre O, radius r . Let $OA = a$.

TO SHOW: the chord through A perpendicular to OA is the shortest such chord.

PROOF:

Step 1. Any chord through A has some perpendicular distance d from O, and its length is $\ell = 2\sqrt{r^2 - d^2}$. So ℓ is smallest exactly when d is **largest**. 1

Step 2. Let M be the foot of the perpendicular from O to a chord through A. Then M lies on that chord, and $\triangle OMA$ is right-angled at M (or M coincides with A). Since the hypotenuse is the longest side,

$$d = OM \leq OA = a.$$

1

Step 3. So the largest possible value of d is a , and it is attained exactly when M coincides with A — that is, when OA itself is the perpendicular from O to the chord, i.e. when the chord is **perpendicular to OA**.

$\therefore$ the shortest chord through A is the one perpendicular to OA, and its length is $2\sqrt{r^2 - a^2}$.

1

The longest chord through A is, of course, the diameter through A — there $d = 0$.

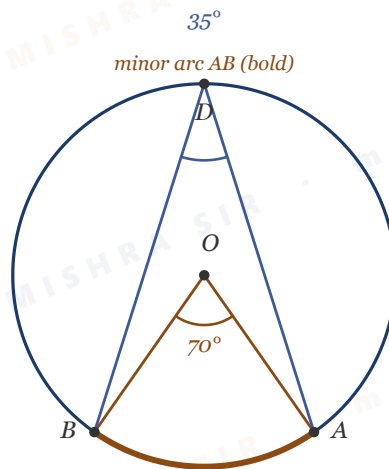
B Angles in a circle — Q2, Q6, Q20, Q24

Halve when you step from the centre to the circle

Theorem 9: the angle an arc makes at the **centre** is twice the angle it makes at any point on the **circle** outside that arc. The special case — a diameter subtends 180° at the centre, hence 90° at the circle — is the single most used fact in Board papers.

Q2. An arc subtends 70° at the centre. What angle does it subtend at a point on the circle?

1 mark



The bold minor arc AB subtends 70° at the centre O (drawn at true size) and 35° at the point D on the circle outside that arc

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SOLUTION:

Step 1 — Quote the theorem. By Theorem 9, the angle at the centre is **twice** the angle at any point of the circle outside the arc.

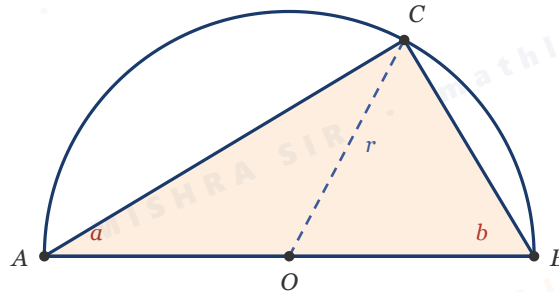
Step 2 — Halve the central angle. $\angle ADB = \frac{1}{2} \times 70^\circ = 35^\circ$.

The arc subtends 35° at a point on the circle.

1

Q6. AB is the diameter of a circle and C lies on the circumference. What is $\angle ACB$? Explain.

2 marks



AB is a diameter and C lies on the circle — the angle in a semicircle is a right angle

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SOLUTION: $\angle ACB = 90^\circ$.

1

REASONING:

Step 1 — The central angle. AB is a diameter, so A, O and B are collinear and the arc AB not containing C subtends a **straight angle** at the centre: $\angle AOB = 180^\circ$.

Step 2 — Halve it. By Theorem 9 the same arc subtends half that angle at C:

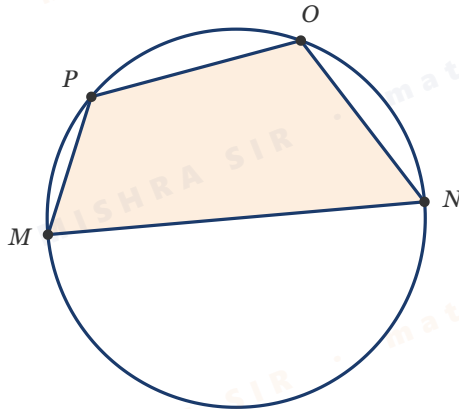
$$\angle ACB = \frac{1}{2} \times 180^\circ = 90^\circ.$$

1

Step 3 — Name the result. This is the corollary on page 110 — **the angle in a semicircle is a right angle.**

Q20. Quadrilateral MNOP is inscribed in a circle. MN is a diameter. What can you say about $\angle MOP$ and $\angle MNP$?

2 marks



Cyclic quadrilateral MNOP — N and O lie on the same arc cut off by the chord MP

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

SOLUTION: $\angle MOP = \angle MNP$.

1

REASONING: Both angles stand on the same chord MP. Since the vertices are in the cyclic order M, N, O, P, the points N and O lie on the **same arc** determined by MP — that is, on the same side of MP. Angles in the same segment are equal (Theorem 9), so $\angle MOP = \angle MNP$.

1

USING THE DIAMETER AS WELL: since MN is a diameter, the angle it subtends at any other point of the circle is 90° . Hence

$$\angle MON = 90^\circ \quad \text{and} \quad \angle MPN = 90^\circ.$$

So MNOP has right angles at O and P when measured against the diameter MN.

Q24. How would you use Fig. 5.30 to justify that the angle in a semicircle is 90° ?

3 marks

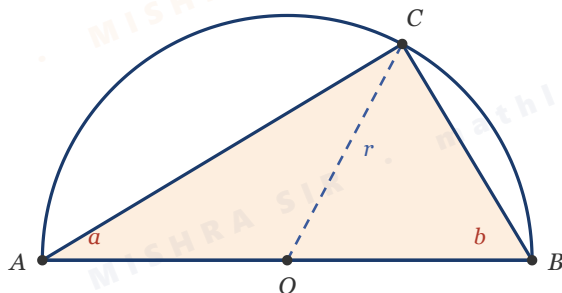


Fig. 5.30 schematic — O is the midpoint of the diameter and OC is drawn; $OA = OB = OC = r$ creates two isosceles triangles

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SETTING UP: AB is the diameter, O its midpoint (so O is the centre), and C is a point on the semicircle. Join OC . Let $\angle CAB = a$ and $\angle CBA = b$.

PROOF:

Step 1. $OA = OC$ (both radii), so $\triangle OAC$ is isosceles and $\angle OCA = \angle OAC = a$.

1

Step 2. $OB = OC$ (both radii), so $\triangle OBC$ is isosceles and $\angle OCB = \angle OBC = b$.

1

Step 3. The angle at C of $\triangle ABC$ is $\angle ACB = \angle OCA + \angle OCB = a + b$.

Now apply the angle sum of $\triangle ABC$:

$$a + b + (a + b) = 180^\circ \Rightarrow 2(a + b) = 180^\circ \Rightarrow a + b = 90^\circ$$

$$\therefore \angle ACB = 90^\circ$$

1

Why this proof is worth knowing: it uses nothing beyond “two radii make an isosceles triangle” and the angle sum of a triangle — no Theorem 9 required. Examiners like it for exactly that reason.

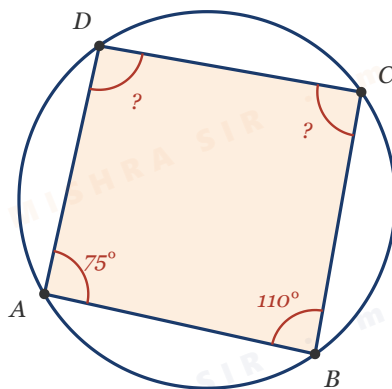
C Cyclic quadrilaterals — Q7, Q8, Q10, Q11, Q21, Q26

The one rule, and its converse

Theorem 11: in a cyclic quadrilateral, each pair of opposite angles adds to 180° . **Theorem 12 (converse):** if one pair of opposite angles of a quadrilateral adds to 180° , the quadrilateral is cyclic. Note carefully: *adjacent* angles have no such relation.

Q7. ABCD is a cyclic quadrilateral. If $\angle A = 75^\circ$, find $\angle C$. If $\angle B = 110^\circ$, find $\angle D$.

2 marks



Cyclic quadrilateral ABCD with the two given angles marked — A and C are opposite, as are B and D

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SOLUTION:

Step 1 — Quote the rule. In a cyclic quadrilateral, each pair of **opposite** angles is supplementary (Theorem 11).

Step 2 — Use the first pair. $\angle A + \angle C = 180^\circ \Rightarrow \angle C = 180^\circ - 75^\circ = 105^\circ$.

1

Step 3 — Use the second pair. $\angle B + \angle D = 180^\circ \Rightarrow \angle D = 180^\circ - 110^\circ = 70^\circ$.

1

Step 4 — Check. $75 + 110 + 105 + 70 = 360^\circ \checkmark$

$\angle C = 105^\circ$ and $\angle D = 70^\circ$.

Q8. PQRS is inscribed in a circle. $\angle P = (2x + 10)^\circ$ and $\angle R = (3x - 20)^\circ$. Find x , $\angle P$ and $\angle R$.

3 marks

SOLUTION:

Step 1 — Spot the opposite pair. In quadrilateral PQRS the vertices P and R are **opposite**, so $\angle P + \angle R = 180^\circ$. 1

Step 2 — Form the equation. $(2x + 10) + (3x - 20) = 180$.

Step 3 — Solve for x . $5x - 10 = 180 \Rightarrow 5x = 190 \Rightarrow x = 38$. 1

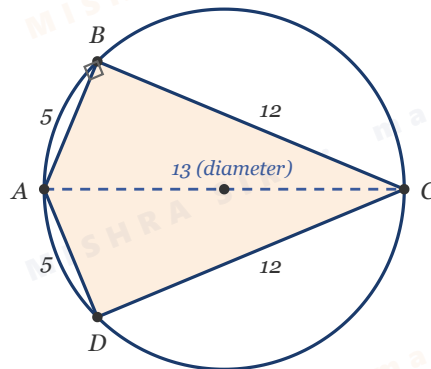
Step 4 — Substitute back. $\angle P = 2(38) + 10 = 86^\circ$ and $\angle R = 3(38) - 20 = 94^\circ$. 1

Step 5 — Check. $86^\circ + 94^\circ = 180^\circ \checkmark$

$x = 38$, $\angle P = 86^\circ$, $\angle R = 94^\circ$.

Q10. A cyclic quadrilateral has sides 5, 5, 12, 12 units. Find its area.

4 marks



The equal sides are adjacent, so the quadrilateral is a kite. Being cyclic, its axis of symmetry AC is a diameter and the angles at B and D are right angles.

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STEP 1 — IDENTIFY THE SHAPE. The two pairs of equal sides are **adjacent** (5, 5 then 12, 12), so ABCD is a **kite** with $AB = AD = 5$ and $BC = DC = 12$. **1**

STEP 2 — USE THE CYCLIC CONDITION. By symmetry $\angle B = \angle D$. Being opposite angles of a cyclic quadrilateral, $\angle B + \angle D = 180^\circ$.

$$\therefore 2\angle B = 180^\circ \Rightarrow \angle B = \angle D = 90^\circ \quad \mathbf{1}$$

STEP 3 — SPLIT INTO TWO RIGHT TRIANGLES. The diagonal AC divides the kite into $\triangle ABC$ and $\triangle ADC$, each right-angled with legs 5 and 12.

$$\text{Area of each} = \frac{1}{2} \times 5 \times 12 = 30 \text{ sq units.} \quad \mathbf{1}$$

$$\text{Total area} = 2 \times 30 = 60 \text{ square units.} \quad \mathbf{1}$$

CHECK BY BRAHMAGUPTA'S FORMULA (for a cyclic quadrilateral with sides a, b, c, d and

$$s = \frac{a+b+c+d}{2}:$$

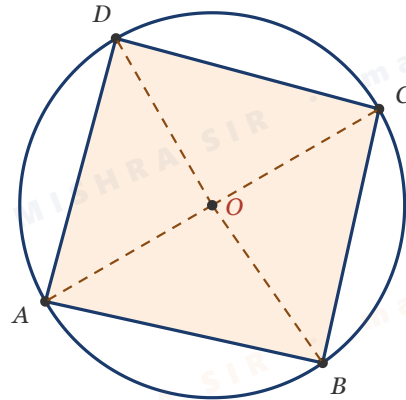
$$s = \frac{5 + 5 + 12 + 12}{2} = 17$$

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)} = \sqrt{12 \times 12 \times 5 \times 5} = 12 \times 5 = 60 \checkmark$$

A bonus fact: since $\angle B = 90^\circ$, the diagonal AC subtends a right angle at B — so AC is a **diameter**. Its length is $\sqrt{5^2 + 12^2} = 13$, giving a circumradius of 6.5 units.

Q11.* Without drawing the circumcircle, how can we tell whether its centre lies inside or outside a cyclic quadrilateral?

3 marks



Here every side subtends less than 90° at the opposite vertices, so the centre O falls inside the quadrilateral

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THE TEST: look at each **side** and the angle it subtends at one of the two remaining vertices.

- If that inscribed angle is **less than 90°** , the corresponding central angle is less than 180° , so the centre lies on the same side of that chord as the quadrilateral. **1**
- If it is **greater than 90°** , the central angle exceeds 180° , and the centre lies on the other side of that chord — hence **outside** the quadrilateral.
- If it is exactly 90° , that side is a diameter and the centre lies **on** it. **1**

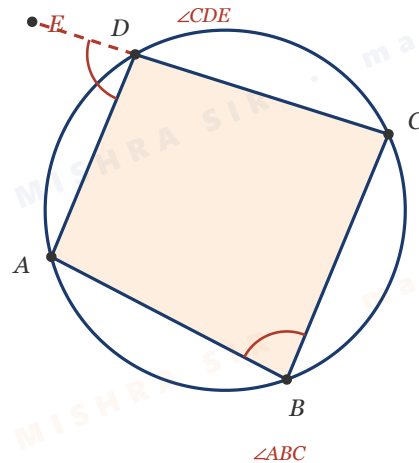
CONCLUSION: the centre lies **inside** the quadrilateral precisely when every side subtends an angle less than 90° at the opposite vertices — equivalently, when each of the four arcs cut off by the sides is smaller than a semicircle.

BEST PRACTICAL WAY: measure the four angles of the quadrilateral, work out the inscribed angle standing on each side, and apply the test above. No circle needs to be drawn at all. **1**

A quick check on the extreme case: in a rectangle every side subtends less than 90° at the far vertices except the diagonals, and indeed the centre sits exactly at the intersection of the diagonals — inside. In a very “flat” cyclic quadrilateral one side becomes almost a diameter, and the centre slips outside.

Q21. Explain why the exterior angle of a cyclic quadrilateral at any vertex equals the interior opposite angle (e.g. $\angle CDE = \angle ABC$).

2 marks



equal angles are marked with the same arc

Cyclic quadrilateral ABCD with side CD produced to E. The equal angles $\angle CDE$ and $\angle ABC$ carry the same arc marking

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GIVEN: Cyclic quadrilateral ABCD, with side CD produced to a point E.

TO SHOW: $\angle CDE = \angle ABC$.

PROOF:

Step 1 — Linear pair at D. C, D and E are collinear, so $\angle ADC + \angle CDE = 180^\circ$. 1

Step 2 — Cyclic quadrilateral. $\angle ADC + \angle ABC = 180^\circ$ (opposite angles, Theorem 11).

Step 3 — Equate. Both sums equal 180° , so $\angle ADC + \angle CDE = \angle ADC + \angle ABC$.

Step 4 — Cancel. $\angle CDE = \angle ABC$. 1

A very useful shortcut in Board questions: the exterior angle of a cyclic quadrilateral equals the interior opposite angle — not the adjacent one. It saves a step whenever a side is produced.

Q26.* How would you use Fig. 5.31 to justify that the opposite angles of a cyclic quadrilateral add to 180° ?

4 marks

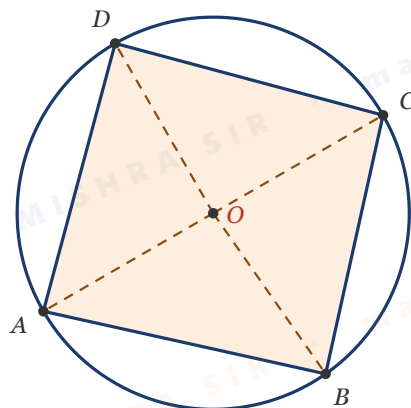


Fig. 5.31 schematic — the centre O is joined to all four vertices, creating four isosceles triangles (each pair of joining segments is a radius)

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari, Part I*.

SETTING UP: Join O to A , B , C and D . Since $OA = OB = OC = OD = r$, each of $\triangle OAB$, $\triangle OBC$, $\triangle OCD$, $\triangle ODA$ is isosceles, so the two base angles in each are equal. Call them:
 $\angle OAB = \angle OBA = \alpha$, $\angle OBC = \angle OCB = \beta$, $\angle OCD = \angle ODC = \gamma$, $\angle ODA = \angle OAD = \delta$.

1

PROOF:

Step 1. Read off the four angles of the quadrilateral:

$$\angle A = \alpha + \delta, \angle B = \alpha + \beta, \angle C = \beta + \gamma, \angle D = \gamma + \delta.$$

1

Step 2. The angles of a quadrilateral add to 360° :

$$(\alpha + \delta) + (\alpha + \beta) + (\beta + \gamma) + (\gamma + \delta) = 360^\circ$$

$$2(\alpha + \beta + \gamma + \delta) = 360^\circ \Rightarrow \alpha + \beta + \gamma + \delta = 180^\circ$$

1

Step 3. Now add a pair of opposite angles:

$$\angle A + \angle C = (\alpha + \delta) + (\beta + \gamma) = \alpha + \beta + \gamma + \delta = 180^\circ$$

$$\text{Similarly } \angle B + \angle D = (\alpha + \beta) + (\gamma + \delta) = 180^\circ.$$

1

Hence the opposite angles of a cyclic quadrilateral are supplementary.

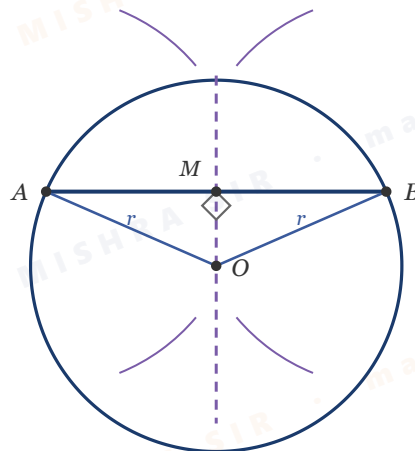
D Proofs about chords and the centre — Q5, Q12, Q16, Q17, Q25

Everything here starts from one sentence

The centre is equidistant from the two ends of every chord. That single fact places the centre on the perpendicular bisector of every chord, and it is the engine behind Q5, Q12, Q16, Q17 and Q25.

Q5. Prove that the perpendicular bisector of a chord passes through the centre of the circle.

3 marks



$OA = OB \rightarrow O$ lies on the perpendicular bisector of AB

$OA = OB$, both radii — so O is equidistant from A and B and must lie on the perpendicular bisector of AB
(constructed with the purple arcs)

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari*, Part I.

GIVEN: A circle with centre O and a chord AB . Let ℓ be the perpendicular bisector of AB .

TO SHOW: O lies on ℓ .

PROOF:

Step 1. A and B lie on the circle, so $OA = OB = r$ — O is equidistant from A and B . 1

Step 2. The perpendicular bisector of a segment is precisely the **locus** of all points equidistant from its two endpoints (used repeatedly in Section 5.3). 1

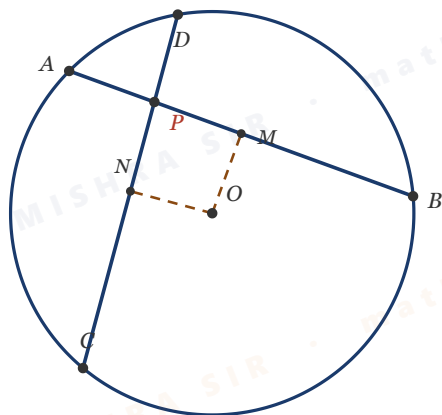
Step 3. Since O is equidistant from A and B , O belongs to that locus.

$\therefore$ the perpendicular bisector of AB passes through the centre O . 1

This is why the circumcentre is found by intersecting perpendicular bisectors — Theorem 1 is the same idea applied to three chords at once.

Q12.* Two chords of equal length intersect. Show that the segments of one chord equal the corresponding segments of the other.

4 marks



Equal chords AB and CD meeting at P; M and N are their midpoints, and $OM = ON$ because equal chords are equidistant from the centre

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GIVEN: Chords AB and CD of a circle with centre O, with $AB = CD$, intersecting at a point P.

TO SHOW: $AP = CP$ and $BP = DP$.

PROOF:

Step 1. Draw $OM \perp AB$ and $ON \perp CD$. Since $AB = CD$, by **Theorem 6** the two chords are equidistant from the centre:

$$OM = ON.$$

1

Step 2. By **Theorem 5**, M and N are the midpoints of AB and CD, so

$$AM = \frac{AB}{2} = \frac{CD}{2} = CN.$$

1

Step 3. Consider the right triangles $\triangle OMP$ and $\triangle ONP$:

$$\angle OMP = \angle ONP = 90^\circ; OP = OP \text{ (common hypotenuse); } OM = ON \text{ (Step 1).}$$

By **RHS congruence**, $\triangle OMP \cong \triangle ONP$, so $MP = NP$.

1

Step 4. Adding and subtracting along each chord:

$$AP = AM + MP = CN + NP = CP$$

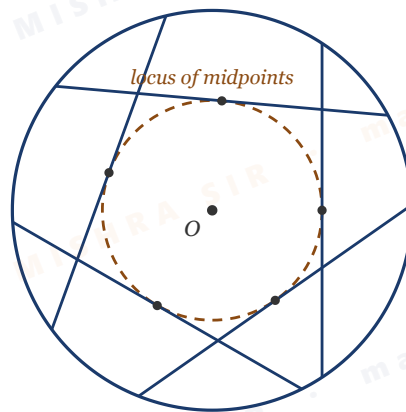
$$BP = BM - MP = DN - NP = DP$$

Hence the segments of one chord equal the corresponding segments of the other.

Depending on where P falls, one pair may need a subtraction instead of an addition — the equality of the two segment lengths is unaffected.

Q16.* Consider all chords of a circle of a fixed length. What shape is formed by the midpoints of all these chords?

3 marks



Chords of one fixed length drawn in every direction — all their midpoints lie on a smaller circle concentric with the original

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SOLUTION: A circle concentric with the given circle.

1

REASONING: Let the fixed chord length be ℓ and the radius be r . For any such chord, the line from the centre O to its midpoint M is perpendicular to the chord (Theorem 4), so $\triangle OMA$ is right-angled and

$$OM = \sqrt{r^2 - \left(\frac{\ell}{2}\right)^2}$$

1

This value depends only on r and ℓ , both of which are fixed — so **every** such midpoint lies at the same distance from O . The set of points at a fixed distance from a fixed point is a circle.

$\therefore$ the midpoints trace a circle with centre O and radius $\sqrt{r^2 - \left(\frac{\ell}{2}\right)^2}$.

1

TWO EDGE CASES: if $\ell = 2r$ the chords are diameters, the radius of the locus is 0, and the “circle” shrinks to the single point O . If ℓ is very small, the locus is almost the original circle itself.

Q17.* In a circle with centre O , chords AB and AC are congruent. Explain why the centre lies on the bisector of $\angle BAC$.

3 marks

GIVEN: Circle with centre O ; chords AB and AC with $AB = AC$.

TO SHOW: AO bisects $\angle BAC$.

PROOF 1 (CONGRUENCE): Consider $\triangle OAB$ and $\triangle OAC$.

$OA = OA$ (common)

$OB = OC$ (both radii)

1

$AB = AC$ (given)

By **SSS congruence**, $\triangle OAB \cong \triangle OAC$.

1

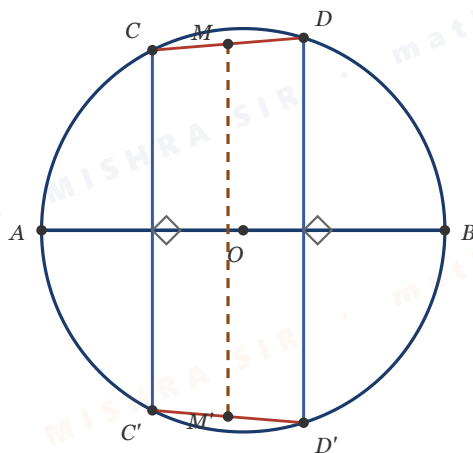
$\therefore \angle OAB = \angle OAC$ (corresponding parts), so AO bisects $\angle BAC$ — the centre lies on the angle bisector.

1

PROOF 2 (DISTANCES): By Theorem 6, equal chords are equidistant from the centre, so O is at the same perpendicular distance from line AB as from line AC . The locus of points equidistant from two intersecting lines is the pair of angle bisectors — and O lies inside $\angle BAC$, so it is on the bisector of $\angle BAC$ itself.

Q25.* Chords CC' and DD' are drawn perpendicular to a diameter AB . Prove that the segment MM' joining the midpoints of CD and $C'D'$ is perpendicular to AB .

4 marks



CC' and DD' are perpendicular to the diameter AB ; M is the midpoint of CD and M' the midpoint of $C'D'$.

MM' comes out perpendicular to AB .

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SET-UP: Take O as origin with the diameter AB along the horizontal axis, and let the radius be r .

$CC' \perp AB$, so CC' is a vertical chord — say it meets AB at the point with coordinate p . Its endpoints are

$$C = \left(p, \sqrt{r^2 - p^2}\right) \text{ and } C' = \left(p, -\sqrt{r^2 - p^2}\right). \quad 1$$

Similarly, DD' is vertical at coordinate q :

$$D = \left(q, \sqrt{r^2 - q^2}\right) \text{ and } D' = \left(q, -\sqrt{r^2 - q^2}\right). \quad 1$$

FINDING THE MIDPOINTS:

$$M = \text{midpoint of } CD = \left(\frac{p+q}{2}, \frac{\sqrt{r^2 - p^2} + \sqrt{r^2 - q^2}}{2}\right)$$

$$M' = \text{midpoint of } C'D' = \left(\frac{p+q}{2}, -\frac{\sqrt{r^2 - p^2} + \sqrt{r^2 - q^2}}{2}\right) \quad 1$$

CONCLUSION: M and M' have the **same** first coordinate, $\frac{p+q}{2}$. A segment joining two points with equal horizontal coordinates is vertical, and AB is horizontal.

$$\therefore MM' \perp AB. \quad 1$$

The geometric reason behind the algebra: reflection in the diameter AB is a symmetry

joining a point to its mirror image is perpendicular to the mirror line — and here the mirror is AB .

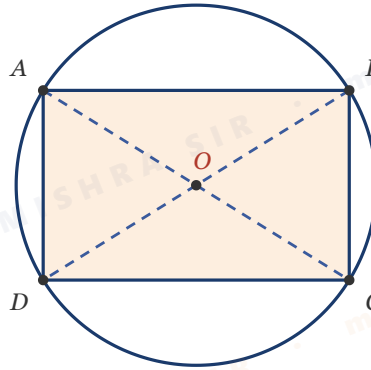
E Inscribed figures — Q14, Q15, Q19

Treat every side as a chord

When a polygon is inscribed in a circle, each of its sides is a chord and each of its vertices is a point on the circle. So all the chord and angle theorems apply directly — there is nothing new to learn, only something new to notice.

Q14.* Show that a rectangle is the only parallelogram that can be inscribed in a circle.

3 marks



A parallelogram inscribed in a circle is forced to have all four angles equal to 90° — it must be a rectangle, and its diagonals are diameters

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari, Part I*.

GIVEN: A parallelogram ABCD inscribed in a circle.

TO SHOW: ABCD is a rectangle.

PROOF:

Step 1. ABCD is a parallelogram, so opposite angles are **equal**:

$$\angle A = \angle C.$$

1

Step 2. ABCD is cyclic, so opposite angles are **supplementary** (Theorem 11):

$$\angle A + \angle C = 180^\circ.$$

1

Step 3. Substituting Step 1 into Step 2:

$$\angle A + \angle A = 180^\circ \Rightarrow 2\angle A = 180^\circ \Rightarrow \angle A = 90^\circ.$$

The same argument on the other pair gives $\angle B = \angle D = 90^\circ$.

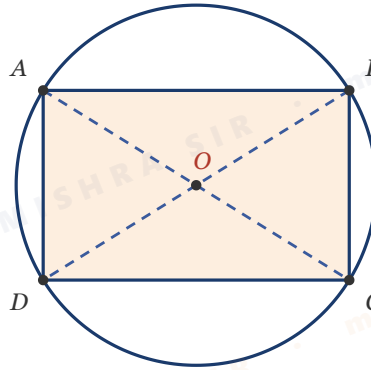
A parallelogram with all angles 90° is a **rectangle**.

1

Hence the only parallelogram that can be inscribed in a circle is a rectangle (a square being the special case).

Q15.* Show that if a rectangle is inscribed in a circle, the point of intersection of its diagonals is the centre.

3 marks



Each angle of the rectangle is 90° , so each diagonal subtends a right angle at the opposite vertices — both diagonals are diameters

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GIVEN: Rectangle ABCD inscribed in a circle with centre O; its diagonals AC and BD meet at P.

TO SHOW: $P = O$.

PROOF:

Step 1. $\angle ABC = 90^\circ$ (angle of a rectangle). A chord subtending 90° at a point of the circle must be a **diameter** (the converse of the semicircle corollary). Hence **AC is a diameter**. 1

Step 2. Likewise $\angle BAD = 90^\circ$, so **BD is also a diameter**. 1

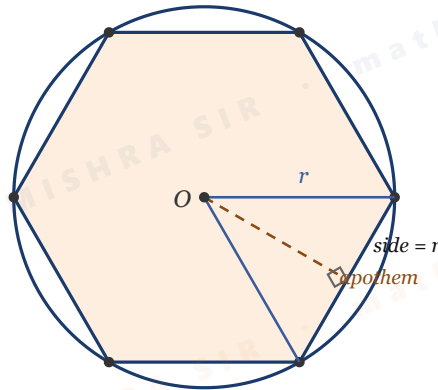
Step 3. Every diameter passes through the centre, so O lies on AC and O lies on BD. But two distinct lines meet in exactly one point, and AC and BD meet at P.

$\therefore P = O$ — **the diagonals intersect at the centre of the circle**. 1

Consistency check: the diagonals of a rectangle are equal and bisect each other, so P is equidistant from all four vertices — exactly the property of the centre.

Q19.* A regular hexagon is inscribed in a circle of radius r . Find the side length and the distance of each side from the centre.

3 marks



Regular hexagon inscribed in a circle — joining the centre to two adjacent vertices gives an equilateral triangle, so side = r

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PART 1 — THE SIDE LENGTH. Join the centre O to two adjacent vertices P and Q . The six central angles are equal and fill a full turn, so

$$\angle POQ = \frac{360^\circ}{6} = 60^\circ. \quad \boxed{1}$$

In $\triangle POQ$, $OP = OQ = r$ (radii), so the base angles are equal and each is $\frac{180^\circ - 60^\circ}{2} = 60^\circ$. All three angles are 60° , so the triangle is **equilateral**.

$\therefore$ **side of the hexagon** = $PQ = r$. 1

PART 2 — THE DISTANCE OF EACH SIDE FROM THE CENTRE (THE APOTHEM). Each side is a

chord of length r , so using $d = \sqrt{r^2 - \left(\frac{\ell}{2}\right)^2}$:

$$d = \sqrt{r^2 - \left(\frac{r}{2}\right)^2} = \sqrt{r^2 - \frac{r^2}{4}} = \sqrt{\frac{3r^2}{4}} = \frac{\sqrt{3}}{2} r \approx 0.866 r \quad \boxed{1}$$

Sanity check: the apothem must be less than r but not much less, since the hexagon nearly fills the circle — and $0.866 r$ fits that picture.

⚠ Common Mistakes in the End-of-Chapter Exercises

Not halving the diameter (Q3). The question gives 26 cm as the *diameter*. Every formula in this chapter uses the radius — write $r = 13$ before doing anything else.

Leaving the answer as the half-chord (Q1, Q4). $\sqrt{r^2 - d^2}$ is half. Double it.

Adding instead of subtracting in Q18. The chords are on the *same* side, so $d_2 - d_1 = 7$. On opposite sides it would be $d_2 + d_1 = 7$, and the radius would come out different.

Pairing adjacent angles in a cyclic quadrilateral (Q7, Q8). Only *opposite* angles are supplementary. In PQRS the opposite pairs are P–R and Q–S.

Treating 5, 5, 12, 12 as a trapezium (Q10). The equal sides are adjacent, so it is a kite. Brahmagupta's formula is the safe cross-check.

Using the exterior angle with the wrong interior angle (Q21). The exterior angle equals the interior *opposite* angle, never the adjacent one.

Forgetting to name the theorem. In every proof question, the marks sit on the justification — write “by Theorem 5”, “by RHS congruence”, “angles in the same segment”. A correct answer with no reason loses most of its marks.

◆ END OF CHAPTER 5 — END-OF-CHAPTER EXERCISES ◆

CHAPTER 6

Measuring Space: Perimeter and Area

Exercise Set 6.1 · Exercise Set 6.2 · Exercise Set 6.3 · End-of-Chapter Exercises

EXERCISE SET 6.1

Method Map — Perimeter & Circumference

Every question in Exercise Set 6.1 uses one of these four tools (take $\pi = \frac{22}{7}$ unless stated otherwise)

Tool	Formula	Used in
Circumference of a circle	$C = 2\pi r = \pi d$	Q1, Q2, Q6, Q8
Length of an arc (angle θ at centre)	$\ell = \frac{\theta}{360^\circ} \times 2\pi r$	Q3, Q4, Q7
Perimeter of a sector	$\ell + 2r$ (arc + two radii)	Q4
Composite shapes	Walk the boundary once; add only the pieces actually on the border (half circle = πr , quarter = $\frac{\pi r}{2}$)	Q5, Q7

Boundary-walk rule: imagine the insect from the chapter walking around the edge. Dotted lines (diameters, triangle sides used only for construction) are *not* part of the perimeter — never add them.

A Direct use of $C = 2\pi r$ — Q1, Q2

Q1. Perimeter of a circle is 44 cm — find its radius

1 mark

SOLUTION:

$$2\pi r = 44 \Rightarrow 2 \times \frac{22}{7} \times r = 44 \Rightarrow r = \frac{44 \times 7}{44} = 7.$$

1

Radius = 7 cm.

Q2. Circumference (correct to 3 significant figures) for $r = 7$ cm, 10 cm, 12 cm**3 marks****SOLUTION:** Using $C = 2\pi r = \frac{44}{7}r$:

$$(i) r = 7 : C = \frac{44}{7} \times 7 = 44 \rightarrow \mathbf{44.0 \text{ cm}}$$

1

$$(ii) r = 10 : C = \frac{440}{7} = 62.857 \dots \rightarrow \mathbf{62.9 \text{ cm}}$$

1

$$(iii) r = 12 : C = \frac{528}{7} = 75.428 \dots \rightarrow \mathbf{75.4 \text{ cm}}$$

1

"3 significant figures" means three meaningful digits — so 44 must be written as 44.0.

B Arcs and sectors — Q3, Q4**Q3. Arc length: (i) $r = 3.5$ cm, $\theta = 60^\circ$ (ii) $r = 6.3$ m, $\theta = 120^\circ$** **2 marks****SOLUTION:** $\ell = \frac{\theta}{360} \times 2\pi r$.

$$(i) \ell = \frac{60}{360} \times 2 \times \frac{22}{7} \times 3.5 = \frac{1}{6} \times 22 = \frac{11}{3} \approx \mathbf{3.67 \text{ cm}}$$

1

$$(ii) \ell = \frac{120}{360} \times 2 \times \frac{22}{7} \times 6.3 = \frac{1}{3} \times 39.6 = \mathbf{13.2 \text{ m}}$$

1**Q4. Perimeter of a sector — $r = 14$ cm, angle 75°** **2 marks****SOLUTION:**

$$\text{Arc} = \frac{75}{360} \times 2 \times \frac{22}{7} \times 14 = \frac{5}{24} \times 88 = \frac{55}{3} \approx 18.33 \text{ cm.}$$

1

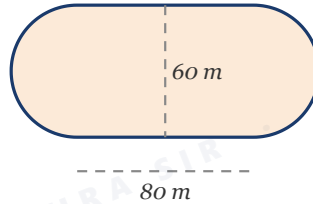
$$\text{Perimeter of sector} = \text{arc} + \text{two radii} = \frac{55}{3} + 28 = \frac{139}{3} \approx \mathbf{46.33 \text{ cm}} \left(= 46\frac{1}{3} \text{ cm} \right).$$

1**C Composite boundaries (Fig. 6.14 i-ix) — Q5****One method for all nine parts**

Walk the border once and list its pieces: straight segments at full length, semicircular arcs as πr , quarter arcs as $\frac{\pi r}{2}$. Dashed lines are construction only. Then substitute $\pi = \frac{22}{7}$.

Q5 (i). Running-track shape — straights 80 m, width 60 m

2 marks



Schematic of Fig. 6.14 (i) — two 80 m straights + two semicircular ends of diameter 60 m (not to scale)

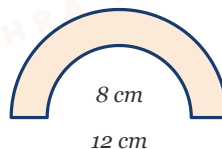
Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

SOLUTION: Border = two straights + two semicircles of diameter 60 m (together one full circle).

$$P = 2(80) + \pi(60) = 160 + \frac{22}{7} \times 60 = 160 + \frac{1320}{7} \quad \boxed{1} = \frac{2440}{7} \approx \mathbf{348.57 \text{ m.}} \quad \boxed{1}$$

Q5 (ii). Semicircular arch — outer base 12 cm, inner opening 8 cm

2 marks



Schematic of Fig. 6.14 (ii) — outer semicircle ($d = 12$), inner semicircle ($d = 8$), two 2 cm base strips

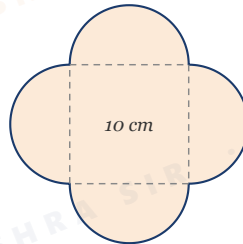
Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

SOLUTION: Border = outer semicircle + inner semicircle + two base strips of $\frac{12 - 8}{2} = 2 \text{ cm}$ each.

$$P = \pi(6) + \pi(4) + 2(2) = 10\pi + 4 = \frac{220}{7} + 4 \quad \boxed{1} = \frac{248}{7} \approx \mathbf{35.43 \text{ cm.}} \quad \boxed{1}$$

Q5 (iii). Four semicircles on the sides of a square of side 10 cm

2 marks



Schematic of Fig. 6.14 (iii) — a semicircle bulges outward on each side of the (dashed) square

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari*, Part I.

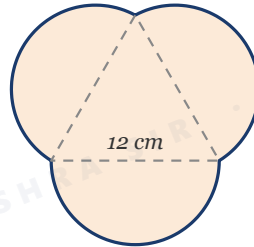
SOLUTION: Border = 4 semicircles of radius 5 cm; the square itself is construction only.

$$P = 4 \times \pi(5) = 20\pi = \frac{440}{7} \approx 62.86 \text{ cm.}$$

2

Q5 (iv). Trefoil — three semicircles on the sides of an equilateral triangle of side 12 cm

2 marks



Schematic of Fig. 6.14 (iv) — semicircles drawn outward on each side of the dashed triangle meet at its vertices

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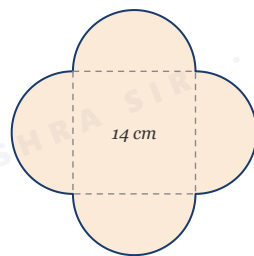
SOLUTION: Border = 3 semicircles of radius 6 cm.

$$P = 3 \times \pi(6) = 18\pi = \frac{396}{7} \approx \mathbf{56.57 \text{ cm.}}$$

2

Q5 (v). Four semicircles on the sides of a square of side 14 cm

2 marks



Schematic of Fig. 6.14 (v) — same construction as (iii), side 14 cm

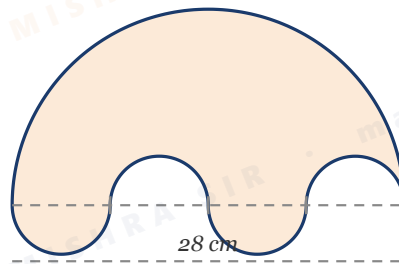
Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

SOLUTION: $P = 4 \times \pi(7) = 28\pi = \frac{22}{7} \times 28 = \mathbf{88 \text{ cm.}}$

2

Q5 (vi). Large semicircle on a 28 cm base with a wavy base of four small semicircles

3 marks



Schematic of Fig. 6.14 (vi) — big semicircle ($d = 28$) above; along the base, four semicircles of diameter 7 alternate below–above–below–above

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari*, Part I.

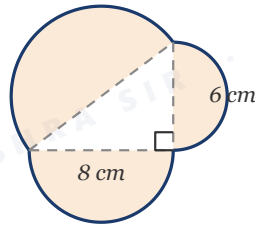
SOLUTION: The base 28 cm is divided into 4 equal parts, so each small semicircle has diameter 7 cm ($r = 3.5$). 1

$$P = \pi(14) + 4 \times \pi(3.5) = 14\pi + 14\pi = 28\pi \quad \text{1} = 88 \text{ cm.} \quad \text{1}$$

Note the surprise: the wavy bottom has exactly the same length as the big arc on top.

Q5 (vii). Semicircles on all three sides of a right triangle with legs 8 cm and 6 cm

3 marks



Schematic of Fig. 6.14 (vii) — the dashed right triangle (8, 6, hypotenuse 10) is construction; the border is three semicircular arcs

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

SOLUTION: Hypotenuse = $\sqrt{8^2 + 6^2} = \sqrt{100} = 10$ cm.

1

Border = semicircles of radii 4, 3 and 5: $P = \pi(4) + \pi(3) + \pi(5) = 12\pi$

1

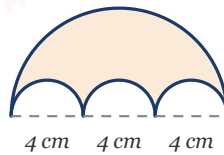
$= \frac{264}{7} \approx 37.71$

cm.

1

Q5 (viii). Big semicircle on 12 cm with three 4 cm semicircles along the base

2 marks



Schematic of Fig. 6.14 (viii) — the dashed diameter is not part of the border

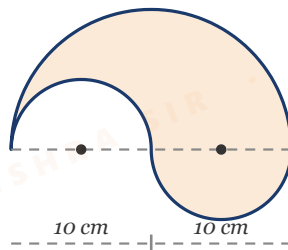
Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

SOLUTION: $P = \pi(6) + 3 \times \pi(2) = 6\pi + 6\pi = 12\pi$ **1** $= \frac{264}{7} \approx 37.71 \text{ cm}$ — equal to (vii)!

1

Q5 (ix). S-shaped (yin–yang style) figure on a 20 cm base (10 cm + 10 cm)

2 marks



Schematic of Fig. 6.14 (ix) — one big semicircle ($d = 20$) and two small semicircles ($d = 10$), one curving up, one curving down

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

SOLUTION: $P = \pi(10) + \pi(5) + \pi(5) = 20\pi$ **1** $= \frac{440}{7} \approx 62.86 \text{ cm}$ **1**

D Applications and ratio — Q6, Q7, Q8

Q6. Car tyre of diameter 56 cm: (i) distance per revolution (ii) revolutions in 10 km

3 marks

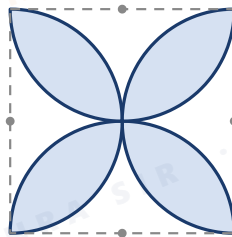
SOLUTION:

(i) One revolution covers one circumference: $C = \pi d = \frac{22}{7} \times 56 = 176 \text{ cm.}$ 1

(ii) 10 km = 10,00,000 cm. 1 Number of revolutions = $\frac{10,00,000}{176} = 5681.8 \dots \approx 5682$
revolutions (5681 complete revolutions and most of the next). 1

Q7. Total perimeter of all petals in the two flowers (Fig. 6.15 A, B)

4 marks



square side 14 cm

Schematic of Fig. 6.15 A — semicircles drawn inside the square (side 14 cm) with the sides as diameters; each petal is bounded by two 90° arcs of radius 7

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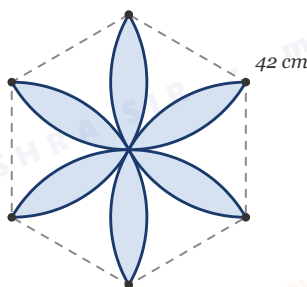
SOLUTION (I) — SQUARE, SIDE 14 CM:

Each semicircle has its centre at a midpoint of a side, radius 7 cm. Two adjacent semicircles cross at a corner of the square and at its centre, so each petal is bounded by **two quarter-circle (90°) arcs of radius 7**.

1

One arc = $\frac{1}{4} \times 2 \times \frac{22}{7} \times 7 = 11$ cm $\rightarrow$ one petal = 22 cm $\rightarrow$ 4 petals = $4 \times 22 = 88$ cm.

1



Schematic of Fig. 6.15 B — arcs of radius 42 cm centred at the vertices of the regular hexagon; each petal runs from the centre to a vertex

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

SOLUTION (II) — REGULAR HEXAGON, SIDE 42 CM:

In a regular hexagon the distance from the centre O to each vertex equals the side (42 cm). An arc centred at vertex V_1 with radius 42 passes through both O and the neighbouring vertex V_0 , and triangle V_1OV_0 is equilateral — so each petal arc subtends 60° at its centre. 1

One arc = $\frac{60}{360} \times 2 \times \frac{22}{7} \times 42 = 44$ cm $\rightarrow$ one petal = 88 cm $\rightarrow$ 6 petals = $6 \times 88 = 528$ cm.

1

Q8. Perimeters of two circles are in the ratio 5 : 4 — ratio of radii?

1 mark

SOLUTION: $\frac{C_1}{C_2} = \frac{2\pi r_1}{2\pi r_2} = \frac{r_1}{r_2}$. Perimeter is directly proportional to radius, so the ratio of radii is also 5 : 4. 1

⚠ Common Mistakes in This Exercise

Adding dashed lines to the perimeter. In Q5 the diameters, triangle sides and squares shown dashed are construction lines — the insect never walks on them.

Forgetting the two radii of a sector (Q4). Perimeter of a sector = arc + $2r$, not just the arc.

Unit slips in Q6. Convert 10 km to cm (10,00,000 cm) before dividing by 176 cm.

Significant figures (Q2). 44 written to 3 s.f. is 44.0 — the trailing zero is required.

Wrong petal angle in Q7. Square flower $\rightarrow 90^\circ$ arcs (radius 7); hexagon flower $\rightarrow 60^\circ$ arcs (radius 42). Identify the centre of each arc before choosing θ .

◆ END OF EXERCISE SET 6.1 ◆

EXERCISE SET 6.2

Method Map — Areas of Straight-line Figures

Tools for Exercise Set 6.2

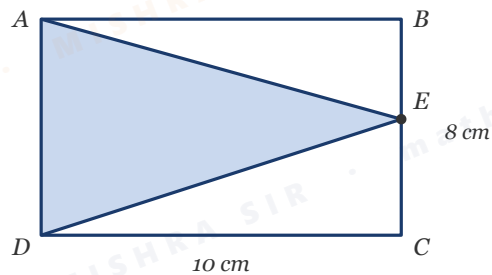
Tool	Statement	Used in
Rectangle / parallelogram / triangle	ab ; base $\times$ height; $\frac{1}{2}$ base $\times$ height	Q1, Q2, Q5, Q10
Heron's formula	$s = \frac{a+b+c}{2}$, $A = \sqrt{s(s-a)(s-b)(s-c)}$	Q3, Q4
Same base, same parallels	Triangles on the same (or equal) base and between the same parallels have equal areas	Q6, Q7, Q9, Q11
Median property	A median divides a triangle into two triangles of equal area	Q9, Q11
Midpoint theorem	Segment joining midpoints of two sides is parallel to the third side and half of it	Q8

Height means perpendicular height. In every area formula the height is measured at right angles to the chosen base — this single idea powers all the proof questions below.

A Direct computation — Q1 to Q5

Q1. Area of triangle ADE in Fig. 6.31 (rectangle $10\text{ cm} \times 8\text{ cm}$, E on BC)

2 marks



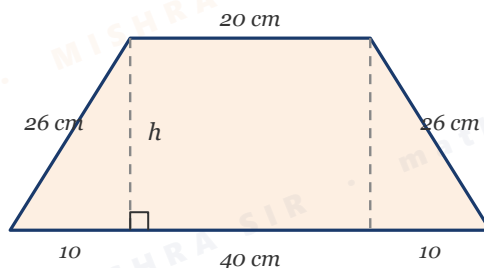
Schematic of Fig. 6.31 — triangle ADE inside rectangle ABCD

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

SOLUTION: Take AD as the base: $AD = 8\text{ cm}$. Wherever E lies on BC, its perpendicular distance from side AD equals the full width of the rectangle, 10 cm . 1

$$\text{ar}(\triangle ADE) = \frac{1}{2} \times 8 \times 10 = \mathbf{40\text{ cm}^2} \text{ — exactly half the rectangle.}$$

1

Q2. Trapezium with parallel sides 40 cm, 20 cm and equal slant sides 26 cm**3 marks**

Drop perpendiculars from the ends of the shorter side — the base splits as 10 + 20 + 10

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SOLUTION: The trapezium is isosceles, so the two right triangles at the ends have equal bases

$$= \frac{40 - 20}{2} = 10 \text{ cm.} \quad 1$$

$$\text{Height } h = \sqrt{26^2 - 10^2} = \sqrt{676 - 100} = \sqrt{576} = 24 \text{ cm.} \quad 1$$

$$\text{Area} = \frac{1}{2}(40 + 20) \times 24 = \frac{1}{2} \times 60 \times 24 = \mathbf{720 \text{ cm}^2}. \quad 1$$

Q3. Triangle with sides 8 cm, 11 cm and perimeter 32 cm**3 marks**

SOLUTION: Third side = $32 - 8 - 11 = 13 \text{ cm}$; $s = \frac{32}{2} = 16 \text{ cm.}$ 1

By Heron's formula: $A = \sqrt{16(16 - 8)(16 - 11)(16 - 13)} = \sqrt{16 \times 8 \times 5 \times 3} = \sqrt{1920}$ 1

$$= \sqrt{64 \times 30} = 8\sqrt{30} \approx \mathbf{43.8 \text{ cm}^2}. \quad 1$$

Q4. Triangular plot, sides in ratio 3 : 5 : 7, perimeter 300 m**3 marks**

SOLUTION: $3x + 5x + 7x = 300 \Rightarrow x = 20$; sides are 60 m, 100 m, 140 m; $s = 150 \text{ m.}$ 1

$$A = \sqrt{150(150 - 60)(150 - 100)(150 - 140)} = \sqrt{150 \times 90 \times 50 \times 10} = \sqrt{67,50,000} \quad 1$$

$$= 1500\sqrt{3} \approx \mathbf{2598 \text{ m}^2}. \quad 1$$

Q5. Rhombus: one diagonal twice the other, area 128 cm^2 — shorter diagonal?

2 marks

SOLUTION: Let the diagonals be d and $2d$. Area of a rhombus $= \frac{1}{2}d_1d_2$:

$$\frac{1}{2} \cdot d \cdot 2d = 128 \Rightarrow d^2 = 128 \quad 1 \Rightarrow d = \sqrt{128} = 8\sqrt{2} \approx 11.3 \text{ cm.} \quad 1$$

B Equal-area reasoning — Q6 to Q11

Q6. ABCD is a parallelogram; P, Q are any points on AB. Ratio $\text{ar}(\triangle PCD) : \text{ar}(\triangle QCD)$?

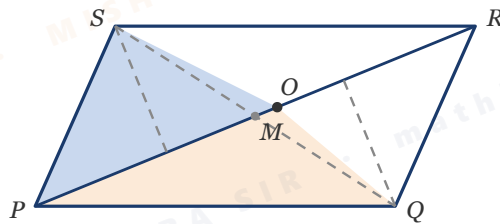
2 marks

SOLUTION: Both triangles stand on the same base CD, and both apexes P, Q lie on line AB, which is parallel to CD. So both have the same height — the distance between the parallels AB and CD. 1

Hence $\text{ar}(\triangle PCD) = \text{ar}(\triangle QCD)$, i.e. the ratio is **1 : 1**, wherever P and Q lie on AB. 1

Q7. O lies on diagonal PR of parallelogram PQRS. Prove $\text{ar}(\triangle PSO) = \text{ar}(\triangle PQO)$

3 marks



S and Q are equidistant from diagonal PR because M, the midpoint of SQ, lies on PR

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

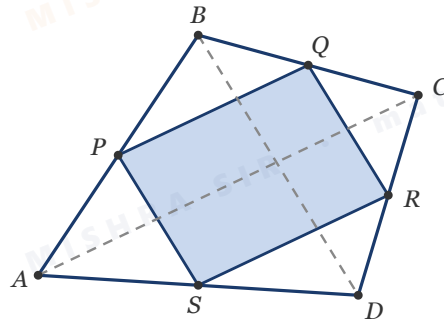
PROOF: Both triangles share the base **PO** (part of line PR), so it is enough to show S and Q are at equal perpendicular distances from line PR. 1

The diagonals of a parallelogram bisect each other, so the midpoint M of SQ lies on PR. Drop perpendiculars SX and QY to PR: in $\triangle SMX$ and $\triangle QMY$, $SM = QM$, $\angle SMX = \angle QMY$ (vertically opposite), $\angle SXM = \angle QYM = 90^\circ$ — so the triangles are congruent (AAS) and $SX = QY$. 1

Therefore $\text{ar}(\triangle PSO) = \frac{1}{2} PO \cdot SX = \frac{1}{2} PO \cdot QY = \text{ar}(\triangle PQO)$. 1

Q8. Midpoints of the sides of a 4-gon are joined in order — prove the parallelogram formed has half the area of the 4-gon

4 marks



P, Q, R, S are midpoints; the dashed diagonals AC and BD drive the proof

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PROOF: Let the 4-gon be ABCD with midpoints P (of AB), Q (BC), R (CD), S (DA). The area outside PQRS consists of the four corner triangles BPQ, DRS, APS, CQR. 1

Corner pair 1 (use diagonal AC): In $\triangle ABC$, P and Q are midpoints of AB, BC, so $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$; $\triangle BPQ$ is similar to $\triangle BAC$ with ratio $\frac{1}{2}$, hence $\text{ar}(\triangle BPQ) = \frac{1}{4} \text{ar}(\triangle BAC)$. Likewise $\text{ar}(\triangle DRS) = \frac{1}{4} \text{ar}(\triangle DAC)$. Adding: these two corners = $\frac{1}{4} \text{ar}(ABCD)$. 1

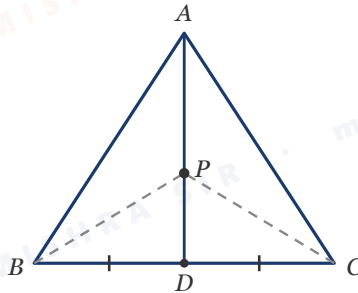
Corner pair 2 (use diagonal BD): in the same way

$$\text{ar}(\triangle APS) + \text{ar}(\triangle CQR) = \frac{1}{4} \text{ar}(\triangle ABD) + \frac{1}{4} \text{ar}(\triangle CBD) = \frac{1}{4} \text{ar}(ABCD). \quad 1$$

Total cut away = $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ of ABCD, so $\text{ar}(PQRS) = \frac{1}{2} \text{ar}(ABCD)$. ■ 1

Q9. AD is a median of $\triangle ABC$ and P is any point on AD. Show $\text{ar}(\triangle ABP) = \text{ar}(\triangle ACP)$

3 marks



Schematic of Fig. 6.32 — subtract the small equal pair from the big equal pair

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

PROOF: AD is a median, so $\text{ar}(\triangle ABD) = \text{ar}(\triangle ACD)$ (equal bases $BD = DC$, same apex A).

1

PD is also a median of $\triangle PBC$ (same D!), so $\text{ar}(\triangle PBD) = \text{ar}(\triangle PCD)$.

1

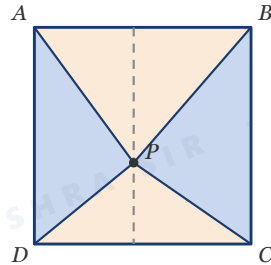
Subtracting, $\text{ar}(\triangle ABP) = \text{ar}(\triangle ABD) - \text{ar}(\triangle PBD) = \text{ar}(\triangle ACD) - \text{ar}(\triangle PCD) = \text{ar}(\triangle ACP)$.

■

1

Q10. P is any point inside square ABCD, joined to all four vertices. Ratio of red region ($\Delta PAB + \Delta PCD$) to green region ($\Delta PBC + \Delta PDA$)?

3 marks



Schematic of Fig. 6.33 — the dashed line through P shows the two heights that add up to the side

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

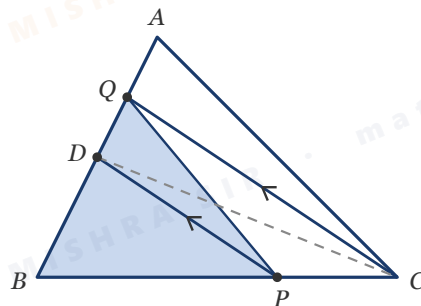
SOLUTION: Let the side of the square be a , and let the perpendicular distances of P from AB and CD be h_1 and h_2 . Since $AB \parallel CD$, $h_1 + h_2 = a$. 1

$\text{ar}(\Delta PAB) + \text{ar}(\Delta PCD) = \frac{1}{2}ah_1 + \frac{1}{2}ah_2 = \frac{1}{2}a(h_1 + h_2) = \frac{a^2}{2}$ — half the square, wherever P is. 1

By the same argument on sides BC and DA, the green pair is also $\frac{a^2}{2}$. Ratio = **1 : 1**. 1

Q11. D is the midpoint of AB; P lies on BC; Q on AB with $CQ \parallel PD$. Prove $\text{ar}(\triangle BPQ) = \frac{1}{2} \text{ar}(\triangle ABC)$

4 marks



Schematic of Fig. 6.34 — the parallels $CQ \parallel PD$ let us slide the apex without changing area (CD dashed is the median used at the end)

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PROOF: Triangles $\triangle PDQ$ and $\triangle PDC$ stand on the same base PD , and their apexes Q, C lie on line QC , which is parallel to PD — so $\text{ar}(\triangle PDQ) = \text{ar}(\triangle PDC)$. 1

Now build $\triangle BPQ$ from two parts:

$$\text{ar}(\triangle BPQ) = \text{ar}(\triangle BPD) + \text{ar}(\triangle PDQ) = \text{ar}(\triangle BPD) + \text{ar}(\triangle PDC).$$

1

But $\triangle BPD$ and $\triangle PDC$ together make $\triangle BDC$: so $\text{ar}(\triangle BPQ) = \text{ar}(\triangle BDC)$. 1

1

Finally CD is a median of $\triangle ABC$ (D is the midpoint of AB), so $\text{ar}(\triangle BDC) = \frac{1}{2} \text{ar}(\triangle ABC)$.

$$\text{Hence } \text{ar}(\triangle BPQ) = \frac{1}{2} \text{ar}(\triangle ABC). \blacksquare$$

1

⚠ Common Mistakes in This Exercise

Using the slant side as the height (Q2). 26 cm is a slant side; the height 24 cm must come from Pythagoras.

Heron without the semi-perimeter. Always compute s first and subtract each side from s — a wrong s sinks the whole answer.

“Same base, same parallels” quoted without naming both. In Q6, Q7, Q11 state the common

base *and* the pair of parallels — that sentence carries the marks.

In Q9, forgetting that D serves two triangles. The trick is that AD is a median of $\triangle ABC$ *and* PD is a median of $\triangle PBC$.

◆ END OF EXERCISE SET 6.2 ◆

EXERCISE SET 6.3



Method Map — Area of a Circle, Sector and Segment

Tools for Exercise Set 6.3 (take $\pi = \frac{22}{7}$ unless the question fixes another value)

Region	Area	Used in
Circle	πr^2	Q2, Q4, Q5, Q8–Q10
Sector of angle θ	$\frac{\theta}{360^\circ} \times \pi r^2$	Q1, Q3, Q4, Q6
Minor segment	sector – triangle (same chord)	Q5, Q7
Major sector / segment	circle – minor part	Q4, Q5
Inscribed regular polygons	split into triangles from the centre / use the diagonal	Q8, Q9, Q10

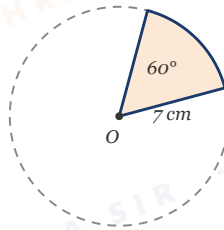
Clock conversion: the minute hand sweeps 360° in 60 minutes, i.e. **6° per minute**. An equilateral triangle of side a has area $\frac{\sqrt{3}}{4}a^2$ — needed in Q5, Q7, Q8, Q10.

A

Sectors — Q1 to Q4, Q6

Q1. Sector of radius 7 cm and angle 60°

2 marks



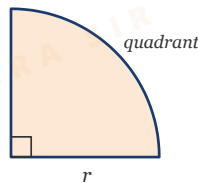
A 60° sector is one-sixth of the circle

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

SOLUTION: $A = \frac{60}{360} \times \frac{22}{7} \times 7^2 = \frac{1}{6} \times 154$ 1 $= \frac{77}{3} \approx 25.67 \text{ cm}^2$. 1

Q2. Quadrant of a circle whose circumference is 44 cm

2 marks



A quadrant is a 90° sector — one-quarter of the circle

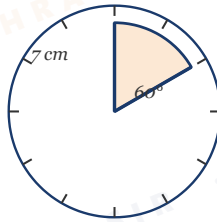
Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

SOLUTION: $2\pi r = 44 \Rightarrow r = 7 \text{ cm}$. 1

Quadrant $= \frac{1}{4} \pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 49 = \frac{154}{4} = 38.5 \text{ cm}^2$. 1

Q3. Minute hand 7 cm long — area swept in 10 minutes

2 marks



In 10 minutes the hand turns $10 \times 6^\circ = 60^\circ$

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SOLUTION: Angle swept = $10 \times 6^\circ = 60^\circ$.

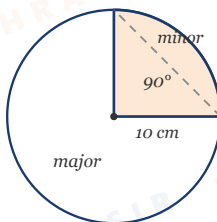
1

$$A = \frac{60}{360} \times \frac{22}{7} \times 49 = \frac{77}{3} \approx \mathbf{25.67 \text{ cm}^2}.$$

1

Q4. Chord of a circle of radius 10 cm subtends 90° : area of (i) minor sector (ii) major sector ($\pi \approx 3.14$)

3 marks



Minor sector 90° , major sector 270° — together they make the whole circle

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SOLUTION: Circle area = $3.14 \times 100 = 314 \text{ cm}^2$.

1

(i) Minor sector = $\frac{90}{360} \times 314 = \mathbf{78.5 \text{ cm}^2}$.

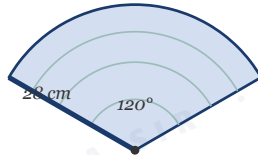
1

(ii) Major sector = $314 - 78.5 = \mathbf{235.5 \text{ cm}^2}$ $\left(= \frac{270}{360} \times 314 \right)$.

1

Q6. Two non-overlapping wipers, blade 28 cm, each sweeping 120° — total area cleaned

2 marks



Each wiper cleans a 120° sector of radius 28 cm

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SOLUTION: One wiper: $\frac{120}{360} \times \frac{22}{7} \times 28^2 = \frac{1}{3} \times \frac{22}{7} \times 784 = \frac{2464}{3} \text{ cm}^2$.

1

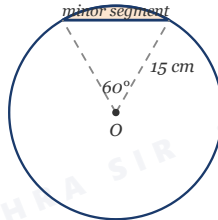
Two wipers (no overlap): $2 \times \frac{2464}{3} = \frac{4928}{3} \approx \mathbf{1642.67 \text{ cm}^2}$.

1

B Segments — Q5, Q7

Q5. Chord of a circle of radius 15 cm subtends 60° — minor and major segments ($\pi \approx 3.14$, $\sqrt{3} \approx 1.73$)

4 marks



Minor segment = 60° sector – equilateral triangle on the chord

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SOLUTION: Sector (60°): $\frac{60}{360} \times 3.14 \times 225 = \frac{706.5}{6} = 117.75 \text{ cm}^2$. 1

The triangle formed by the two radii and the chord has two sides 15 with a 60° angle between

them, so it is **equilateral** of side 15: area = $\frac{\sqrt{3}}{4} \times 225 = \frac{1.73 \times 225}{4} = 97.3125 \text{ cm}^2$. 1

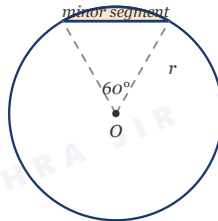
Minor segment = $117.75 - 97.3125 \approx 20.44 \text{ cm}^2$. 1

Major segment = circle – minor = $3.14 \times 225 - 20.44 = 706.5 - 20.44 \approx 686.06 \text{ cm}^2$. 1

Q7*. Chord subtending 60° in a circle of radius r — show the minor segment

has area $r^2 \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4} \right)$

3 marks



Same picture as Q5, now in letters

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PROOF: Sector of $60^\circ = \frac{60}{360} \pi r^2 = \frac{\pi r^2}{6}$.

1

The radii–chord triangle is isosceles with vertex angle 60° , hence equilateral with side r : area

$= \frac{\sqrt{3}}{4} r^2$.

1

Minor segment $= \frac{\pi r^2}{6} - \frac{\sqrt{3}}{4} r^2 = r^2 \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4} \right)$. ■

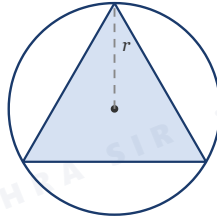
1

C Inscribed polygons — Q8, Q9, Q10 (starred)

Q8*. Equilateral triangle inscribed in a circle of radius r — show area(triangle) :

$$\text{area(circle)} = \frac{3\sqrt{3}}{4\pi} \approx 0.413$$

4 marks



Join the centre to the vertices: three isosceles triangles with vertex angle 120°

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PROOF: Joining the centre O to the three vertices splits the triangle into 3 congruent isosceles triangles with sides r, r and central angle 120° . 1

In one such triangle, drop the perpendicular from O to the side: it bisects the 120° angle into two 30° – 60° – 90° halves, giving half-side = $\frac{\sqrt{3}}{2}r$ and distance $\frac{r}{2}$. So the side of the equilateral triangle is $a = \sqrt{3}r$ 1 and each small triangle has area $\frac{1}{2} \cdot \sqrt{3}r \cdot \frac{r}{2} = \frac{\sqrt{3}}{4}r^2$.

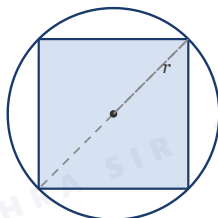
$$\text{Triangle area} = 3 \times \frac{\sqrt{3}}{4}r^2 = \frac{3\sqrt{3}}{4}r^2. \quad \text{1}$$

$$\text{Ratio} = \frac{3\sqrt{3}r^2/4}{\pi r^2} = \frac{3\sqrt{3}}{4\pi} \approx \frac{5.196}{12.566} \approx \mathbf{0.413}. \quad \text{1}$$

Q9*. Square inscribed in a circle of radius r — show $\text{area}(\text{square}) : \text{area}(\text{circle}) =$

$$\frac{2}{\pi} \approx 0.637$$

3 marks



The diagonal of the square is a diameter

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PROOF: The diagonal of the inscribed square passes through the centre, so diagonal $= 2r$.

1

If the side is a , then $a\sqrt{2} = 2r \Rightarrow a = r\sqrt{2}$, so square area $= a^2 = 2r^2$.

1

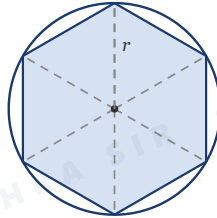
$$\text{Ratio} = \frac{2r^2}{\pi r^2} = \frac{2}{\pi} \approx \mathbf{0.637. \blacksquare}$$

1

Q10*. Regular hexagon inscribed in a circle of radius r — show the ratio is

$$\frac{3\sqrt{3}}{2\pi} \approx 0.827; \text{ why exactly twice Q8?}$$

4 marks



Radii to the vertices cut the hexagon into six equilateral triangles of side r

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PROOF: The six radii to the vertices divide the hexagon into 6 triangles, each with two sides r and central angle 60° — so each is equilateral of side r , area $\frac{\sqrt{3}}{4}r^2$. 1

$$\text{Hexagon area} = 6 \times \frac{\sqrt{3}}{4}r^2 = \frac{3\sqrt{3}}{2}r^2; \text{ ratio} = \frac{3\sqrt{3}}{2\pi} \approx \mathbf{0.827}. \blacksquare \quad \text{1}$$

Why double Q8: hexagon area $\frac{3\sqrt{3}}{2}r^2$ is exactly twice the inscribed triangle's $\frac{3\sqrt{3}}{4}r^2$. 1

Reason: the triangle uses three alternate vertices of the hexagon; each of its three sides cuts off a small triangle, and these three cut-off corners can be folded in to cover the central triangle exactly — the hexagon is two copies of the triangle in area. 1

⚠ Common Mistakes in This Exercise

Mixing the arc formula with the sector formula. Arc length uses $2\pi r$; sector area uses πr^2 . Both carry the factor $\theta/360$.

Major segment $\neq$ major sector. Major segment = circle – minor segment (Q5); major sector = circle – minor sector (Q4).

Using $\frac{22}{7}$ where the question fixes $\pi = 3.14$. Q4 and Q5 prescribe 3.14 — follow the question's value.

Segment without the triangle. A segment is never $\theta/360 \times \pi r^2$ alone — subtract the radii-chord triangle.

◆ END OF EXERCISE SET 6.3 ◆

END-OF-CHAPTER EXERCISES



Method Map — The Whole Chapter in One Toolbox

End-of-Chapter Exercises mix every idea of Chapter 6 (take $\pi = \frac{22}{7}$ unless stated otherwise)

Idea	Key fact	Used in
Heron / Pythagoras	$s = \frac{a+b+c}{2}$, $A = \sqrt{s(s-a)(s-b)(s-c)}$	Q2–Q6
Circumference	$C = 2\pi r = \pi d$	Q7, Q8, Q9
Dissection principle	Cutting a figure and rearranging the pieces never changes total area	Q1, Q11–Q16, Q20
Scaling	Lengths scaled by $k \Rightarrow$ areas scaled by k^2	Q15, Q18
Sector / segment / lune	segment = sector – triangle; count each region once	Q21–Q27

The chapter's master idea: area is conserved under cutting and rearranging, and a region counted twice must be subtracted once. Almost every starred problem below is this idea in disguise.

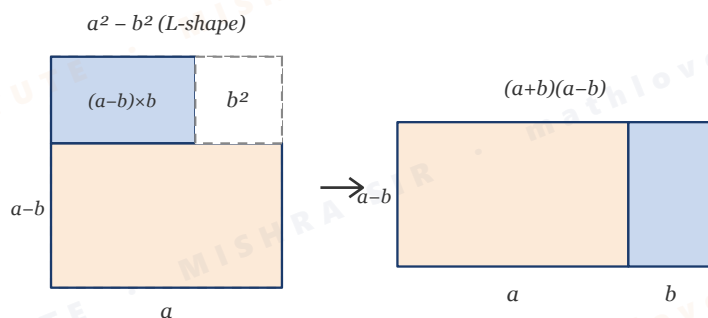
A

Identities, triangles and wheels — Q1 to Q10

Q1. Draw area models for $(a + b)(a - b) = a^2 - b^2$ and $(a + b + c)^2$

4 marks

SOLUTION (FIRST IDENTITY):

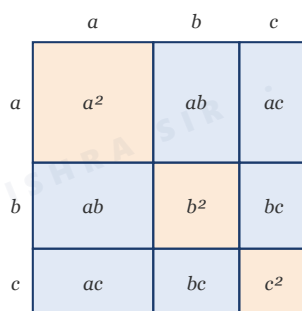


Remove a $b \times b$ corner from an $a \times a$ square; slide the upper $(a-b) \times b$ strip to the right end — the L-shape becomes an $(a+b) \times (a-b)$ rectangle

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Start with a square of side a (area a^2) and remove a small square of side b from one corner. The remaining L-shape has area $a^2 - b^2$. **1** Cut the L into two rectangles — one $a \times (a-b)$, one $(a-b) \times b$ — and re-stand the second piece at the right end. The result is a single rectangle of length $a+b$ and width $a-b$. Same pieces, same area: $(a+b)(a-b) = a^2 - b^2$. **1**

SOLUTION (SECOND IDENTITY):



A square of side $a + b + c$ cut into a 3×3 grid of rectangles

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Divide a square of side $a + b + c$ by two lines each way at distances a and $a + b$. The nine cells

Adding all nine: $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$.

1

Q2. Isosceles triangle: perimeter 40 cm, equal sides 15 cm — area?

2 marks

SOLUTION: Base = $40 - 15 - 15 = 10$ cm; height = $\sqrt{15^2 - 5^2} = \sqrt{200} = 10\sqrt{2}$ cm.

1

Area = $\frac{1}{2} \times 10 \times 10\sqrt{2} = 50\sqrt{2} \approx 70.7$ cm².

1

Q3. Isosceles triangle: base 10 cm, area 60 cm² — the equal sides?

2 marks

SOLUTION: $\frac{1}{2} \times 10 \times h = 60 \Rightarrow h = 12$ cm.

1

Equal side = $\sqrt{5^2 + 12^2} = \sqrt{169} = 13$ cm.

1

Q4. Right triangle of area 54 cm² with one leg 12 cm — perimeter?

2 marks

SOLUTION: $\frac{1}{2} \times 12 \times (\text{other leg}) = 54 \Rightarrow \text{other leg} = 9$ cm.

1

Hypotenuse = $\sqrt{12^2 + 9^2} = \sqrt{225} = 15$ cm; perimeter = $12 + 9 + 15 = 36$ cm.

1

Q5. Sides in ratio 2 : 3 : 4, perimeter 45 cm — area?

3 marks

SOLUTION: $2x + 3x + 4x = 45 \Rightarrow x = 5$: sides 10, 15, 20 cm; $s = 22.5$ cm.

1

$A = \sqrt{22.5 \times 12.5 \times 7.5 \times 2.5} = \sqrt{5273.4375}$

1

$= \frac{75\sqrt{15}}{4} \approx 72.6$ cm².

1

Q6. Triangle with sides 7, 24, 25 cm — area in two different ways

3 marks

SOLUTION — WAY 1 (RIGHT TRIANGLE): $7^2 + 24^2 = 49 + 576 = 625 = 25^2$, so the triangle is right-angled with legs 7 and 24.

1

Area = $\frac{1}{2} \times 7 \times 24 = 84$ cm².

1

WAY 2 (HERON): $s = 28$: $A = \sqrt{28 \times 21 \times 4 \times 3} = \sqrt{7056} = 84$ cm² — the two methods agree.

1

Q7. Bicycle wheel of diameter 60 cm rotates 100 times — distance travelled?

2 marks

SOLUTION: One rotation = $\pi d = \frac{22}{7} \times 60 = \frac{1320}{7}$ cm.

1

100 rotations = $\frac{1,32,000}{7} \approx 18,857$ cm ≈ 188.57 m.

1

Q8. Quadrant of a circle whose circumference is 66 cm**2 marks**

SOLUTION: $2 \times \frac{22}{7} \times r = 66 \Rightarrow r = \frac{66 \times 7}{44} = 10.5 \text{ cm.}$

1

$\text{Quadrant} = \frac{1}{4} \times \frac{22}{7} \times 10.5^2 = \frac{1}{4} \times \frac{22}{7} \times 110.25 = 86.625 \text{ cm}^2.$

1**Q9. Car wheel of outer radius 28 cm: distance in one turn; turns in 1 km****2 marks**

SOLUTION: One turn $= 2 \times \frac{22}{7} \times 28 = 176 \text{ cm.}$

1

1 km = 1,00,000 cm; turns $= \frac{1,00,000}{176} = 568.18 \dots \approx 568 \text{ complete turns}$ (a little more than 568).

1**Q10*. Two rectangles have equal area and equal perimeter — must they be congruent?****3 marks**

SOLUTION — YES. Let the rectangles have sides l, w and l', w' . Equal perimeters give

$l + w = l' + w' = S$; equal areas give $lw = l'w' = P$.

1

Then $\{l, w\}$ and $\{l', w'\}$ are both the pair of roots of the same quadratic $t^2 - St + P = 0$.

1

A quadratic has only one pair of roots, so $\{l, w\} = \{l', w'\}$: the rectangles have the same dimensions and are **congruent**. ■

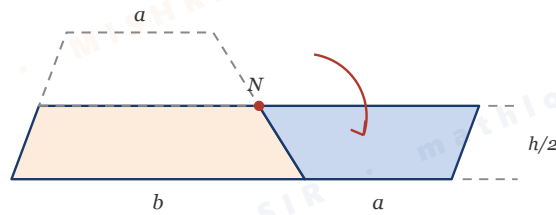
1

(Interesting contrast: for triangles, equal perimeter + equal area does not force congruence — but for rectangles it does.)

B Trapezium, kite and fitting copies — Q11 to Q15

Q11. Using a parallelogram, show that a trapezium has area $\frac{1}{2}(a + b)h$

3 marks



Cut along the mid-height line and rotate the top piece 180° about N (the midpoint of a slant side): a parallelogram of base $a + b$ and height $h/2$ appears

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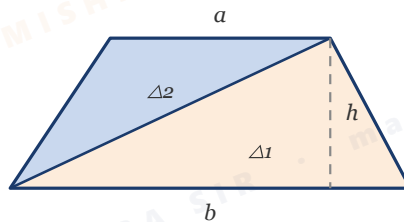
PROOF: Cut the trapezium along the line at half its height, and rotate the top piece by 180° about N , the midpoint of one slant side. Because N is a midpoint, the rotated top edge (length a) lands exactly on the extension of the base line. 1

The two pieces now form a **parallelogram** with base $a + b$ and height $\frac{h}{2}$. 1

Cutting and rotating preserves area, so Trapezium = $(a + b) \times \frac{h}{2} = \frac{1}{2}(a + b)h$. ■ 1

Q12. The same formula by dividing the trapezium into two triangles

2 marks



One diagonal creates a triangle on base b and a triangle on base a — both of height h

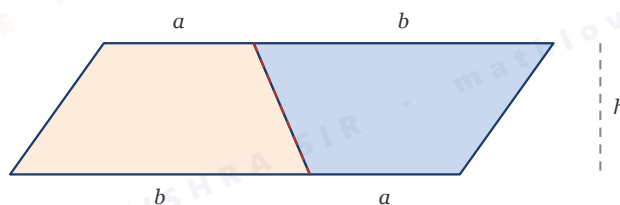
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PROOF: A diagonal splits the trapezium into $\Delta 1$ with base b and $\Delta 2$ with base a ; both have height h , the distance between the parallel sides. 1

$$\text{Area} = \frac{1}{2}bh + \frac{1}{2}ah = \frac{1}{2}(a+b)h. \blacksquare$$
1

Q13. Two identical copies of a trapezium make a parallelogram — deduce the formula

2 marks



The second copy is rotated 180° and joined along a slant side; the base becomes $a + b$

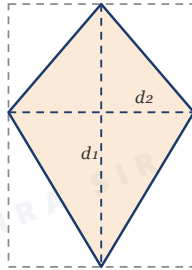
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PROOF: Rotate a copy of the trapezium by 180° and glue it along one slant side. The top of one copy lines up with the bottom of the other (parallel sides!), producing a parallelogram of base $a + b$ and height h . 1

$$\text{Two trapeziums} = (a+b)h \Rightarrow \text{one trapezium} = \frac{1}{2}(a+b)h. \blacksquare$$
1

Q14. Area of a kite = $\frac{1}{2} \times$ product of diagonals — (i) by algebra (ii) by geometry

3 marks



The diagonals are perpendicular; the dashed rectangle $d_2 \times d_1$ encloses the kite

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

(I) ALGEBRA: The axis diagonal d_1 splits the kite into two triangles, each with base d_1 and heights x and $d_2 - x$ (the two parts of the other diagonal). Area

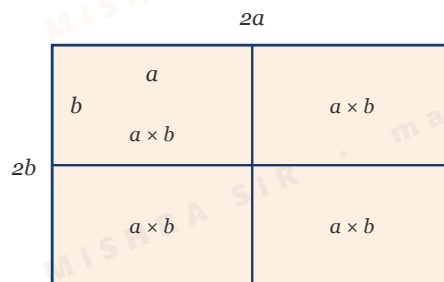
$$= \frac{1}{2}d_1x + \frac{1}{2}d_1(d_2 - x) = \frac{1}{2}d_1d_2. \quad 1$$

(II) GEOMETRY: Enclose the kite in the rectangle with sides parallel to the diagonals (sides d_1 and d_2). The diagonals divide the picture into four small rectangles, and in each one the kite occupies exactly half (a diagonal of that small rectangle). 1

$$\text{So kite} = \frac{1}{2} \times \text{rectangle} = \frac{1}{2}d_1d_2. \quad 1$$

Q15. Fitting copies: (i) rectangle scaled $\times 2$ (ii) triangle scaled $\times 2$ (iii) triangle scaled $\times 3$

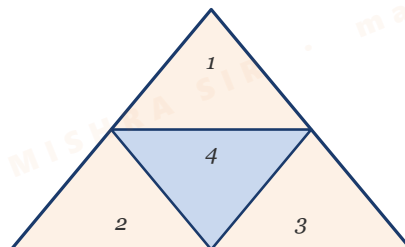
4 marks



(i) A $2a \times 2b$ rectangle is a 2×2 grid of $a \times b$ rectangles

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(i) PQRS has area $2a \times 2b = 4ab = 4 \text{ ar}(\text{ABCD})$. **1** And yes — cutting PQRS by its two midlines gives a 2×2 grid of exact $a \times b$ copies: 4 copies fit.

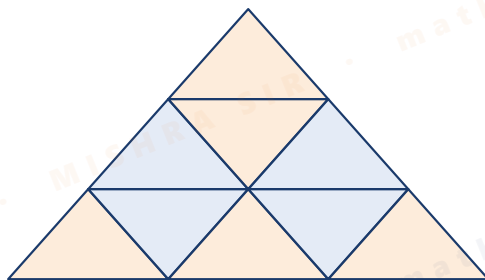


(ii) Joining midpoints cuts the double triangle into 4 copies — the middle one is rotated 180°

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(ii) Scaling every side by 2 scales the area by $2^2 = 4$ (Heron's formula: every factor $s, s - a, s - b, s - c$ doubles, so the product under the root grows 2^4 and the root grows 2^2).

1 Joining the midpoints of $\triangle PQR$ cuts it into 4 triangles, each with sides a, b, c (midpoint theorem): 4 copies fit — three "upright" and the middle one rotated 180° . **1**



(iii) Rows of 1, 3 and 5 small triangles: 9 copies, alternating in orientation

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari, Part I*.

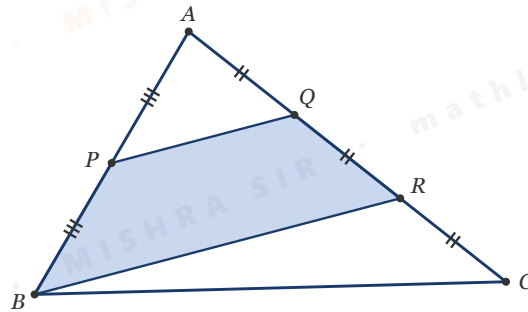
(iii) Scale factor 3 $\Rightarrow$ area factor $3^2 = 9$. Dividing each side into 3 and drawing lines parallel to the sides tiles $\triangle PQR$ with rows of 1, 3, 5 small triangles — 9 congruent copies of $\triangle ABC$ (alternate ones rotated 180°).

1

C Fraction puzzles — Q16 to Q20

Q16*. Fig. 6.43: what fraction of the triangle is shaded? Fig. 6.44: what fraction of the square is shaded?

4 marks



Schematic of Fig. 6.43 — P is the midpoint of AB (triple ticks); Q, R divide AC into three equal parts (double ticks)

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SOLUTION (FIG. 6.43): Take $\text{ar}(\triangle ABC) = 1$. The unshaded part consists of $\triangle APQ$ at the top and $\triangle BRC$ at the bottom.

$\triangle APQ$ shares angle A with $\triangle ABC$, with $AP = \frac{1}{2} AB$ and $AQ = \frac{1}{3} AC$, so

$$\text{ar}(\triangle APQ) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}.$$

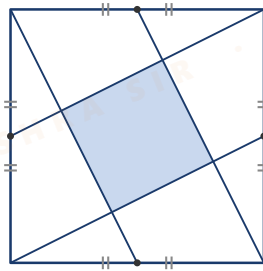
1

$\triangle BRC$ and $\triangle BAC$ stand on bases RC and AC along the same line, with the same apex B :

$$\text{ar}(\triangle BRC) = \frac{RC}{AC} = \frac{1}{3}.$$

1

$$\text{Shaded fraction} = 1 - \frac{1}{6} - \frac{1}{3} = \frac{1}{2}.$$



Schematic of Fig. 6.44 — each vertex is joined to the midpoint of a non-adjacent side, going around the square

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SOLUTION (FIG. 6.44): Take the square of side 1. The four vertex-to-midpoint lines form two parallel pairs (each line rises 1 unit across 2, slope $\pm\frac{1}{2}$ or ∓ 2), so the middle region is a square.

1

Distance between the parallel pair through $(0, 0)$ and through $(0, \frac{1}{2})$: the segment of length $\frac{1}{2}$ between them is the hypotenuse-leg of a right triangle similar to the $(2, 1, \sqrt{5})$ triangle of the cevian, so the perpendicular gap is $\frac{1/2}{\sqrt{5}/2} = \frac{1}{\sqrt{5}}$. The inner square has side $\frac{1}{\sqrt{5}}$ and area $\frac{1}{5}$:

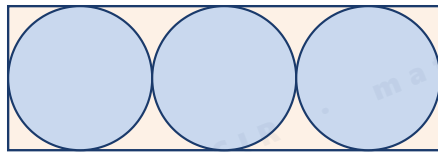
the shaded fraction is $\frac{1}{5}$.

1

Check by dissection: each corner triangle slides along its cevian to fill the neighbouring notch, splitting the big square into 5 tilted squares congruent to the shaded one.

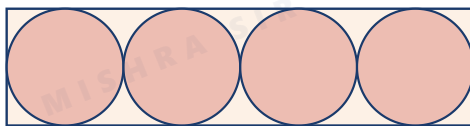
Q17. What fraction of each rectangle is covered by the circles (Figs. 6.45, 6.46)?

3 marks



Three equal circles in a snug $6r \times 2r$ rectangle

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Four equal circles in a snug $8r \times 2r$ rectangle

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SOLUTION: Let each circle have radius r .

Three circles: rectangle = $6r \times 2r = 12r^2$, circles = $3\pi r^2$; fraction = $\frac{3\pi r^2}{12r^2} = \frac{\pi}{4}$. 1

Four circles: rectangle = $8r \times 2r = 16r^2$, circles = $4\pi r^2$; fraction = $\frac{4\pi r^2}{16r^2} = \frac{\pi}{4}$. 1

Both fractions equal $\frac{\pi}{4} = \frac{22}{7} \times \frac{1}{4} = \frac{11}{14} \approx 0.785$. 1

Q18. Conjecture for n circles in a row — test for 10, 20, 50 circles, then prove it

3 marks

CONJECTURE: the covered fraction is always $\frac{\pi}{4}$, no matter how many circles are fitted in a row.

1

TEST: $n = 10 : \frac{10\pi r^2}{20r \cdot 2r} = \frac{\pi}{4}; n = 20 : \frac{20\pi r^2}{80r^2} = \frac{\pi}{4}; n = 50 : \frac{50\pi r^2}{200r^2} = \frac{\pi}{4}$.

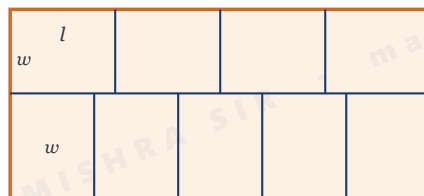
1

PROOF: n touching circles of radius r need a rectangle of exactly $2nr \times 2r = 4nr^2$; the circles cover $n\pi r^2$. Fraction $= \frac{n\pi r^2}{4nr^2} = \frac{\pi}{4}$ — n cancels. Equivalently, each circle sits in its own $2r \times 2r$ square and fills $\pi/4$ of it. ■

1

Q19*. Nine identical rectangles (4 across the top, 5 standing below) form a rectangle of area 72 cm^2 — perimeter of each small rectangle?

4 marks



Schematic of Fig. 6.47 — the top row holds 4 rectangles lying down; the bottom row holds 5 standing up

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SOLUTION: Let each small rectangle be $l \times w$ ($l > w$). The top row's width is $4l$; the bottom row's width is $5w$; both equal the big rectangle's width: $4l = 5w$.

1

Nine equal rectangles share the 72 cm^2 : $lw = 8$.

1

From $w = \frac{4l}{5}$: $\frac{4l^2}{5} = 8 \Rightarrow l^2 = 10 \Rightarrow l = \sqrt{10}$, $w = \frac{4\sqrt{10}}{5}$.

1

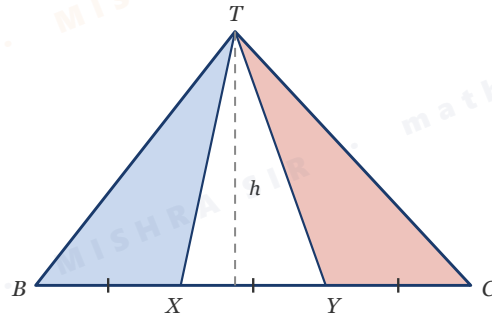
Perimeter $= 2(l + w) = 2\left(\sqrt{10} + \frac{4\sqrt{10}}{5}\right) = \frac{18\sqrt{10}}{5} \approx 11.4 \text{ cm}$.

Check: big rectangle $= 4l \times (l + w) = 4\sqrt{10} \times \frac{9\sqrt{10}}{5} = \frac{360}{5} \times \dots = 72 \checkmark$

1

Q20*. Cevians to the points of trisection: show the blue and red triangles are equal, and cut one to cover the other

4 marks



Schematic of Fig. 6.48 — X, Y trisect BC ; both shaded triangles have base $BC/3$ and apex T

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EQUALITY: $BX = YC = \frac{BC}{3}$, and both triangles have the same apex T , hence the same height h above line BC . 1

$$\text{ar}(\triangle TBX) = \frac{1}{2} \cdot \frac{BC}{3} \cdot h = \text{ar}(\triangle TYC). \blacksquare$$

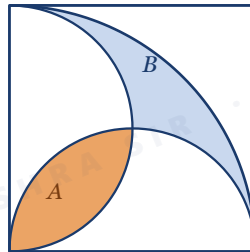
1

DISSECTION (ONE WORKABLE RECIPE): ① Cut the blue triangle along its midline parallel to BC and rotate the top piece 180° about the midpoint of TX — the blue triangle becomes a parallelogram with base BX and height $\frac{h}{2}$. 1 ② A parallelogram can be re-cut into any other parallelogram of the same base and height: slice it by lines perpendicular to the base and slide each strip sideways until the slant matches that of the red triangle's half-parallelogram; ③ reverse step ① on the red side. Since every step is cut-and-slide, the blue pieces exactly cover the red triangle. 1

D Circle-region puzzles — Q21 to Q27 (all starred)

Q21*. Quarter circle centred at a vertex of a square + two semicircles on the adjacent sides: show regions A and B are equal (Fig. 6.49)

4 marks



Schematic of Fig. 6.49 — A is the overlap of the two semicircles; B lies between the quarter arc and the semicircle arcs

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PROOF: Take the square of side 2, and let O be the corner where the two diameters meet. The quarter circle has centre O and radius 2; each semicircle has radius 1. Every point of each semicircle is within distance 2 of O (the farthest points are the opposite vertices, exactly on the quarter arc), so both semicircles S_1, S_2 lie inside the quarter disc Q, and by the figure

$$B = Q - (S_1 \cup S_2), A = S_1 \cap S_2.$$

1

$$\text{Areas: } Q = \frac{1}{4}\pi(2)^2 = \pi; S_1 = S_2 = \frac{1}{2}\pi(1)^2 = \frac{\pi}{2}, \text{ so } S_1 + S_2 = \pi = Q.$$

1

$$\text{Now } \text{ar}(S_1 \cup S_2) = S_1 + S_2 - \text{ar}(S_1 \cap S_2) = \pi - A.$$

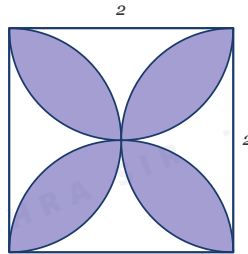
1

$$\text{Therefore } B = Q - (\pi - A) = \pi - \pi + A = A. \blacksquare$$

1

Q22*. Four semicircles inside a square of side 2 make a 4-petalled flower — its perimeter and area (Fig. 6.50)

4 marks



Schematic of Fig. 6.50 — semicircles of radius 1 on all four sides; all four arcs pass through the centre

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AREA: The four semicircles together have area $4 \times \frac{\pi(1)^2}{2} = 2\pi$. They cover the whole square (side 2, area 4), and the points covered twice are exactly the petals. **1**

Counting every covered point: $2\pi = 4 + \text{ar(flower)} \Rightarrow \text{ar(flower)} = 2\pi - 4 \approx \mathbf{2.28 \text{ sq. units.}}$

1

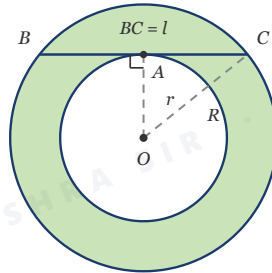
PERIMETER: Adjacent semicircles cross at a corner-of-side point and at the centre; each petal is bounded by two 90° arcs of radius 1, of length $\frac{2\pi(1)}{4} = \frac{\pi}{2}$ each. **1**

Perimeter $= 4 \times 2 \times \frac{\pi}{2} = 4\pi \approx \mathbf{12.57 \text{ units.}}$

1

Q23*. Chord BC of the outer circle touches the inner circle at A; $BC = l$. Show the ring's area is $\frac{1}{4} \pi l^2$ (Fig. 6.51)

3 marks



Schematic of Fig. 6.51 — $OA \perp BC$ (radius to point of contact), and A is the midpoint of the chord

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PROOF: A tangent is perpendicular to the radius at the point of contact, so $OA \perp BC$; a perpendicular from the centre bisects a chord, so A is the midpoint of BC and $AC = \frac{l}{2}$. 1

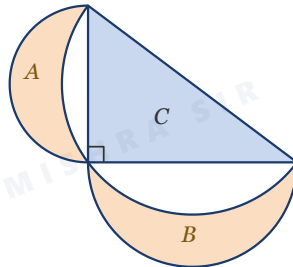
In right $\triangle OAC$: $R^2 = r^2 + \left(\frac{l}{2}\right)^2 \Rightarrow R^2 - r^2 = \frac{l^2}{4}$. 1

Green ring = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2) = \frac{1}{4} \pi l^2$. ■ 1

Remarkably, the answer does not depend on the two radii separately — only on the chord.

Q24*. Semicircles on all sides of a right triangle: show $\text{Area}(A) + \text{Area}(B) = \text{Area}(C)$ (Fig. 6.52)

4 marks



Schematic of Fig. 6.52 — A and B are the crescents (lunes) on the legs; C is the triangle. The semicircle on the hypotenuse passes through the right-angle vertex

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PROOF: Let the legs be a, b and the hypotenuse c . The semicircle drawn on the hypotenuse towards the triangle passes exactly through the right-angle vertex (angle in a semicircle is 90°), so it consists of the triangle C plus two circular segments — one sitting on each leg. 1

Each lune = (semicircle on that leg) – (the segment on that leg):

$$A + B = \frac{\pi}{2} \left(\frac{a}{2} \right)^2 + \frac{\pi}{2} \left(\frac{b}{2} \right)^2 - (\text{seg}_a + \text{seg}_b) = \frac{\pi}{8} (a^2 + b^2) - (\text{seg}_a + \text{seg}_b). \quad 1$$

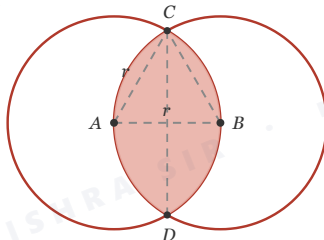
By Pythagoras $a^2 + b^2 = c^2$, so $\frac{\pi}{8} (a^2 + b^2) = \frac{\pi}{8} c^2 = \text{area of the semicircle on the hypotenuse} = C + \text{seg}_a + \text{seg}_b. \quad 1$

Therefore $A + B = C + \text{seg}_a + \text{seg}_b - (\text{seg}_a + \text{seg}_b) = C. \quad 1$

These are the famous lunes of Hippocrates (c. 440 BCE) — curved regions whose total area equals a straight-sided one.

Q25*. Two circles of radius r pass through each other's centres — area of the common (lens) region (Fig. 6.53)

4 marks



Schematic of Fig. 6.53 — $AB = AC = BC = r$, so $\angle CAB = 60^\circ$ and the lens chord CD subtends 120° at each centre

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari, Part I*.

SOLUTION: Let the centres be A, B and the intersection points C, D . Since C lies on both circles, $AC = BC = r$; also $AB = r$. So $\triangle ABC$ is equilateral and $\angle CAB = 60^\circ$; by symmetry $\angle CAD = 120^\circ$. 1

The lens is two equal circular segments on chord CD . One segment (in circle A) = sector of 120° — $\triangle ACD$.

$$\text{Sector} = \frac{120}{360} \pi r^2 = \frac{\pi r^2}{3}. \quad \text{1}$$

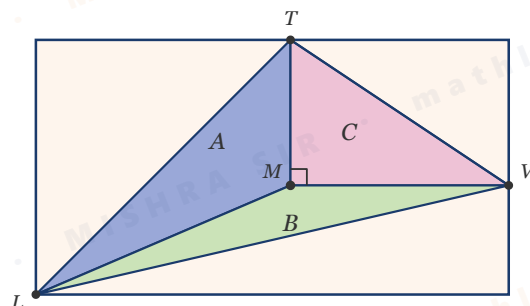
For $\triangle ACD$: $CD \perp AB$ with $CD = 2 \times \frac{\sqrt{3}}{2} r = \sqrt{3} r$ and the distance from A to CD is $\frac{r}{2}$ (half of AB), so its area = $\frac{1}{2} \times \sqrt{3} r \times \frac{r}{2} = \frac{\sqrt{3}}{4} r^2$. 1

$$\text{Lens} = 2 \left(\frac{\pi r^2}{3} - \frac{\sqrt{3}}{4} r^2 \right) = \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right) r^2 = \frac{(4\pi - 3\sqrt{3})}{6} r^2 \approx 1.23 r^2. \quad \text{1}$$

Q26*. Three triangles A, B, C inside a rectangle: show the rectangle's area is

$$\frac{2(A + C)(B + C)}{C} \quad \text{(Fig. 6.54)}$$

4 marks



Schematic of Fig. 6.54 — T on the top side, V on the right side, M inside with TM vertical and MV horizontal; L is the opposite corner

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PROOF: Let the rectangle be $w \times h$ with L at the bottom-left corner $(0, 0)$, $T = (t, h)$ on the top side, and M directly below T with $M = (t, m)$, $V = (w, m)$ on the right side ($TM \perp MV$, as drawn).

1

With base TM (length $h - m$): $A = \frac{1}{2}(h - m)t$ (apex L at distance t) and $C = \frac{1}{2}(h - m)(w - t)$ (apex V at distance $w - t$). With base MV (length $w - t$): $B = \frac{1}{2}(w - t)m$.

1

Adding: $A + C = \frac{1}{2}(h - m)w$ and $B + C = \frac{1}{2}(w - t)h$.

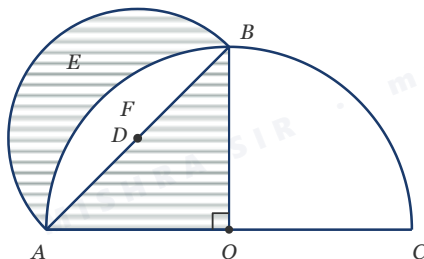
1

Therefore $\frac{2(A + C)(B + C)}{C} = \frac{2 \cdot \frac{1}{2}(h - m)w \cdot \frac{1}{2}(w - t)h}{\frac{1}{2}(h - m)(w - t)} = wh = \text{area of the rectangle.} \blacksquare$

1

Q27*. Quarter circle, semicircle and triangle (Fig. 6.55): show the two shaded regions are equal

4 marks



Schematic of Fig. 6.55 — $OA = OB = OC = r$; arc AFB is a quarter of the circle centred at O ; AEB is the semicircle on chord AB ; D is its centre. Shaded: the lune between the two arcs, and $\triangle AOB$

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari, Part I*.

PROOF: Let $OA = OB = OC = r$ with $OB \perp AC$. Then $\text{ar}(\triangle AOB) = \frac{1}{2}r \cdot r = \frac{r^2}{2}$, and

$$AB = \sqrt{r^2 + r^2} = r\sqrt{2}. \quad \boxed{1}$$

$$\text{Semicircle on } AB: \text{radius } \frac{r\sqrt{2}}{2}, \text{ area} = \frac{1}{2}\pi \left(\frac{r\sqrt{2}}{2}\right)^2 = \frac{\pi r^2}{4}. \quad \boxed{1}$$

$$\text{Segment between chord } AB \text{ and the quarter arc } AFB = \text{quarter disc} - \text{triangle} = \frac{\pi r^2}{4} - \frac{r^2}{2}.$$

1

Lune = semicircle – segment = $\frac{\pi r^2}{4} - \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right) = \frac{r^2}{2} = \text{ar}(\triangle AOB)$. The two shaded regions are equal — and, beautifully, the answer contains no π at all. ■ **1**

⚠ Common Mistakes in This Exercise

Scaling areas linearly (Q15, Q18). Doubling every length multiplies area by 4, not 2 — lengths scale by k , areas by k^2 .

Assuming "4 times the area" automatically means "4 copies fit" (Q15). It happens to be true here, but only the explicit tilings (grid / midpoint triangles) prove it.

Double-counting overlaps (Q21, Q22). When regions overlap, use: total painted = sum of pieces – overlap. This one line solves both problems.

In Q24, subtracting the triangle instead of the segments. The hypotenuse-semicircle = triangle + two segments; keep the bookkeeping of each region explicit.

Q19: forgetting the second equation. The picture gives $4l = 5w$; the area gives $lw = 8$. Both are needed.

◆ END OF CHAPTER 6 — END-OF-CHAPTER EXERCISES ◆

CHAPTER 7

The Mathematics of Maybe: Introduction to Probability

Exercise Sets 7.1 – 7.4 + End-of-Chapter

EXERCISE SETS 7.1 – 7.4 + END-OF-CHAPTER

Weightage & Placement of This Chapter

Where Chapter 7 sits in Class 9 Mathematics

⚠ **Honest note on marks:** Class 9 is a **school-conducted** examination, so there is no single official chapter-wise mark table the way there is for Class X. The figures below reflect the standard structure of the NCERT *Ganita Manjari* Part I and typical Class 9 annual-paper practice. **Please confirm the exact split against your own school's question-paper blueprint** before planning revision time.

Detail	Value
Textbook	Ganita Manjari, Grade 9, Part I (new NCERT, 2026–27)
Chapter number	Chapter 7 — "The Mathematics of Maybe"
Broad area	Statistics & Probability
Sub-topics	Randomness · Probability scale · Experimental probability · Theoretical probability · Statistical data & sampling · Sample space · Events · Tree diagrams
Exercise sets	7.1 (1 Q) · 7.2 (6 Q) · 7.3 (3 Q) · 7.4 (2 Q) · End-of-Chapter (16 Q) = 28 questions
Typical annual-exam weight	Approximately 5–6 marks (school-dependent)
Why it matters beyond marks	This is the direct foundation for Class 10 Probability and Class 11–12 Probability & Statistics. It is also the most scoring chapter in the paper — the arithmetic is easy, and marks are lost almost entirely to careless counting rather than to difficulty.

Predicted Question Types

Format	Marks	Likely Topic
MCQ / Fill in the blanks	1	$P(\text{impossible event}) = 0$; $P(\text{certain event}) = 1$; range $0 \leq P(E) \leq 1$; name for the set of all outcomes
Very Short Answer	1	Sample size of a die/coin experiment; probability of an even number on a die; probability from a word's letters
Short Answer	2	Write the sample space for a two-step experiment; experimental probability from given trial data
Short Answer	3	Sample survey $\rightarrow$ estimate the count for a whole population; comparing experimental vs theoretical probability
Long Answer	4–5	Tree diagram questions — two coins, two dice, drawing balls with and without replacement
Competency / Case Study	4–5 (sub-parts)	Tyre-life frequency table; survey of favourite clubs/fruits; spinner problems
HOTS (starred ★ questions)	3–5	Sum of two dice being a prime greater than 5; drawing without replacement; identifying a valid sample space; geometric probability (dye on a rectangle)

✦ The **starred (*) questions** (End-of-Chapter Q9–Q16) are explicitly marked as higher-order. Long answers are drawn from exactly this pool — and Q10, Q12 and Q13 are almost certain to appear in some form because they test the tree diagram, which is the flagship skill of this chapter.

All Key Definitions & Formulas

Probability: A measurement of the likelihood of an event — just as length measures distance, probability measures how confident we are that something will happen.

Random event / randomness: A situation where you **know all the possible outcomes but cannot say which one will definitely occur** (tossing a coin, rolling a die). You can say what *could* happen, not what *will* happen.

Random experiment (trial): Something you can **repeat**, where the result may differ each time and cannot be known in advance.

Outcome: A single result of an experiment. **Sample space (S):** The set of **all** possible outcomes.

Sample size $n(S)$: The number of elements in S.

Two rules for a valid sample space: (1) it must include **every** possible outcome; (2) **no outcome may be listed more than once**. Both rules are tested directly in End-of-Chapter Q15.

Event (E): Any single outcome or combination of outcomes — that is, a **subset of the sample space**.

Example: for a die, $E = \text{'number greater than 4'} = \{5, 6\}$.

The probability scale: $0 \leq P(E) \leq 1$. $P = 0$ means **impossible**; $P = 1$ means **certain**; $P = 0.5$ means **equally likely** (even chance); below 0.5 is **less likely**, above 0.5 is **more likely**.

Experimental probability (also called **relative frequency**):

$$\text{Experimental Probability} = \frac{\text{Number of times the event occurred}}{\text{Total number of trials}}$$

Based on **actual collected data**, not on assumptions.

Theoretical probability:

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$$

Assumes all outcomes are **equally likely**. Needs **no experiment and no data**.

Law of Large Numbers: As the number of trials increases, the **experimental probability tends to get closer to the theoretical probability**. This is why 12 rolls of a die may give a badly off answer while 6000 rolls will not.

Gambler's Fallacy: The mistaken belief that past results change future chances — e.g. "six heads in a row, so tails must come next". **The coin has no memory**. Each toss is an **independent** event and the probability stays exactly the same.

Fair / unbiased: A coin or die is **fair** if it is symmetrical, so there is no reason for it to favour any side. A **random toss** means it falls freely without bias or interference.

Population and sample: The **population** is the whole group being studied; a **sample** is the smaller group actually surveyed. **Sampling** is the process of choosing that smaller group. A larger and more representative sample gives a more reliable estimate.

Estimating a population count from a sample:

$$\text{Estimated count} = \text{Probability from the sample} \times \text{Population size}$$

Tree diagram: A branching picture that lists all outcomes of a **multi-step experiment**. Every complete **path from start to end is one outcome**. Probabilities are written on the branches.

Subjective probability: A likelihood based on personal judgement of evidence ("it's very hot, so it may rain") rather than on data or on counting equally likely outcomes.

The Fundamental Principle of Counting — and Why You May Use It

Statement. If one action can be done in m ways, and for **each** of those a second action can be done in n ways, then the two actions together can be done in $m \times n$ ways. For three steps it extends to $m \times n \times p$, and so on.

✦ **This is NOT permutations and combinations.** It is simply a description of the tree diagram you have drawn: the first step gives m branches, each of those splits into n more, so the tree ends in $m \times n$ tips — and each tip is one outcome. You are allowed to use it in Class 9 because it is **counting the tree**, not applying a formula from a later class. If you ever feel unsure, **draw the tree and count** — the answer is identical and the marks are the same.

Worked illustration. Tossing a coin (2 outcomes) and rolling a die (6 outcomes) together:
Total outcomes = $2 \times 6 = 12$. The tree has 2 first branches, each splitting into 6, giving 12 tips: H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6 ✓

The important extension — "with" vs "without" replacement.

- **With replacement** (the item is put back): the second step has the **same** number of options as the first. Two draws from 4 balls $\rightarrow 4 \times 4 = 16$ outcomes.
- **Without replacement** (the item is kept aside): the second step has **one fewer** option. Two draws from 4 balls $\rightarrow 4 \times 3 = 12$ outcomes.

This single distinction is the most examinable idea in the whole chapter.

Experiment	Reasoning	Sample size $n(S)$
Toss 1 coin	—	2
Toss 2 coins	2×2	4
Toss 3 coins	$2 \times 2 \times 2$	8
Roll 1 die	—	6
Roll 2 dice	6×6	36
Die + coin together	6×2	12
2 draws from 4 balls, with replacement	4×4	16

2 draws from 4 balls, **without** replacement

$$4 \times 3$$

12

3 snacks $\times$ 2 drinks

$$3 \times 2$$

6

💡 Shortcuts & Exam-Writing Tips

1. **Always write the sample space first.** Even when the question does not ask for it, listing S makes the denominator obvious and earns method marks. Most lost marks in this chapter come from a wrong denominator, not wrong division.
2. **Probability can never exceed 1 or fall below 0.** If your answer comes out as $\frac{7}{5}$ or a negative number, you have miscounted — check immediately rather than writing it down.
3. **"At least one" means "one or more".** For two coins, 'at least one head' is {HH, HT, TH} — three outcomes, **not** one. Read this phrase slowly; it is deliberately placed to catch quick readers.
4. **"Exactly two heads" $\neq$ "at least two heads".** For three coins, *exactly* two gives {HHT, HTH, THH} = 3 outcomes, while *at least* two also includes HHH = 4 outcomes.
5. **Order matters when you list outcomes of a two-step experiment.** HT and TH are **different** outcomes. Merging them into "one head and one tail" halves your count and produces a wrong denominator.
6. **Draw the tree neatly and label the branches with probabilities.** In a 4–5 mark question the diagram itself carries marks, independent of the final answer. Label the first stage, the second stage, and the outcome column.
7. **For "without replacement", change the denominator at the second stage.** From 9 balls, the second draw is out of **8**, not 9. Forgetting this is the single most common error in the starred questions.
8. **Estimating a population figure:** find the probability from the sample first, then multiply by the population. Do not divide by the sample size twice.
9. **Simplify fractions, then give the decimal or percentage too** if the question mentions a percentage. Writing " $\frac{8}{30} = \frac{4}{15} \approx 0.267$ or 26.7%" is safer than choosing just one form.
10. **Experimental and theoretical probability need not agree** — and saying so earns marks. The reason is always the same: too few trials. Mention the **Law of Large Numbers** by name.
11. **Beware the Gambler's Fallacy in worded questions.** "The die has shown 6 three times — what is the probability of another 6?" The answer is still $\frac{1}{6}$. Nothing changes.
12. **Equally likely is an assumption, not a fact.** A bag with 3 red and 7 blue marbles does **not** give equally likely colours. Questions asking "are these equally likely?" want you to spot exactly this.
13. **End with a sentence.** "Hence the probability of drawing a green sweet is $\frac{4}{15}$." A bare fraction routinely loses the final half mark.

■ NCERT Solutions — Exercise Set 7.1

Approach for This Exercise

This exercise is about the **probability scale**, not calculation. For each event decide where it sits between 0 and 1, attach the correct label, and — most importantly — **give a reason**. The reason carries the mark; the label alone does not.

Q1. Rank the following events on a scale from 0 (Impossible) to 1 (Certain). Label each event: Impossible, less likely, equally likely (even chance), more likely, certain. Give reasons.

4 marks

(i) The next Monday will come after Sunday. (ii) It will snow in Mumbai in July. (iii) An elephant will walk through your classroom today. (iv) You will greet at least one friend at school tomorrow.

SOLUTION:

Event	Value	Label
(i) Monday comes after Sunday	1	Certain
(ii) Snow in Mumbai in July	0	Impossible
(iii) Elephant walks through the classroom	Close to 0	Less likely
(iv) Greeting at least one friend at school	Close to 1	More likely

(i) Certain — probability 1. The days of the week follow a fixed, unchanging order, and Monday always follows Sunday. There is no possible outcome in which this fails, so the event is certain.

1 mark

(ii) Impossible — probability 0. Mumbai has a tropical climate and July falls in its monsoon season, when temperatures remain far above freezing. Snowfall requires sub-zero conditions, which simply do not occur there, so the event cannot happen.

1 mark

(iii) Less likely — probability very close to 0. Elephants are not found in classrooms and schools are enclosed buildings, so this is extremely unlikely. However — and this is the point of the question — it is **not strictly impossible**, since an elephant could conceivably stray in from a nearby procession or forest area.

1 mark

(iv) More likely — probability close to 1. Students normally meet several friends during a school day and greeting at least one is the usual outcome. It is not certain, though, since one might be absent or the school might be closed.

1 mark

Examiner's eye: parts (ii) and (iii) look similar but are deliberately different. Snow in Mumbai is ruled out by **physical impossibility** (probability exactly 0); an elephant in the classroom is merely **extremely improbable** (probability very small but greater than 0). Writing "impossible" for (iii) is the trap this question is built around.

 NCERT Solutions — Exercise Set 7.2 (6 Questions)

 Approach for This Exercise

This set is about **experimental probability** and **sampling**. The routine: (1) find the total number of trials or the sample size — this is your denominator; (2) find how many times the event occurred — this is your numerator; (3) simplify. For population estimates, find the probability from the sample **first**, then multiply by the population size.

Q1. A teacher mixes a large bag of sweets and randomly selects a sample of 30 sweets: 10 red, 8 green, 7 yellow, 5 blue. (i) Calculate the probability that a randomly picked sweet from the sample is green. (ii) If there are 600 sweets in the bag, estimate how many are likely to be yellow.

4 marks

GIVEN: Sample of 30 sweets — 10 red, 8 green, 7 yellow, 5 blue.

CHECK FIRST: $10 + 8 + 7 + 5 = 30$ ✓ — the counts add up to the sample size, so no data is missing.

SOLUTION:

(i) Probability that a sweet is green.

Step 1: Number of favourable outcomes (green sweets) = 8

Step 2: Total number in the sample = 30 1 mark

Step 3: $P(\text{green}) = \frac{8}{30} = \frac{4}{15} \approx 0.267$ or about 26.7% 1 mark

(ii) Estimating the number of yellow sweets in 600.

METHOD 1 — Probability first, then multiply (recommended)

Step 4: From the sample, $P(\text{yellow}) = \frac{7}{30}$ 1 mark

Step 5: Estimated yellow sweets = $P(\text{yellow}) \times \text{total} = \frac{7}{30} \times 600$
 $= 7 \times 20 = 140$ 1 mark

METHOD 2 — Scale factor

The bag is $\frac{600}{30} = 20$ times as large as the sample. So every colour should appear about **20 times as often**:

Yellow = $7 \times 20 = 140$ ✓ (and as a check: red = 200, green = 160, blue = 100; total
 $= 200 + 160 + 140 + 100 = 600$ ✓)

Which method to use: Method 2 is quicker and self-checking — once you have the scale factor 20, every colour follows in one multiplication and the four answers must add to 600. Use **Method 1** when the question asks for the probability anyway (as part (i) does here), since you have already done half the work.

Hence $P(\text{green}) = \frac{4}{15}$, and about 140 of the 600 sweets are likely to be yellow.

⚠ Note the word "estimate". This is a prediction from a sample, not an exact count — the real bag may contain slightly more or fewer yellow sweets.

Q2. A random sample of 40 students is asked about their favourite club: 14 Science, 11 Arts, 9 Sports, 6 Debate. The school has 800 students. (i) Probability that a randomly chosen student from the sample prefers the Arts Club? (ii) Estimate how many students in the whole school prefer the Sports Club.

4 marks

GIVEN: Sample size 40; Science 14, Arts 11, Sports 9, Debate 6; population 800.

CHECK FIRST: $14 + 11 + 9 + 6 = 40$ ✓

SOLUTION:

(i) Probability of preferring the Arts Club.

Step 1: Favourable outcomes = 11; total = 40

1 mark

Step 2: $P(\text{Arts}) = \frac{11}{40} = 0.275$ or 27.5%

(This fraction cannot be simplified, since 11 is prime and does not divide 40.)

1 mark

(ii) Estimating Sports Club members in the whole school.

METHOD 1 — Probability first, then multiply

Step 3: $P(\text{Sports}) = \frac{9}{40}$

1 mark

Step 4: Estimated number = $\frac{9}{40} \times 800 = 9 \times 20 = 180$

1 mark

METHOD 2 — Scale factor

The school is $\frac{800}{40} = 20$ times the sample, so Sports = $9 \times 20 = 180$ ✓

(Check: Science = 280, Arts = 220, Sports = 180, Debate = 120; total = 800 ✓)

Which method to use: Method 2, for the same reason as Q1 — the scale factor makes all four estimates fall out at once and the total acts as a built-in check on your arithmetic.

Hence $P(\text{Arts}) = \frac{11}{40} = 0.275$, and about 180 students in the school are likely to prefer the Sports Club.

Q3. Toss a coin 20 times and record the result each time. (i) How many heads? (ii) How many tails? (iii) Calculate the experimental probability of getting heads. (iv) If you toss the coin once more, what is the probability of getting tails?

4 marks

NATURE OF THIS QUESTION: This is a **hands-on activity** — your own numbers will differ from anyone else's, and that is the point. A specimen set of results is used below so you can see the correct **method and presentation**.

SPECIMEN RECORD (20 TOSSES):

Outcome	Tally	Frequency
Heads (H)		12
Tails (T)		8
Total	—	20

(i) Number of heads = 12 ½ mark

(ii) Number of tails = 8 (check: $12 + 8 = 20$ ✓) ½ mark

(iii) **Experimental probability of heads:**

$$= \frac{\text{Number of times heads occurred}}{\text{Total number of trials}} = \frac{12}{20} = \frac{3}{5} = 0.6 \text{ or } 60\% \quad \text{1.5 marks}$$

(iv) **Probability of tails on the next toss.**

This part is **not** asking for experimental probability. The next toss is a fresh, **independent** event, and for a fair coin:

$$P(\text{tails}) = \frac{1}{2} = 0.5 \quad \text{1.5 marks}$$

⚠ This is the trap in the question. Many students answer $\frac{8}{20} = 0.4$ for part (iv), reasoning that tails came up 8 times out of 20. That is the **Gambler's Fallacy**. The coin has no memory of the previous 20 tosses; each new toss is independent, so the theoretical probability of tails remains exactly $\frac{1}{2}$. Parts (iii) and (iv) are placed side by side deliberately, to test whether you can tell **experimental** probability (from data already collected) apart from **theoretical** probability (for a future fair toss).

Note: your 12 heads out of 20 differs from the theoretical 10 out of 20 simply because 20 trials is a small number. By the **Law of Large Numbers**, tossing 500 or 5000 times would bring the experimental value much closer to 0.5.

Q4. Toss a paper cup into the air 100 times. After each toss record whether the cup lands on its bottom, upside down on its top, or on its side. Assign probabilities to the outcomes using experimental probability.

4 marks

NATURE OF THIS QUESTION: Again a **practical activity**; specimen results are shown so the method is clear.

WHY THIS EXPERIMENT IS INTERESTING: Unlike a coin or a die, a paper cup is **not symmetrical**. There is therefore **no way to calculate a theoretical probability** here — the three outcomes are certainly **not equally likely**. The only way to assign probabilities is to **perform the experiment and collect data**. This is precisely why experimental probability matters.

1 mark

SPECIMEN RECORD (100 TOSSES):

Landing position	Frequency	Experimental probability
On its bottom	22	$\frac{22}{100} = 0.22$
Upside down on its top	16	$\frac{16}{100} = 0.16$
On its side	62	$\frac{62}{100} = 0.62$
Total	100	1.00

2 marks

Observations to write in your answer:

- Each probability is found as $\frac{\text{frequency of that landing}}{100}$.
- The three probabilities **must add up to 1** ($0.22 + 0.16 + 0.62 = 1.00$ ✓) — this is a compulsory check, because the cup must land in one of the three ways.
- Landing on its **side** is by far the most likely, because the side has the largest surface area and the lowest centre of gravity when resting.
- These values are **estimates**. Repeating the experiment would give slightly different figures, and more tosses would give more reliable ones.

1 mark

Exam tip: if this appears in a written paper, you will be given the frequency table. The marks then sit in three places: dividing by the correct total, showing that the probabilities sum to 1, and explaining **why theoretical probability cannot be used** (the cup is not symmetrical, so the outcomes are not equally likely).

Q5. What is the probability of getting an even number when rolling a fair 6-sided die?

2 marks

GIVEN: A fair 6-sided die.

SOLUTION:

Step 1 — Write the sample space:

$S = \{1, 2, 3, 4, 5, 6\}$, so $n(S) = 6$ **½ mark**

Step 2 — Write the event:

$E = \{\text{even numbers}\} = \{2, 4, 6\}$, so $n(E) = 3$ **½ mark**

Step 3 — Apply the formula:

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2} = 0.5 \text{ or } 50\% \quad \text{1 mark}$$

Hence the probability of getting an even number is $\frac{1}{2}$.

Note: the word **fair** matters — it is what allows us to assume all six faces are equally likely. Without it, we could not use theoretical probability at all.

Q6. Suppose you roll a 6-sided die 12 times and get a '3' three times. (i) What is the experimental probability of rolling a '3'? (ii) What is the theoretical probability? (iii) Why might these probabilities be different? What would you expect if you rolled the die 60, 600 or 6000 times?

5 marks

GIVEN: 12 trials; a '3' occurred 3 times.

SOLUTION:

(i) Experimental probability. This uses the actual data:

$$P_{\text{exp}}(3) = \frac{\text{Number of times 3 occurred}}{\text{Total trials}} = \frac{3}{12} = \frac{1}{4} = 0.25 \text{ or } 25\%$$

1.5 marks

(ii) Theoretical probability. This uses equally likely outcomes and ignores the data entirely:

$$S = \{1, 2, 3, 4, 5, 6\}, n(S) = 6; \text{favourable} = \{3\}, \text{so } n(E) = 1$$

$$P_{\text{theo}}(3) = \frac{1}{6} \approx 0.167 \text{ or about } 16.7\%$$

1.5 marks

(iii) Why they differ, and what happens with more trials.

Reason for the difference: 12 rolls is a very small number of trials. In a short run, random variation can easily push the observed frequency away from the expected one. Theoretically we would expect a '3' about $\frac{1}{6} \times 12 = 2$ times; getting it 3 times instead is an entirely ordinary fluctuation and does **not** mean the die is unfair.

1 mark

What to expect as trials increase:

Number of rolls	Expected number of 3s	Expected experimental probability
12	2	may vary a lot (here 0.25)
60	10	closer to 0.167
600	100	closer still
6000	1000	very close to 0.167

As the number of trials increases, the experimental probability gets closer and closer to the theoretical probability of $\frac{1}{6}$. This is the **Law of Large Numbers**.

1 mark

How to earn the full mark on part (iii): name the **Law of Large Numbers** explicitly, and say clearly that the difference is caused by **too few trials**, not by a biased die. Answers that only say "because it is random" typically receive half credit.

— End of Exercise Set 7.2 —

NCERT Solutions — Exercise Set 7.3 (3 Questions)

Approach for This Exercise

These questions are about writing the **sample space** correctly. Two rules govern everything: include **every** outcome, and list **none twice**. For two-step experiments, work systematically — fix the first item, run through all the second items, then move on. Random listing is how outcomes get missed.

Q1. When a single 6-sided die is rolled, what is the total number of possible outcomes in the sample space?

1 mark

SOLUTION:

The die can show any one of its six faces:

$$S = \{1, 2, 3, 4, 5, 6\}$$

Hence the total number of possible outcomes is $n(S) = 6$.

1 mark

Q2. For the following experiments write down the sample space S . (i) Rolling a die and tossing a coin together. (ii) Choosing a random integer between -5 and $+5$. (iii) A box containing 5 green and 7 red balls. One ball is drawn at random.

5 marks

SOLUTION:

(i) Rolling a die and tossing a coin together.

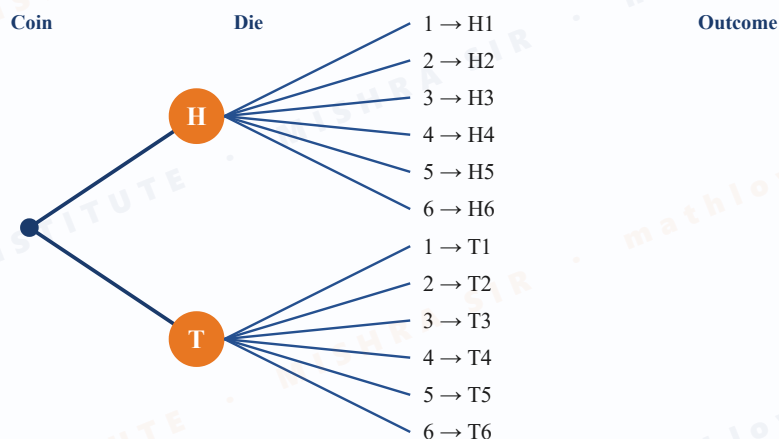
Each outcome must record **both** results — the coin face and the die number. Work systematically: fix H , run through 1 to 6; then fix T , run through 1 to 6.

$$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$$

$$n(S) = 2 \times 6 = 12$$

2 marks

	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6



Tree diagram — tossing a coin and rolling a die together (12 outcomes)

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari, Part I*.

(ii) Choosing a random integer between -5 and $+5$.

The word "between" is **ambiguous**, so state your reading before answering — examiners accept either provided the interpretation is written down.

READING 1 — "between" excludes the end values (strict reading)

$$S = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}, \quad n(S) = 9$$

READING 2 — "between" includes -5 and $+5$ (inclusive reading)

$$S = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}, n(S) = 11$$

2 marks

Which to write: the strict reading (9 integers) is the safer default, because in ordinary English "between A and B" does not include A and B. But write one line stating your assumption — e.g. "taking 'between' to exclude -5 and $+5$ " — and you protect the mark either way. **Do not forget 0**: it is an integer and belongs in the list. Omitting zero is by far the most common error here.

(iii) A box containing 5 green and 7 red balls; one ball is drawn.

What the sample space is depends on **what you are recording**, and both answers are correct for their own question.

If you record only the COLOUR (the usual reading)

$$S = \{\text{Green, Red}\}, n(S) = 2$$

Important caution: these two outcomes are **not equally likely**, since there are 7 red balls and only 5 green.

$$\text{So } P(\text{Green}) = \frac{5}{12}, \text{ not } \frac{1}{2}.$$

If the balls are treated as distinguishable (numbered)

$$S = \{G_1, G_2, G_3, G_4, G_5, R_1, R_2, R_3, R_4, R_5, R_6, R_7\}, n(S) = 5 + 7 = 12$$

Here all 12 outcomes **are** equally likely.

1 mark

Which to write: for this question, $S = \{\text{Green, Red}\}$ is the expected answer, since the box is described only by colour. But **add the caution sentence** that the two outcomes are not equally likely — that single line shows the examiner you understand the difference between listing a sample space and assuming equal likelihood, and it is exactly the distinction End-of-Chapter Q3(iv) tests.

Q3. In a village fair there are 3 snacks: Samosa, Pakora, Bhaji. For drinks, villagers can choose Chai or Lassi. (i) List the sample space of all possible snack and drink combinations. (ii) List the event 'Selecting Samosa as a snack.'

4 marks

GIVEN: 3 snacks and 2 drinks; one of each is chosen.

SOLUTION:

(i) The sample space.

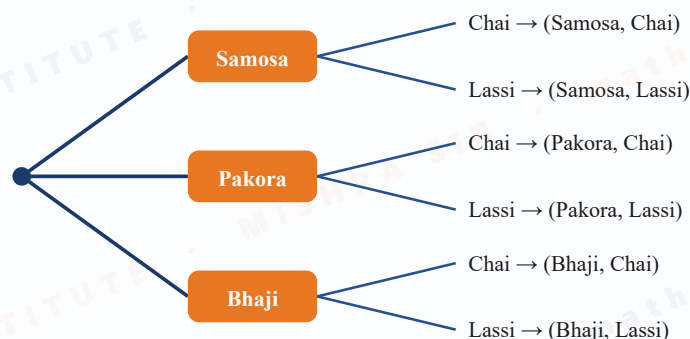
By the Fundamental Principle of Counting, the number of combinations is $3 \times 2 = 6$. Listing them systematically — take each snack in turn and pair it with both drinks:

$$S = \{(\text{Samosa, Chai}), (\text{Samosa, Lassi}), (\text{Pakora, Chai}), (\text{Pakora, Lassi}), (\text{Bhaji, Chai}), (\text{Bhaji, Lassi})\}$$

$$n(S) = 6$$

2 marks

Snack ↓ / Drink →	Chai	Lassi
Samosa	(Samosa, Chai)	(Samosa, Lassi)
Pakora	(Pakora, Chai)	(Pakora, Lassi)
Bhaji	(Bhaji, Chai)	(Bhaji, Lassi)



Tree diagram — 3 snacks $\times$ 2 drinks = 6 combinations

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari, Part I*.

(ii) The event 'Selecting Samosa as a snack'.

An event is a **subset** of the sample space. We keep every outcome whose snack is Samosa — regardless of the drink:

$$E = \{(\text{Samosa, Chai}), (\text{Samosa, Lassi})\}, \quad n(E) = 2$$

2 marks

Bonus (if probability is asked): $P(E) = \frac{2}{6} = \frac{1}{3}$

⚠ **Common error:** writing $E = \{\text{Samosa}\}$. The event must be described using outcomes **from the sample space**, and every outcome in this sample space is a *pair*. "Samosa" alone is not an element of S , so it cannot be an element of E .

— End of Exercise Set 7.3 —

NCERT Solutions — Exercise Set 7.4 (2 Questions)

Approach for This Exercise

This is the **tree diagram** exercise — the flagship skill of the chapter. Draw the first stage, then split **every** first-stage branch into the full second stage, and write the outcome at each tip. In a 4–5 mark question the diagram itself carries marks, so draw it neatly and label all three columns: Stage 1, Stage 2, Outcome.

Q1. Basket A has one apple and two oranges. Basket B has one banana and one mango. You randomly pick one fruit from each basket. (i) Draw a tree diagram showing all possible pairs. (ii) List the sample space. (iii) What is the probability of picking one apple and one banana?

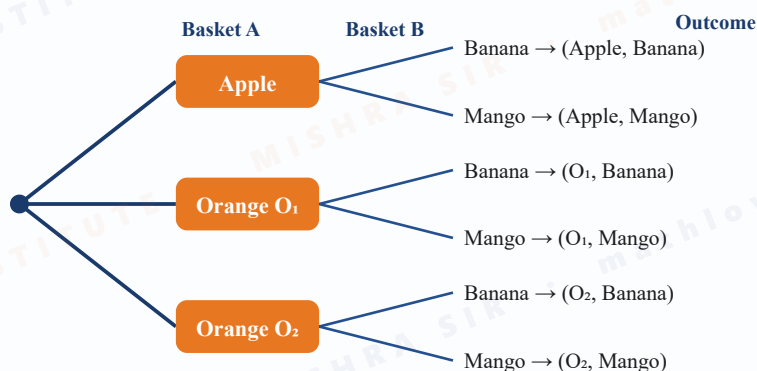
5 marks

GIVEN: Basket A = {Apple, Orange, Orange} — 3 fruits; Basket B = {Banana, Mango} — 2 fruits.

CRITICAL FIRST OBSERVATION: Basket A contains **two** oranges. To keep all outcomes **equally likely**, they must be treated as two separate fruits, written O_1 and O_2 . If you merge them into a single "Orange", the sample space shrinks to 4 outcomes and every probability comes out wrong.

SOLUTION:

(i) Tree diagram. Basket A gives 3 branches; each splits into the 2 fruits of Basket B, so $3 \times 2 = 6$ tips.



Tree diagram — one fruit from each basket (6 equally likely outcomes)

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

2 marks

(ii) The sample space.

$S = \{(Apple, Banana), (Apple, Mango), (O_1, Banana), (O_1, Mango), (O_2, Banana), (O_2, Mango)\}$

$n(S) = 3 \times 2 = 6$

1.5 marks

(iii) Probability of one apple and one banana.

Favourable outcomes: only (Apple, Banana), so $n(E) = 1$

$$P(\text{Apple and Banana}) = \frac{n(E)}{n(S)} = \frac{1}{6} \approx 0.167$$

1.5 marks

Hence the probability of picking one apple and one banana is $\frac{1}{6}$.

METHOD 2 — Multiply along the branch

$$P(\text{Apple from A}) = \frac{1}{3} \text{ and } P(\text{Banana from B}) = \frac{1}{2}$$

$$\text{Following that single path through the tree: } \frac{1}{3} \times \frac{1}{2} = \frac{1}{6} \checkmark$$

Which method to use: Method 1 (count the tips) is the safer Class 9 answer, because it is exactly what the question's tree diagram is for, and it needs no rule about multiplying along branches. Method 2 is a good check and becomes the standard technique in Class 10 — but if you use it, make sure the tree is drawn first, since the marks for parts (i) and (ii) depend on it.

⚠ The trap: if you had merged the two oranges, you would get $n(S) = 4$ and answer $\frac{1}{4}$ — which is wrong. Two identical-looking objects still count separately when we need equally likely outcomes.

Q2. A box contains 3 red pens, 4 black pens and 2 green pens. You pick a pen without looking and put it back. Then your friend does the same. (i) What are the possible outcomes of the pen colours? Draw a tree diagram. (ii) Use the tree diagram to find the probability that both of you pick pens of the same colour.

5 marks

GIVEN: 3 red + 4 black + 2 green = 9 pens. The first pen is **replaced** before the second pick.

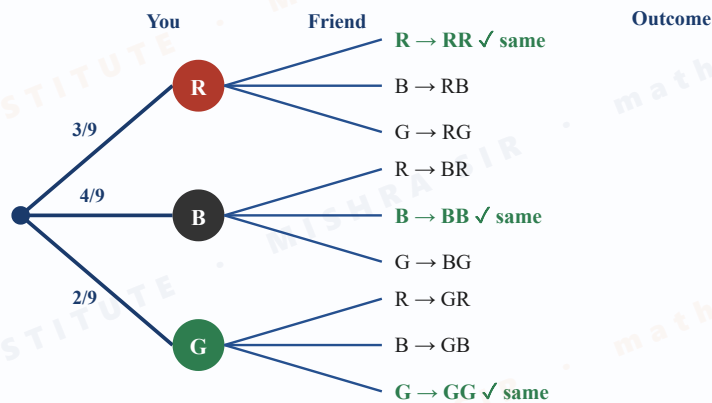
WHY "PUT IT BACK" MATTERS: because the pen is returned, the box is identical for the second pick — still 9 pens. So the second stage has the **same** probabilities as the first. (If it were not replaced, the second denominator would become 8.)

SOLUTION:

(i) Possible outcomes. Each person's pen can be Red (R), Black (B) or Green (G), so by the Fundamental Principle of Counting there are $3 \times 3 = 9$ colour outcomes:

$$S = \{RR, RB, RG, BR, BB, BG, GR, GB, GG\}, \quad n(S) = 9$$

1 mark



Tree diagram — two picks with replacement (9 colour outcomes; the 3 matching ones marked ✓)

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

(ii) Probability that both pick the same colour.

⚠ Important: the 9 colour outcomes are **not equally likely**, because there are different numbers of pens of each colour. So we **cannot** simply answer $\frac{3}{9}$. We must use the branch probabilities.

Step 1 — Probability for a single pick:

$$P(R) = \frac{3}{9} = \frac{1}{3}, \quad P(B) = \frac{4}{9}, \quad P(G) = \frac{2}{9}$$

$$(\text{Check: } \frac{3}{9} + \frac{4}{9} + \frac{2}{9} = \frac{9}{9} = 1 \checkmark)$$

1 mark

Step 2 — Multiply along each of the three matching paths:

$$P(RR) = \frac{3}{9} \times \frac{3}{9} = \frac{9}{81}$$

$$P(BB) = \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$$

$$P(GG) = \frac{2}{9} \times \frac{2}{9} = \frac{4}{81}$$

1.5 marks

Step 3 — Add the three (they are the only ways to match):

$$P(\text{same colour}) = \frac{9}{81} + \frac{16}{81} + \frac{4}{81} = \frac{29}{81} \approx 0.358 \text{ or about } 35.8\%$$

1.5 marks

Hence the probability that both pick pens of the same colour is $\frac{29}{81}$.

METHOD 2 — Count equally likely pen-pairs instead of colours

Treat the 9 pens as distinguishable. Two picks with replacement give $9 \times 9 = 81$ equally likely outcomes.

Both red: $3 \times 3 = 9$ ways | Both black: $4 \times 4 = 16$ ways | Both green: $2 \times 2 = 4$ ways

$$\text{Total matching} = 9 + 16 + 4 = 29, \text{ so } P = \frac{29}{81} \checkmark$$

Which method to use: Method 2 is conceptually the safest — every one of the 81 outcomes really is equally likely, so the plain $\frac{\text{favourable}}{\text{total}}$ formula applies without any special rule, and the denominator 81 explains itself. **Method 1 matches what the question asks for** (it says "use the tree diagram"), so write Method 1 as your main answer and keep Method 2 as the check.

Whichever you use, the mark is lost by answering $\frac{3}{9} = \frac{1}{3}$ — that would only be right if red, black and green pens were equal in number.

— End of Exercise Set 7.4 —

NCERT Solutions — End-of-Chapter Exercises (16 Questions)

Approach for This Exercise

Eight of the sixteen questions are **starred (★)** higher-order questions, and your annual-paper long answers will come from here. Two habits carry you through nearly all of them: (1) **write the sample space or draw the tree before calculating** — the denominator then cannot be wrong; (2) for "without replacement", **reduce the second denominator by one**. Remember: no permutations or combinations are needed anywhere below.

Q1. Fill in the blanks. (i) The probability of an impossible event is _____. (ii) The set of all possible outcomes of a random experiment is called the _____. (iii) The probability of an event that is certain to happen is _____. (iv) Tossing a fair coin has a probability of _____ for getting heads.

4 marks

SOLUTION:

(i) **0** — an impossible event has no favourable outcomes, so $P = \frac{0}{n(S)} = 0$.

1 mark

(ii) **sample space** (denoted by S); the number of its elements is the sample size $n(S)$.

1 mark

(iii) **1** — for a certain event every outcome is favourable, so $P = \frac{n(S)}{n(S)} = 1$.

1 mark

(iv) $\frac{1}{2}$ (that is, 0.5 or 50%) — the sample space is $\{H, T\}$ with $n(S) = 2$, and heads is 1 favourable outcome.

1 mark

Remember the boundary rule: $0 \leq P(E) \leq 1$ always. Parts (i) and (iii) fix the two ends of the probability scale, and every other probability you ever calculate must fall between them.

Q2. In a survey of 50 students, 15 said they liked football. The number of students who like football is 15, and the _____ (frequency / relative frequency) is _____ (fill in the fraction or decimal).

2 marks

SOLUTION:

Step 1 — Distinguish the two words:

• **Frequency** = the raw count of how many times something occurred = **15**

• **Relative frequency** = that count divided by the total number of trials

1 mark

*Step 2 — Since 15 is already given as the number of students, the blank asks for the **relative frequency**:*

$$\text{Relative frequency} = \frac{15}{50} = \frac{3}{10} = 0.3 \text{ or } 30\%$$

1 mark

Hence the blanks are: relative frequency, and $\frac{3}{10} = 0.3$.

Note: relative frequency and **experimental probability** are two names for the same quantity — the chapter uses them interchangeably.

Q3. Which of the following experiments have equally likely outcomes? Explain. (i) A driver attempts to start a car; it starts or does not start. (ii) Tossing a fair coin once. (iii) Rolling a fair 6-sided die. (iv) Choosing a marble from a bag with 3 red and 7 blue marbles. (v) A baby is born; it is a boy or a girl.

5 marks

THE TEST TO APPLY: *outcomes are equally likely only when there is **no reason for any one to occur more often than another**. Merely having two possible outcomes does **not** make them equally likely — that is the misconception this question is built to expose.*

SOLUTION:

(i) NOT equally likely. *A car in working order starts almost every time. Starting and not starting are the only two outcomes, but they are nowhere near equal in chance — the outcome depends on the battery, fuel and engine condition, not on symmetry.*

1 mark

(ii) Equally likely. *A fair coin is symmetrical, so there is no reason to favour heads over tails.*

$$P(H) = P(T) = \frac{1}{2}.$$

1 mark

(iii) Equally likely. *A fair die is a symmetrical cube, so each of the six faces has the same chance: $P = \frac{1}{6}$ each.*

1 mark

(iv) NOT equally likely. *The bag has 10 marbles but the colours are unevenly distributed:*

$$P(\text{red}) = \frac{3}{10} = 0.3 \text{ while } P(\text{blue}) = \frac{7}{10} = 0.7$$

Blue is much more likely simply because there are more blue marbles.

1 mark

(v) Approximately equally likely. *A baby being a boy or a girl is very close to an even chance, and for school purposes we treat it as $\frac{1}{2}$ each. Strictly speaking, real birth records show a very slight excess of boys, so the outcomes are **almost** — not exactly — equally likely.*

1 mark

The pattern worth learning: equal likelihood comes from **physical symmetry** (a fair coin, a fair die) or from **equal numbers** of each type. Parts (i) and (iv) fail on both counts. Note also that the word "**fair**" appears in (ii) and (iii) and is absent from the others — that word is the examiner's signal.

Q4. Write the sample space and calculate the probability. (i) Two coins tossed — P(at least one head)? (ii) Cards numbered 1 to 10, one drawn — P(even number)? (iii) A die rolled once — P(number greater than 4)? (iv) A bag with 3 red, 2 blue, 1 green ball — P(not red)? (v) Three coins tossed — P(exactly two heads)?

10 marks

SOLUTION:

(i) Two coins — at least one head.

$$S = \{HH, HT, TH, TT\}, n(S) = 2 \times 2 = 4$$

"At least one head" means **one or more** heads, so we include every outcome containing an H:

$$E = \{HH, HT, TH\}, n(E) = 3$$

$$P(E) = \frac{3}{4} = 0.75$$

2 marks

⚠ The only outcome excluded is TT. Answering $\frac{2}{4}$ means you have read "exactly one head" instead of "at least one".

(ii) Ten cards numbered 1 to 10 — even number.

$$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}, n(S) = 10$$

$$E = \{2, 4, 6, 8, 10\}, n(E) = 5$$

$$P(E) = \frac{5}{10} = \frac{1}{2} = 0.5$$

2 marks

(iii) A die — number greater than 4.

$$S = \{1, 2, 3, 4, 5, 6\}, n(S) = 6$$

"Greater than 4" excludes 4 itself: $E = \{5, 6\}, n(E) = 2$

$$P(E) = \frac{2}{6} = \frac{1}{3} \approx 0.333$$

2 marks

⚠ If the question had said "4 or more", the answer would be $\frac{3}{6} = \frac{1}{2}$. Read the inequality carefully.

(iv) Bag with 3 red, 2 blue, 1 green — not red.

$$\text{Total balls} = 3 + 2 + 1 = 6, \text{ so } n(S) = 6$$

"Not red" means blue or green: $n(E) = 2 + 1 = 3$

$$P(\text{not red}) = \frac{3}{6} = \frac{1}{2} = 0.5$$

2 marks

Check using the complement: $P(\text{red}) = \frac{3}{6} = \frac{1}{2}$, so $P(\text{not red}) = 1 - \frac{1}{2} = \frac{1}{2} \checkmark$

(v) Three coins — exactly two heads.

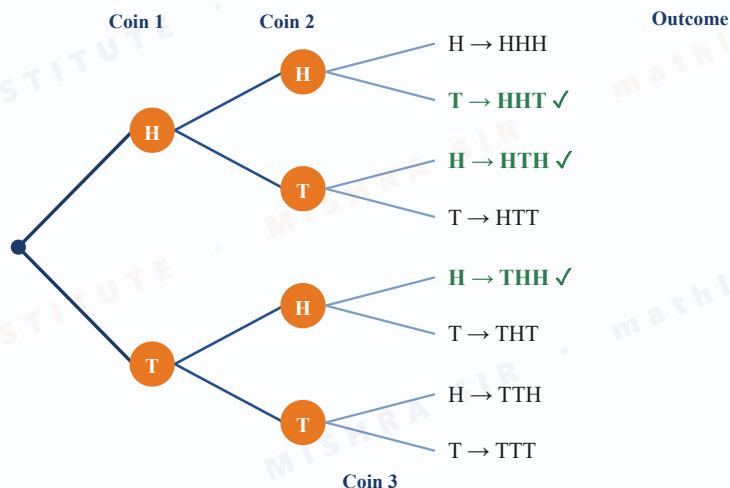
$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}, n(S) = 2 \times 2 \times 2 = 8$$

"Exactly two heads" means two heads and one tail — **not** three heads:

$$E = \{HHT, HTH, THH\}, n(E) = 3$$

$$P(E) = \frac{3}{8} = 0.375$$

2 marks



Tree diagram — three coins, 8 outcomes; the 3 with exactly two heads are ticked

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari, Part I*.

Hence: (i) $\frac{3}{4}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{3}$ (iv) $\frac{1}{2}$ (v) $\frac{3}{8}$.

Q5. A bag has 3 candies: strawberry, lemon and mint. One is picked at random. What is the probability of picking a strawberry candy?

2 marks

SOLUTION:

$S = \{\text{Strawberry, Lemon, Mint}\}$, $n(S) = 3$ 1 mark

$E = \{\text{Strawberry}\}$, $n(E) = 1$

$P(\text{strawberry}) = \frac{1}{3} \approx 0.333$ 1 mark

Hence the probability of picking a strawberry candy is $\frac{1}{3}$.

Note: here the three candies are **one each**, so the outcomes genuinely are equally likely — unlike the pen box of Exercise 7.4 Q2, where the counts differed.

Q6. A child has 2 shirts (red and blue) and 3 types of pants (jeans, khakis, shorts). List all possible outfits of one shirt and one pair of pants. Display your answer in a table format.

3 marks

SOLUTION:

By the Fundamental Principle of Counting: $2 \times 3 = 6$ outfits.

1 mark

Shirt ↓ / Pants →	Jeans	Khakis	Shorts
Red	(Red, Jeans)	(Red, Khakis)	(Red, Shorts)
Blue	(Blue, Jeans)	(Blue, Khakis)	(Blue, Shorts)

2 marks

The six outfits are: (Red, Jeans), (Red, Khakis), (Red, Shorts), (Blue, Jeans), (Blue, Khakis), (Blue, Shorts).

Hence there are 6 possible outfits.

Why a table works so well here: a two-step experiment with m and n options fits exactly into an $m \times n$ grid, and a grid makes it impossible to miss an outcome or repeat one — the two rules a sample space must satisfy. Use a **table** for two stages and a **tree** for three or more.

Q7. A tyre company records distances before replacement in 1000 cases: Less than 4000 km → 20; 4001 to 9000 → 210; 9001 to 14000 → 325; More than 14000 → 445. Find the probability that a randomly chosen tyre lasts: (i) less than 4000 km, (ii) between 4000 and 14000 km, (iii) more than 14000 km.

4 marks

GIVEN: A frequency table over 1000 cases.

CHECK FIRST: $20 + 210 + 325 + 445 = 1000$ ✓ — the frequencies account for every case.

NOTE: these are **experimental (statistical) probabilities**, obtained from recorded data rather than from equally likely outcomes.

SOLUTION:

(i) Less than 4000 km.

Favourable cases = 20

$$P = \frac{20}{1000} = \frac{1}{50} = 0.02 \text{ or } 2\%$$

1 mark

(ii) Between 4000 and 14000 km.

This covers **two** class intervals — 4001 to 9000 **and** 9001 to 14000:

Favourable cases = $210 + 325 = 535$

$$P = \frac{535}{1000} = \frac{107}{200} = 0.535 \text{ or } 53.5\%$$

1.5 marks

(iii) More than 14000 km.

Favourable cases = 445

$$P = \frac{445}{1000} = \frac{89}{200} = 0.445 \text{ or } 44.5\%$$

1.5 marks

Final check: $0.02 + 0.535 + 0.445 = 1.000$ ✓ — the three categories cover every tyre, so their probabilities must total 1. Always run this check; it catches a missed interval instantly.

⚠ **The trap is part (ii).** It spans **two rows** of the table, and students who read only one row answer 0.21 or 0.325. Whenever a range in the question is wider than a single class interval, add the frequencies before dividing.

Q8. The letters of the word 'PEACE' are placed on cards. Leela draws a card without looking. (i) What is the probability that it is a P, E or C? (ii) What is the probability that it is not an E?

3 marks

GIVEN: Cards showing P, E, A, C, E.

CRITICAL OBSERVATION: the letter **E** appears twice. There are **5 cards**, not 4 — so $n(S) = 5$, and E is twice as likely as any other letter.

SOLUTION:

Letter	P	E	A	C	Total
Number of cards	1	2	1	1	5

(i) Probability of P, E or C.

Favourable cards: $P(1) + E(2) + C(1) = 4$

1 mark

$$P(P, E \text{ or } C) = \frac{4}{5} = 0.8 \text{ or } 80\%$$

0.5 mark

⚠ Answering $\frac{3}{5}$ means counting E only once. The question asks for the probability of drawing a **card**, and two cards carry an E.

(ii) Probability that it is not an E.

METHOD 1 — Count the favourable cards directly

Cards that are not E: $P, A, C = 3$

$$P(\text{not E}) = \frac{3}{5} = 0.6 \text{ or } 60\%$$

1 mark

METHOD 2 — Use the complement

$$P(E) = \frac{2}{5}, \text{ so } P(\text{not E}) = 1 - \frac{2}{5} = \frac{3}{5} \checkmark$$

0.5 mark

Which method to use: Method 1 for a small sample space like this — counting 3 cards is immediate and needs no rule. Method 2 becomes clearly better when the "not" set is large: for "not a 1" on a die it is far quicker to compute $1 - \frac{1}{6}$ than to count five faces. Learn both and pick by which side is smaller to count.

Hence (i) $\frac{4}{5}$ and (ii) $\frac{3}{5}$.

★ Q9. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, all equally likely. What is the probability that it points at (i) 8? (ii) an odd number? (iii) a number greater than 2? (iv) a number less than 9? (v) a multiple of 3? 5 marks

GIVEN: $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $n(S) = 8$, all equally likely.

SOLUTION:

(i) Pointing at 8.

$$E = \{8\}, n(E) = 1 \rightarrow P = \frac{1}{8} = 0.125$$

1 mark

(ii) An odd number.

$$E = \{1, 3, 5, 7\}, n(E) = 4 \rightarrow P = \frac{4}{8} = \frac{1}{2} = 0.5$$

1 mark

(iii) A number greater than 2.

"Greater than 2" excludes 2 itself: $E = \{3, 4, 5, 6, 7, 8\}$, $n(E) = 6$

$$P = \frac{6}{8} = \frac{3}{4} = 0.75$$

1 mark

Check by complement: numbers not greater than 2 are $\{1, 2\}$, so $1 - \frac{2}{8} = \frac{6}{8} \checkmark$

(iv) A number less than 9.

Every number on the spinner (1 to 8) is less than 9, so $E = S$ and $n(E) = 8$

$$P = \frac{8}{8} = 1 \text{ — this is a certain event.}$$

1 mark

(v) A multiple of 3.

Multiples of 3 within 1 to 8 are 3 and 6 (since $9 > 8$): $E = \{3, 6\}$, $n(E) = 2$

$$P = \frac{2}{8} = \frac{1}{4} = 0.25$$

1 mark

Part (iv) is the point of this question. It is placed here to check that you recognise a **certain event** when it is disguised in ordinary wording. Nothing needs calculating — every outcome qualifies, so $P = 1$. Similarly, had it asked for "a number greater than 8", the answer would be $P = 0$, an impossible event.

Hence: (i) $\frac{1}{8}$ (ii) $\frac{1}{2}$ (iii) $\frac{3}{4}$ (iv) 1 (v) $\frac{1}{4}$.

★ Q10. A basket contains 4 red balls and 5 blue balls. One ball is drawn and laid aside, and a second ball is drawn. Draw a tree diagram to represent the possible outcomes and probabilities. Hence find (i) P(a red ball then a blue ball), (ii) P(2 blue balls).

6 marks

GIVEN: 4 red + 5 blue = 9 balls. The first ball is **laid aside** — this is drawing **without replacement**.

THE KEY IDEA: because the first ball is not returned, only **8** balls remain for the second draw. So every second-stage denominator is **8**, not 9. Moreover, the numerator depends on what was drawn first.

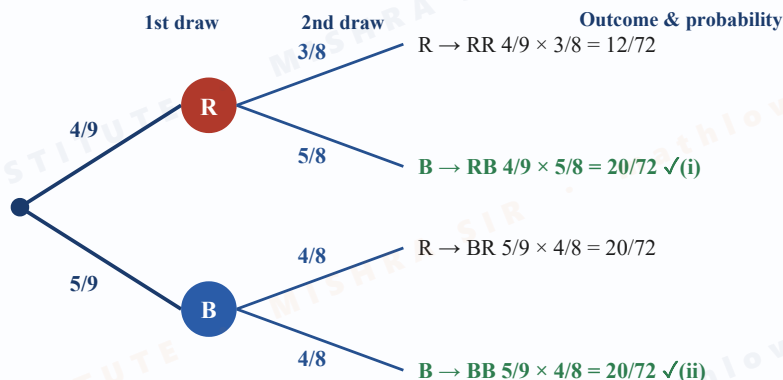
SOLUTION:

First draw: $P(R) = \frac{4}{9}$, $P(B) = \frac{5}{9}$

Second draw — four cases, worked out from what is left:

First ball	Balls remaining	P(Red next)	P(Blue next)
Red	3 red, 5 blue (8 total)	$\frac{3}{8}$	$\frac{5}{8}$
Blue	4 red, 4 blue (8 total)	$\frac{4}{8}$	$\frac{4}{8}$

2 marks



Tree diagram — two draws without replacement (note the second-stage denominators are all 8)

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

2 marks

(i) Probability of a red ball then a blue ball.

Follow the single path $R \rightarrow B$ and multiply the branch probabilities:

$$P(RB) = \frac{4}{9} \times \frac{5}{8} = \frac{20}{72} = \frac{5}{18} \approx 0.278$$

1 mark

(ii) Probability of 2 blue balls.

Follow the path $B \rightarrow B$:

METHOD 2 — Count equally likely ordered pairs

Treat the 9 balls as distinguishable. Two draws without replacement give $9 \times 8 = 72$ equally likely ordered pairs.

$$\text{Red then blue: } 4 \times 5 = 20 \text{ ways} \rightarrow P = \frac{20}{72} = \frac{5}{18} \checkmark$$

$$\text{Blue then blue: } 5 \times 4 = 20 \text{ ways} \rightarrow P = \frac{20}{72} = \frac{5}{18} \checkmark$$

Which method to use: the question says "use the tree diagram", so **Method 1 must be your written answer** — and the diagram itself carries about a third of the marks. **Method 2 is the better check**, because the 72 ordered pairs are genuinely equally likely, so the plain $\frac{\text{favourable}}{\text{total}}$ rule applies with no multiplication rule needed.

Full check: all four paths must total 1 — $\frac{12}{72} + \frac{20}{72} + \frac{20}{72} + \frac{20}{72} = \frac{72}{72} = 1 \checkmark$

⚠ The single biggest error: writing $\frac{5}{9} \times \frac{5}{9}$ for part (ii). That would be right only *with* replacement. The words "laid aside" change the second denominator to 8 and the blue count to 4.

★ Q11. I throw a pair of 6-sided dice. Write down an event that has a probability of 0 and an outcome that has a probability of 1.

3 marks

GIVEN: Two dice thrown; each shows 1 to 6, so $n(S) = 6 \times 6 = 36$.

RANGE OF POSSIBLE SUMS: the smallest is $1 + 1 = 2$ and the largest is $6 + 6 = 12$. Every sum obtained must lie between 2 and 12.

1 mark

SOLUTION:

An event with probability 0 (impossible event). Any event that can never occur, for example:

- "The sum of the two dice is **13**" — impossible, since the maximum sum is 12.
- "The sum of the two dice is **1**" — impossible, since the minimum sum is 2.
- "A die shows the number **7**" — impossible, since a 6-sided die has only 1 to 6.

For any of these, $n(E) = 0$, so $P(E) = \frac{0}{36} = 0$.

1 mark

An event with probability 1 (certain event). Any event that always occurs, for example:

- "The sum of the two dice lies **between 2 and 12 (inclusive)**" — every one of the 36 outcomes satisfies this.
- "Each die shows a number from 1 to 6."

For these, $n(E) = 36$, so $P(E) = \frac{36}{36} = 1$.

1 mark

How to be sure of full marks: do not merely name the event — **add the one-line reason** ("because the greatest possible sum is 12"). The examiner is testing whether you understand *why* the probability is 0 or 1, not whether you can recall an example. Also note that any single answer suffices; the alternatives above are given so you can see the pattern.

★ Q12. Write the sample space and calculate the probability. (i) Two dice rolled — P(sum is a prime number greater than 5)? (ii) A bag with 4 red, 3 green, 2 blue balls; two drawn without replacement — P(both of different colours)? (iii) Three coins tossed — P(first coin shows heads AND exactly two heads in total)? (iv) A four-digit number formed from digits 1, 2, 3, 4 with no repetition — P(the number is even)? (v) A 3-question multiple-choice test with 4 options each, all guessed — P(exactly 2 correct)?

12 marks

SOLUTION:

(i) Two dice — sum is a prime number greater than 5.

Step 1: $n(S) = 6 \times 6 = 36$

Step 2: Possible sums run from 2 to 12. The primes in that range are 2, 3, 5, 7, 11. Those **greater than 5** are 7 and 11.

1 mark

Step 3: List the favourable outcomes:

Sum 7: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) — **6 ways**

Sum 11: (5, 6), (6, 5) — **2 ways**

$n(E) = 6 + 2 = 8$

1 mark

Step 4: $P = \frac{8}{36} = \frac{2}{9} \approx 0.222$

1 mark

⚠ Do not include 9 — it is not prime ($9 = 3 \times 3$). Do not include 5 — it is not greater than 5.

(ii) 4 red, 3 green, 2 blue; two drawn without replacement — both different colours.

Step 1: Total = $4 + 3 + 2 = 9$ balls. Without replacement, ordered pairs = $9 \times 8 = 72$, all equally likely.

1 mark

Step 2: It is far easier to count the **same-colour** cases and subtract:

Both red: $4 \times 3 = 12$ | Both green: $3 \times 2 = 6$ | Both blue: $2 \times 1 = 2$

Same colour total = $12 + 6 + 2 = 20$

1 mark

Step 3: Different colours = $72 - 20 = 52$

$P = \frac{52}{72} = \frac{13}{18} \approx 0.722$

1 mark

Check: $P(\text{same}) = \frac{20}{72} = \frac{5}{18}$, and $\frac{13}{18} + \frac{5}{18} = 1$ ✓

(iii) Three coins — first coin heads AND exactly two heads in total.

Step 1: $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$, $n(S) = 8$

Step 2: Both conditions must hold together. Outcomes with exactly two heads are HHT, HTH, THH; of these, the ones starting with H are HHT and HTH.

$E = \{HHT, HTH\}$, $n(E) = 2$

1.5 marks

Step 3: $P = \frac{2}{8} = \frac{1}{4} = 0.25$

1 mark

⚠ THH has exactly two heads but fails the first condition; HHH starts with H but has three heads. Both conditions apply at once.

(iv) Four-digit numbers from 1, 2, 3, 4 with no repetition — the number is even.

Step 1: Count the sample space by the Fundamental Principle of Counting. The first digit can be any of 4; then 3 remain; then 2; then 1:

$$n(S) = 4 \times 3 \times 2 \times 1 = 24$$

1 mark

Step 2: A number is even when its **last digit** is even — here 2 or 4, giving **2 choices** for the last place. The remaining three digits can then be arranged in $3 \times 2 \times 1 = 6$ ways:

$$n(E) = 2 \times 6 = 12$$

1 mark

$$\text{Step 3: } P = \frac{12}{24} = \frac{1}{2} = 0.5$$

0.5 mark

Sense-check: two of the four digits are even and two odd, so by symmetry exactly half the arrangements should end in an even digit ✓

(v) Three MCQs, 4 options each, all guessed — exactly 2 correct.

Step 1: Each question has 4 options, so $n(S) = 4 \times 4 \times 4 = 64$ equally likely guess-patterns.

1 mark

Step 2: "Exactly 2 correct" means **one question is wrong**. There are **3 choices** for which question is the wrong one, and for that question there are **3 wrong options** to pick. The other two must each be the single correct option (1 way each):

$$n(E) = 3 \times 3 = 9$$

1 mark

$$\text{Step 3: } P = \frac{9}{64} \approx 0.141$$

0.5 mark

Which question is wrong	Pattern (✓ = correct)	Ways
Question 1	X ✓ ✓	$3 \times 1 \times 1 = 3$
Question 2	✓ X ✓	$1 \times 3 \times 1 = 3$
Question 3	✓ ✓ X	$1 \times 1 \times 3 = 3$
Total favourable		9

Note on part (v): this is solved entirely by **listing the three cases in a table** — no combinations formula is used or needed. The common wrong answer is $\frac{3}{64}$, which forgets that the wrong question can be answered wrongly in **3 different ways**, not one.

Hence: (i) $\frac{2}{9}$ (ii) $\frac{13}{18}$ (iii) $\frac{1}{4}$ (iv) $\frac{1}{2}$ (v) $\frac{9}{64}$.

★ Q13. A box contains 4 balls numbered 1 to 4. Record a sample space using a tree diagram for:

(i) a ball is drawn and recorded, then returned, and a second ball is drawn and recorded; (ii) a ball is drawn and recorded, and without replacing it a second is drawn; (iii) What are the sizes of these two sample spaces?

6 marks

GIVEN: Balls numbered 1, 2, 3, 4. Two draws, done two different ways.

THE WHOLE POINT OF THIS QUESTION is to compare **with replacement** against **without replacement** side by side.

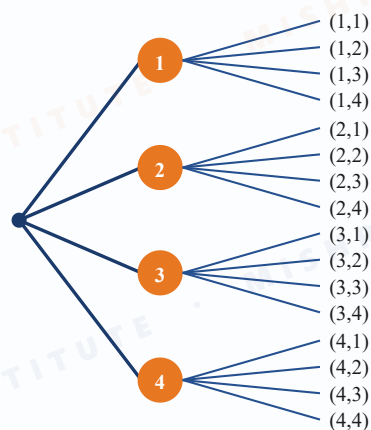
SOLUTION:

(i) WITH replacement. The ball goes back, so the second draw again has all 4 balls available. Every first-stage branch splits into 4.

1st ↓ / 2nd →	1	2	3	4
1	(1,1)	(1,2)	(1,3)	(1,4)
2	(2,1)	(2,2)	(2,3)	(2,4)
3	(3,1)	(3,2)	(3,3)	(3,4)
4	(4,1)	(4,2)	(4,3)	(4,4)

$n(S_1) = 4 \times 4 = 16$ — the shaded diagonal shows the repeats, which are possible **only** with replacement.

2 marks



WITH replacement

$4 \times 4 = 16$ outcomes

Repeats like (1,1) ARE possible

(i) Tree diagram — two draws with replacement

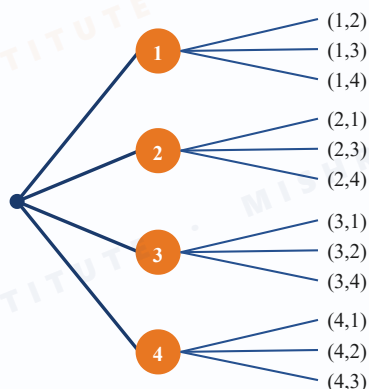
Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

(ii) **WITHOUT replacement.** The first ball is kept aside, so only 3 balls remain. Each first-stage branch splits into 3, and the repeated pairs disappear.

$$S_2 = \{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)\}$$

$$n(S_2) = 4 \times 3 = 12$$

2 marks



WITHOUT replacement

$$4 \times 3 = 12 \text{ outcomes}$$

Repeats like (1,1) are NOT possible

(ii) Tree diagram — two draws without replacement

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

(iii) **Sizes of the two sample spaces.**

$$n(S_1) = 16 \text{ (with replacement) and } n(S_2) = 12 \text{ (without replacement)}$$

2 marks

The difference is exactly 4 — precisely the four repeated pairs (1, 1), (2, 2), (3, 3), (4, 4), which can occur only when the ball is returned.

The one line to memorise from this question: with replacement the second stage keeps the **same** number of options; without replacement it has **one fewer**. Every "with/without replacement" question in Class 9 and Class 10 turns on this single sentence.

★ Q14. List the elements of a sample space for the simultaneous tossing of a coin and drawing of a card from a set of 6 cards numbered 1 through 6.

3 marks

GIVEN: A coin (2 outcomes) and 6 numbered cards (6 outcomes), done together.

SOLUTION:

Step 1: Each outcome must record **both** results, so it is written as a pair.

Step 2: By the Fundamental Principle of Counting, $n(S) = 2 \times 6 = 12$.

1 mark

Step 3: List systematically — fix H and run through all six cards, then fix T:

$S = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$

2 marks

Coin ↓ / Card →	1	2	3	4	5	6
H	(H,1)	(H,2)	(H,3)	(H,4)	(H,5)	(H,6)
T	(T,1)	(T,2)	(T,3)	(T,4)	(T,5)	(T,6)

Hence the sample space has 12 elements, as listed above.

Note: this is the same structure as Exercise 7.3 Q2(i) (die and coin) — only the names change. Recognising a familiar structure under new wording is a large part of scoring well in probability.

★ Q15. Three coins are tossed and the number of heads is recorded. Which of the following lists is a sample space for this experiment? Why do the other lists fail to qualify? (i) {1, 2, 3} (ii) {0, 1, 2} (iii) {0, 1, 2, 3, 4} (iv) {0, 1, 2, 3}

4 marks

READ CAREFULLY: we are recording the **number of heads**, not the sequence of coins. So the outcomes are numbers, not strings like HHT.

STEP 1 — WORK OUT WHAT IS ACTUALLY POSSIBLE. Tossing three coins, the number of heads can be:

Number of heads	Which outcomes give it	Possible?
0	TTT	Yes
1	HTT, THT, TTH	Yes
2	HHT, HTH, THH	Yes
3	HHH	Yes
4	— none —	No

So the complete, non-repeating list is {0, 1, 2, 3}.

1 mark

Answer: list (iv) {0, 1, 2, 3} is the correct sample space.

1 mark

STEP 2 — WHY THE OTHERS FAIL. A sample space must include **every** possible outcome and **nothing** impossible.

(i) {1, 2, 3} — **fails**. It **omits 0**. Getting no heads at all (TTT) is perfectly possible, so leaving it out makes the list incomplete.

0.5 mark

(ii) {0, 1, 2} — **fails**. It **omits 3**. Getting three heads (HHH) is possible, so the list is again incomplete.

0.5 mark

(iii) {0, 1, 2, 3, 4} — **fails**. It **includes 4**, which is impossible — you cannot obtain 4 heads from only 3 coins.

A sample space must not contain outcomes that can never occur.

1 mark

Why this question is worth studying closely: it tests the two defining rules of a sample space from both directions at once — (i) and (ii) fail by **leaving something out**, while (iii) fails by **putting something impossible in**. Note also that the four outcomes 0, 1, 2, 3 are **not equally likely** (one head can happen in 3 ways, but zero heads in only 1). A valid sample space does **not** have to consist of equally likely outcomes.

★ Q16. Suppose you drop a dye at random on the rectangular region shown, measuring 3 m by 2 m. What is the probability that it will land inside the circle with a diameter of 1 m?

4 marks

GIVEN: A rectangle 3 m × 2 m containing a circle of diameter 1 m.

THE IDEA — GEOMETRIC PROBABILITY: when a point lands at random on a region, every bit of area is equally likely, so probability is measured by **area** instead of by counting outcomes:

$$P = \frac{\text{Area of the favourable region}}{\text{Area of the whole region}}$$

1 mark

SOLUTION:

Step 1 — Area of the rectangle (total region):

$$A_{\text{rect}} = \text{length} \times \text{breadth} = 3 \times 2 = 6 \text{ m}^2$$

1 mark

Step 2 — Area of the circle (favourable region). The **diameter** is 1 m, so the radius is

$$r = \frac{1}{2} = 0.5 \text{ m}$$

$$A_{\text{circle}} = \pi r^2 = \pi \left(\frac{1}{2}\right)^2 = \frac{\pi}{4} \text{ m}^2$$

1 mark

Step 3 — Form the ratio:

$$P = \frac{A_{\text{circle}}}{A_{\text{rect}}} = \frac{\pi/4}{6} = \frac{\pi}{24}$$

1 mark

Step 4 — As a decimal, taking $\pi \approx 3.14$:

$$P \approx \frac{3.14}{24} \approx 0.131 \text{ or about } 13.1\%$$

(Using $\pi = \frac{22}{7}$: $P = \frac{22}{7 \times 24} = \frac{22}{168} = \frac{11}{84} \approx 0.131$ — the same value.)

Hence the probability is $\frac{\pi}{24} \approx 0.131$.

⚠ **The trap is the radius.** The question gives the **diameter** as 1 m. Using $r = 1$ gives $\pi/6 \approx 0.524$ — more than half the rectangle, which is visibly absurd from the figure. Whenever an answer looks too large against the picture, check the radius first.

Sense-check that costs nothing: the circle is clearly a small part of the rectangle, so the answer must be well under 0.5. The value 0.131 passes; 0.524 does not.

— End of End-of-Chapter Exercises —



Bonus — Solved In-Text "Think and Reflect" & Example Items

Why this section matters

The **Think and Reflect** boxes are not decoration. They are exactly where school question-setters find 1-mark and 2-mark conceptual questions, because guide books rarely solve them. All of them, plus the worked Examples, are covered below.

A Randomness, Fairness and the Probability Scale

A1. (p.156) In a cricket match, a coin is tossed to decide which team bats first, and this is considered a fair method. Can you explain why?

2 marks

SOLUTION:

A coin toss is considered fair for three connected reasons:

- 1. The coin is unbiased.** *It is symmetrical, so there is no reason for it to land on one side more often than the other. Heads and tails are **equally likely**, each with probability $\frac{1}{2}$.*
- 2. Neither team can influence the outcome.** *The result depends on chance alone, not on skill, strength, reputation or the umpire's judgement. Both captains therefore have exactly the same chance.*
- 3. The outcome cannot be predicted in advance.** *This unpredictability is precisely what removes any possibility of favouritism.*

1 mark

1 mark

The insight worth carrying away: randomness is usually thought of as a nuisance, but here it is **deliberately useful**. When we want a decision that nobody can control or complain about, we choose a genuinely random method. The same logic underlies lucky draws and lotteries.

A2. (p.157) Ask a friend to predict the outcome of a ₹1 coin you toss. Your friend could guess heads or tails but could not know for certain. What does this illustrate?

2 marks

SOLUTION:

*This illustrates the meaning of **randomness**. Two things are true at once, and both matter:*

- 1. All possible results are known in advance** — *the coin must land heads or tails, so the sample space $S = \{H, T\}$ is completely known.*
- 2. Each individual trial is unpredictable** — *knowing the sample space does not tell you which outcome will actually occur on this particular toss.*

1 mark

1 mark

Summary sentence for the exam: *in a random experiment you can say what **could** happen, but not what **will** happen.*

A3. (p.163) If I have rolled a 4 on a die 8 times in succession, what is the probability of rolling a 4 again? Explain.

2 marks

SOLUTION:

The probability is still $\frac{1}{6} \approx 0.167$, assuming the die is fair.

1 mark

Reason: each roll is an **independent** event. The die has **no memory** of the eight previous rolls, and its six faces remain equally likely every single time. Believing that a different number is now "due" is the

Gambler's Fallacy.

1 mark

What probability actually tells you: it does not predict the next outcome; it predicts the **long-run** pattern. Over thousands of rolls, about one-sixth will be 4s — but that says nothing about roll number nine.

A4. (p.164, Example 6) In Snakes and Ladders you have rolled three 6s in a row. You think, "there is no way I will get a 6 again." Why is this reasoning wrong?

2 marks

SOLUTION:

This reasoning is the **Gambler's Fallacy**. Each roll of the die is an **independent event**, so the probability of a 6 is always:

$$P(6) = \frac{1}{6} \approx 0.166 \text{ or about } 16.6\%$$

1 mark

It does **not** change based on what happened in previous turns. The die does not "remember" the three 6s and does not compensate for them.

1 mark

Key Idea: in games of chance the Gambler's Fallacy tricks people into believing a pattern exists, when in fact randomness has no memory.

A5. (p.161, Examples 3 & 4) (a) Rolling a standard 6-sided die, what is $P(\text{getting a 4})$? (b)

A letter is picked at random from the word 'PROBABILITY'. What is $P(\text{picking the letter B})$? 3 marks

SOLUTION:

(a) Rolling a 4.

Favourable outcomes = 1 (only the number 4); possible outcomes = 6

$$P(4) = \frac{1}{6} = 0.1666\ldots \approx 0.167 \text{ or } 16.7\%$$

1 mark

(b) Picking the letter B from PROBABILITY.

Step 1 — Count the letters carefully: P, R, O, B, A, B, I, L, I, T, Y — that is 11 letters.

Step 2 — Count the Bs: there are 2 (positions 4 and 6).

1 mark

$$\text{Step 3 — } P(B) = \frac{2}{11} = 0.1818\ldots \approx 0.182 \text{ or } 18.2\%$$

1 mark

⚠ Word-based probability questions always turn on careful counting. Count the total letters and the repeats **before** writing anything else — miscounting PROBABILITY as 10 letters, or seeing only one B, is where the marks go. Write the letters out in a row and tick them off.

B Sample Spaces, Events and Tree Diagrams

B1. (p.167) In Example 1 we used the sample space {Rain, No Rain}. What if we want to include drizzle, light rain and heavy rain? What does this teach us about choosing a sample space? 3 marks

SOLUTION:

*The original sample space $S = \{\text{Rain, No Rain}\}$ records only **whether** it rains, with $n(S) = 2$.*

1 mark

*If we also care about **how much** it rains, the sample space must be expanded:*

$$S = \{\text{No Rain, Drizzle, Light Rain, Heavy Rain}\}, \quad n(S) = 4$$

1 mark

The lesson: a sample space is not fixed by the situation alone — it must be **detailed enough to suit the specific question being asked**. If the question is "will I need an umbrella?", two outcomes suffice.

If it is "will the match be abandoned?", the finer four-outcome space is needed.

1 mark

Note: both lists remain valid sample spaces, since each includes every possibility without repetition.

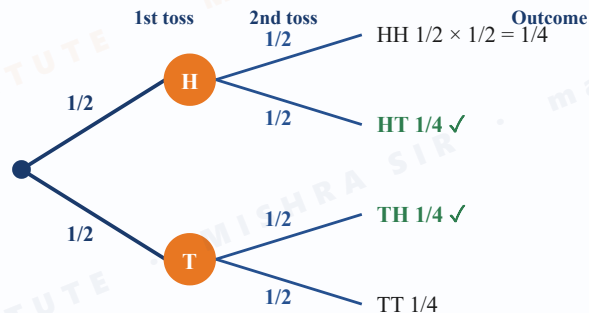
*The choice between them depends on the **level of detail the problem requires**.*

B2. (p.169) A fair coin is tossed twice. (a) Show that $P(HH) = \frac{1}{4}$ using a tree diagram. (b)

Can you calculate the probability of getting one head and one tail?

3 marks

SOLUTION:



Tree diagram — a fair coin tossed twice (4 equally likely outcomes)

Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

(a) Probability of HH.

$S = \{HH, HT, TH, TT\}$, $n(S) = 4$; favourable = $\{HH\}$, $n(E) = 1$

$$P(HH) = \frac{1}{4} = 0.25 \text{ or } 25\%$$

1 mark

(b) Probability of one head and one tail.

This can happen in **two** ways — head first then tail, or tail first then head:

$E = \{HT, TH\}$, $n(E) = 2$

$$P = \frac{2}{4} = \frac{1}{2} = 0.5 \text{ or } 50\%$$

2 marks

⚠ This is the most common error in the whole chapter. Students answer $\frac{1}{4}$, treating "one head and one tail" as a single outcome. But **HT and TH are different outcomes** — the order in which they occur differs — so both must be counted. Notice that "one head and one tail" is **twice** as likely as HH, even though all three look superficially similar.

B3. (p.166) Write the sample space and sample size for: (a) whether it rains tomorrow, (b) the result of a match for one team, (c) tossing two coins.

3 marks

SOLUTION:

(a) $S = \{\text{Rain, No Rain}\}$, $n(S) = 2$ 1 mark

(b) $S = \{\text{Win, Lose, Draw}\}$, $n(S) = 3$

Note that a draw is a genuine third possibility in most sports — omitting it makes the sample space incomplete. 1 mark

(c) $S = \{HH, HT, TH, TT\}$, $n(S) = 2 \times 2 = 4$ 1 mark

Coin 1	Coin 2	Outcome
H	H	HH
H	T	HT
T	H	TH
T	T	TT

B4. (p.167) For each experiment, write the sample space and the given event. (a) Two coins; event: 'at least one coin shows Head.' (b) A 6-sided die; event: 'the number rolled is greater than 4.' (c) A basket of apple, banana, orange; event: 'picking a fruit that is yellow.'

3 marks

SOLUTION:

(a) $S = \{HH, HT, TH, TT\}$; $E = \{HH, HT, TH\}$, $n(E) = 3$, so $P(E) = \frac{3}{4}$ 1 mark

(b) $S = \{1, 2, 3, 4, 5, 6\}$; $E = \{5, 6\}$, $n(E) = 2$, so $P(E) = \frac{2}{6} = \frac{1}{3}$ 1 mark

(c) $S = \{\text{Apple, Banana, Orange}\}$; $E = \{\text{Banana}\}$, $n(E) = 1$, so $P(E) = \frac{1}{3}$ 1 mark

Reminder: in every case the event E is a **subset** of S — each of its members must be an element already present in the sample space.

B5. (p.162, Example 5) A survey of 50 students gives: 20 mango, 15 apples, 10 bananas, 5 grapes. (a) $P(\text{a randomly chosen student likes mango})$? (b) For a school of 1500 students, how many mangoes should be bought? (c) How could the estimate be made more reliable?

4 marks

CHECK FIRST: $20 + 15 + 10 + 5 = 50 \checkmark$

SOLUTION:

(a) $P(\text{mango}) = \frac{20}{50} = \frac{2}{5} = 0.4 \text{ or } 40\%$

1 mark

(b) $\text{Estimated mangoes} = 0.4 \times 1500 = 600$

1.5 marks

(Scale-factor check: the school is $1500 \div 50 = 30$ times the sample, so mango = $20 \times 30 = 600 \checkmark$)

(c) **Making the estimate more reliable.** Two distinct improvements, and both earn marks:

1. **Increase the sample size** — survey 100 or more students instead of 50. A larger sample reduces the effect of random variation.

2. **Make the sample more representative** — the 50 students all came from **one class**, whose tastes may not reflect the whole school. Include students from **different classes and grades** so the sample mirrors the population.

1.5 marks

Both points are needed for full marks. Students usually mention only "survey more people" and miss the second, which is the more important idea: a **large but biased** sample is still a bad sample. Size and representativeness are separate requirements, and the term for this whole process is **sampling**.

— End of Solved In-Text Items —

Master Table — Questions Solvable by More Than One Method

At a glance: where alternatives exist, and which one to actually use

✦ **Marks are awarded for any correct method.** The "easiest" column indicates which route has fewer steps and fewer chances to slip — it is not a rule about what is allowed. Note, though, that when a question explicitly says **"use the tree diagram"** (Ex 7.4 Q2, EOC Q10), the tree method must be your written answer, because the diagram itself carries marks.

Question	Method 1	Method 2	Easiest / Recommended
Ex 7.2 Q1, Q2	Find the probability, then multiply by the population	Scale factor (population $\div$ sample), then multiply each category	Method 2 — self-checking, all categories at once

Ex 7.4 Q1(iii)	Count the tips of the tree: 1 out of 6	Multiply along the branch: $\frac{1}{3} \times \frac{1}{2}$	Method 1 — matches the Class 9 approach
Ex 7.4 Q2(ii)	Multiply along branches, add the 3 matching paths	Count equally likely pen- pairs: 29 out of 81	M1 as the answer (question demands the tree); M2 as the check
EOC Q4(iv)	Count 'not red' directly: 3 out of 6	Complement: $1 - P(\text{red})$	Method 1 here; complement wins when the 'not' set is large
EOC Q8(ii)	Count the non-E cards: 3 out of 5	Complement: $1 - \frac{2}{5}$	Method 1 — only 3 cards to count
EOC Q9(iii)	List {3, 4, 5, 6, 7, 8} directly	Complement: $1 - \frac{2}{8}$	Either; complement is faster as the list grows
EOC Q10	Tree diagram, multiply along each path	Count ordered pairs: $9 \times 8 = 72$ equally likely	M1 as the answer (question demands the tree); M2 as the check
EOC Q12(ii)	Count 'different colours' directly (52 cases)	Count 'same colour' (20) and subtract from 72	Method 2 — far fewer cases to count
EOC Q16	Areas as exact fractions: $\frac{\pi/4}{6} = \frac{\pi}{24}$	Substitute $\pi = \frac{22}{7}$ early, giving $\frac{11}{84}$	Method 1 — keep π symbolic, approximate only at the end

The one general rule worth memorising: to find $P(\text{not } A)$, count whichever side is **smaller**. If the "not" group is small, count it directly; if it is large, count A instead and use $1 - P(A)$. This single habit saves time on nearly every probability paper.

Quick Revision Checklist — The Night Before

#	Can you do this without looking?	Where it was covered
1	State $P(E) = \frac{\text{favourable}}{\text{possible}}$ and the range $0 \leq P(E) \leq 1$	Formula panel · EOC Q1
2	Tell experimental from theoretical probability, and say when each applies	Ex 7.2 Q3 · Q6
3	Name the Law of Large Numbers and explain why 12 trials mislead	Ex 7.2 Q6
4	Spot and explain the Gambler's Fallacy	Ex 7.2 Q3(iv) · A3 · A4

5	Write a sample space that omits nothing and repeats nothing	Ex 7.3 · EOC Q15
6	Use $m \times n$ counting and justify it as counting the tree	Counting principle panel
7	Draw a labelled tree with Stage 1, Stage 2 and Outcome columns	Ex 7.4 · EOC Q10 · Q13
8	Change the second denominator for without replacement	EOC Q10 · Q12(ii) · Q13
9	Handle "at least one" and "exactly two" correctly	EOC Q4(i) · Q4(v) · B2
10	Remember that HT and TH are different outcomes	B2 · Tip 5
11	Estimate a population count from a sample	Ex 7.2 Q1 · Q2 · B5
12	Say why a sample must be large and representative	B5
13	Decide whether outcomes are genuinely equally likely	EOC Q3 · Ex 7.3 Q2(iii)
14	Do a geometric-probability question using areas (and halve the diameter!)	EOC Q16
15	Check that the probabilities of all categories add to 1	Ex 7.2 Q4 · EOC Q7

CHAPTER 8

Predicting What Comes Next: Sequences and Progressions

Exercise Sets 8.1 – 8.3 + End-of-Chapter

EXERCISE SETS 8.1 – 8.3 + END-OF-CHAPTER

Weightage & Placement of This Chapter

Where Chapter 8 sits in Class 9 Mathematics

⚠ **Honest note on marks:** Class 9 is a **school-conducted** examination (not a CBSE board exam), so there is no single official chapter-wise mark table the way there is for Class X. The figures below reflect the standard unit structure of the NCERT *Ganita Manjari* Part I and typical Class 9 annual-paper practice.

Please confirm the exact split against the question-paper blueprint circulated by your own school before planning revision time.

Detail	Value
Textbook	Ganita Manjari, Grade 9, Part I (new NCERT, 2026–27)
Chapter number	Chapter 8 — the last chapter of Part I
Broad area	Algebra / Patterns
Sub-topics	Sequences · Explicit rule · Recursive rule · Virahāṅka–Fibonacci · AP · Sum of first n naturals · GP · Fractals
Exercise sets	8.1 (6 Q) · 8.2 (7 Q) · 8.3 (7 Q) · End-of-Chapter (15 Q) = 35 questions
Typical annual-exam weight	Approximately 5–7 marks (school-dependent)
Why it matters beyond marks	This is the direct foundation for Class 10 Arithmetic Progressions and Class 11 Sequences & Series. Weakness here compounds for two full years.



Predicted Question Types

Format	Marks	Likely Topic
MCQ / Assertion-Reason	1	Identify AP vs GP; state common difference or common ratio; is a given sequence an AP?
Very Short Answer	1	Write the first 3 terms given t_n ; find t_{10} of a simple AP; write a recursive rule
Short Answer	2	Find the n^{th} term of an AP/GP; check whether a given number is a term; find a triangular number
Short Answer	3	Two-condition AP problems (given t_p and t_q , find a , d and another term) — the single most repeated type
Long Answer	4–5	"How many multiples/2-digit/3-digit numbers..." + their sum; bouncing-ball GP; salary-increment AP
Competency / Case Study	4–5 (sub-parts)	Fractal patterns (Sierpiński triangle / square carpet), growing tile patterns, bacteria growth — read the stage table, find explicit AND recursive rules
HOTS (starred ★ questions)	3–5	Expressing 100 as a sum of consecutive naturals; GP with given sum & product; proving x, y, z are in GP

★ The **starred (*) questions** in the textbook are explicitly marked as higher-order. Examiners draw the 4–5 mark long answers from exactly this pool. Do not skip them.

All Key Formulas & Definitions

Sequence: An ordered list of numbers. Each number in the list is called a **term**. A sequence may be **finite** (6, 12, 24, 48, 96) or **infinite** (1, 2, 3, 4, ...).

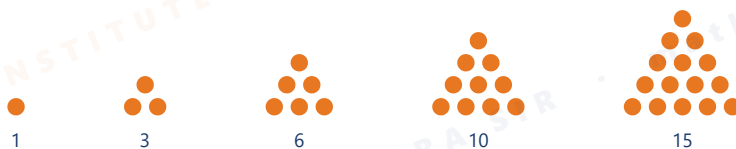
Notation: $t_1, t_2, t_3, \dots$ — the subscript matches the position. $t_4 = 7$ means "the term in the 4th position is 7". Other letters s_n, u_n, V_n are used when more than one sequence is under discussion.

Explicit formula: Uses the **position number** n to calculate the term directly. Example: $u_n = 2n - 1$ gives the odd numbers. Advantage — you can find the 1000th term **without** knowing the 999th.

Recursive formula: Gives a term using **previous** term(s). It **must** include the starting value(s).
Example: $t_1 = 1, t_n = t_{n-1} + 3$ for $n \geq 2$.

Triangular numbers: 1, 3, 6, 10, 15, ... with $t_n = \frac{n(n+1)}{2}$. Each term is the sum of natural numbers up to that position.

Triangular numbers — $t_n = n(n+1)/2$



Square numbers — $t_n = n^2$



Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook *Ganita Manjari, Part I*.

Each triangular number is the previous one plus the next natural number (1, +2, +3, +4, ...); each square number is the previous one plus the next odd number (1, +3, +5, +7, ...).

Square numbers: 1, 4, 9, 16, ... with $t_n = n^2$. Each term is the sum of the odd numbers up to that position.

Virahāṅka–Fibonacci sequence: $V_1 = 1$, $V_2 = 2$, $V_n = V_{n-1} + V_{n-2}$ for $n \geq 3$, giving 1, 2, 3, 5, 8, 13, 21, 34, ... First recorded by **Virahāṅka** (7th century CE) in *Vṛttajātisamuchaya*, studied further by Gopāla (c. 1135 CE) and Hemachandra (c. 1150 CE), and later by Fibonacci (c. 1200 CE).

Arithmetic Progression (AP): Consecutive terms differ by a **constant d** (the **common difference**). General form: a , $a + d$, $a + 2d$, $a + 3d$, ...

n^{th} term of an AP: $t_n = a + (n-1)d$ Recursive rule of an AP: $t_1 = a$, $t_n = t_{n-1} + d$ for $n \geq 2$.

Finding d : $d = t_2 - t_1 = t_3 - t_2 = \dots$ Also useful: $d = \frac{t_q - t_p}{q - p}$ for any two terms.

Sum of the first n natural numbers: $S_n = \frac{n(n+1)}{2}$ — first recorded in Āryabhaṭa's *Āryabhaṭīya*, Chapter 2, Verse 19.

Sum of a block of consecutive naturals: $p + (p + 1) + \dots + q = S_q - S_{p-1}$

Geometric Progression (GP): Each term is the previous term $\times$ a **constant** r (the **common ratio**).

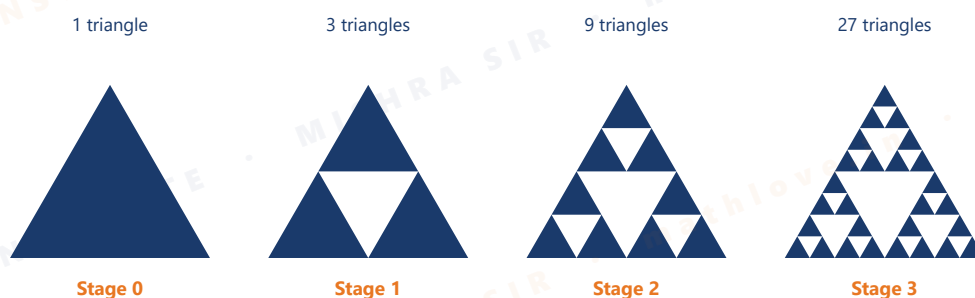
General form: $a, ar, ar^2, ar^3, \dots$

n^{th} term of a GP: $t_n = ar^{n-1}$ **Recursive rule of a GP:** $t_1 = a, t_n = r t_{n-1}$ for $n \geq 2$.

Finding r : $r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \dots$ (divide, never subtract).

Graph test (very examinable): Plotting (n, t_n) for an AP gives points on a **straight line**. For a GP the points **do not** lie on a straight line — they form a curve that rises steeply (if $r > 1$) or falls towards 0 (if $0 < r < 1$).

Sierpiński triangle: Black triangles $t_n = 3^n$; shaded area $s_n = \left(\frac{3}{4}\right)^n$. Number of triangles increases rapidly while the total area shrinks towards 0.



Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

Sierpiński triangle, Stages 0–3. Triangles: $t_n = 3^n$ (a GP with $r = 3$); shaded area: $s_n = (3/4)^n$, which shrinks towards 0.

Sierpiński square carpet: Red squares $t_n = 8^n$; red area $s_n = \left(\frac{8}{9}\right)^n$.

Fractal: A shape or pattern that repeats itself at different scales — zoom in on a small part and it looks like the whole (trees, cauliflower, snowflakes, coastlines).

Shortcuts & Exam-Writing Tips

1. **First test every sequence twice.** Subtract consecutive terms $\rightarrow$ constant? It's an AP. Divide consecutive terms $\rightarrow$ constant? It's a GP. Check at least **two** gaps/ratios before declaring; one gap proves nothing.
2. **Never write " $t_n = a + nd$ ".** The bracket is $(n - 1)$, not n . This single slip is the most common source of lost marks in the whole chapter.
3. **"Is X a term?" is always the same procedure:** set $t_n = X$, solve for n . If n is a **natural number** $\rightarrow$ yes, and state *which* term. If n is a fraction or negative $\rightarrow$ no, and say **why** ("since n must be a natural number"). Examiners award a mark for that justification sentence.
4. **Two-condition AP questions:** write both conditions as equations in a and d , then **subtract** them — a cancels instantly and d falls out in one line. Never substitute first.
5. **Counting-type questions** ("how many 2-digit numbers are divisible by 3?"): find the **smallest and largest** such numbers, then use $n = \frac{\text{last} - \text{first}}{d} + 1$. The "+1" is forgotten constantly — it is worth a mark on its own.
6. **Sums of consecutive numbers** must be routed through $S_n = \frac{n(n+1)}{2}$. For $12 + 15 + \dots + 99$, factor out the 3 first: $3(4 + 5 + \dots + 33) = 3(S_{33} - S_3)$.
7. **A recursive rule is incomplete without its seed.** Writing only " $t_n = t_{n-1} + 4$ " earns partial credit at best — you must also write $t_1 = 1$ and the range $n \geq 2$.
8. **Salary / bacteria / bounce word problems:** read carefully whether the question asks for the **term number** or the **number of years/hours elapsed**. "After how many years" is usually $n - 1$, not n .
9. **GP with $r^2 = k$ has TWO answers.** Whenever squaring or an even power appears, write both $r = +$ and $r = -$ cases and present both progressions unless the question rules one out.
10. **Fractal questions:** always build a small stage-table (Stage 0, 1, 2, 3) before hunting for the rule. The rule then reads straight off the table. Note the stage numbering starts at **0**, so the formula is 3^n and not 3^{n-1} .
11. **Show the substitution line.** " $t_{10} = 3 + (10 - 1) \times 5$ " before " $= 48$ " — method marks are awarded for the substitution even if the arithmetic slips.
12. **When two methods exist, pick the one with fewer fractions.** Several questions below are solved by more than one route and labelled **Method 1 / Method 2**, with a green box naming the easier one. In the exam **any correct method earns full marks** — so choose the one you can execute without slipping, not the one that looks cleverer.
13. **Always end with a sentence,** not a bare number: "Hence the 29th term is 64." Bare numerical answers routinely lose the final $\frac{1}{2}$ mark.

NCERT Solutions — Exercise Set 8.1 (6 Questions)

Approach for This Exercise

This set is purely about **reading a rule and using it**. For an **explicit** rule, substitute $n = 1, 2, 3, \dots$ directly. For a **recursive** rule, you must build terms one at a time, in order — you cannot jump. For "is X a term?" questions, set the rule equal to X and solve for n ; n must come out a **natural number**.

Q1. Find the first five terms of the sequence in which the n^{th} term is given by (i) $t_n = 3n - 4$, (ii)

$t_n = 2 - 5n$, (iii) $t_n = n^2 - 2n + 3$, for $n \geq 1$.

3 marks

GIVEN: Three explicit rules for t_n .

TO FIND: The first five terms of each sequence.

SOLUTION:

(i) $t_n = 3n - 4$ — substitute $n = 1, 2, 3, 4, 5$:

$$t_1 = 3(1) - 4 = -1$$

$$t_2 = 3(2) - 4 = 2$$

$$t_3 = 3(3) - 4 = 5$$

$$t_4 = 3(4) - 4 = 8$$

$$t_5 = 3(5) - 4 = 11$$

First five terms: -1, 2, 5, 8, 11

1 mark

(ii) $t_n = 2 - 5n$:

$$t_1 = 2 - 5 = -3, \quad t_2 = 2 - 10 = -8, \quad t_3 = 2 - 15 = -13, \quad t_4 = 2 - 20 = -18, \quad t_5 = 2 - 25 = -23$$

First five terms: -3, -8, -13, -18, -23

1 mark

(iii) $t_n = n^2 - 2n + 3$:

$$t_1 = 1 - 2 + 3 = 2$$

$$t_2 = 4 - 4 + 3 = 3$$

$$t_3 = 9 - 6 + 3 = 6$$

$$t_4 = 16 - 8 + 3 = 11$$

$$t_5 = 25 - 10 + 3 = 18$$

First five terms: 2, 3, 6, 11, 18

1 mark

Observation (worth noting): (i) and (ii) are APs (differences 3 and -5), but (iii) is **not** an AP — its differences 1, 3, 5, 7 keep changing.

Q2. Find the 10^{th} and 15^{th} terms of the sequence $t_n = 5n - 3$ for $n \geq 1$.

2 marks

GIVEN: $t_n = 5n - 3$.

TO FIND: t_{10} and t_{15} .

SOLUTION:

Step 1: Put $n = 10$: $t_{10} = 5(10) - 3 = 50 - 3 = 47$

1 mark

Step 2: Put $n = 15$: $t_{15} = 5(15) - 3 = 75 - 3 = 72$

1 mark

Hence $t_{10} = 47$ and $t_{15} = 72$.

Q3. Determine whether 97 and 172 are terms of the sequence $t_n = 5n - 3$ for $n \geq 1$.

2 marks

GIVEN: $t_n = 5n - 3$.

TO FIND: Whether 97 and 172 occur as terms.

SOLUTION:

For 97: Set $t_n = 97$

$$5n - 3 = 97$$

$$5n = 100$$

$$n = 20$$

Since $n = 20$ is a natural number, **97 is a term of the sequence — it is the 20th term.**

1 mark

For 172: Set $t_n = 172$

$$5n - 3 = 172$$

$$5n = 175$$

$$n = 35$$

Since $n = 35$ is also a natural number, **172 is a term of the sequence — it is the 35th term.**

1 mark

Hence both 97 and 172 are terms, occupying positions 20 and 35 respectively.

⚠ Note: Do not assume one of the two must fail just because the question pairs them. Always solve both and report honestly.

Q4. Which term of the sequence $t_n = 5n - 3$ for $n \geq 1$ is 607?

2 marks

GIVEN: $t_n = 5n - 3$ and a term of value 607.

TO FIND: The position n .

SOLUTION:

Step 1: $5n - 3 = 607$

½ mark

Step 2: $5n = 607 + 3 = 610$

½ mark

Step 3: $n = \frac{610}{5} = 122$

½ mark

Since 122 is a natural number, the value is attained.

Hence 607 is the 122nd term of the sequence.

½ mark

Q5. A sequence is given by the recursive rule $t_1 = -5$, $t_{n+1} = t_n + 3$ for $n \geq 1$. Find the first five terms. Is 52 a term of this sequence? If so, which term is it?

3 marks

GIVEN: $t_1 = -5$, $t_{n+1} = t_n + 3$.

TO FIND: First five terms; and whether 52 is a term.

SOLUTION:

Part 1 — building the terms. Each term is 3 more than the one before:

$$t_1 = -5$$

$$t_2 = t_1 + 3 = -5 + 3 = -2$$

$$t_3 = t_2 + 3 = -2 + 3 = 1$$

$$t_4 = t_3 + 3 = 1 + 3 = 4$$

$$t_5 = t_4 + 3 = 4 + 3 = 7$$

First five terms: -5, -2, 1, 4, 7

1 mark

Part 2 — testing whether 52 occurs. There are two ways.

METHOD 1 — Convert to an explicit rule, then solve

Since every term exceeds the previous by the constant 3, this is an **AP** with $a = -5$ and $d = 3$.

$$t_n = a + (n - 1)d = -5 + (n - 1)(3) = 3n - 8$$

1 mark

$$\text{Set } t_n = 52: 3n - 8 = 52 \Rightarrow 3n = 60 \Rightarrow n = 20$$

Since $n = 20$ is a natural number, **52 is the 20th term.**

1 mark

METHOD 2 — Count the steps directly

To climb from $t_1 = -5$ up to 52 the total rise is $52 - (-5) = 57$. Each step adds 3, so the number of steps is

$$\frac{57}{3} = 19.$$

19 steps after the first term lands on the $(19 + 1)^{\text{th}} = 20^{\text{th}}$ term ✓

METHOD 3 — Simply keep listing

-5, -2, 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, 49, 52 — counting, 52 is the 20th ✓

Which method to use: Method 1 is the best habit — it converts a recursive rule into a formula that answers *any* follow-up ("find the 200th term", "is 100 a term?") instantly. Method 2 is a good quick mental check. Avoid **Method 3** in an exam: it works here only because the answer is small, and it collapses completely if the question asks about the 500th term.

Q6. Let $T_1 = 1$, $T_2 = 2$, $T_3 = 4$, and $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ for $n \geq 4$. Find T_4 , T_5 , T_6 , T_7 and T_8 .

3 marks

GIVEN: Three seed values and a rule using the **previous three terms**.

TO FIND: T_4 to T_8 .

SOLUTION:

This sequence needs three starting values because each new term adds up the three terms just before it. Work strictly in order:

$$T_4 = T_3 + T_2 + T_1 = 4 + 2 + 1 = 7$$

$$T_5 = T_4 + T_3 + T_2 = 7 + 4 + 2 = 13$$

1 mark

$$T_6 = T_5 + T_4 + T_3 = 13 + 7 + 4 = 24$$

$$T_7 = T_6 + T_5 + T_4 = 24 + 13 + 7 = 44$$

1 mark

$$T_8 = T_7 + T_6 + T_5 = 44 + 24 + 13 = 81$$

1 mark

Hence $T_4 = 7$, $T_5 = 13$, $T_6 = 24$, $T_7 = 44$, $T_8 = 81$.

Note: The full sequence 1, 2, 4, 7, 13, 24, 44, 81, ... is called the **tribonacci** sequence — the three-term cousin of the Virahāṅka–Fibonacci sequence. It is neither an AP nor a GP.

— End of Exercise Set 8.1 —

■ NCERT Solutions — Exercise Set 8.2 (7 Questions)

🔑 Approach for This Exercise

Every question here is an **arithmetic progression**. The routine is fixed: (1) identify a and d ; (2) write $t_n = a + (n - 1)d$; (3) substitute or solve. For questions with two given terms, form two equations and **subtract** to eliminate a . For any **sum**, remember this chapter gives you only $S_n = \frac{n(n+1)}{2}$ for natural numbers — reduce the problem to that form.

Q1. Find the 10th and 26th terms of the AP: 3, 8, 13, 18,

2 marks

GIVEN: AP: 3, 8, 13, 18, ...

TO FIND: t_{10} and t_{26} .

SOLUTION:

Step 1: First term $a = 3$.

Step 2: Common difference $d = 8 - 3 = 5$ (check: $13 - 8 = 5$, $18 - 13 = 5$ ✓ — confirmed AP).

Step 3: $t_n = a + (n - 1)d = 3 + (n - 1)5$

METHOD 1 — Direct formula (recommended)

Step 4: $t_{10} = 3 + (10 - 1) \times 5 = 3 + 9 \times 5 = 3 + 45 = 48$ **1 mark**

Step 5: $t_{26} = 3 + (26 - 1) \times 5 = 3 + 25 \times 5 = 3 + 125 = 128$ **1 mark**

METHOD 2 — Jump from a term already found

Once $t_{10} = 48$ is known, t_{26} is 16 steps further, and each step adds $d = 5$:

$t_{26} = t_{10} + 16d = 48 + 16 \times 5 = 48 + 80 = 128$ ✓

METHOD 3 — Build the explicit rule first

$t_n = 3 + (n - 1)5 = 5n - 2$. Then $t_{10} = 50 - 2 = 48$ and $t_{26} = 130 - 2 = 128$ ✓

Which method to use: **Method 1** for a two-part question like this — it is the standard expected route and needs no prior result. **Method 3** becomes the best choice the moment a question asks for *three or more* terms, since simplifying to $5n - 2$ once makes every later substitution a single multiplication.

Hence $t_{10} = 48$ and $t_{26} = 128$.

Q2. Which term of the AP: 21, 18, 15, ... is -81 ? Also, is 0 a term of this AP? Give reasons for your answer.

3 marks

GIVEN: AP: 21, 18, 15, ...

TO FIND: The position of -81 ; and whether 0 occurs.

SOLUTION:

Step 1: $a = 21$, $d = 18 - 21 = -3$ (a **decreasing** AP).

Step 2: $t_n = 21 + (n - 1)(-3) = 21 - 3n + 3 = 24 - 3n$

1 mark

Part (a) — locating -81 :

$$24 - 3n = -81$$

$$-3n = -81 - 24 = -105$$

$$n = \frac{-105}{-3} = 35$$

Since 35 is a natural number, -81 is the **35th term of the AP.**

1 mark

Part (b) — testing 0 :

$$24 - 3n = 0$$

$$3n = 24$$

$$n = 8$$

Since $n = 8$ is a natural number, **yes, 0 is a term of this AP — it is the 8th term.**

Alternative for part (b) — count down by hand

The AP decreases by 3 each time: 21, 18, 15, 12, 9, 6, 3, 0 — 0 appears as the 8th term ✓

Which method to use: Use the **formula method** as your written answer — it is general and earns the reasoning mark. Listing works here only because 0 arrives quickly; it would be useless for part (a), where -81 is the 35th term. Listing is best kept as a **10-second verification** of a formula answer, not as the answer itself.

Reason: the value of n obtained is a positive whole number, which is exactly the condition for a value to occupy a genuine position in the sequence. (Verification: 21, 18, 15, 12, 9, 6, 3, **0** — the 8th term is indeed 0.)

1 mark

Q3. Find the n^{th} term of the AP: 11, 8, 5, 2, ... Write the recursive rule for this AP.

2 marks

GIVEN: AP: 11, 8, 5, 2, ...

TO FIND: The explicit rule t_n and the recursive rule.

SOLUTION:

Step 1: $a = 11$; $d = 8 - 11 = -3$ (check: $5 - 8 = -3$, $2 - 5 = -3$ ✓).

Step 2 — **Explicit rule:**

$$t_n = a + (n - 1)d = 11 + (n - 1)(-3)$$

$$t_n = 11 - 3n + 3$$

$$t_n = 14 - 3n \text{ (check: } t_1 = 14 - 3 = 11 \text{ ✓)}$$

1 mark

Step 3 — **Recursive rule:** since each term is 3 less than the previous one,

$$t_1 = 11, \quad t_n = t_{n-1} - 3 \text{ for } n \geq 2$$

1 mark

Reminder: the recursive answer is only complete if the seed value $t_1 = 11$ and the range $n \geq 2$ are both written.

Q4. An AP consists of 50 terms in which the 3rd term is 12 and the last term is 106. Find the 29th term.

3 marks

GIVEN: An AP of 50 terms with $t_3 = 12$ and $t_{50} = 106$.

TO FIND: t_{29} .

SOLUTION:

Step 1 — Convert both facts into equations using $t_n = a + (n - 1)d$:

$$t_3 = a + 2d = 12 \quad \dots\dots(1)$$

$$t_{50} = a + 49d = 106 \quad \dots\dots(2)$$

1 mark

METHOD 1 — Eliminate a by subtracting the equations

Step 2 — Subtract (1) from (2) so that a cancels:

$$(a + 49d) - (a + 2d) = 106 - 12$$

$$47d = 94$$

$$d = \frac{94}{47} = 2$$

1 mark

Step 3 — Substitute $d = 2$ into (1):

$$a + 2(2) = 12 \Rightarrow a + 4 = 12 \Rightarrow a = 8$$

$$\text{Step 4 — Required term: } t_{29} = a + 28d = 8 + 28 \times 2 = 8 + 56 = \mathbf{64}$$

1 mark

METHOD 2 — Gap formula, then step across from a known term

Between the 3rd and 50th terms there are $50 - 3 = 47$ steps, so

$$d = \frac{t_{50} - t_3}{50 - 3} = \frac{106 - 12}{47} = \frac{94}{47} = 2$$

Now travel from $t_3 = 12$ to t_{29} , which is $29 - 3 = 26$ steps further:

$$t_{29} = t_3 + 26d = 12 + 26 \times 2 = 12 + 52 = \mathbf{64} \checkmark$$

*Notice this method **never needs a at all.***

Which method to use: Method 2 is easier and faster here, because the question asks only for one particular term and a is never wanted. Use **Method 1** when the question also asks for the first term, the AP itself, or its n^{th} term — then a has to be found anyway.

Hence the 29th term of the AP is 64.

Q5. How many 2-digit numbers are divisible by 3? What is the sum of all these 2-digit numbers?

4 marks

GIVEN: All two-digit numbers divisible by 3.

TO FIND: How many there are, and their total.

SOLUTION:

Part 1 — How many?

Step 1: The smallest 2-digit multiple of 3 is 12 (since 9 is one digit); the largest is 99 (since 102 has three digits).

Step 2: These form the AP 12, 15, 18, ..., 99 with $a = 12$, $d = 3$, last term $t_n = 99$.

1 mark

Step 3: $12 + (n - 1)3 = 99$

$$(n - 1)3 = 87$$

$$n - 1 = 29$$

$$n = 30$$

Hence there are 30 two-digit numbers divisible by 3.

1 mark

Part 2 — Their sum. This chapter provides only the sum of the first n **natural** numbers, so the sum must be routed through $S_n = \frac{n(n+1)}{2}$. Two ways do this.

METHOD 1 — Take 3 common, then use $S_q - S_{p-1}$

$$12 + 15 + 18 + \dots + 99 = 3(4 + 5 + 6 + \dots + 33)$$

1 mark

Now $4 + 5 + \dots + 33 = S_{33} - S_3$:

$$S_{33} = \frac{33 \times 34}{2} = \frac{1122}{2} = 561$$

$$S_3 = \frac{3 \times 4}{2} = 6$$

$$\text{So } 4 + 5 + \dots + 33 = 561 - 6 = 555$$

Therefore the required sum $= 3 \times 555 = 1665$

1 mark

METHOD 2 — Āryabhaṭa's pairing (average $\times$ number of terms)

Section 8.5 notes Āryabhaṭa's second rule: the sum equals the **average of the first and last terms**, multiplied by the number of terms. Writing the sum forwards and backwards:

$$S = 12 + 15 + \dots + 96 + 99$$

$$S = 99 + 96 + \dots + 15 + 12$$

Adding, each of the 30 pairs totals $12 + 99 = 111$:

$$2S = 111 \times 30 = 3330$$

$$S = \frac{3330}{2} = 1665 \checkmark$$

Which method to use: Method 2 is far quicker — one multiplication and one halving, with no factoring out and no two separate S_n evaluations. It is also the method the chapter itself derives on page 183. Prefer

Method 1 only if your teacher has specifically asked you to demonstrate the $S_q - S_{p-1}$ technique, since Method 2's pairing argument should be shown in the two-line reversed-sum form to earn full method marks.

Hence there are 30 such numbers, and their sum is 1665.

Q6. Harish started work at an annual salary of ₹5,00,000 and received an increment of ₹20,000 each year. After how many years did his income reach ₹7,00,000?

3 marks

GIVEN: Starting salary ₹5,00,000; fixed annual increment ₹20,000.

TO FIND: The number of years after which the salary becomes ₹7,00,000.

SOLUTION:

Step 1: The yearly salaries form an AP: 5,00,000; 5,20,000; 5,40,000; ...

Here $a = 5,00,000$ and $d = 20,000$.

1 mark

METHOD 1 — AP n^{th} -term formula

Step 2: Let the salary be ₹7,00,000 in the n^{th} year.

$$a + (n - 1)d = 7,00,000$$

$$5,00,000 + (n - 1)(20,000) = 7,00,000$$

$$\text{Step 3: } (n - 1)(20,000) = 2,00,000$$

$$n - 1 = \frac{2,00,000}{20,000} = 10, \text{ so } n = 11$$

1 mark

METHOD 2 — Direct "total rise $\div$ yearly rise"

$$\text{Total increase needed} = 7,00,000 - 5,00,000 = 2,00,000$$

Each year adds ₹20,000, so the number of increments required is

$$\frac{2,00,000}{20,000} = 10 \text{ increments} = \mathbf{10 \text{ years}} \checkmark$$

Which method to use: Method 2 is the easier and safer one for this question — it answers "after how many years" directly, so you never have to remember to convert $n = 11$ back to $n - 1 = 10$. That conversion is exactly where most students lose the mark. Use Method 1 if the question instead asks "in which year" or "which term".

Step 4 — Interpret carefully. $n = 11$ means the salary is ₹7,00,000 in his 11th year of service. The number of years that have passed since he started is $n - 1$.

Hence his income reached ₹7,00,000 after 10 years.

1 mark

⚠ Trap: The question asks "after how many years", not "which term". Writing 11 here is the classic error — the 1st term corresponds to year zero of increments.

Q7. A child arranges marbles in rows so that the first row has 1 marble, the second has 2 marbles, the third has 3, and so on up to 25 rows. How many marbles does the child use in all?

2 marks

GIVEN: Rows contain 1, 2, 3, ..., 25 marbles.

TO FIND: The total number of marbles.

SOLUTION:

Step 1: Total = $1 + 2 + 3 + \dots + 25$, which is exactly the sum of the first 25 natural numbers.

1 mark

METHOD 1 — Standard formula (recommended)

Step 2: Using $S_n = \frac{n(n+1)}{2}$ with $n = 25$:

$$S_{25} = \frac{25 \times 26}{2} = \frac{650}{2} = 325$$

1 mark

METHOD 2 — Pairing from both ends

Write the sum forwards and backwards and add:

$$S = 1 + 2 + \dots + 24 + 25$$

$$S = 25 + 24 + \dots + 2 + 1$$

Each of the 25 pairs totals 26, so $2S = 26 \times 25 = 650$ and $S = 325$ ✓

Which method to use: Method 1 — it is a single substitution into a formula you must know anyway.

Method 2 is really just the derivation of that formula, so use it only if a question explicitly says "without using the formula" or "show the method".

Hence the child uses 325 marbles in all.

Note: 325 is the 25th **triangular number** — the marbles literally form a triangle, which is why the triangular-number formula applies.

— End of Exercise Set 8.2 —

NCERT Solutions — Exercise Set 8.3 (7 Questions)

 Approach for This Exercise

These are **geometric progressions**. Find r by **dividing** consecutive terms (never subtracting), then use $t_n = ar^{n-1}$. For "which term is X?" questions, express both sides as powers of the **same base** and compare exponents — this is far safer than trial-and-error division. Since no GP sum formula exists in this chapter, any total must be found by listing and adding the individual terms.

Q1. Find the 12th term of a GP with common ratio 2, whose 8th term is 192.

3 marks

GIVEN: GP with $r = 2$ and $t_8 = 192$.

TO FIND: t_{12} .

SOLUTION:

METHOD 1 — Step forward from the known term

Step 1: Note that t_{12} is 4 steps beyond t_8 . Each step multiplies by r , so:

$$t_{12} = t_8 \times r^4 \quad \text{1 mark}$$

$$\text{Step 2: } t_{12} = 192 \times 2^4 = 192 \times 16 \quad \text{1 mark}$$

$$\text{Step 3: } 192 \times 16 = 3072$$

Hence the 12th term is 3072. 1 mark

METHOD 2 — Find a first, then apply the formula

$$t_8 = ar^7 = 192 \Rightarrow a(2^7) = 192 \Rightarrow a(128) = 192 \Rightarrow a = \frac{192}{128} = \frac{3}{2}$$

$$\text{Then } t_{12} = ar^{11} = \frac{3}{2} \times 2^{11} = \frac{3}{2} \times 2048 = 3072 \checkmark$$

Which method to use: Method 1 is clearly easier — it is three lines, uses only whole numbers, and never requires you to compute 2^7 or 2^{11} . Method 2 forces you through the fraction $a = \frac{3}{2}$ and two large powers of 2, so there is more to get wrong for no extra credit. Reach for Method 2 only when the question actually asks for the first term or the general n^{th} term.

Q2. Find the 10th and n^{th} terms of the GP: 5, 25, 125,

2 marks

GIVEN: GP: 5, 25, 125, ...

TO FIND: t_{10} and t_n .

SOLUTION:

$$\text{Step 1: } a = 5; r = \frac{25}{5} = 5 \text{ (check: } \frac{125}{25} = 5 \checkmark \text{ — confirmed GP).}$$

Step 2 — n^{th} term:

$$t_n = ar^{n-1} = 5 \times 5^{n-1} = 5^{1+n-1}$$

$$t_n = 5^n \quad \text{1 mark}$$

Step 3 — 10th term:

$$t_{10} = 5^{10} = 9\,765\,625 \quad \text{1 mark}$$

Hence $t_n = 5^n$ and $t_{10} = 9,765,625$.

★ Q3. A sequence is given by the recursive rule $t_1 = 2$, $t_{n+1} = 3t_n - 2$ for $n \geq 1$. Which term of the sequence is 730?

3 marks

GIVEN: $t_1 = 2$, $t_{n+1} = 3t_n - 2$.

TO FIND: The position of the value 730.

SOLUTION:

This rule is **not** a plain GP (there is a "-2" attached), so no direct formula applies — we generate terms in order until 730 appears.

$$t_1 = 2$$

$$t_2 = 3(2) - 2 = 6 - 2 = 4$$

$$t_3 = 3(4) - 2 = 12 - 2 = 10$$

$$t_4 = 3(10) - 2 = 30 - 2 = 28$$

1 mark

$$t_5 = 3(28) - 2 = 84 - 2 = 82$$

$$t_6 = 3(82) - 2 = 246 - 2 = 244$$

$$t_7 = 3(244) - 2 = 732 - 2 = 730$$

1 mark

Hence 730 is the 7th term of the sequence.

1 mark

Insight (for the curious): the terms 2, 4, 10, 28, 82, 244, 730 are each exactly $3^{n-1} + 1$. Indeed

$$3^6 + 1 = 729 + 1 = 730, \text{ confirming } n = 7.$$

Q4. Which term of the GP: 2, 6, 18, ... is 4374? Write the explicit formula as well as the recursive formula for the n^{th} term.

4 marks

GIVEN: GP: 2, 6, 18, ...

TO FIND: Position of 4374; explicit and recursive rules.

SOLUTION:

Step 1: $a = 2$; $r = \frac{6}{2} = 3$ (check: $\frac{18}{6} = 3$ ✓).

Step 2 — **Explicit formula:** $t_n = 2 \times 3^{n-1}$ 1 mark

Step 3 — **Recursive formula:** $t_1 = 2$, $t_n = 3t_{n-1}$ for $n \geq 2$ 1 mark

Step 4 — **Locating 4374:**

$$2 \times 3^{n-1} = 4374$$

$$3^{n-1} = \frac{4374}{2} = 2187$$
 1 mark

METHOD 1 — Match powers of the same base (recommended)

Step 5 — Express 2187 as a power of 3:

$$3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81, 3^5 = 243, 3^6 = 729, 3^7 = 2187$$

$$\text{So } 3^{n-1} = 3^7 \Rightarrow n-1 = 7 \Rightarrow n = 8$$

METHOD 2 — Divide repeatedly by r

Starting at 4374 and dividing by 3 until we reach the first term 2:

$$4374 \div 3 = 1458 \text{ (1), } 486 \text{ (2), } 162 \text{ (3), } 54 \text{ (4), } 18 \text{ (5), } 6 \text{ (6), } 2 \text{ (7)}$$

It took 7 divisions to come back to t_1 , so 4374 sits 7 places after the first term — the **8th term** ✓

Which method to use: Method 1, without hesitation. Matching exponents is exact and works just as easily for huge values (see End-of-Chapter Q10, where the term is 131072). Method 2 is a fair backup if you cannot spot the power quickly, but with a large number it means many divisions and one arithmetic slip destroys the answer.

Hence 4374 is the 8th term of the GP. 1 mark

Q5. A ball is dropped from a height of 80 metres. After hitting the ground, it bounces back to 60% of the height from which it fell, and continues in this way. (i) What height does the ball reach after the 5th bounce? (ii) What is the total vertical distance travelled by the time it hits the ground for the 6th time?

5 marks

GIVEN: Drop height 80 m; each bounce reaches 60% (i.e. 0.6 times) of the previous height.

TO FIND: (i) height after the 5th bounce; (ii) total vertical distance up to the 6th ground contact.

SOLUTION:

Part (i) — Height after the 5th bounce.

Step 1: The bounce heights form a GP. After the 1st bounce the height is $80 \times 0.6 = 48$ m, so $a = 48$ and

$r = 0.6$. 1 mark

Step 2: Listing the bounce heights:

1st bounce: $80 \times 0.6 = 48$ m

2nd bounce: $48 \times 0.6 = 28.8$ m

3rd bounce: $28.8 \times 0.6 = 17.28$ m

4th bounce: $17.28 \times 0.6 = 10.368$ m

5th bounce: $10.368 \times 0.6 = 6.2208$ m 1 mark

Equivalently, $t_5 = 80 \times (0.6)^5 = 80 \times 0.07776 = 6.2208$ m.

Hence after the 5th bounce the ball reaches a height of 6.2208 m (≈ 6.22 m).

1 mark

Part (ii) — Total vertical distance up to the 6th impact.

Step 3 — Account for the motion carefully. The ball:

- falls 80 m $\rightarrow$ **1st** hit,
- rises 48 m and falls 48 m $\rightarrow$ **2nd** hit,
- rises 28.8 and falls 28.8 $\rightarrow$ **3rd** hit,
- rises 17.28 and falls 17.28 $\rightarrow$ **4th** hit,
- rises 10.368 and falls 10.368 $\rightarrow$ **5th** hit,
- rises 6.2208 and falls 6.2208 $\rightarrow$ **6th** hit.

So the first 5 bounce heights are each counted **twice** (once up, once down), plus the initial 80 m drop counted once. 1 mark

Step 4 — Add the bounce heights:

$$48 + 28.8 + 17.28 + 10.368 + 6.2208 = 110.6688 \text{ m}$$

Step 5 — Total distance:

$$= 80 + 2(110.6688)$$

$$= 80 + 221.3376$$

$$= \mathbf{301.3376 \text{ m}}$$
 1 mark

Hence the total vertical distance travelled is 301.3376 m (≈ 301.34 m).

⚠ Trap: Forgetting to double the bounce heights (the ball goes up and comes back down), or wrongly doubling the initial 80 m drop, are the two errors that cost the most marks in this question.

Q6. Which term of the sequence $2, 2\sqrt{2}, 4, \dots$ is 128?

3 marks

GIVEN: Sequence $2, 2\sqrt{2}, 4, \dots$

TO FIND: The position of 128.

SOLUTION:

Step 1 — Confirm it is a GP by dividing:

$$\frac{t_2}{t_1} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$\frac{t_3}{t_2} = \frac{4}{2\sqrt{2}} = \frac{4}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{4} = \sqrt{2}$$

The ratio is constant, so it is a GP with $a = 2$ and $r = \sqrt{2}$. 1 mark

METHOD 1 — Convert everything to powers of 2 (recommended)

Step 2 — Since $\sqrt{2} = 2^{1/2}$:

$$t_n = ar^{n-1} = 2 \times (2^{1/2})^{n-1} = 2^1 \times 2^{\frac{n-1}{2}} = 2^{1+\frac{n-1}{2}} = 2^{\frac{n+1}{2}}$$
 1 mark

Step 3 — Set this equal to $128 = 2^7$:

$$2^{\frac{n+1}{2}} = 2^7$$

$$\text{Comparing exponents (same base): } \frac{n+1}{2} = 7 \Rightarrow n+1 = 14 \Rightarrow n = 13$$

METHOD 2 — Spot that alternate terms double

List the terms: $2, 2\sqrt{2}, 4, 4\sqrt{2}, 8, 8\sqrt{2}, 16, \dots$

The **whole numbers** sit at positions 1, 3, 5, 7, ... and are 2, 4, 8, 16, ... — i.e. 2^k sits at position $2k - 1$.

Since $128 = 2^7$, take $k = 7$: position $= 2(7) - 1 = 13$ ✓

Which method to use: Method 1 is the more reliable one and is what the marking scheme expects —

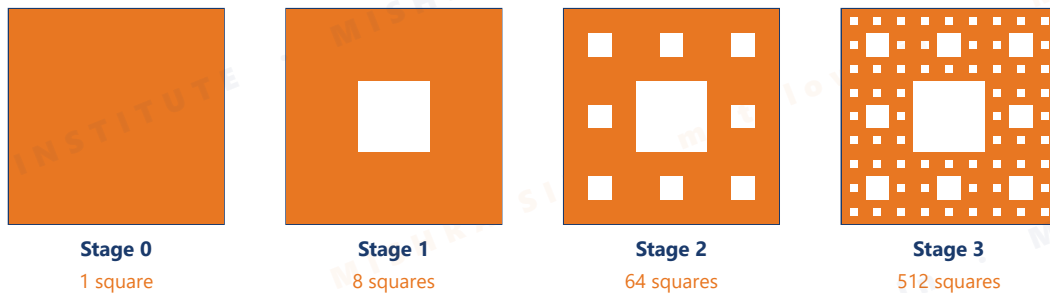
writing $\sqrt{2}$ as $2^{1/2}$ turns an awkward surd GP into ordinary index work. Method 2 is a neat sanity check and is fine if you can see the pattern instantly, but it depends on noticing a structure rather than on a general technique, so it will not transfer to a GP like $3, 3\sqrt{3}, 9, \dots$ as cleanly.

Hence 128 is the 13th term of the sequence.

1 mark

Q7. Fig. 8.12 shows Stages 0 to 3 of the Sierpiński square carpet. Stage 0 is a square sheet. Each side is trisected and the points of trisection of opposite sides joined, giving nine smaller squares; the centre square is removed and the 8 outer squares retained. The process repeats on each retained square. (i) How many red squares in Stages 0 to 3? (ii) Predict the number in Stages 4 and 5. (iii) Find a rule for the number of red squares at the n^{th} stage — explicit and recursive. (iv) If the Stage-0 area is 1 square unit, find the red area at Stages 1, 2, 3, 4, 5, the explicit and recursive rules, and what happens as n increases.

6 marks



Note: This is an MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is not clear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

Fig. 8.12 — Sierpiński square carpet, Stages 0–3. Each square is trisected and the centre removed, leaving 8 of 9.
Squares: $t_n = 8^n$; shaded area: $s_n = (8/9)^n$.

GIVEN: The Sierpiński square carpet construction — each shaded square is replaced by 8 smaller squares, each of area $\frac{1}{9}$ of it.

TO FIND: Counts, areas, and rules at any stage.

SOLUTION:

(i) Number of red squares in Stages 0 to 3.

Stage 0: the whole sheet is one red square $\rightarrow 1$

Stage 1: divided into 9, centre removed $\rightarrow 8$

Stage 2: each of the 8 becomes 8 $\rightarrow 8 \times 8 = 64$

Stage 3: $64 \times 8 = 512$ 1 mark

(ii) Stages 4 and 5.

Stage 4: $512 \times 8 = 4096$

Stage 5: $4096 \times 8 = 32768$ 1 mark

(iii) Rule for the number of red squares.

The counts 1, 8, 64, 512, 4096, 32768 form a **GP with common ratio 8**. Writing them as powers of 8:

$$1 = 8^0, 8 = 8^1, 64 = 8^2, 512 = 8^3, 4096 = 8^4, 32768 = 8^5$$

The exponent matches the stage number, so:

Explicit formula: $t_n = 8^n$

Recursive formula: $t_0 = 1$, $t_n = 8t_{n-1}$ for $n \geq 1$ 1 mark

(iv) Area of the red region.

At each stage, 9 equal parts are formed and 8 are kept, so the red area becomes $\frac{8}{9}$ of the previous stage's red area.

Stage 0: 1 square unit

Stage 1: $\frac{8}{9}$ square units

Stage 2: $\frac{8}{9} \times \frac{8}{9} = \left(\frac{8}{9}\right)^2 = \frac{64}{81}$

Stage 3: $\left(\frac{8}{9}\right)^3 = \frac{512}{729}$ 1 mark

Stage 4: $\left(\frac{8}{9}\right)^4 = \frac{4096}{6561}$

Stage 5: $\left(\frac{8}{9}\right)^5 = \frac{32768}{59049}$ 1 mark

Explicit formula: $s_n = \left(\frac{8}{9}\right)^n$

Recursive formula: $s_0 = 1$, $s_n = \frac{8}{9} s_{n-1}$ for $n \geq 1$

Behaviour as n increases: since $\frac{8}{9} < 1$, each stage multiplies the area by a number smaller than 1, so the red area keeps decreasing and gets closer and closer to 0 (without ever actually becoming 0).

1 mark

The striking conclusion: the number of red squares grows extremely fast (8^n) while the total red area shrinks towards zero — exactly the same paradoxical behaviour as the Sierpiński triangle, and the defining signature of a fractal.

— End of Exercise Set 8.3 —

NCERT Solutions — End-of-Chapter Exercises (15 Questions)

Approach for This Exercise

This is the hardest set, and 11 of the 15 questions are **starred (★)** higher-order questions — your annual-paper long answers will come from here. Two habits carry you through almost all of them: (1) turn every given fact into an **equation in a and d** (or a and r) and then **subtract/divide** to eliminate one unknown; (2) for GP problems where a square or an even power appears, expect **two** valid answers and present both.

Q1. Find the 31st term of an AP whose 11th term is 38 and 16th term is 73.

3 marks

GIVEN: $t_{11} = 38$, $t_{16} = 73$.

TO FIND: t_{31} .

SOLUTION:

Step 1 — Form equations using $t_n = a + (n - 1)d$:

$$a + 10d = 38 \dots\dots(1)$$

$$a + 15d = 73 \dots\dots(2)$$

1 mark

METHOD 1 — Solve for a and d

Step 2 — Subtract (1) from (2):

$$5d = 73 - 38 = 35 \Rightarrow d = 7$$

1 mark

Step 3 — Substitute into (1): $a + 10(7) = 38 \Rightarrow a + 70 = 38 \Rightarrow a = -32$

$$\text{Step 4 — } t_{31} = a + 30d = -32 + 30(7) = -32 + 210 = \mathbf{178}$$

1 mark

METHOD 2 — Gap formula, then step across (no a needed)

$$d = \frac{t_{16} - t_{11}}{16 - 11} = \frac{73 - 38}{5} = \frac{35}{5} = 7$$

From $t_{16} = 73$ to t_{31} is $31 - 16 = 15$ steps:

$$t_{31} = t_{16} + 15d = 73 + 15(7) = 73 + 105 = \mathbf{178} \checkmark$$

Which method to use: Method 2 is easier — it skips finding a altogether (and here $a = -32$ is negative, which is where sign errors creep in). Switch to Method 1 whenever the question also wants the first term, the full AP, or the general n^{th} term.

Hence the 31st term is 178.

Q2. Determine the AP whose third term is 16 and whose 7th term exceeds the 5th term by 12.

3 marks

GIVEN: $t_3 = 16$ and $t_7 - t_5 = 12$.

TO FIND: The AP.

SOLUTION:

Step 1 — Use the second condition first (it contains only d):

$$t_7 - t_5 = (a + 6d) - (a + 4d) = 2d$$

$$\text{So } 2d = 12$$

$$d = 6$$

1 mark

Step 2 — Use the first condition:

$$t_3 = a + 2d = 16$$

$$a + 2(6) = 16$$

$$a + 12 = 16$$

$$a = 4$$

1 mark

Step 3 — Write out the AP with $a = 4$, $d = 6$:

4, 10, 16, 22, 28, ...

Hence the required AP is 4, 10, 16, 22, 28, ... , with $t_n = 6n - 2$.

1 mark

(Check: $t_3 = 6(3) - 2 = 16$ ✓, and $t_7 - t_5 = 40 - 28 = 12$ ✓)

★ Q3. How many three-digit numbers are divisible by 7?

3 marks

GIVEN: Three-digit numbers divisible by 7.

TO FIND: How many there are.

SOLUTION:

Step 1 — **Smallest** three-digit multiple of 7: divide 100 by 7 $\rightarrow 7 \times 14 = 98$ (two digits), $7 \times 15 = 105$. So the first is **105**.

Step 2 — **Largest** three-digit multiple of 7: $7 \times 142 = 994$, and $7 \times 143 = 1001$ (four digits). So the last is **994**. 1 mark

Step 3 — These form the AP 105, 112, 119, ..., 994 with $a = 105$, $d = 7$, $t_n = 994$.

½ mark

METHOD 1 — AP n^{th} -term formula

Step 4 — Solve:

$$105 + (n - 1)7 = 994$$

$$(n - 1)7 = 994 - 105 = 889$$

$$n - 1 = \frac{889}{7} = 127$$

$$n = 128$$

1 mark

METHOD 2 — Counting multiples ("last $\div$ 7 minus $99 \div 7$ ")

The multiples of 7 up to 994 are 7×1 to 7×142 , so there are 142 of them.

The multiples of 7 up to 99 (the two-digit-and-below ones) are 7×1 to 7×14 , so there are 14.

Three-digit ones = $142 - 14 = \mathbf{128}$ ✓

METHOD 3 — Gap-count formula

$$n = \frac{\text{last} - \text{first}}{d} + 1 = \frac{994 - 105}{7} + 1 = 127 + 1 = 128 \checkmark$$

Which method to use: Method 3 is the fastest and is worth memorising as a one-liner for every "how many multiples" question — but you must never forget the +1. **Method 1** is the safest for full marks, since writing out a , d and $t_n = 994$ shows the examiner your reasoning explicitly. Method 2 is an excellent independent check.

Hence there are **128** three-digit numbers divisible by 7.

½ mark

★ Q4. How many multiples of 4 lie between 10 and 250?

3 marks

GIVEN: Multiples of 4 strictly between 10 and 250.

TO FIND: Their count.

SOLUTION:

Step 1 — Smallest multiple of 4 greater than 10: $4 \times 2 = 8$ (too small), $4 \times 3 = 12$.

Step 2 — Largest multiple of 4 less than 250: $4 \times 62 = 248$ (since $4 \times 63 = 252 > 250$).

1 mark

Step 3 — AP: 12, 16, 20, ..., 248 with $a = 12$, $d = 4$, $t_n = 248$.

½ mark

METHOD 1 — AP n^{th} -term formula

Step 4 — Solve:

$$12 + (n - 1)4 = 248$$

$$(n - 1)4 = 236 \Rightarrow n - 1 = 59 \Rightarrow n = 60$$

1 mark

METHOD 2 — Count the multipliers

The multiples run from $4 \times 3 = 12$ to $4 \times 62 = 248$, so the multiplier takes every whole value from 3 to 62.

$$\text{Count} = 62 - 3 + 1 = 60 \checkmark$$

Which method to use: Method 2 is the quicker one for multiples of a small number like 4 — you simply count the multipliers. Use **Method 1** when the numbers do not divide neatly, or when the examiner has asked you to treat the list as an AP. Both need the same care with the +1.

Hence 60 multiples of 4 lie between 10 and 250.

½ mark

★ Q5. Find a GP for which the sum of the first two terms is -4 and the fifth term is 4 times the third term.

4 marks

GIVEN: $t_1 + t_2 = -4$ and $t_5 = 4t_3$.

TO FIND: The GP.

SOLUTION:

Let the first term be a and the common ratio r .

Step 1 — Use the second condition (it eliminates a at once):

$$ar^4 = 4(ar^2)$$

Dividing both sides by ar^2 (valid, since $a \neq 0$ and $r \neq 0$ in a GP):

$$r^2 = 4$$

$r = 2$ or $r = -2$ — **two cases must be considered.**

1 mark

Step 2 — Use the first condition: $a + ar = -4$, i.e. $a(1 + r) = -4$.

Case I: $r = 2$

$$a(1 + 2) = -4$$

$$3a = -4$$

$$a = -\frac{4}{3}$$

$$\text{GP: } -\frac{4}{3}, -\frac{8}{3}, -\frac{16}{3}, -\frac{32}{3}, \dots$$

1 mark

$$(\text{Check: } -\frac{4}{3} - \frac{8}{3} = -\frac{12}{3} = -4 \checkmark)$$

Case II: $r = -2$

$$a(1 - 2) = -4$$

$$-a = -4$$

$$a = 4$$

$$\text{GP: } 4, -8, 16, -32, 64, \dots$$

1 mark

$$(\text{Check: } 4 + (-8) = -4 \checkmark; \text{ and } t_5 = 64 = 4 \times 16 = 4t_3 \checkmark)$$

Hence there are two such GPs: $-\frac{4}{3}, -\frac{8}{3}, -\frac{16}{3}, \dots$ (with $r = 2$) and $4, -8, 16, -32, \dots$ (with $r = -2$).

1 mark

★ Q6. Find all possible ways of expressing 100 as the sum of consecutive natural numbers.

5 marks

GIVEN: The number 100.

TO FIND: Every way of writing it as a sum of consecutive natural numbers.

SOLUTION:

Step 1 — Set up. Suppose 100 is the sum of k consecutive natural numbers starting from p :

$$100 = p + (p + 1) + (p + 2) + \cdots + (p + k - 1)$$

Step 2 — Reduce to the natural-number sum formula. Using $S_n = \frac{n(n+1)}{2}$:

$$100 = S_{p+k-1} - S_{p-1}$$

A neater equivalent form: the sum of k consecutive numbers is k times their average, and the average is

$$\frac{\text{first} + \text{last}}{2}. \text{ So}$$
$$100 = \frac{k(p + (p + k - 1))}{2}$$

$$200 = k(2p + k - 1) \quad \text{1 mark}$$

Step 3 — Read off the conditions. k must be a factor of 200, and $2p + k - 1 = \frac{200}{k}$ gives

$$p = \frac{1}{2} \left(\frac{200}{k} - k + 1 \right), \text{ which must be a positive whole number.}$$

This forces k and $\frac{200}{k}$ to have **opposite parity** (one odd, one even). 1 mark

Step 4 — Test the factors of 200 (1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200):

• $k = 1$: $p = \frac{1}{2}(200 - 1 + 1) = 100 \rightarrow$ the single term **100** itself (trivial case).

• $k = 2$: $200/2 = 100$, both even $\rightarrow$ rejected.

• $k = 4$: $200/4 = 50$, both even $\rightarrow$ rejected.

• $k = 5$: $200/5 = 40$; $p = \frac{1}{2}(40 - 5 + 1) = \frac{36}{2} = 18 \checkmark$

• $k = 8$: $200/8 = 25$; $p = \frac{1}{2}(25 - 8 + 1) = \frac{18}{2} = 9 \checkmark$

• $k = 10, 20, 40, 50, 100, 200$: all give $200/k$ even alongside even k , or $p \leq 0 \rightarrow$ rejected.

• $k = 25$: $p = \frac{1}{2}(8 - 25 + 1) = -8 \rightarrow$ negative, rejected. Same for $k = 40$. 1 mark

Step 5 — Write out the solutions:

(a) $k = 5$, starting at 18: $18 + 19 + 20 + 21 + 22 = 100 \checkmark$ 1 mark

(b) $k = 8$, starting at 9: $9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 = 100 \checkmark$

$$(\text{Verify: } S_{16} - S_8 = \frac{16 \times 17}{2} - \frac{8 \times 9}{2} = 136 - 36 = 100 \checkmark)$$

Hence, apart from the trivial single-term "sum" 100 itself, there are exactly two ways:

18 + 19 + 20 + 21 + 22 and 9 + 10 + $\cdots$ + 16.

1 mark

METHOD 2 — Trial by number of terms

Try each possible run-length k and see whether 100 splits evenly:

$k = 2$: consecutive pair summing to 100 $\rightarrow$ average 50, but the two numbers would be 49.5 and 50.5 $\rightarrow$ not whole. $\times$

$k = 3$: average = $100/3$, not a whole number $\rightarrow X$ (for **odd** k , the middle term must be $100/k$).

$k = 4$: average 25, terms would be 23.5, 24.5, 25.5, 26.5 $\rightarrow X$

$k = 5$: average = 20, whole $\rightarrow$ middle term 20, terms 18, 19, 20, 21, 22 $\checkmark$

$k = 6$: average = $100/6 \rightarrow X$ $k = 7$: $100/7 \rightarrow X$

$k = 8$: sum of 8 terms = $8 \times (\text{average of the middle pair}) = 100 \rightarrow$ average 12.5, giving 9 to 16 $\checkmark$

Beyond $k = 13$ the smallest possible sum $1+2+\dots+14=105 > 100$, so the search stops.

bonus check

Which method to use: Method 1 is the proper one — the equation $200 = k(2p + k - 1)$ plus the parity rule *proves* you have found every possibility, which is exactly what "find all possible ways" demands.

Method 2 is easier to think through and is fine for finding the answers, but on its own it only checks cases; you must add the stopping argument (that $1+2+\dots+14 > 100$) to show the list is complete.

Best exam strategy: **find the answers with Method 2, then justify completeness with Method 1's factor argument.**

★ Q7. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria originally, how many will be present at the end of the 2^{nd} hour, the 4^{th} hour, and the n^{th} hour?

3 marks

GIVEN: Initial count 30; the count doubles each hour.

TO FIND: Count after 2 hours, 4 hours, and n hours.

SOLUTION:

Step 1 — Build the pattern. At hour 0 (the start) there are 30 bacteria; each hour multiplies the count by 2:

End of 1st hour: $30 \times 2 = 60$

End of 2nd hour: $60 \times 2 = 120$ or directly $30 \times 2^2 = 120$

1 mark

Step 2 — End of 4th hour:

$30 \times 2^4 = 30 \times 16 = 480$

1 mark

Step 3 — General rule. The counts 30, 60, 120, 240, 480, ... form a GP with $a = 30$ and $r = 2$, and the exponent of 2 equals the number of hours elapsed:

At the end of the n^{th} hour the count is 30×2^n .

1 mark

⚠ **Note on indexing:** here the formula is 30×2^n , not $30 \times 2^{n-1}$, because the **starting** count of 30 corresponds to hour **0**, not hour 1. Always anchor the exponent to what the question calls "the start".

★ Q8. The sum of the 4th and 8th terms of an AP is 24, and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

4 marks

GIVEN: $t_4 + t_8 = 24$ and $t_6 + t_{10} = 44$.

TO FIND: t_1, t_2, t_3 .

SOLUTION:

Step 1 — Express each condition in a and d :

$$t_4 + t_8 = (a + 3d) + (a + 7d) = 2a + 10d = 24$$

Dividing by 2: $a + 5d = 12 \dots\dots(1)$

1 mark

$$t_6 + t_{10} = (a + 5d) + (a + 9d) = 2a + 14d = 44$$

Dividing by 2: $a + 7d = 22 \dots\dots(2)$

1 mark

METHOD 1 — Subtract the two equations

Step 2 — Subtract (1) from (2):

$$2d = 22 - 12 = 10 \Rightarrow d = 5$$

1 mark

Step 3 — Substitute into (1): $a + 5(5) = 12 \Rightarrow a + 25 = 12 \Rightarrow a = -13$

METHOD 2 — Use the "middle term" shortcut

In any AP, $t_4 + t_8 = 2t_6$ (the two terms are symmetric about the 6th). So $2t_6 = 24 \Rightarrow t_6 = 12$.

Similarly $t_6 + t_{10} = 2t_8$, so $2t_8 = 44 \Rightarrow t_8 = 22$.

Now t_8 is 2 steps after t_6 : $2d = 22 - 12 = 10 \Rightarrow d = 5$

And $t_1 = t_6 - 5d = 12 - 25 = -13$ ✓

Which method to use: Method 1 is the more dependable — it needs no special property and works even when the given terms are not symmetric. Method 2 is elegant and quick, but only applies when the two terms sum symmetrically about a middle term (as they happen to here), so treat it as a bonus shortcut rather than your default.

Step 4 — First three terms:

$$t_1 = -13, \quad t_2 = -13 + 5 = -8, \quad t_3 = -8 + 5 = -3$$

Hence the first three terms are $-13, -8, -3$.

1 mark

(Check: $t_4 = 2, t_8 = 22$, sum = 24 ✓; $t_6 = 12, t_{10} = 32$, sum = 44 ✓)

★ Q9. Find the smallest value of n such that the sum of the first n natural numbers is greater than 1,000.

3 marks

GIVEN: $S_n = \frac{n(n+1)}{2} > 1000$

TO FIND: The smallest such natural number n .

SOLUTION:

Step 1 — Set up the inequality:

$$\frac{n(n+1)}{2} > 1000$$

$$n(n+1) > 2000$$

1 mark

METHOD 1 — Estimate with square roots, then test (recommended)

Step 2 — Since $n(n+1)$ is a little more than n^2 , and $44^2 = 1936$, $45^2 = 2025$, the answer lies near 44 or 45.

Test both:

For $n = 44$: $44 \times 45 = 1980$, which is **not** greater than 2000 $\rightarrow$ fails.

For $n = 45$: $45 \times 46 = 2070 > 2000 \rightarrow$ succeeds.

1 mark

METHOD 2 — Solve the quadratic boundary

Solve $n^2 + n - 2000 = 0$:

$$n = \frac{-1 + \sqrt{1 + 8000}}{2} = \frac{-1 + \sqrt{8001}}{2}$$

Since $89^2 = 7921$ and $90^2 = 8100$, $\sqrt{8001} \approx 89.4$, giving $n \approx \frac{88.4}{2} = 44.2$.

As n must be a natural number **strictly greater** than 44.2, the smallest value is $n = 45$ ✓

Which method to use: Method 1 — it needs only two multiplications and no surds, and Class 9 questions of this type are always set so the answer sits within one or two of an obvious square. Method 2 is more general but drags in an awkward $\sqrt{8001}$ that must then be approximated, adding risk for no gain.

Step 3 — Verify against the original requirement:

$$S_{44} = \frac{1980}{2} = 990, \text{ and } 990 < 1000 \quad \times$$

$$S_{45} = \frac{2070}{2} = 1035, \text{ and } 1035 > 1000 \quad \checkmark$$

Hence the smallest value is $n = 45$, for which the sum is 1035.

1 mark

★ Q10. Which term of the GP: 2, 8, 32, ... is 131072? Write the explicit formula as well as the recursive formula for the n^{th} term.

4 marks

GIVEN: GP: 2, 8, 32, ...

TO FIND: Position of 131072; explicit and recursive rules.

SOLUTION:

Step 1: $a = 2$; $r = \frac{8}{2} = 4$ (check: $\frac{32}{8} = 4$ ✓).

Step 2 — **Explicit formula:** $t_n = 2 \times 4^{n-1}$

1 mark

Step 3 — **Recursive formula:** $t_1 = 2$, $t_n = 4t_{n-1}$ for $n \geq 2$

1 mark

Step 4 — Locate 131072:

$$2 \times 4^{n-1} = 131072$$

$$4^{n-1} = \frac{131072}{2} = 65536$$

1 mark

Step 5 — Write 65536 as a power of 4:

$$4^1 = 4, 4^2 = 16, 4^3 = 64, 4^4 = 256, 4^5 = 1024, 4^6 = 4096, 4^7 = 16384, 4^8 = 65536$$

$$\text{So } n - 1 = 8 \Rightarrow n = 9$$

Hence 131072 is the 9th term of the GP.

1 mark

★ Q11. The sum of the first three terms of a GP is $\frac{13}{12}$ and their product is -1 . Find the common ratio and the terms.

5 marks

GIVEN: Three GP terms with sum $\frac{13}{12}$ and product -1 .

TO FIND: The common ratio and the three terms.

SOLUTION:

Step 1 — Choose a smart symmetric form. When the product of three GP terms is given, take them as

$\frac{a}{r}, a, ar$ — the r then cancels in the product.

1 mark

Step 2 — Use the product condition:

$$\frac{a}{r} \times a \times ar = a^3 = -1$$

$$a = -1$$

1 mark

Step 3 — Use the sum condition with $a = -1$:

$$\frac{-1}{r} + (-1) + (-1)r = \frac{13}{12}$$

$$-\frac{1}{r} - 1 - r = \frac{13}{12}$$

$$-\left(\frac{1}{r} + r\right) = \frac{13}{12} + 1 = \frac{25}{12}$$

$$\frac{1}{r} + r = -\frac{25}{12}$$

1 mark

Step 4 — Clear the fraction by multiplying throughout by $12r$:

$$12 + 12r^2 = -25r$$

$$12r^2 + 25r + 12 = 0$$

Step 5 — Factorise (split the middle term: $25 = 16 + 9$, since $16 \times 9 = 144 = 12 \times 12$):

$$12r^2 + 16r + 9r + 12 = 0$$

$$4r(3r + 4) + 3(3r + 4) = 0$$

$$(3r + 4)(4r + 3) = 0$$

$$r = -\frac{4}{3} \quad \text{or} \quad r = -\frac{3}{4}$$

1 mark

Step 6 — Find the terms. Take $r = -\frac{3}{4}$ with $a = -1$:

$$\frac{a}{r} = \frac{-1}{-3/4} = \frac{4}{3}, \quad a = -1, \quad ar = (-1)\left(-\frac{3}{4}\right) = \frac{3}{4}$$

Terms: $\frac{4}{3}, -1, \frac{3}{4}$

$$\text{Check — sum: } \frac{4}{3} - 1 + \frac{3}{4} = \frac{16 - 12 + 9}{12} = \frac{13}{12} \quad \checkmark \quad \text{product: } \frac{4}{3} \times (-1) \times \frac{3}{4} = -1 \quad \checkmark$$

Taking $r = -\frac{4}{3}$ simply reverses the same three numbers: $\frac{3}{4}, -1, \frac{4}{3}$.

Hence the common ratio is $-\frac{3}{4}$ (or $-\frac{4}{3}$ for the reversed order), and the terms are $\frac{4}{3}, -1, \frac{3}{4}$.

1 mark

METHOD 2 — Using the ordinary form a, ar, ar^2

Product: $a \cdot ar \cdot ar^2 = a^3 r^3 = (ar)^3 = -1$, so $ar = -1$, i.e. the **middle term is -1** .

$$\text{Sum: } a + ar + ar^2 = \frac{13}{12} \text{, and since } ar = -1 \text{ this gives } a + ar^2 = \frac{13}{12} + 1 = \frac{25}{12}.$$

From $ar = -1$ we get $a = -\frac{1}{r}$, so $-\frac{1}{r} - r = \frac{25}{12}$, leading to the same quadratic $12r^2 + 25r + 12 = 0$ and the same terms.

Which method to use: Method 1 (the symmetric form $\frac{a}{r}, a, ar$) is decidedly easier. The r 's cancel in the product immediately, handing you $a = -1$ in a single line. Method 2 reaches the same place but only after an extra rearrangement. **Rule of thumb: whenever a question gives you the PRODUCT of three terms in GP, always choose $\frac{a}{r}, a, ar$.** (The parallel rule for three terms in AP is to take them as $a - d, a, a + d$ when their SUM is given.)

★ Q12. If the 4^{th} , 10^{th} and 16^{th} terms of a GP are x, y and z respectively, prove that x, y, z are in GP.

3 marks

GIVEN: In a GP with first term a and common ratio r : $t_4 = x, t_{10} = y, t_{16} = z$.

TO PROVE: x, y, z are themselves in GP.

SOLUTION:

Step 1 — Recall the test: three numbers x, y, z are in GP if and only if $\frac{y}{x} = \frac{z}{y}$, i.e. $y^2 = xz$.

1 mark

Step 2 — Write each term using $t_n = ar^{n-1}$:

$$x = t_4 = ar^3$$

$$y = t_{10} = ar^9$$

$$z = t_{16} = ar^{15}$$

1 mark

Step 3 — Compute the two sides separately.

$$\text{LHS: } y^2 = (ar^9)^2 = a^2r^{18}$$

$$\text{RHS: } xz = (ar^3)(ar^{15}) = a^2r^{3+15} = a^2r^{18}$$

Step 4 — Compare:

$$y^2 = a^2r^{18} = xz$$

Since $y^2 = xz$, the numbers x, y, z satisfy the condition for a GP.

Hence x, y, z are in GP, with common ratio r^6 . Q.E.D.

1 mark

Insight: the positions 4, 10, 16 are themselves in AP (common difference 6). Terms of a GP picked at equally spaced positions always form another GP — the new common ratio is r raised to that spacing.

★ Q13. The sum of the first three terms of a geometric progression is 26, and the sum of their squares is 364. Find the terms of the GP.

5 marks

GIVEN: $a + ar + ar^2 = 26$ and $a^2 + a^2r^2 + a^2r^4 = 364$.

TO FIND: The three terms.

SOLUTION:

Step 1 — Write both conditions in factored form:

$$a(1 + r + r^2) = 26 \dots\dots(1)$$

$$a^2(1 + r^2 + r^4) = 364 \dots\dots(2)$$

1 mark

Step 2 — **Key algebraic identity.** Notice that

$$(1 + r + r^2)(1 - r + r^2) = 1 + r^2 + r^4$$

So equation (2) becomes

$$a(1 + r + r^2) \cdot a(1 - r + r^2) = 364$$

Substituting from (1): $26 \times a(1 - r + r^2) = 364$

$$a(1 - r + r^2) = \frac{364}{26} = 14 \dots\dots(3)$$

1 mark

Step 3 — **Subtract (3) from (1):**

$$a(1 + r + r^2) - a(1 - r + r^2) = 26 - 14$$

$$a(2r) = 12$$

$$ar = 6 \dots\dots(4)$$

Add (1) and (3):

$$a(2 + 2r^2) = 26 + 14 = 40$$

$$a(1 + r^2) = 20, \text{ i.e. } a + ar^2 = 20 \dots\dots(5)$$

1 mark

Step 4 — From (4), $a = \frac{6}{r}$. Substitute into (5):

$$\frac{6}{r} + 6r = 20$$

Multiply throughout by r : $6 + 6r^2 = 20r$

$$6r^2 - 20r + 6 = 0$$

$$\text{Divide by 2: } 3r^2 - 10r + 3 = 0$$

1 mark

Step 5 — Factorise (split $-10r = -9r - r$):

$$3r^2 - 9r - r + 3 = 0$$

$$3r(r - 3) - 1(r - 3) = 0$$

$$(r - 3)(3r - 1) = 0$$

$$r = 3 \quad \text{or} \quad r = \frac{1}{3}$$

Step 6 — Find the terms. For $r = 3$: from (4), $a = \frac{6}{3} = 2$, giving 2, 6, 18.

For $r = \frac{1}{3}$: $a = 18$, giving 18, 6, 2 — the same three numbers in reverse order.

$$\text{Check: } 2 + 6 + 18 = 26 \quad \checkmark \quad 4 + 36 + 324 = 364 \quad \checkmark$$

METHOD 2 — Brute substitution (not recommended)

From (1), $a = \frac{26}{1+r+r^2}$. Substituting this into (2) gives

$$\frac{676(1+r^2+r^4)}{(1+r+r^2)^2} = 364, \text{ which expands to a quartic in } r \text{ before it simplifies.}$$

It does eventually reduce to $3r^2 - 10r + 3 = 0$, but only after heavy algebra.

Which method to use: Method 1, without question. Spotting the identity

$(1+r+r^2)(1-r+r^2) = 1+r^2+r^4$ converts the second condition into a simple division $364 \div 26 = 14$, and then adding and subtracting the two equations delivers $ar = 6$ and $a(1+r^2) = 20$ in two lines. Method

2 is mathematically valid but realistically un-finishable under exam time pressure. **Memorise that identity**

— it is the entire key to this question.

★ Q14. Suppose $P_1 = 1$, $P_2 = 2$ and for $n \geq 2$, $P_n = P_1 + P_2 + \dots + P_{n-1} + 1$. Find $P_1, P_2, \dots, P_8$. Can you find a simpler recursive formula for P_n ? Can you give an explicit formula?

5 marks

GIVEN: $P_1 = 1$, $P_2 = 2$, and each later term is "the sum of all previous terms, plus 1".

TO FIND: P_1 to P_8 ; a simpler recursive rule; an explicit rule.

SOLUTION:

Step 1 — Compute term by term:

$$P_1 = 1$$

$$P_2 = 2$$

$$P_3 = P_1 + P_2 + 1 = 1 + 2 + 1 = 4$$

$$P_4 = P_1 + P_2 + P_3 + 1 = 1 + 2 + 4 + 1 = 8$$

1 mark

$$P_5 = 1 + 2 + 4 + 8 + 1 = 16$$

$$P_6 = 1 + 2 + 4 + 8 + 16 + 1 = 32$$

$$P_7 = 1 + 2 + 4 + 8 + 16 + 32 + 1 = 64$$

$$P_8 = 1 + 2 + 4 + 8 + 16 + 32 + 64 + 1 = 128$$

1 mark

Hence $P_1, \dots, P_8 = 1, 2, 4, 8, 16, 32, 64, 128$.

Step 2 — **Simpler recursive formula.** From the list, each term (from P_3 onwards) is exactly **double** the one before. Why? Because

$$P_n = (P_1 + \dots + P_{n-2}) + P_{n-1} + 1 = \underbrace{(P_1 + \dots + P_{n-2} + 1)}_{= P_{n-1}} + P_{n-1} = 2P_{n-1}$$

Simpler recursive rule: $P_1 = 1$, $P_2 = 2$, $P_n = 2P_{n-1}$ for $n \geq 3$.

1.5 marks

Step 3 — **Explicit formula.** From P_2 onwards the sequence is a GP with first term 2 and ratio 2:

$$P_2 = 2^1, P_3 = 2^2, P_4 = 2^3, \dots \text{ — the exponent is always } n - 1.$$

Explicit rule: $P_n = 2^{n-1}$ for $n \geq 1$.

1.5 marks

(Check: $P_1 = 2^0 = 1$ ✓, $P_8 = 2^7 = 128$ ✓ — the formula covers the very first term too.)

★ Q15. Suppose $W_1 = 1$, $W_2 = 2$ and for $n > 2$, $W_n = W_1 + W_2 + \cdots + W_{n-2} + 2$. Find $W_1, W_2, \dots, W_8$. Do you recognise this sequence?

4 marks

GIVEN: $W_1 = 1$, $W_2 = 2$, and each later term is the sum of all terms up to two places before it, plus 2.

TO FIND: W_1 to W_8 , and to identify the sequence.

SOLUTION:

△ **Read the rule carefully:** the sum stops at W_{n-2} , **not** W_{n-1} . This one index is the whole question.

Step 1 — Compute term by term:

$$W_1 = 1$$

$$W_2 = 2$$

$$W_3 = W_1 + 2 = 1 + 2 = 3$$

$$W_4 = W_1 + W_2 + 2 = 1 + 2 + 2 = 5$$

1 mark

$$W_5 = W_1 + W_2 + W_3 + 2 = 1 + 2 + 3 + 2 = 8$$

$$W_6 = W_1 + W_2 + W_3 + W_4 + 2 = 1 + 2 + 3 + 5 + 2 = 13$$

$$W_7 = 1 + 2 + 3 + 5 + 8 + 2 = 21$$

$$W_8 = 1 + 2 + 3 + 5 + 8 + 13 + 2 = 34$$

1 mark

Hence $W_1, \dots, W_8 = 1, 2, 3, 5, 8, 13, 21, 34$.

1 mark

Step 2 — **Recognising the sequence.** Observe that $3 = 1 + 2$, $5 = 2 + 3$, $8 = 3 + 5$, $13 = 5 + 8$, $21 = 8 + 13$, $34 = 13 + 21$ — every term is the sum of the previous two.

This is the Virahāṅka–Fibonacci sequence, satisfying $W_n = W_{n-1} + W_{n-2}$ — the very sequence introduced in Section 8.3 of this chapter, first written down by **Virahāṅka** in the 7th century CE in his work

Vṛttajāṭisamuchaya, in the context of Prakrit metre and poetry.

1 mark

Why the two rules agree: $W_n = (W_1 + \cdots + W_{n-3} + 2) + W_{n-2} = W_{n-1} + W_{n-2}$, since the bracketed part is precisely W_{n-1} .

— End of End-of-Chapter Exercises —

📌 Bonus — Solved In-Text "Exercise" & "Think and Reflect" Items

Why this section matters

The small **Exercise** and **Think and Reflect** boxes scattered through the chapter are **not** optional decoration. School question-setters routinely lift 1-mark and 2-mark questions straight out of them because they are not covered in most guide books. All of them are solved below.

A Sections 8.1 – 8.3 (Sequences, Explicit & Recursive Rules)

A1. (p.176) Consider the sequence 1, 4, 7, 10, 13, ... Predict the next four terms. Also derive the first 10 terms of the sequence obtained by adding all terms up to a given term.

3 marks

SOLUTION:

The difference between consecutive terms is 3, so it is an AP with $a = 1$, $d = 3$, and $t_n = 3n - 2$.

Next four terms: 16, 19, 22, 25.

1 mark

Now the sequence of running totals (partial sums):

1st: 1

2nd: $1 + 4 = 5$

3rd: $1 + 4 + 7 = 12$

4th: $12 + 10 = 22$

5th: $22 + 13 = 35$

6th: $35 + 16 = 51$

7th: $51 + 19 = 70$

8th: $70 + 22 = 92$

9th: $92 + 25 = 117$

10th: $117 + 28 = 145$

First 10 terms of the sum-sequence: 1, 5, 12, 22, 35, 51, 70, 92, 117, 145.

2 marks

(These are the **pentagonal numbers**, given by $\frac{n(3n-1)}{2}$ — worth knowing, though not required.)

A2. (p.176) Write t_5 , t_6 , t_7 and t_8 for the sequence of triangular numbers.

2 marks

SOLUTION: Using $t_n = \frac{n(n+1)}{2}$:

$$t_5 = \frac{5 \times 6}{2} = 15$$

$$t_6 = \frac{6 \times 7}{2} = 21$$

$$t_7 = \frac{7 \times 8}{2} = 28$$

$$t_8 = \frac{8 \times 9}{2} = 36$$

Hence $t_5 = 15$, $t_6 = 21$, $t_7 = 28$, $t_8 = 36$.

2 marks

A3. (p.177) Using $u_n = 2n - 1$, find the 53rd, 108th and 1170th terms of the odd-number sequence.

2 marks

SOLUTION:

$$u_{53} = 2(53) - 1 = 106 - 1 = \mathbf{105}$$

$$u_{108} = 2(108) - 1 = 216 - 1 = \mathbf{215}$$

$$u_{1170} = 2(1170) - 1 = 2340 - 1 = \mathbf{2339}$$

2 marks

This is the whole point of an explicit formula — the 1170th term is obtained without knowing the 1169th.

A4. (p.177) For $s_n = 5n - 2$: write the first 6 terms; find the 100th and 1000th terms. Also, why must n be a natural number?

3 marks

SOLUTION:

First six terms: 3, 8, 13, 18, 23, 28

1 mark

$$s_{100} = 5(100) - 2 = \mathbf{498}$$

$$s_{1000} = 5(1000) - 2 = \mathbf{4998}$$

1 mark

Why n must be a natural number: n represents the **position** of a term in the list — 1st, 2nd, 3rd, and so on. A position cannot be 94.6, or -3 , or $\frac{1}{2}$; there is no "94.6th term". So whenever solving $s_n = k$ produces a non-natural n , the honest conclusion is that k simply **does not appear** in the sequence.

1 mark

A5. (p.178) Consider $t_n = 3n - 7$. (i) Find the 1st, 2nd, 3rd, 12th, 18th and 50th terms. (ii) Which term is 332? (iii) Is 557 a term? Why or why not?

4 marks

SOLUTION:

$$(i) t_1 = 3 - 7 = -4; t_2 = 6 - 7 = -1; t_3 = 9 - 7 = 2;$$

$$t_{12} = 36 - 7 = 29; t_{18} = 54 - 7 = 47; t_{50} = 150 - 7 = \mathbf{143}$$

2 marks

$$(ii) 3n - 7 = 332 \Rightarrow 3n = 339 \Rightarrow n = 113.$$

Hence 332 is the 113th term.

1 mark

$$(iii) 3n - 7 = 557 \Rightarrow 3n = 564 \Rightarrow n = 188.$$

Since 188 is a natural number, **yes — 557 is a term of the sequence; it is the 188th term.**

1 mark

⚠ Note: the phrasing "why or why not" tempts students into assuming the answer must be "no".

Always compute first, then justify whatever you actually find. Here 564 is divisible by 3, so the answer is affirmative.

A6. (p.178, Example 3 follow-up) For $u_1 = 1$, $u_n = 2u_{n-1} + 3$ ($n \geq 2$), is 133 a term of this sequence?

2 marks

SOLUTION: Generate terms until we pass 133:

$$u_1 = 1$$

$$u_2 = 2(1) + 3 = 5$$

$$u_3 = 2(5) + 3 = 13$$

$$u_4 = 2(13) + 3 = 29$$

$$u_5 = 2(29) + 3 = 61$$

$$u_6 = 2(61) + 3 = 125$$

$$u_7 = 2(125) + 3 = 253$$

1 mark

The terms jump from 125 straight to 253, skipping over 133 entirely. Since the sequence is strictly increasing, no later term can equal 133 either.

Hence 133 is NOT a term of this sequence.

1 mark

(In fact $u_n = 2^{n+1} - 3$, and $133 + 3 = 136$ is not a power of 2 — confirming the same conclusion.)

A7. (p.179) Write the next two terms of the Virahāṅka–Fibonacci sequence 1, 2, 3, 5, 8, 13, 21, 34, ...

1 mark

SOLUTION: Each term is the sum of the previous two:

$$\text{Next term} = 21 + 34 = 55$$

$$\text{Following term} = 34 + 55 = 89$$

Hence the next two terms are 55 and 89.

1 mark

B Section 8.4 – 8.5 (Arithmetic Progressions & Sum of Naturals)

B1. (p.182) Verify that these are APs and write their n^{th} terms. What do you observe on plotting the ordered pairs? (i) 2, 5, 8, 11, ... (ii) -5, -1, 3, 7, ...

3 marks

SOLUTION:

(i) Differences: $5 - 2 = 3$, $8 - 5 = 3$, $11 - 8 = 3$ — constant, so it is an AP with $a = 2$, $d = 3$.

$$t_n = 2 + (n - 1)3 = 2 + 3n - 3 = 3n - 1$$

1 mark

(ii) Differences: $-1 - (-5) = 4$, $3 - (-1) = 4$, $7 - 3 = 4$ — constant, so it is an AP with

$$a = -5, d = 4.$$

$$t_n = -5 + (n - 1)4 = -5 + 4n - 4 = 4n - 9$$

1 mark

Observation on plotting: the ordered pairs (n, t_n) — that is, $(1, 2)$, $(2, 5)$, $(3, 8)$, $(4, 11)$ and $(1, -5)$, $(2, -1)$, $(3, 3)$, $(4, 7)$ — lie on a straight line in each case. This happens for every AP, because $t_n = dn + (a - d)$ is a linear expression in n , and the slope of the line equals the common difference d .

1 mark

B2. (p.182) Using $t_n = a + (n - 1)d$, find the n^{th} term of: (i) $\frac{1}{2}, \frac{5}{2}, \frac{9}{2}, \frac{13}{2}, \dots$ (ii) 1.5, 3.5, 5.5, 7.5, ... Also write the recursive rules.

4 marks

SOLUTION:

(i) $a = \frac{1}{2}; d = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$

$$t_n = \frac{1}{2} + (n - 1)(2) = \frac{1}{2} + 2n - 2 = 2n - \frac{3}{2}$$

Writing over a common denominator: $t_n = \frac{4n - 3}{2}$

1 mark

(Check: $t_1 = \frac{1}{2} \checkmark$, $t_4 = \frac{13}{2} \checkmark$)

Recursive rule: $t_1 = \frac{1}{2}$, $t_n = t_{n-1} + 2$ for $n \geq 2$.

1 mark

(ii) $a = 1.5; d = 3.5 - 1.5 = 2$

$$t_n = 1.5 + (n - 1)(2) = 1.5 + 2n - 2 = 2n - 0.5$$

1 mark

(Check: $t_1 = 1.5 \checkmark$, $t_4 = 7.5 \checkmark$)

Recursive rule: $t_1 = 1.5$, $t_n = t_{n-1} + 2$ for $n \geq 2$.

1 mark

B3. (p.185) Use $S_n = \frac{n(n+1)}{2}$ to find S_{20} , S_{50} and S_{1000} .

3 marks

SOLUTION:

$$S_{20} = \frac{20 \times 21}{2} = \frac{420}{2} = 210$$

1 mark

$$S_{50} = \frac{50 \times 51}{2} = \frac{2550}{2} = 1275$$

1 mark

$$S_{1000} = \frac{1000 \times 1001}{2} = \frac{1001000}{2} = 500500$$

1 mark

B4. (p.185) Using $t_n = \frac{n(n+1)}{2}$, find the 10^{th} , 17^{th} and 80^{th} triangular numbers.

3 marks

SOLUTION:

$$t_{10} = \frac{10 \times 11}{2} = 55$$

$$t_{17} = \frac{17 \times 18}{2} = \frac{306}{2} = 153$$

$$t_{80} = \frac{80 \times 81}{2} = \frac{6480}{2} = 3240$$

3 marks

Remember: the n^{th} triangular number and the sum of the first n natural numbers are the same quantity — one formula serves both.

B5. (p.183, Example 5 follow-up) A taxi charges a fixed booking fee of ₹200 plus ₹40 per km. If the person travels 10 km, what is the total fare?

2 marks

SOLUTION:

Step 1: The fares 240, 280, 320, ... form an AP with $a = 240$, $d = 40$, and $t_n = 200 + 40n$ where n is the distance in km.

1 mark

Step 2: For $n = 10$:

$$t_{10} = 200 + 40(10) = 200 + 400 = 600, \text{ i.e. ₹600}$$

1 mark

Hence the total fare for 10 km is ₹600.

C Section 8.6 (Geometric Progressions & Fractals)

C1. (p.187, Examples 6–8) Are these GPs? If so, state the common ratio. (i) 1, 2, 4, 8, 16, ... (ii) 1, 3, 9, 27, 81, ... (iii) 1, -1, 1, -1, 1, ...

3 marks

SOLUTION: Divide consecutive terms in each case.

$$(i) \frac{2}{1} = 2, \frac{4}{2} = 2, \frac{8}{4} = 2, \frac{16}{8} = 2 \text{ — constant.}$$

Yes, a GP with $r = 2$; $t_n = 2^{n-1}$.

1 mark

$$(ii) \frac{3}{1} = 3, \frac{9}{3} = 3, \frac{27}{9} = 3, \frac{81}{27} = 3 \text{ — constant.}$$

Yes, a GP with $r = 3$; $t_n = 3^{n-1}$.

1 mark

$$(iii) \frac{-1}{1} = -1, \frac{1}{-1} = -1, \frac{-1}{1} = -1 \text{ — constant.}$$

Yes, a GP with $r = -1$; $t_n = (-1)^{n-1}$. The terms simply alternate in sign without growing — a perfectly valid GP.

1 mark

C2. (p.188) Check whether these are GPs and find their n^{th} terms. (i) 2, 10, 50, 250, ... (ii)

4, $\frac{8}{3}$, $\frac{16}{9}$, $\frac{32}{27}$, ... (iii) 3, $\frac{-3}{2}$, $\frac{3}{4}$, $\frac{-3}{8}$, ...

3 marks

SOLUTION:

(i) $\frac{10}{2} = 5$, $\frac{50}{10} = 5$, $\frac{250}{50} = 5$ — constant.

GP with $a = 2$, $r = 5$. $t_n = 2 \times 5^{n-1}$

1 mark

(ii) $\frac{8/3}{4} = \frac{8}{3} \times \frac{1}{4} = \frac{2}{3}$; $\frac{16/9}{8/3} = \frac{16}{9} \times \frac{3}{8} = \frac{2}{3}$; $\frac{32/27}{16/9} = \frac{32}{27} \times \frac{9}{16} = \frac{2}{3}$ — constant.

GP with $a = 4$, $r = \frac{2}{3}$. $t_n = 4 \left(\frac{2}{3}\right)^{n-1}$

1 mark

(iii) $\frac{-3/2}{3} = -\frac{1}{2}$; $\frac{3/4}{-3/2} = \frac{3}{4} \times \left(-\frac{2}{3}\right) = -\frac{1}{2}$; $\frac{-3/8}{3/4} = -\frac{1}{2}$ — constant.

GP with $a = 3$, $r = -\frac{1}{2}$. $t_n = 3 \left(-\frac{1}{2}\right)^{n-1}$

1 mark

All three are geometric progressions. Note that in (ii) and (iii) the terms are shrinking because

$|r| < 1$ — a GP does not have to grow.

C3. (p.188) Find a recursive rule for the formula $t_n = 3 \times 10^{n-1}$, which generates 3, 30, 300, 3000, ...

2 marks

SOLUTION:

Step 1: Here $a = 3$ and $r = 10$, since each term is 10 times the previous one.

1 mark

Step 2: Therefore

$t_1 = 3$, $t_n = 10 t_{n-1}$ for $n \geq 2$

1 mark

(Check: $t_2 = 10 \times 3 = 30$ ✓, $t_3 = 10 \times 30 = 300$ ✓)

C4. (p.189) For the Sierpiński triangle: (a) how many black triangles in Stages 0–3? (b) predict Stages 4 and 5. (c) find a rule for the n^{th} stage. (d) if the Stage-0 black area is 1 sq unit, find the black area at Stages 1–5, a rule for stage n , and describe what happens as n increases.

5 marks

SOLUTION:

(a) At each stage every black triangle is replaced by **three** smaller black triangles:

Stage 0: 1; Stage 1: 3; Stage 2: 9; Stage 3: 27

1 mark

(b) Stage 4: $27 \times 3 = 81$; Stage 5: $81 \times 3 = 243$

1 mark

(c) The counts 1, 3, 9, 27, 81, 243 are all powers of 3: $3^0, 3^1, 3^2, 3^3, 3^4, 3^5$, and the exponent matches the stage number.

Explicit rule: $t_n = 3^n$. **Recursive rule:** $t_0 = 1$, $t_n = 3t_{n-1}$ for $n \geq 1$.

1 mark

(d) At each stage the triangle is divided into 4 equal parts and the central one removed, so $\frac{3}{4}$ of the area survives:

Stage 1: $\frac{3}{4}$ Stage 2: $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$ Stage 3: $\left(\frac{3}{4}\right)^3 = \frac{27}{64}$

Stage 4: $\left(\frac{3}{4}\right)^4 = \frac{81}{256}$ Stage 5: $\left(\frac{3}{4}\right)^5 = \frac{243}{1024}$

1 mark

Explicit rule: $s_n = \left(\frac{3}{4}\right)^n$. **Recursive rule:** $s_0 = 1$, $s_n = \frac{3}{4}s_{n-1}$ for $n \geq 1$.

As n increases: since $\frac{3}{4} < 1$, the black area keeps shrinking and approaches 0, while the number of black triangles 3^n grows without bound.

1 mark

C5. (p.186, 8.6.2) Why do the plotted points of a GP not lie on a straight line, unlike those of an AP? Explain with the pattern 3, 6, 12, 24, 48.

2 marks

SOLUTION:

For an **AP**, moving one step to the right always **adds** the same amount d — a constant rise for a constant run, which is exactly the definition of a straight line.

1 mark

For a **GP**, moving one step right **multiplies** by r , so the rise itself keeps changing. For 3, 6, 12, 24, 48 the successive jumps are 3, 6, 12, 24 — increasing every time. Plotting

(1, 3), (2, 6), (3, 12), (4, 24), (5, 48) therefore gives a curve that gets steeper and steeper, not a line.

1 mark

Exam use: "the points lie on a straight line" is sufficient justification that a sequence is an AP; "the points lie on a curve rising ever more steeply" indicates a GP with $r > 1$; "a curve falling towards 0" indicates a GP with $0 < r < 1$.

Master Table — Questions Solvable by More Than One Method

At a glance: where alternatives exist, and which one to actually use

★ **Marks are awarded for any correct method.** The "easiest" column is guidance on which route has the fewest fractions, fewest steps, and fewest chances to slip — not a rule about what is "allowed". If you can execute your preferred method cleanly, use it.

Question	Method 1	Method 2 (and 3)	Easiest / Recommended
Ex 8.1 Q5	Convert recursive → explicit rule, then solve	Count steps: $(\text{rise} \div d) + 1$ · M3: list all terms	Method 1 — transfers to any follow-up question
Ex 8.2 Q1	Direct formula $a + (n - 1)d$	Jump from a known term · M3: simplify to $5n - 2$ first	Method 1 (M3 if 3+ terms wanted)
Ex 8.2 Q2(b)	Set $t_n = 0$ and solve	Count down the terms by hand	Method 1 — listing only as a check
Ex 8.2 Q4	Two equations, subtract to find d , then a	Gap formula $d = \frac{t_q - t_p}{q - p}$, step across — no a needed	Method 2 — skips a entirely
Ex 8.2 Q5 (sum)	Take 3 common, then $S_{33} - S_3$	Āryabhata pairing: average × number of terms	Method 2 — one multiply, one halve
Ex 8.2 Q6	AP n^{th} -term formula, then convert $n \rightarrow n - 1$	Total rise ÷ yearly rise	Method 2 — avoids the n vs $n - 1$ trap
Ex 8.2 Q7	$S_n = \frac{n(n+1)}{2}$	Pairing from both ends	Method 1 — single substitution
Ex 8.3 Q1	Step forward: $t_{12} = t_8 \times r^4$	Find a first, then ar^{11}	Method 1 — no fractions, no large powers
Ex 8.3 Q4	Match powers of the same base	Divide repeatedly by r and count	Method 1 — exact, scales to huge numbers

Ex 8.3 Q6	Write $\sqrt{2} = 2^{1/2}$, compare exponents	Spot that whole-number terms sit at odd positions	Method 1 — general and expected
EOC Q1	Two equations, solve for a and d	Gap formula, then step from t_{16}	Method 2 — $a = -32$ is a sign-error risk
EOC Q3	AP formula on 105 ... 994	Count multiples: $142 - 14 \cdot M3$: $\frac{\text{last} - \text{first}}{d} + 1$	M3 fastest; M1 safest for full marks
EOC Q4	AP formula on 12 ... 248	Count the multipliers: $62 - 3 + 1$	Method 2 — quicker for small divisors
EOC Q6	$200 = k(2p + k - 1)$ with the parity rule	Trial by run-length k , plus a stopping argument	Find with M2, prove completeness with M1
EOC Q8	Subtract the two equations	Middle-term property $t_4 + t_8 = 2t_6$	Method 1 — always applies; M2 is situational
EOC Q9	Estimate via squares, test $n = 44, 45$	Solve $n^2 + n - 2000 = 0$ with the quadratic formula	Method 1 — avoids $\sqrt{8001}$
EOC Q11	Symmetric form $\frac{a}{r}, a, ar$	Ordinary form a, ar, ar^2	Method 1 — product given $\Rightarrow$ always symmetric form
EOC Q13	Identity $(1 + r + r^2)(1 - r + r^2) = 1 + r^2 + r^4$	Substitute a from (1) into (2) $\rightarrow$ quartic	Method 1 — M2 is un-finishable in exam time

Two shortcuts worth memorising from this table: (1) if three terms in **GP** have their **product** given, take them as $\frac{a}{r}, a, ar$; (2) if three terms in **AP** have their **sum** given, take them as $a - d, a, a + d$. In both cases one unknown cancels immediately.

Quick Revision Checklist — The Night Before

#	Can you do this without looking?	Where it was covered
1	State $t_n = a + (n - 1)d$ and $t_n = ar^{n-1}$ correctly, with $(n - 1)$ not n	Formula panel
2	Decide AP vs GP by testing differences, then ratios	Tip 1 · C1 · C2
3	Answer "is X a term?" and justify with the natural-number condition	Ex 8.1 Q3 · A4 · A5
4	Solve a two-condition AP by subtracting equations	Ex 8.2 Q4 · EOC Q1 · Q8
5	Count "how many multiples/2-digit/3-digit..." without dropping the +1	Ex 8.2 Q5 · EOC Q3 · Q4
6	Use $S_n = \frac{n(n+1)}{2}$ and $S_q - S_{p-1}$ for a block of consecutive numbers	Ex 8.2 Q5 · Q7 · B3
7	Write a complete recursive rule (seed + rule + range)	Ex 8.2 Q3 · Ex 8.3 Q4 · C3
8	Match powers of the same base to locate a GP term	Ex 8.3 Q4 · Q6 · EOC Q10
9	Remember that $r^2 = k$ gives two progressions	EOC Q5
10	Use the symmetric form $\frac{a}{r}, a, ar$ when a product is given	EOC Q11
11	Prove three terms are in GP via $y^2 = xz$	EOC Q12
12	Build a stage-table for a fractal, then read off $3^n, 8^n, \left(\frac{3}{4}\right)^n, \left(\frac{8}{9}\right)^n$	Ex 8.3 Q7 · C4
13	Recall the Virahāṅka–Fibonacci rule and its 7th-century Indian origin	Formula panel · EOC Q15
14	Explain the straight-line (AP) vs curve (GP) graph test	B1 · C5
15	For each multi-method question, know which route you will actually write	Master Table above

End of Compiled Volume

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