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Class 9 — Mathematics
Study Notes · Formula Sheets · Expected Questions

Companion volume to *MLI Class 9 Maths — Ganita Manjari Part I, NCERT Solutions (Compiled)*
Based on *Ganita Manjari, Part I*, NCERT 2026 · CBSE Subject Code 041

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What this volume is. The main MLI Maths volume solves every NCERT exercise of *Ganita Manjari Part I*. This companion adds the two things a solutions book cannot give you: **Part A — concept notes and a formula sheet** for each chapter, and **Part C — expected questions** of the kind actually set in school half-yearly and annual papers, each solved step-wise with the marks shown. Used together, the two volumes are meant to remove the need for any second guide or market notes.

How answers are written. Every solution is **step-wise**, with the formula stated before it is used, the substitution shown, and the final answer boxed in the answer strip. Answer length is matched to the marks: 1 mark = the answer alone; 2 marks = formula + one substitution step; 3 marks = formula + working + conclusion; 5 marks = full construction/derivation with every step justified. Diagrams are redrawn schematics — correct in construction, not to scale.

A note on accuracy. Parts of this material have been organised with the help of AI-based tools and thereafter checked against the NCERT text and the CBSE curriculum. An occasional slip may still remain — treat the written working as the reference and bring anything doubtful to your teacher.

⚠ This volume covers Part I only — it is NOT the whole examinable syllabus

NCERT has so far released only **Ganita Manjari Part I (8 chapters)**. The official CBSE Class IX Mathematics curriculum for 2026–27 is built on **6 units / 15 chapters**, so these 8 chapters cover **only about half of the 80-mark annual paper**. Still fully examinable but awaiting Part II: **Linear Equations in Two Variables**; **Euclid's axioms and postulates, Lines & Angles, Triangles (congruence), Quadrilaterals**; **Surface Areas and Volumes**; **Statistics**. MLI will issue a Part II companion as soon as NCERT releases the textbook.

Revised unit weightage, 2026–27 (total 80): Number System 7 · Algebra 20 · Coordinate Geometry 4 · Geometry 25 · Mensuration 14 · Statistics & Probability 10. Internal Assessment 20 (Pen-Paper Test & Multiple Assessment 10, Portfolio 5, Lab Practical 5). The older 10/20/4/27/13/6 split no longer applies.

✓ How to use this document — and what it does not replace

This volume is written so that you do **not** need a second guide or market notes for the chapters it covers. Four honest points, so nothing surprises you later:

- **This volume covers *Ganita Manjari Part I* only.** NCERT has not yet released Part II, so Linear Equations in Two Variables, Euclid's axioms, Lines & Angles, Triangles, Quadrilaterals, Surface Areas and Volumes, and Statistics are **not here** — yet they are fully examinable. See the red banner above. MLI will issue a Part II companion as soon as the book is out.
- **Keep your NCERT *Ganita Manjari* open beside this.** Every figure here is an **MLI-redrawn schematic** — correct in construction but **not drawn to scale**, and not a copy of any textbook figure. Each one carries a note saying so. Never measure a length or an angle off these diagrams; always work from the given data.
- **Constructions are meant to be drawn by hand.** Where a question asks for a construction or a proof, draw the figure yourself on plain paper — a proof submitted without a figure loses a mark, and copying a printed figure teaches you nothing about the construction.
- **If anything here disagrees with your printed NCERT book, the book wins.** Tell your faculty so this volume can be corrected. Parts of this material were organised with AI assistance and then checked; every numerical answer has been worked through, but an occasional slip may still remain.

📜 Contents

Every chapter also carries a Challenge Set — 3 HOTS questions of 3–5 marks each, solved, at above-average level. 25 questions, 116 marks in all.

CHAPTER 1

Orienting Yourself: The Use of Coordinates

Concept Notes · Formula Sheet · Expected Questions

PART A — CONCEPT NOTES

📍 1.1 The Cartesian system

- Two perpendicular number lines — the horizontal **x-axis** and the vertical **y-axis** — meet at the **origin O(0, 0)**.
- A point is written as an **ordered pair (x, y)**: x is the **abscissa** (distance from the y-axis), y is the **ordinate** (distance from the x-axis). Order matters — (3, 5) and (5, 3) are different points.
- The axes divide the plane into four **quadrants**, numbered anticlockwise from the upper right.

Quadrant	Sign of x	Sign of y	Example
I	+	+	(4, 3)
II	−	+	(−4, 3)
III	−	−	(−4, −3)
IV	+	−	(4, −3)

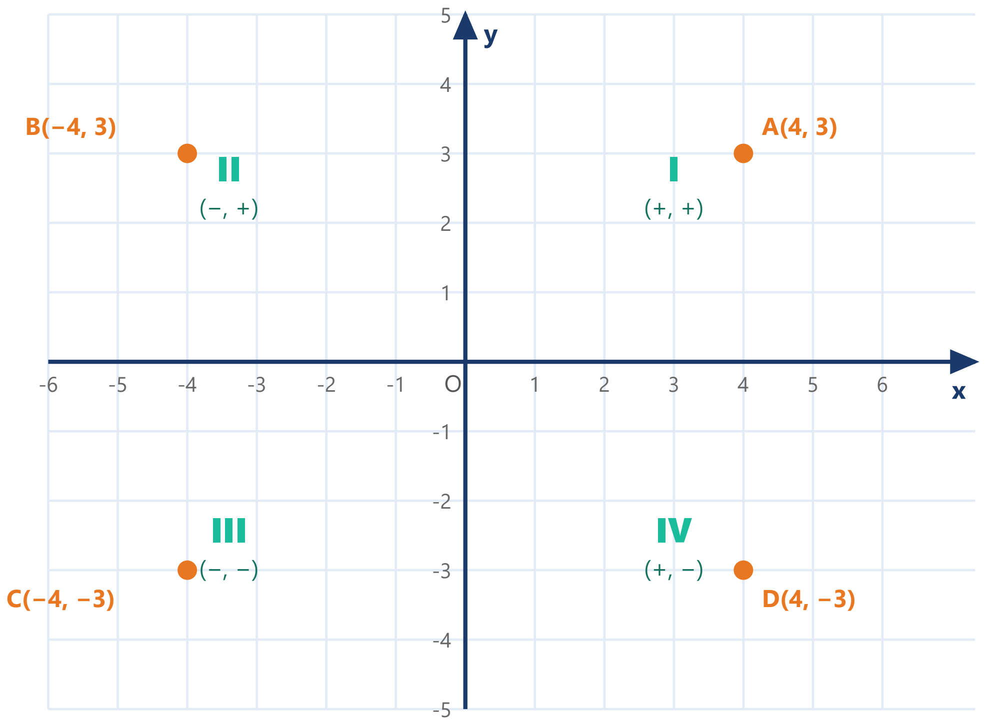


Fig. 1.1 — The four quadrants, their sign conventions, and one point plotted in each

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. If any part of this figure is unclear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.*

Points on the axes are in no quadrant. Any point on the x-axis has the form (a, 0); any point on the y-axis has the form (0, b). This single line is worth a full mark almost every year.

📏 1.2 Distance and midpoint

Distance formula. $PQ = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$ — it is simply Pythagoras' theorem applied to the horizontal and vertical gaps between the two points.

Distance from the origin. $OP = \sqrt{(x^2 + y^2)}$

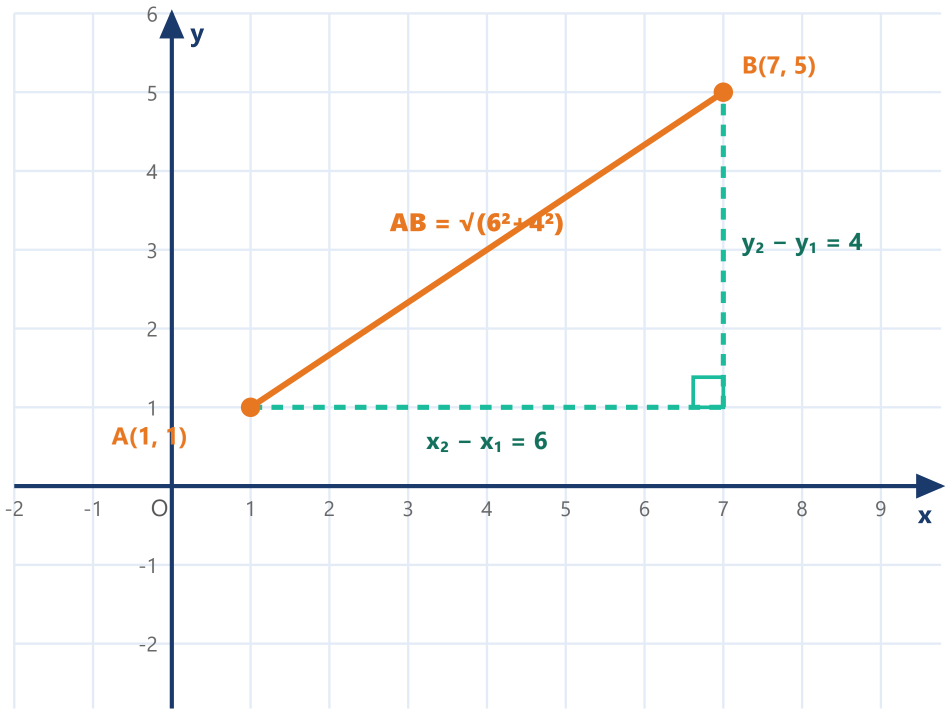
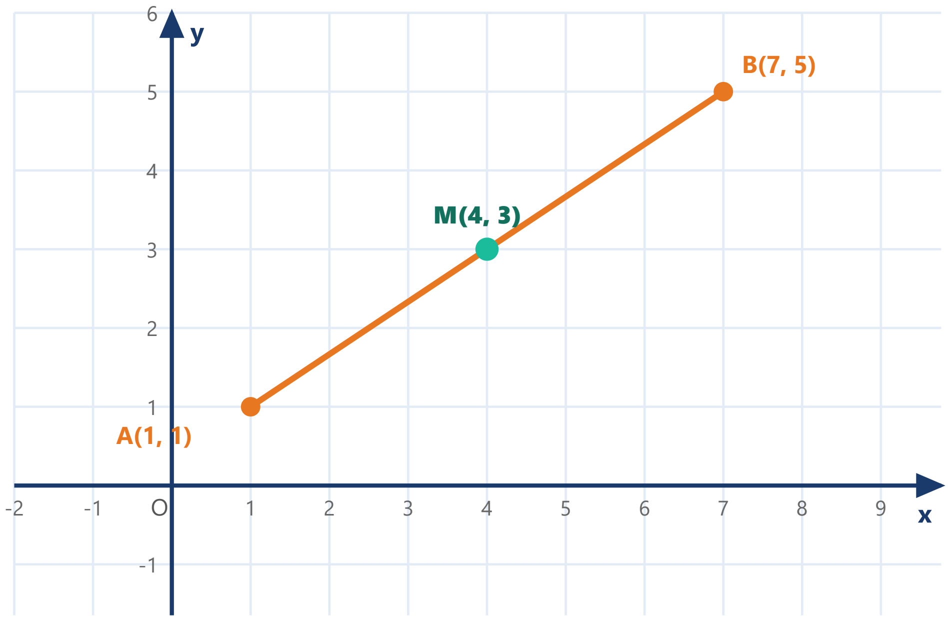


Fig. 1.2 — The distance formula is Pythagoras: the horizontal gap and the vertical gap are the two legs

Note: *MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is unclear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.*

Midpoint formula. $M = ((x_1+x_2)/2 , (y_1+y_2)/2)$ — the average of the two x's and the average of the two y's.



M = ((1+7)/2 , (1+5)/2) = (4, 3) — the average of the x's and the average of the y's

Fig. 1.3 — The midpoint of a segment

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. If any part of this figure is unclear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.*

What each is used to prove

To show that...	Use...
Three points are **collinear**	AB + BC = AC (the two shorter distances add to the longest)
A triangle is **isosceles**	Two of the three sides are equal
A triangle is **equilateral**	All three sides are equal
A triangle is **right-angled**	The Pythagorean relation holds for the three squared sides
A quadrilateral is a **parallelogram**	The diagonals have the same midpoint (they bisect each other)
A quadrilateral is a **rhombus**	All four sides equal but the diagonals are unequal
A quadrilateral is a **square**	All four sides equal *and* both diagonals equal

⚠ 1.3 Where marks are lost

- Subtracting in a mixed order — write $(x_2 - x_1)$ and $(y_2 - y_1)$ using the **same** point as "point 1" throughout.
- Forgetting that a squared difference is always positive, and then dropping a minus sign into the final root.
- Leaving $\sqrt{50}$ instead of simplifying to $5\sqrt{2}$ — the surd must be reduced to lowest form.
- In a "find the fourth vertex" question, using the distance formula instead of the far quicker **equal-midpoints-of-diagonals** property.

> **Weightage.** Coordinate Geometry is Unit III — **4 marks of 80**, 6 teaching periods, the smallest unit of the year. Because the skill set is tiny (plot, distance, midpoint), it is the most efficient unit in the whole syllabus to score full marks in.

PART C — EXPECTED QUESTIONS (SOLVED)

E1 — 1 Mark

The point $(-7, 0)$ lies

(i) in quadrant II (ii) in quadrant III (iii) on the x-axis (iv) on the y-axis

(iii) on the x-axis

E2 — 1 Mark

The distance of the point $P(-6, 8)$ from the origin is

(i) 2 (ii) 10 (iii) 14 (iv) $\sqrt{2}$

(ii) 10

WORKING $OP = \sqrt{(-6)^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$.

E3 — 1 Mark

If the midpoint of AB is the origin and A is $(5, -3)$, then B is

(i) $(5, -3)$ (ii) $(-5, 3)$ (iii) $(-3, 5)$ (iv) $(0, 0)$

(ii) $(-5, 3)$

E4 **1 Mark**

Assertion (A): The points (2, 3) and (3, 2) represent the same location.

Reason (R): (x, y) is an ordered pair.

(i) Both true, R explains A (ii) Both true, R does not explain A (iii) A false, R true (iv) A true, R false

(iii) A is false, R is true

REASON Because the pair is *ordered*, interchanging the coordinates gives a different point — so R is true and it is exactly what makes A false.

E5 **2 Marks**

Find the distance between A(3, −2) and B(−1, 1).

Step 1 B $= \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]} = \sqrt{[(-1 - 3)^2 + (1 - (-2))^2]}$

Step 2 $\sqrt{[(-4)^2 + 3^2]} = \sqrt{(16 + 9)} = \sqrt{25}$

AB = 5 units

E6 **2 Marks**

The midpoint of the segment joining (a, 4) and (6, b) is (5, 7). Find a and b.

Step 1 coordinate: $(a + 6)/2 = 5 \Rightarrow a + 6 = 10 \Rightarrow a = 4$

Step 2 coordinate: $(4 + b)/2 = 7 \Rightarrow 4 + b = 14 \Rightarrow b = 10$

a = 4, b = 10

E7 **3 Marks**

Show that the points A(1, 7), B(4, 2) and C(−1, −1) are the vertices of an isosceles triangle.

Step 1 B $= \sqrt{[(4-1)^2 + (2-7)^2]} = \sqrt{(9 + 25)} = \sqrt{34}$

Step 2 C $= \sqrt{[(-1-4)^2 + (-1-2)^2]} = \sqrt{(25 + 9)} = \sqrt{34}$

Step 3 A $= \sqrt{[(1-(-1))^2 + (7-(-1))^2]} = \sqrt{(4 + 64)} = \sqrt{68}$

AB = BC = √34 while CA = √68, so exactly two sides are equal — △ABC is isosceles.

E8 **3 Marks**

Find the value of k for which the distance between A(−4, 1) and B(k, 6) is 13 units.

Step 1 $B^2 = (k - (-4))^2 + (6 - 1)^2 = (k + 4)^2 + 25$

Step 2 iven $AB = 13$, so $(k + 4)^2 + 25 = 169 \Rightarrow (k + 4)^2 = 144$

Step 3 $+ 4 = \pm 12 \Rightarrow k = 8$ or $k = -16$

$k = 8$ or $k = -16$ (both values are valid — do not discard one)

E9 **3 Marks**

Prove that A(1, 2), B(4, 3) and C(7, 4) are collinear.

Step 1 $B = \sqrt{[(4-1)^2 + (3-2)^2]} = \sqrt{(9+1)} = \sqrt{10}$

Step 2 $C = \sqrt{[(7-4)^2 + (4-3)^2]} = \sqrt{(9+1)} = \sqrt{10}$

Step 3 $C = \sqrt{[(7-1)^2 + (4-2)^2]} = \sqrt{(36+4)} = \sqrt{40} = 2\sqrt{10}$

$AB + BC = \sqrt{10} + \sqrt{10} = 2\sqrt{10} = AC$. Since the two shorter distances add exactly to the longest, B lies on AC and the points are collinear.

E10 **5 Marks**

Three vertices of a parallelogram ABCD are A(1, 2), B(4, 3) and C(6, 6). Find the fourth vertex D, and verify your answer by showing that AB = DC.

KEY IDEA In a parallelogram the diagonals bisect each other, so **midpoint of AC = midpoint of BD**. This is far quicker than using the distance formula four times.

Step 1 Midpoint of AC = $((1+6)/2, (2+6)/2) = (7/2, 4)$

Step 2 Let D = (x, y). Midpoint of BD = $((4+x)/2, (3+y)/2)$

Step 3 Equate x-parts: $(4 + x)/2 = 7/2 \Rightarrow 4 + x = 7 \Rightarrow x = 3$

Step 4 Equate y-parts: $(3 + y)/2 = 4 \Rightarrow 3 + y = 8 \Rightarrow y = 5$

Step 5 Check: $AB = \sqrt{[(4-1)^2+(3-2)^2]} = \sqrt{10}$ and $DC = \sqrt{[(6-3)^2+(6-5)^2]} = \sqrt{10}$. Equal, as required.

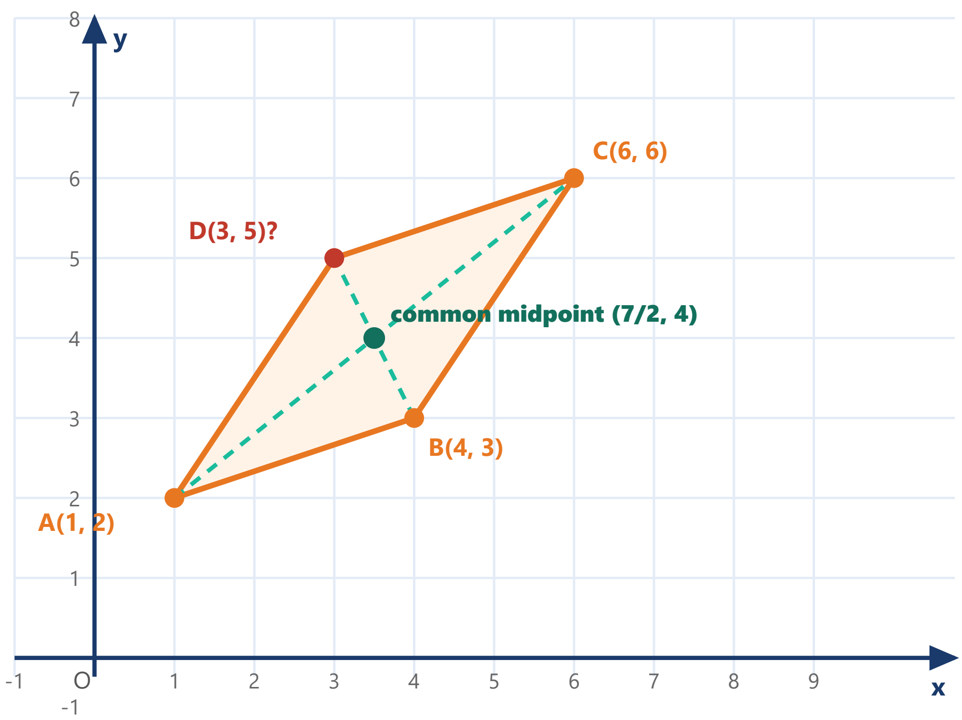


Fig. 1.4 — Finding the fourth vertex: the diagonals of a parallelogram share the same midpoint

Note: *MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is unclear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.*

D = (3, 5)

E11 **5 Marks**

Case study. A school ground is mapped on graph paper with the flagpole at the origin. The gate is at G(−6, 0), the library at L(0, 8), the canteen at C(−6, 8) and a bench at B, the midpoint of GL.

(a) Name the quadrant in which C lies. (1)

(b) Find the coordinates of the bench B. (1)

(c) Find the distance the peon walks from the gate G to the library L. (2)

(d) Is GLC a right-angled triangle? Justify. (1)

Step 1) C(−6, 8) has x negative and y positive — **Quadrant II**.

Step 2) B = ((−6+0)/2, (0+8)/2) = **(−3, 4)**.

Step 3) GL = $\sqrt{[(0-(-6))^2 + (8-0)^2]} = \sqrt{(36 + 64)} = \sqrt{100}$ = **10 units**.

Step 4) GC = $\sqrt{[0^2 + 8^2]} = 8$ and CL = $\sqrt{[6^2 + 0^2]} = 6$. Then $GC^2 + CL^2 = 64 + 36 = 100 = GL^2$.

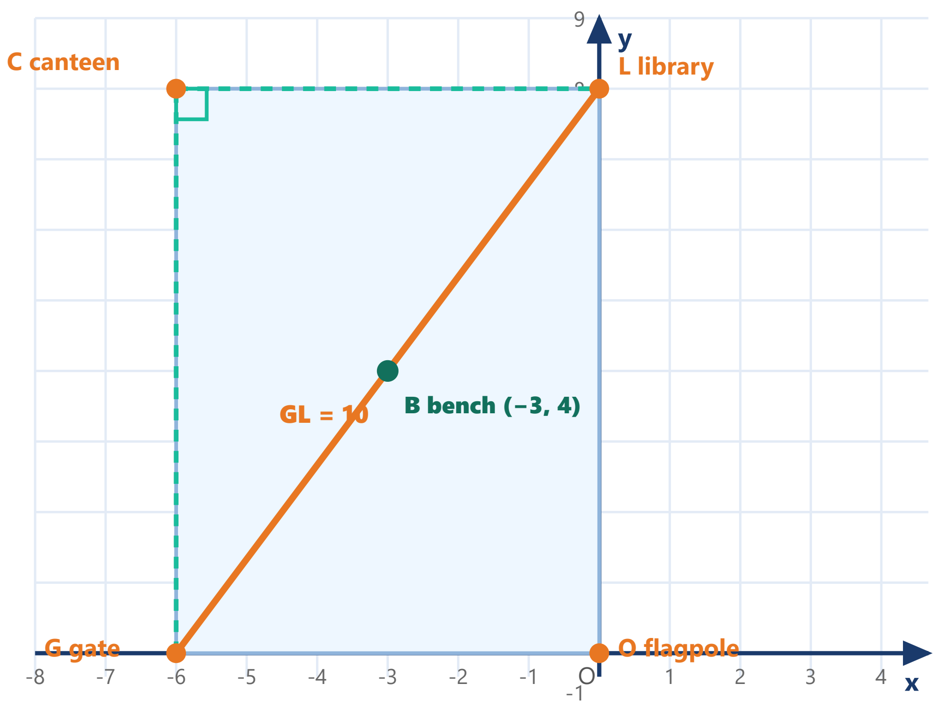


Fig. 1.5 — The school ground mapped on a grid (case study E11)

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Yes — the Pythagorean relation holds, so △GLC is right-angled at C.

CHALLENGE SET — ABOVE-AVERAGE / HOTS

Read this before you start. Every question below is **3 to 5 marks** and needs **more than one idea** joined together, or reasoning run *backwards* from the answer. None of them can be finished by putting numbers into a single formula. If a question takes you under two minutes, check your working — you have probably missed a condition.

H1

4 Marks

Find the point on the **x-axis** that is equidistant from A(7, 6) and B(−3, 4).

Why this is above average: You are not given the point — you must first express it in a restricted form, then square both distances to remove the roots. Squaring before comparing is the step most students miss.

Step 1 ny point on the x-axis has the form P(x, 0) — this is the condition you must impose first.

Step 2 $A^2 = (x - 7)^2 + (0 - 6)^2 = x^2 - 14x + 49 + 36$

Step 3 $B^2 = (x + 3)^2 + (0 - 4)^2 = x^2 + 6x + 9 + 16$

Step 4 quidistant ⇒ $PA^2 = PB^2 \Rightarrow -14x + 85 = 6x + 25$

Step 5 $0 = 20x \Rightarrow x = 3$. Check: $PA = \sqrt{(16 + 36)} = \sqrt{52}$ and $PB = \sqrt{(36 + 16)} = \sqrt{52}$. ✓

The point is P(3, 0)

H2

5 Marks

Show that A(−3, −2), B(5, −2), C(7, 4) and D(−1, 4) form a **parallelogram**, and then decide whether it is a rectangle. Justify both answers.

Why this is above average: Two separate tests are needed and the second one decides the answer. Stopping after proving "parallelogram" loses half the marks.

Step 1 $B = \sqrt{[(5+3)^2 + 0^2]} = 8$ and $DC = \sqrt{[(7+1)^2 + 0^2]} = 8$, so AB = DC.

Step 2 $D = \sqrt{[(-1+3)^2 + (4+2)^2]} = \sqrt{(4 + 36)} = \sqrt{40}$ and $BC = \sqrt{[(7-5)^2 + (4+2)^2]} = \sqrt{40}$, so AD = BC.

Step 3 oth pairs of opposite sides are equal, so **ABCD is a parallelogram**.

Step 4 ow test the diagonals. $AC = \sqrt{[(7+3)^2 + (4+2)^2]} = \sqrt{(100 + 36)} = \sqrt{136}$.

Step 5 $D = \sqrt{[(-1-5)^2 + (4+2)^2]} = \sqrt{(36 + 36)} = \sqrt{72}$. Since $\sqrt{136} \neq \sqrt{72}$, the diagonals are unequal.

ABCD is a parallelogram but not a rectangle, because a rectangle must have equal diagonals.

H3

3 Marks

Find the point on the **y-axis** equidistant from $P(5, -2)$ and $Q(-3, 2)$.

Step 1 point on the y-axis has the form $R(0, y)$.

Step 2 $P^2 = 25 + (y + 2)^2$ and $RQ^2 = 9 + (y - 2)^2$

Step 3 $5 + y^2 + 4y + 4 = 9 + y^2 - 4y + 4 \Rightarrow 29 + 4y = 13 - 4y \Rightarrow 8y = -16 \Rightarrow y = -2$

The point is $R(0, -2)$

H4

5 Marks

A right-angled triangle has its vertices at $O(0, 0)$, $B(8, 0)$ and $C(8, 6)$.

(a) Verify that it is right-angled and name the right angle. (2)

(b) Find its perimeter and area. (2)

(c) Find the coordinates of the point where the perpendicular from O meets BC extended, if any, and explain your answer. (1)

Why this is above average: Part (c) looks like a construction problem but is settled by noticing the sides are parallel to the axes. Recognising that saves a page of algebra.

Step 1) OB is horizontal (both $y = 0$) and BC is vertical (both $x = 8$), so $OB \perp BC$. The right angle is at **B**.

Step 2) $OB = 8$, $BC = 6$, $OC = \sqrt{(64 + 36)} = 10$. Perimeter $= 8 + 6 + 10 =$ **24 units**.

Step 3 rea $= \frac{1}{2} \times OB \times BC = \frac{1}{2} \times 8 \times 6 =$ **24 square units**.

Step 4) BC lies along the vertical line $x = 8$. The perpendicular from $O(0, 0)$ to a vertical line is the horizontal line $y = 0$.

Step 5 meets $x = 8$ at $(8, 0)$ — which is the vertex B itself, since the triangle is already right-angled there.

(a) right-angled at B (b) perimeter 24, area 24 (c) (8, 0), i.e. the point B

CHAPTER 2

Introduction to Linear Polynomials

Concept Notes · Formula Sheet · Expected Questions

PART A — CONCEPT NOTES

📝 2.1 What a polynomial is

- A **polynomial in one variable x** is an expression of the form $a_n x^n + \dots + a_1 x + a_0$, where the coefficients are real numbers and every exponent is a **whole number**.
- So √x, 1/x and x^(3/2) are **not** polynomials — a negative or fractional power disqualifies the expression at once.
- **Degree** = the highest power of the variable that carries a non-zero coefficient.

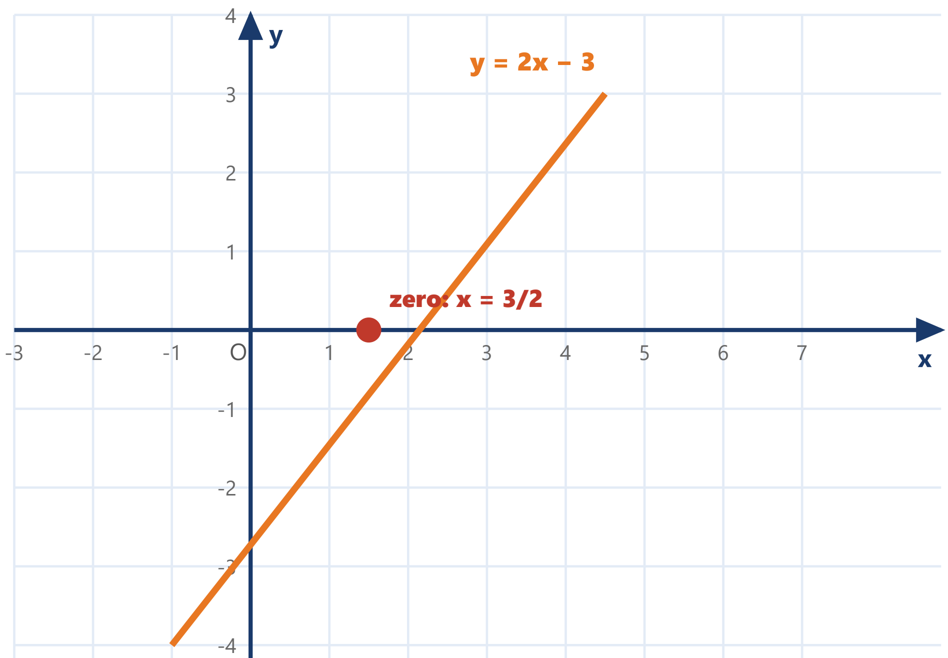
By degree	Name	Example
0	Constant polynomial	7
1	**Linear**	$3x - 5$
2	Quadratic	$x^2 - 4x + 1$
3	Cubic	$2x^3 + x - 6$

By number of terms	Name	Example
1	Monomial	$5x^3$
2	Binomial	$x + 7$
3	Trinomial	$x^2 + x + 1$

The zero polynomial (the constant 0) is the one polynomial whose degree is **not defined**. A non-zero constant such as 7 has degree 0. Examiners set this distinction every year.

🎯 2.2 Zeroes of a polynomial

- A number k is a **zero** of p(x) if **p(k) = 0**.
- For a linear polynomial **ax + b** (a ≠ 0), the single zero is $\boxed{x = -b/a}$.
- A polynomial of degree n has **at most n** zeroes. A linear polynomial has exactly one.
- Geometrically, the zero is the value of x where the graph of y = p(x) crosses the x-axis.



The zero of p(x) = 2x − 3 is where its graph cuts the x-axis: −b/a = 3/2

Fig. 2.1 — A linear polynomial has exactly one zero, where its straight-line graph meets the x-axis

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Factor theorem. (x − a) is a factor of p(x) **if and only if** p(a) = 0. Use it in both directions — to test a factor, and to find an unknown coefficient when a factor is given.

Remainder theorem. When p(x) is divided by (x − a), the remainder is **p(a)**. For a divisor (ax + b), substitute x = −b/a.

⚠ 2.3 Where marks are lost

- Calling the degree of the zero polynomial "0" — it is **not defined**.
- When the divisor is $(x + 3)$, substituting $x = 3$ instead of $x =$ **-3**.
- Writing "$3x^2 + 2\sqrt{x}$" as a polynomial — the $\sqrt{x}$ term rules it out.
- Stopping at $p(a) = 0$ without the concluding sentence "hence $(x - a)$ is a factor of $p(x)$". The conclusion carries a mark.

PART C — EXPECTED QUESTIONS (SOLVED)

E1

1 Mark

The degree of the polynomial 0 is

(i) 0 (ii) 1 (iii) not defined (iv) infinite

(iii) not defined

E2

1 Mark

Which of these is **not** a polynomial?

(i) $x^2 - 3x + 2$ (ii) 5 (iii) $3\sqrt{x} + 1$ (iv) $\sqrt{3}\,x + 1$

(iii) $3\sqrt{x} + 1$

REASON $\sqrt{x}$ is x^(1/2) — a fractional power. Note that (iv) is a perfectly good polynomial: an irrational *coefficient* is allowed, only irrational *powers* are not.

E3

1 Mark

The zero of the linear polynomial $5x + 2$ is

(i) 2/5 (ii) −2/5 (iii) 5/2 (iv) −5/2

(ii) −2/5

E4 **1 Mark**

Assertion (A): $(x - 1)$ is a factor of $x^3 - 3x^2 + 4x - 2$.

Reason (R): $(x - a)$ is a factor of $p(x)$ whenever $p(a) = 0$.

(i) Both true, R explains A (ii) Both true, R does not explain A (iii) A true, R false (iv) A false, R true

(i) Both true, and R is the correct explanation of A

CHECK $p(1) = 1 - 3 + 4 - 2 = 0$.

E5 **2 Marks**

Find the remainder when $p(x) = x^3 + 3x^2 - 5x + 4$ is divided by $(x + 2)$.

Step 1 ivisor $(x + 2) = (x - (-2))$, so by the remainder theorem the remainder is $p(-2)$.

Step 2 $(-2) = (-8) + 3(4) - 5(-2) + 4 = -8 + 12 + 10 + 4$

Remainder = 18

E6 **2 Marks**

Write the coefficient of x^2 and the degree of $p(x) = 7 - 4x + 3x^3 - x^2$.

Step 1 rrange in descending powers: $3x^3 - x^2 - 4x + 7$.

Step 2 ighest power present is 3; the x^2 term is $-1 \cdot x^2$.

Coefficient of x^2 = −1; degree = 3 (a cubic polynomial)

E7 **3 Marks**

Find the value of k if $(x - 2)$ is a factor of $p(x) = 2x^3 - 3x^2 + kx - 6$.

Step 1 y the factor theorem, $(x - 2)$ is a factor $\Rightarrow p(2) = 0$.

Step 2 $(2) = 2(8) - 3(4) + 2k - 6 = 16 - 12 + 2k - 6 = 2k - 2$

Step 3 et $2k - 2 = 0 \Rightarrow k = 1$

$k = 1$

E8

3 Marks

Check whether $(2x - 3)$ is a factor of $p(x) = 4x^3 - 8x^2 + x + 3$.

Step 1 $x - 3 = 0 \Rightarrow x = 3/2$. So test $p(3/2)$.

Step 2 $(3/2) = 4(27/8) - 8(9/4) + 3/2 + 3 = 27/2 - 18 + 3/2 + 3$

Step 3 $(27/2 + 3/2) - 15 = 15 - 15 = 0$

Since $p(3/2) = 0$, $(2x - 3)$ is a factor of $p(x)$.

E9

3 Marks

If $p(x) = x^3 + ax^2 + 2x + 5$ leaves remainder 11 when divided by $(x - 1)$, find a.

Step 1 emainder on division by $(x - 1)$ is $p(1)$.

Step 2 $(1) = 1 + a + 2 + 5 = a + 8$

Step 3 $+ 8 = 11 \Rightarrow a = 3$

$a = 3$

E10

5 Marks

The polynomial $p(x) = x^3 - 6x^2 + 11x - 6$ is known to have $(x - 1)$ as a factor.

(a) Verify this. (1)

(b) Hence factorise p(x) completely. (3)

(c) Write all its zeroes. (1)

Step 1 **)** $p(1) = 1 - 6 + 11 - 6 = 0$, so $(x - 1)$ is a factor.

Step 2 **)** Divide: $x^3 - 6x^2 + 11x - 6 \div (x - 1) = x^2 - 5x + 6$.

Step 3 ctorise the quadratic by splitting the middle term: $x^2 - 5x + 6 = x^2 - 3x - 2x + 6 = x(x - 3) - 2(x - 3) = (x - 3)(x - 2)$.

Step 4 $p(x) = (x - 1)(x - 2)(x - 3)$.

Step 5 **)** Setting each factor to zero gives the zeroes.

$p(x) = (x - 1)(x - 2)(x - 3)$; zeroes are $x = 1, 2$ and 3.

E11

5 Marks

Find a and b if both $(x - 1)$ and $(x + 1)$ are factors of $p(x) = x$&sup4; $+ ax^3 - 3x^2 + 2x + b$.

Step 1 $- 1)$ a factor $\Rightarrow p(1) = 0$: $1 + a - 3 + 2 + b = 0 \Rightarrow a + b = 0$ ...(i)

Step 2 $+ 1)$ a factor $\Rightarrow p(-1) = 0$: $1 - a - 3 - 2 + b = 0 \Rightarrow -a + b = 4$ ...(ii)

Step 3 dd (i) and (ii): $2b = 4 \Rightarrow b = 2$

Step 4 ubstitute in (i): $a + 2 = 0 \Rightarrow a = -2$

Step 5 heck in p(1): $1 - 2 - 3 + 2 + 2 = 0$. ✓

$a = -2, b = 2$

CHALLENGE SET — ABOVE-AVERAGE / HOTS

Read this before you start. Every question below is **3 to 5 marks** and needs **more than one idea** joined together, or reasoning run *backwards* from the answer. None of them can be finished by putting numbers into a single formula. If a question takes you under two minutes, check your working — you have probably missed a condition.

H1

5 Marks

Both $(x - 2)$ and $(x + 1)$ are factors of $p(x) = 2x^3 + ax^2 + bx - 6$. Find a and b, and hence write p(x) in fully factorised form.

Why this is above average: Two unknowns, two conditions, then a division — and the third factor has a leading coefficient of 2, which students routinely write as $(x + 3/2)$ and lose a mark.

Step 1 $- 2)$ a factor $\Rightarrow p(2) = 0$: $16 + 4a + 2b - 6 = 0 \Rightarrow 4a + 2b = -10 \Rightarrow 2a + b = -5$ ...(i)

Step 2 $+ 1)$ a factor $\Rightarrow p(-1) = 0$: $-2 + a - b - 6 = 0 \Rightarrow a - b = 8$ ...(ii)

Step 3 dd (i) and (ii): $3a = 3 \Rightarrow a = 1$. Then from (ii), $b = 1 - 8 = -7$.

Step 4 o $p(x) = 2x^3 + x^2 - 7x - 6$. Dividing by $(x - 2)(x + 1) = x^2 - x - 2$ gives the quotient $2x + 3$.

Step 5 heck: $p(2) = 16 + 4 - 14 - 6 = 0$ ✓ and $p(-1) = -2 + 1 + 7 - 6 = 0$ ✓

$a = 1, b = -7$; $p(x) = (x - 2)(x + 1)(2x + 3)$

H2

5 Marks

The polynomial $p(x) = x^3 - 23x^2 + 142x - 120$ has $(x - 1)$ as a factor. Factorise it completely and write all its zeroes.

> **Why this is above average:** The numbers are large, so guessing the split fails — you must use the product-and-sum rule deliberately.

Step 1 heck: $p(1) = 1 - 23 + 142 - 120 = 0$, so $(x - 1)$ is indeed a factor.

Step 2 ivide: $x^3 - 23x^2 + 142x - 120 \div (x - 1) = x^2 - 22x + 120$.

Step 3 plit the middle term: find two numbers with product 120 and sum −22 — they are −10 and −12.

Step 4 $- 10x - 12x + 120 = x(x - 10) - 12(x - 10) = (x - 10)(x - 12)$

> **$p(x) = (x - 1)(x - 10)(x - 12)$; zeroes are 1, 10 and 12**

H3

4 Marks

When $p(x) = x^3 + 3x^2 - kx + 4$ is divided by $(x - 2)$, the remainder is equal to the value of $p(x)$ at $x = -1$. Find k.

> **Why this is above average:** The condition links two different substitutions instead of setting one expression to zero — the remainder theorem is used twice in one question.

Step 1 emainder on division by $(x - 2)$ is $p(2) = 8 + 12 - 2k + 4 = 24 - 2k$.

Step 2 $(-1) = -1 + 3 + k + 4 = 6 + k$.

Step 3 iven the two are equal: $24 - 2k = 6 + k$

Step 4 $3 = 3k \Rightarrow k = 6$. Check: $p(2) = 24 - 12 = 12$ and $p(-1) = 6 + 6 = 12$. ✓

> **$k = 6$**

CHAPTER 3

The World of Numbers

Concept Notes · Formula Sheet · Expected Questions

PART A — CONCEPT NOTES

🔢 3.1 The number families

Set	Symbol	Members
Natural numbers	N	1, 2, 3, …
Whole numbers	W	0, 1, 2, 3, …
Integers	Z	… −2, −1, 0, 1, 2 …
Rational numbers	Q	p/q with q ≠ 0 and p, q integers
Irrational numbers	—	Cannot be written as p/q — e.g. √2, √3, π
Real numbers	R	All rationals and all irrationals together

The decimal test. A number is **rational** exactly when its decimal expansion is either **terminating** (0.75) or **non-terminating but recurring** (0.333…). It is **irrational** exactly when the expansion is **non-terminating and non-recurring** (1.41421356…).

Between any two rationals there are infinitely many rationals — and infinitely many irrationals too. To insert n rationals between a and b, the quickest route is to write both with denominator (n + 1) and read off the numerators in between.

√ 3.2 Laws of surds and exponents

$\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$ $\sqrt{a} \div \sqrt{b} = \sqrt{(a/b)}$ $(\sqrt{a})^2 = a$

$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$ — the identity behind every rationalisation question.

Rationalising the denominator

- For $1/\sqrt{a}$, multiply numerator and denominator by **√a**.
- For $1/(a + \sqrt{b})$, multiply by the **conjugate** $(a - \sqrt{b})$; for $1/(a - \sqrt{b})$, multiply by $(a + \sqrt{b})$.

Laws of exponents for real bases (a > 0)

$a^m \cdot a^n = a^{(m+n)}$ $a^m \div a^n = a^{(m-n)}$ $(a^m)^n = a^{(mn)}$ $a^0 = 1$ $a^{(-n)} = 1/a^n$ $a^{(1/n)} = \sqrt[n]{a}$

⚠ 3.3 Where marks are lost

- Claiming every square root is irrational — $\sqrt{16} = 4$ is rational. Only the root of a **non-perfect square** is irrational.
- Assuming rational + irrational could be rational. It never is (for a non-zero irrational part).
- Forgetting that irrational × irrational **can** be rational: $\sqrt{2} \times \sqrt{2} = 2$.
- Leaving a surd in the denominator — the answer is not in accepted form until it is rationalised.

Weightage. Number System is Unit I — **7 marks of 80** in the revised 2026–27 scheme (reduced from 10).

PART C — EXPECTED QUESTIONS (SOLVED)

E1 — 1 Mark

Which of the following is irrational?
(i) $\sqrt{49}$ (ii) $0.\overline{3}$ (iii) $\sqrt{12}$ (iv) 22/7

(iii) $\sqrt{12}$

REASON 12 is not a perfect square. Note that 22/7 is a rational *approximation* of π, not π itself.

E2 1 Mark

The value of $(81)^{(-1/4)}$ is

(i) 3 (ii) 1/3 (iii) 9 (iv) −3

(ii) 1/3

E3 1 Mark

Every point on the number line represents

(i) a natural number (ii) a rational number (iii) an irrational number (iv) a real number

(iv) a real number

E4 1 Mark

Assertion (A): The product of two irrational numbers is always irrational.

Reason (R): $\sqrt{2} \times \sqrt{8} = 4$.

(i) Both true, R explains A (ii) A false, R true (iii) A true, R false (iv) Both false

(ii) A is false, R is true

REASON $\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$, a rational number — the very counter-example that makes A false.

E5 2 Marks

Express $0.\overline{36}$ (i.e. 0.363636...) in the form p/q.

Step 1 Let $x = 0.363636\ldots$ Two digits recur, so multiply by 100: $100x = 36.363636\ldots$

Step 2 Subtract: $100x - x = 36 \Rightarrow 99x = 36 \Rightarrow x = 36/99 = 4/11$

$0.\overline{36} = 4/11$

E6

2 Marks

Find four rational numbers between 4/7 and 5/7.

Step 1 o fit 4 numbers, rewrite both with denominator $7 \times 5 = 35$: $4/7 = 20/35$ and $5/7 = 25/35$.

Step 2 he numerators strictly between 20 and 25 are 21, 22, 23, 24.

> **21/35 (= 3/5), 22/35, 23/35 and 24/35**

E7

3 Marks

Rationalise the denominator of $1/(\sqrt{7} - \sqrt{3})$.

Step 1 ultiply numerator and denominator by the conjugate $(\sqrt{7} + \sqrt{3})$.

Step 2 enominator: $(\sqrt{7} - \sqrt{3})(\sqrt{7} + \sqrt{3}) = 7 - 3 = 4$

Step 3 o the expression becomes $(\sqrt{7} + \sqrt{3})/4$

> **$(\sqrt{7} + \sqrt{3})/4$**

E8

3 Marks

Simplify: $(3\sqrt{5} + \sqrt{2})(3\sqrt{5} - \sqrt{2}) + (\sqrt{5} + \sqrt{2})^2$

Step 1 rst bracket pair is of the form $(a + b)(a - b) = a^2 - b^2 = (3\sqrt{5})^2 - (\sqrt{2})^2 = 45 - 2 = 43$

Step 2 econd term: $(\sqrt{5} + \sqrt{2})^2 = 5 + 2 + 2\sqrt{10} = 7 + 2\sqrt{10}$

Step 3 dd: $43 + 7 + 2\sqrt{10} = 50 + 2\sqrt{10}$

> **$50 + 2\sqrt{10}$**

E9

3 Marks

Evaluate: $(64)^{(2/3)} \div (27)^{(-1/3)}$

Step 1 $4)^{(2/3)} = (4^3)^{(2/3)} = 4^2 = 16$

Step 2 $7)^{(-1/3)} = 1/(27)^{(1/3)} = 1/3$

Step 3 $6 \div (1/3) = 16 \times 3 = 48$

> **48**

E10 **5 Marks**

If $a = (3 + \sqrt{5})/2$, find the value of $a^2 + 1/a^2$.

Step 1 $/a = 2/(3 + \sqrt{5})$. Rationalise: $2(3 - \sqrt{5})/((3)^2 - (\sqrt{5})^2) = 2(3 - \sqrt{5})/4 = (3 - \sqrt{5})/2$

Step 2 $a + 1/a = (3 + \sqrt{5})/2 + (3 - \sqrt{5})/2 = 6/2 = 3$

Step 3 se the identity $a^2 + 1/a^2 = (a + 1/a)^2 - 2$

Step 4 $3^2 - 2 = 9 - 2$

$a^2 + 1/a^2 = 7$

Squaring a directly would work but takes three times as long — always look for the $a + 1/a$ shortcut when the given number is a surd fraction.

E11 **5 Marks**

(a) Represent $\sqrt{5}$ on the number line. (3)

(b) State, with reason, whether $\sqrt{5}$ is rational or irrational. (2)

Step 1) Mark O at 0 and A at 2 on the number line, so OA = 2 units.

Step 2 A draw AB perpendicular to the line with AB = 1 unit.

Step 3 Pythagoras, $OB = \sqrt{(2^2 + 1^2)} = \sqrt{5}$. With O as centre and radius OB, draw an arc cutting the number line at P. Then $OP = \sqrt{5}$, so P represents $\sqrt{5}$.

Step 4) 5 is not a perfect square, so $\sqrt{5}$ cannot be written as p/q with integers p, q ($q \neq 0$).

Step 5 decimal expansion 2.2360679... is non-terminating and non-recurring.

$\sqrt{5}$ is irrational.

Note: *the construction above is described in words; draw it yourself on plain paper. For the printed construction figure please refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.*

CHALLENGE SET — ABOVE-AVERAGE / HOTS

Read this before you start. Every question below is **3 to 5 marks** and needs **more than one idea** joined together, or reasoning run *backwards* from the answer. None of them can be finished by putting numbers into a single formula. If a question takes you under two minutes, check your working — you have probably missed a condition.

H1

5 Marks

Simplify by rationalising: $(7 + 3\sqrt{5}) \div (3 + \sqrt{5})$, and state whether the result is rational or irrational, with a reason.

> **Why this is above average:** Rationalising the denominator does not make the number rational — a point almost every student gets wrong. The second part carries its own mark.

Step 1 ultiply numerator and denominator by the conjugate $(3 - \sqrt{5})$.

Step 2 enominator: $(3 + \sqrt{5})(3 - \sqrt{5}) = 9 - 5 = 4$

Step 3 umerator: $(7 + 3\sqrt{5})(3 - \sqrt{5}) = 21 - 7\sqrt{5} + 9\sqrt{5} - 3(5) = 21 - 15 + 2\sqrt{5} = 6 + 2\sqrt{5}$

Step 4 the expression $= (6 + 2\sqrt{5})/4 = (3 + \sqrt{5})/2$

Step 5 his is a rational number $(3/2)$ plus an irrational one $(\sqrt{5}/2)$. A rational plus a non-zero irrational is always **irrational**.

> **$(3 + \sqrt{5})/2$, which is irrational**

H2

5 Marks

If $x = 3 - 2\sqrt{2}$, find the value of $x^2 + 1/x^2$.

> **Why this is above average:** Squaring x directly works but takes three times as long. Spotting that x and 1/x are conjugates whose sum is rational is the intended insight.

Step 1 nd $1/x$ first by rationalising: $1/(3 - 2\sqrt{2}) \times (3 + 2\sqrt{2})/(3 + 2\sqrt{2}) = (3 + 2\sqrt{2})/(9 - 8) = 3 + 2\sqrt{2}$.

Step 2 herefore $x + 1/x = (3 - 2\sqrt{2}) + (3 + 2\sqrt{2}) = 6$ — the surds cancel.

Step 3 se the identity $x^2 + 1/x^2 = (x + 1/x)^2 - 2$

Step 4 $6^2 - 2 = 36 - 2$

> **$x^2 + 1/x^2 = 34$**

H3

4 Marks

Arrange in ascending order without using a calculator: $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{5}$. (Hint: make the indices equal.)

> **Why this is above average:** Different indices cannot be compared directly. Converting to a common index is a technique, not a formula — this is exactly the "apply" band of the paper.

Step 1 he indices are 2, 3 and 4. Their LCM is 12, so rewrite each as a 12th root.

Step 2 $2 = 2^{(1/2)} = 2^{(6/12)} = \sqrt[12]{(2^6)} = \sqrt[12]{64}$

Step 3 $3 = 3^{(1/3)} = 3^{(4/12)} = \sqrt[12]{(3^4)} = \sqrt[12]{81}$

Step 4 $5 = 5^{(1/4)} = 5^{(3/12)} = \sqrt[12]{(5^3)} = \sqrt[12]{125}$

Step 5 ow compare the radicands: $64 < 81 < 125$.

> **$\sqrt{2} < \sqrt[3]{3} < \sqrt[4]{5}$**

CHAPTER 4

Exploring Algebraic Identities

Concept Notes · Formula Sheet · Expected Questions

PART A — CONCEPT NOTES

📝 4.1 The identities you must know cold

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$(x+a)(x+b) = x^2 + (a+b)x + ab$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

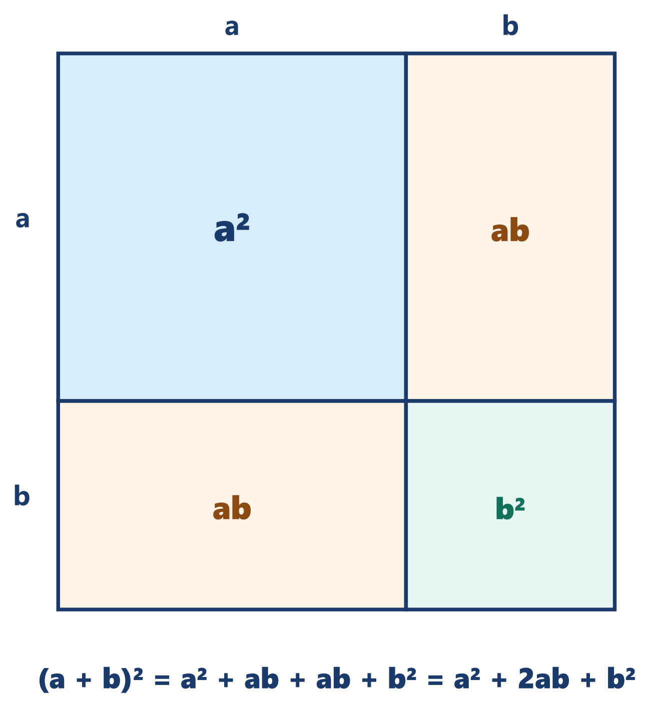


Fig. 4.1 — Why $(a + b)^2$ carries a 2ab term: the square of side (a + b) splits into four regions

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. If any part of this figure is unclear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.*

The one-line corollary that answers whole questions. If $\mathbf{a + b + c = 0}$, then $\mathbf{a^3 + b^3 + c^3 = 3abc}$. Spotting this turns a five-mark problem into two lines.

🔑 4.2 Useful rearrangements

- $\mathbf{a^2 + b^2 = (a + b)^2 - 2ab}$ and $\mathbf{a^2 + b^2 = (a - b)^2 + 2ab}$
- $\mathbf{x^2 + 1/x^2 = (x + 1/x)^2 - 2}$ and $\mathbf{x^2 + 1/x^2 = (x - 1/x)^2 + 2}$
- $\mathbf{x^3 + 1/x^3 = (x + 1/x)^3 - 3(x + 1/x)}$
- $\mathbf{(a + b)^2 - (a - b)^2 = 4ab}$ — a quick way to find ab from the two squares.

Factorising a quadratic by splitting the middle term. For $ax^2 + bx + c$, find two numbers whose **product is a×c** and whose **sum is b**, split bx into those two terms, then group in pairs.

⚠ 4.3 Where marks are lost

- Writing $(a + b)^2 = a^2 + b^2$ — the 2ab term is not optional.
- Mixing up the middle sign in $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$. Memory aid: the **S**ame sign first, **O**pposite sign in the middle, **A**lways **P**lus at the end — "SOAP".
- Forgetting the three cross terms $2ab + 2bc + 2ca$ in $(a + b + c)^2$.
- In factorisation, stopping when one factor can still be factorised further.

PART C — EXPECTED QUESTIONS (SOLVED)

E1

1 Mark

The value of 99^2 using a suitable identity is

(i) 9081 (ii) 9801 (iii) 9081 (iv) 9901

(ii) 9801

WORKING $(100 - 1)^2 = 10000 - 200 + 1 = 9801$.

E2

1 Mark

If $a + b + c = 0$, then $a^3 + b^3 + c^3$ equals

(i) 0 (ii) abc (iii) 3abc (iv) −3abc

(iii) 3abc

E3

1 Mark

Factorise: $4x^2 - 25$

$(2x + 5)(2x - 5)$

E4

2 Marks

Evaluate 103 × 97 without long multiplication.

Step 1 Write as $(100 + 3)(100 - 3)$, which is of the form $(a + b)(a - b) = a^2 - b^2$.

Step 2 $100^2 - 3^2 = 10000 - 9$

9991

E5

2 Marks

If $x + 1/x = 5$, find $x^2 + 1/x^2$.

Step 1 $+ 1/x^2 = (x + 1/x)^2 - 2$

Step 2 $5^2 - 2 = 25 - 2$

23

E6

3 Marks

Expand $(2x - 3y + 4z)^2$.

Step 1 se $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ with $a = 2x$, $b = -3y$, $c = 4z$.

Step 2 quares: $4x^2 + 9y^2 + 16z^2$

Step 3 ross terms: $2(2x)(-3y) = -12xy$; $2(-3y)(4z) = -24yz$; $2(4z)(2x) = 16zx$

$4x^2 + 9y^2 + 16z^2 - 12xy - 24yz + 16zx$

E7

3 Marks

Factorise: $6x^2 + 5x - 6$

Step 1 roduct $a\times c = 6 \times (-6) = -36$; required sum $= +5$. The pair is $+9$ and -4.

Step 2 $x^2 + 9x - 4x - 6$

Step 3 $3x(2x + 3) - 2(2x + 3) = (2x + 3)(3x - 2)$

$(2x + 3)(3x - 2)$

E8

3 Marks

Factorise: $27x^3 + 8y^3$

Step 1 rite as a sum of cubes: $(3x)^3 + (2y)^3$

Step 2 pply $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ with $a = 3x$, $b = 2y$

Step 3 $(3x + 2y)(9x^2 - 6xy + 4y^2)$

$(3x + 2y)(9x^2 - 6xy + 4y^2)$

E9 — 5 Marks

If $x + 1/x = 4$, find (a) $x^2 + 1/x^2$ and (b) $x^3 + 1/x^3$.

Step 1) $x^2 + 1/x^2 = (x + 1/x)^2 - 2 = 16 - 2 =$ **14**

Step 2) Cube the given relation: $(x + 1/x)^3 = x^3 + 1/x^3 + 3(x)(1/x)(x + 1/x)$

Step 3 $= x^3 + 1/x^3 + 3(1)(4)$

Step 4 4 $= x^3 + 1/x^3 + 12$

Step 5 $+ 1/x^3 = 64 - 12 =$ **52**

(a) 14 (b) 52

E10 — 5 Marks

Without actually calculating the cubes, evaluate $(-9)^3 + 4^3 + 5^3$.

Step 1 et $a = -9$, $b = 4$, $c = 5$.

Step 2 heck the sum first: $a + b + c = -9 + 4 + 5 = 0$.

Step 3 nce $a + b + c = 0$, the identity $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ collapses to $a^3 + b^3 + c^3 = 3abc$.

Step 4 abc $= 3 \times (-9) \times 4 \times 5$

Step 5 $3 \times (-180) = -540$

−540

E11 — 5 Marks

Factorise completely: $x^3 - 2x^2 - x + 2$

Step 1 y $p(1) = 1 - 2 - 1 + 2 = 0$, so $(x - 1)$ is a factor.

Step 2 roup instead, which is faster here: $x^2(x - 2) - 1(x - 2)$

Step 3 $(x - 2)(x^2 - 1)$

Step 4 $- 1$ is a difference of squares $= (x + 1)(x - 1)$

Step 5 o not stop at step 3 — the second factor was still factorisable.

$(x - 2)(x + 1)(x - 1)$

CHALLENGE SET — ABOVE-AVERAGE / HOTS

Read this before you start. Every question below is **3 to 5 marks** and needs **more than one idea** joined together, or reasoning run *backwards* from the answer. None of them can be finished by putting numbers into a single formula. If a question takes you under two minutes, check your working — you have probably missed a condition.

H1

4 Marks

If $a + b = 12$ and $ab = 35$, find the value of $a^3 + b^3$ without finding a and b separately.

Why this is above average: The instruction "without finding a and b" forces the identity route; solving the quadratic first earns no method marks.

Step 1 se the identity $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$.

Step 2 earranged: $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

Step 3 $12^3 - 3(35)(12) = 1728 - 1260$

$a^3 + b^3 = 468$

H2

5 Marks

If $x^2 + 1/x^2 = 47$ and x is positive, find (a) $x + 1/x$ and (b) $x^3 + 1/x^3$.

Why this is above average: The sign decision in (a) is a genuine reasoning step, and dropping the ± entirely is treated as a lucky guess.

Step 1 **)** $(x + 1/x)^2 = x^2 + 1/x^2 + 2 = 47 + 2 = 49$

Step 2 $x + 1/x = \pm 7$. Since x is positive, both x and $1/x$ are positive, so $x + 1/x =$ **7**. (The condition "x is positive" is there to let you reject −7.)

Step 3 **)** Cube it: $(x + 1/x)^3 = x^3 + 1/x^3 + 3(x)(1/x)(x + 1/x)$

Step 4 $= x^3 + 1/x^3 + 3(7) \Rightarrow 343 = x^3 + 1/x^3 + 21$

(a) 7 (b) $x^3 + 1/x^3 = 322$

H3 **5 Marks**

(a) Factorise $8x^3 + 27y^3 + 1 - 18xy$. (3)

(b) Hence, without cubing any number, evaluate $20^3 + (-12)^3 + (-8)^3$. (2)

Why this is above average: You must first *recognise* the four-term expression as the $a^3+b^3+c^3-3abc$ pattern — the 18xy term is the only clue — and then reuse the same identity in a numerical setting.

Step 1 **(a)** Write it as a cubes-sum: $(2x)^3 + (3y)^3 + (1)^3 - 3(2x)(3y)(1)$, since $3 \times 2x \times 3y \times 1 = 18xy$.

Step 2 This is exactly $a^3 + b^3 + c^3 - 3abc$ with $a = 2x$, $b = 3y$, $c = 1$.

Step 3 $(2x + 3y + 1)(4x^2 + 9y^2 + 1 - 6xy - 3y - 2x)$

Step 4 **(b)** Let $a = 20$, $b = -12$, $c = -8$. Then $a + b + c = 20 - 12 - 8 = 0$.

Step 5 When $a + b + c = 0$ the identity gives $a^3 + b^3 + c^3 = 3abc = 3(20)(-12)(-8) = 5760$.

(a) $(2x + 3y + 1)(4x^2 + 9y^2 + 1 - 6xy - 3y - 2x)$ (b) 5760

CHAPTER 5

I'm Up and Down, and Round and Round (Circles)

Concept Notes · Theorem List · Expected Questions

PART A — CONCEPT NOTES

5.1 Parts of a circle

Term	Meaning
Radius	Segment from the centre to any point on the circle
Chord	Segment joining any two points on the circle
Diameter	The longest chord — passes through the centre; length = 2r
Arc	A part of the circle; the smaller is the **minor** arc, the larger the **major** arc
Segment	Region between a chord and an arc
Sector	Region between two radii and the arc between them
Semicircle	Either half cut off by a diameter

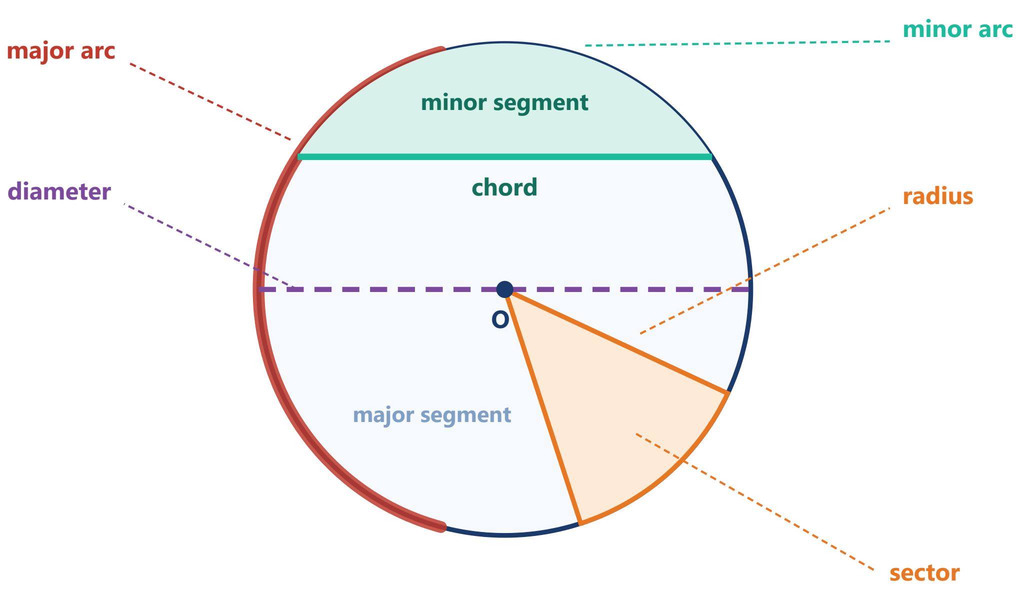


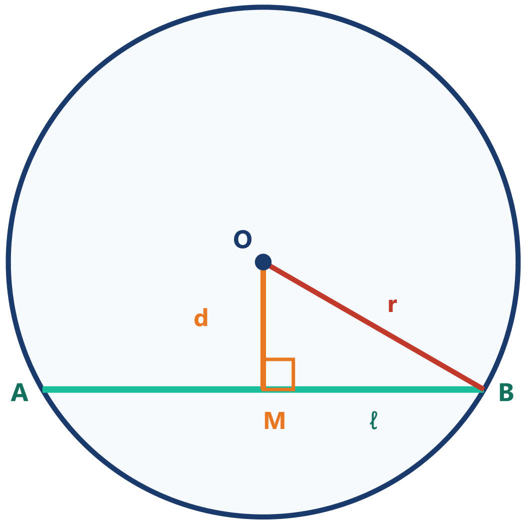
Fig. 5.1 — Parts of a circle: radius, diameter, chord, arc, segment and sector

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. If any part of this figure is unclear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.*

5.2 The theorems — learn the statement, then the use

#	Theorem	Typical use
1	Equal chords of a circle subtend equal angles at the centre (and the converse)	Proving two chords equal
2	The perpendicular from the centre to a chord **bisects** the chord (and the converse)	Finding chord length from the distance to the centre
3	Equal chords are **equidistant** from the centre (and the converse)	Comparing two chords
4	The angle subtended by an arc at the **centre** is **double** the angle it subtends at any point on the remaining part of the circle	The workhorse of every numerical
5	Angles in the **same segment** are equal	Transferring an angle across the figure
6	The angle in a **semicircle** is a right angle	Spotting a 90° instantly when a diameter appears
7	Opposite angles of a **cyclic quadrilateral** are supplementary (sum 180°), and the converse	Finding an unknown angle of a cyclic quadrilateral
8	The exterior angle of a cyclic quadrilateral equals the interior opposite angle	A one-line shortcut for theorem 7 questions

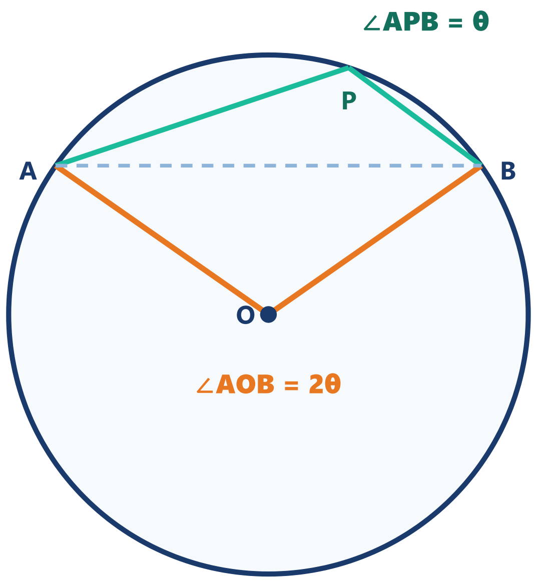
The chord-length relation. If a chord of length 2ℓ is at perpendicular distance d from the centre of a circle of radius r, then by Pythagoras in the half-chord right triangle: $\boxed{r^2 = d^2 + \ell^2}$, so the full chord = $2\sqrt{(r^2 - d^2)}$. Almost every chord numerical reduces to this.



$r^2 = d^2 + \ell^2$ — full chord AB = $2\sqrt{(r^2 - d^2)}$

Fig. 5.2 — The perpendicular from the centre bisects the chord, creating a right triangle

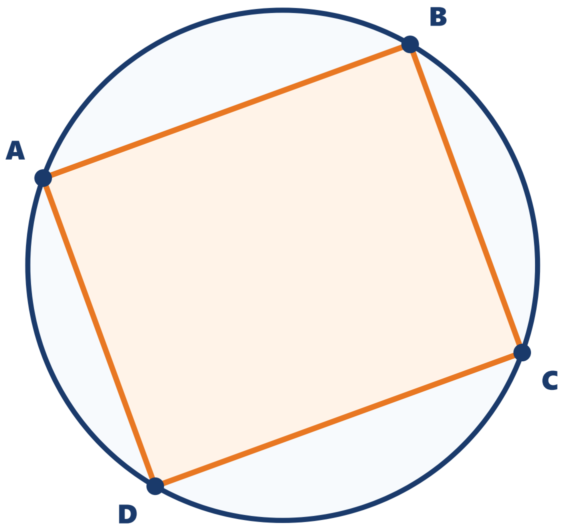
Note: *MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is unclear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.*



The angle at the centre is double the angle at any point P on the remaining arc

Fig. 5.3 — Angle subtended at the centre versus at the circumference

Note: *MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is unclear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.*



$\angle A + \angle C = 180^\circ$ and $\angle B + \angle D = 180^\circ$

Fig. 5.4 — A cyclic quadrilateral: opposite angles are supplementary

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. If any part of this figure is unclear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.*

⚠ 5.3 Where marks are lost

- Using half the chord instead of the full chord (or vice versa) in $r^2 = d^2 + \ell^2$.
- Applying "angle at centre = 2 × angle at circumference" to an angle at the circumference on the **wrong** side of the chord — the point must be on the **remaining** (major) arc.
- In a proof, not writing the reason beside each step. In geometry the reason carries as much weight as the statement.
- Not marking the given data on the figure before starting.

Weightage. Circles sits in Unit IV — Geometry, the largest unit at **25 marks of 80**. Expect one 5-mark proof or long numerical from this chapter.

PART C — EXPECTED QUESTIONS (SOLVED)

E1 1 Mark

The longest chord of a circle is

(i) a radius (ii) an arc (iii) a diameter (iv) a secant

(iii) a diameter

E2 1 Mark

The angle in a semicircle is

(i) 45° (ii) 60° (iii) 90° (iv) 180°

(iii) 90°

E3 1 Mark

In a cyclic quadrilateral ABCD, ∠A = 100°. Then ∠C is

(i) 100° (ii) 80° (iii) 60° (iv) 180°

(ii) 80°

E4 1 Mark

Assertion (A): A chord 8 cm long in a circle of radius 5 cm lies 3 cm from the centre.

Reason (R): The perpendicular from the centre to a chord bisects the chord.

(i) Both true, R explains A (ii) Both true, R does not explain A (iii) A true, R false (iv) A false, R true

(i) Both true, and R is the correct explanation of A

CHECK Half-chord = 4; $d = \sqrt{(5^2 - 4^2)} = 3$ cm. The bisecting property is exactly what makes the half-chord 4.

E5 **2 Marks**

A chord of a circle of radius 29 cm is at a distance of 21 cm from the centre. Find the length of the chord.

Step 1 he perpendicular from the centre bisects the chord, giving a right triangle with hypotenuse $r = 29$ and one leg $d = 21$. Let the half-chord be ℓ.

Step 2 $= \sqrt{(29^2 - 21^2)} = \sqrt{(841 - 441)} = \sqrt{400} = 20$. Full chord $= 2 \times 20$.

Chord = 40 cm

E6 **2 Marks**

An arc subtends 130° at the centre of a circle. Find the angle it subtends at a point on the major arc.

Step 1 ngle at centre = 2 × angle at any point on the remaining (major) arc.

Step 2 equired angle = $130° \div 2$

65°

E7 **3 Marks**

Two chords PQ and RS of a circle of radius 26 cm are equal in length. PQ lies 24 cm from the centre. Find the distance of RS from the centre and the length of each chord.

Step 1 qual chords of a circle are equidistant from the centre, so RS is also **24 cm** from the centre.

Step 2 alf-chord $= \sqrt{(26^2 - 24^2)} = \sqrt{(676 - 576)} = \sqrt{100} = 10$

Step 3 ach chord $= 2 \times 10 = 20$ cm

Distance of RS = 24 cm; each chord = 20 cm

E8 **3 Marks**

In a circle with centre O, AB is a diameter and C is a point on the circle with ∠BAC = 35°. Find ∠ABC.

Step 1 B is a diameter, so ∠ACB = 90° (angle in a semicircle).

Step 2 △ABC the angles sum to 180°: $35° + 90° + \angle ABC = 180°$

Step 3 ABC $= 180° - 125°$

∠ABC = 55°

E9 **3 Marks**

ABCD is a cyclic quadrilateral in which ∠A = (3x + 10)° and ∠C = (2x − 5)°. Find x and both angles.

Step 1 pposite angles of a cyclic quadrilateral are supplementary: ∠A + ∠C = 180°

Step 2 x + 10) + (2x − 5) = 180 ⇒ 5x + 5 = 180 ⇒ 5x = 175 ⇒ x = 35

Step 3 A = 3(35) + 10 = 115°; ∠C = 2(35) − 5 = 65°. Check: 115 + 65 = 180 ✓

x = 35; ∠A = 115°, ∠C = 65°

E10 **5 Marks**

Prove the converse: the line drawn from the centre of a circle to the midpoint of a chord is perpendicular to that chord.

GIVEN A circle with centre O and chord AB. M is the midpoint of AB, so AM = MB. **TO PROVE** OM ⊥ AB. **CONSTRUCTION** Join OA and OB.

Step 1 △OMA and △OMB: OA = OB *(radii of the same circle)*

Step 2 M = OM *(common side)*

Step 3 M = MB *(M is the midpoint, given)*

Step 4 OMA ≅ △OMB *(SSS congruence — three pairs of sides equal)*

Step 5 ∠OMA = ∠OMB *(CPCT)*

Step 6 ut ∠OMA + ∠OMB = 180° *(AMB is a straight line)*, so each is 90°.

Hence OM ⊥ AB — the line from the centre to the midpoint of a chord is perpendicular to it. ■

Note: *draw the figure yourself — circle, centre O, chord AB, foot M, and the two radii OA, OB. A proof without a figure loses a mark. For the printed figure please refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.*

E11 5 Marks

In a circle with centre O, points A, B, C lie on the circle. ∠AOB = 80° and ∠BOC = 120°.

(a) Find the reflex ∠AOC. (1)

(b) Find ∠ABC. (2)

(c) If ABCD is a cyclic quadrilateral with D on the major arc, find ∠ADC. (2)

Step 1 (a) Angles about O sum to 360°, so ∠AOC = 360° − 80° − 120° = 160°. Hence the **reflex ∠AOC = 200°**.

Step 2 (b) B lies on the arc opposite to the 160° angle, so the angle at the centre facing B's arc is the reflex, 200°.

Step 3 ∠ABC = ½ × reflex ∠AOC = ½ × 200° = **100°**

Step 4 (c) ABCD is cyclic, so ∠ABC + ∠ADC = 180°.

Step 5 ∠ADC = 180° − 100° = **80°** (which is also ½ × 160°, confirming the answer).

(a) 200° (b) 100° (c) 80°

CHALLENGE SET — ABOVE-AVERAGE / HOTS

Read this before you start. Every question below is **3 to 5 marks** and needs **more than one idea** joined together, or reasoning run *backwards* from the answer. None of them can be finished by putting numbers into a single formula. If a question takes you under two minutes, check your working — you have probably missed a condition.

H1

5 Marks

Two parallel chords of a circle of radius 25 cm have lengths 48 cm and 14 cm. Find the distance between them, considering **both** possible positions.

> **Why this is above average:** The question has **two** answers. Giving only one — usually the sum — is the single commonest error in circle numericals.

Step 1 or the 48 cm chord: half-chord = 24, so its distance from the centre $d_1 = \sqrt{(25^2 - 24^2)} = \sqrt{(625 - 576)} = \sqrt{49} = 7$ cm.

Step 2 or the 14 cm chord: half-chord = 7, so $d_2 = \sqrt{(25^2 - 7^2)} = \sqrt{(625 - 49)} = \sqrt{576} = 24$ cm.

Step 3 **ase 1 — chords on opposite sides of the centre:** distance = $d_1 + d_2 = 7 + 24 =$ **31 cm**.

Step 4 **ase 2 — chords on the same side of the centre:** distance = $d_2 - d_1 = 24 - 7 =$ **17 cm**.

Step 5 oth are geometrically possible, so both must be stated.

> **31 cm if on opposite sides; 17 cm if on the same side**

H2

4 Marks

In a circle of radius 13 cm, two parallel chords of lengths 24 cm and 10 cm lie on the **same side** of the centre. Find the distance between them.

> **Why this is above average:** The final check — longer chord must lie nearer the centre — is the reasoning mark, not the arithmetic.

Step 1 4 cm chord: half = 12, distance = $\sqrt{(169 - 144)} = \sqrt{25} = 5$ cm.

Step 2) cm chord: half = 5, distance = $\sqrt{(169 - 25)} = \sqrt{144} = 12$ cm.

Step 3 ame side, so subtract: $12 - 5$

Step 4 anity check: the longer chord (24 cm) is nearer the centre (5 cm) — as it must be.

> **Distance between the chords = 7 cm**

H3

5 Marks

In a circle with centre O, AB is a diameter and C is any point on the circle. $CD \perp AB$ with D on AB. If $\angle CAB = 3x$ and $\angle CBA = 2x$, find x, $\angle ACD$ and $\angle BCD$.

Why this is above average: Three triangles are used in turn, and the closing observation about the altitude is the kind of remark that earns the last mark.

Step 1 B is a diameter, so $\angle ACB = 90°$ (angle in a semicircle).

Step 2 $\triangle ABC$: $3x + 2x + 90° = 180° \Rightarrow 5x = 90° \Rightarrow$ $\mathbf{x = 18°}$. So $\angle CAB = 54°$ and $\angle CBA = 36°$.

Step 3 $\triangle ACD$, $\angle ADC = 90°$ ($CD \perp AB$) and $\angle CAD = \angle CAB = 54°$.

Step 4 $\angle ACD = 180° - 90° - 54° =$ $\mathbf{36°}$

Step 5 hen $\angle BCD = \angle ACB - \angle ACD = 90° - 36° =$ $\mathbf{54°}$. Note $\angle ACD = \angle CBA$ and $\angle BCD = \angle CAB$ — the altitude to the hypotenuse swaps the two acute angles.

$x = 18°$, $\angle ACD = 36°$, $\angle BCD = 54°$

CHAPTER 6

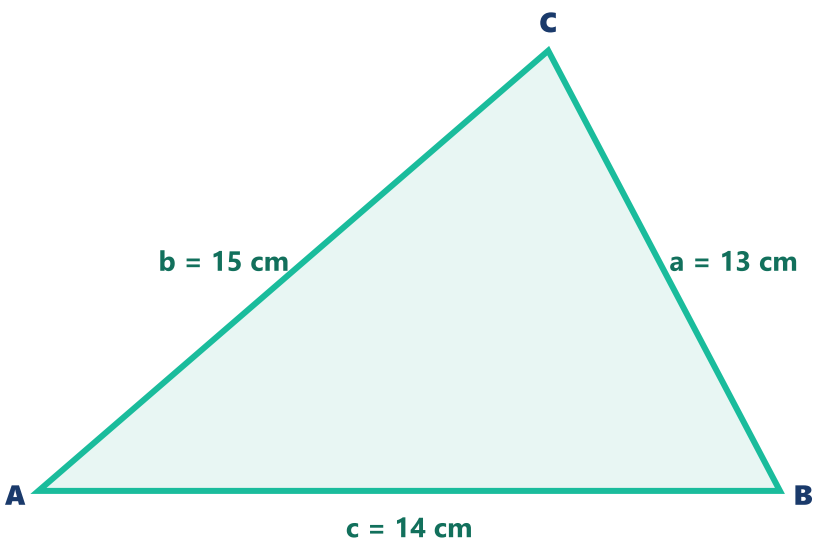
Measuring Space: Perimeter and Area

Concept Notes · Formula Sheet · Expected Questions

PART A — CONCEPT NOTES

📏 6.1 The formula sheet

Shape	Perimeter	Area
Square (side a)	4a	a^2
Rectangle (l, b)	2(l + b)	l × b
Triangle (base b, height h)	sum of sides	½ × b × h
Right triangle (legs a, b)	$a + b + \sqrt{(a^2+b^2)}$	½ab
Equilateral triangle (side a)	3a	$(\sqrt{3}/4)a^2$
Parallelogram	2(a + b)	base × height
Rhombus (diagonals d_1, d_2)	4a	½ d_1d_2
Trapezium (parallel sides a, b; height h)	sum of sides	½(a + b)h
Circle (radius r)	2πr	πr^2



$s = (13+14+15)/2 = 21 \rightarrow$ Area $= \sqrt{(21\times8\times7\times6)} = 84$ cm²

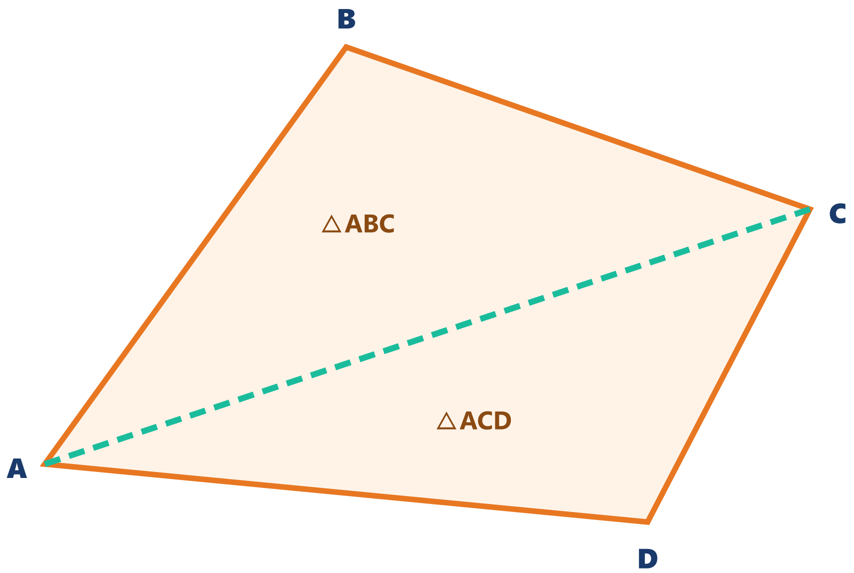
Fig. 6.1 — Heron's formula is used when all three sides are known but no height is given

__Note:__ MLI-redrawn schematic — correct in construction but __not drawn to scale__, and not a reproduction of any textbook figure. If any part of this figure is unclear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.

Heron's formula — for a triangle whose three **sides** a, b, c are known but whose height is not:

$s = (a + b + c)/2$ $\text{Area} = \sqrt{[s(s-a)(s-b)(s-c)]}$

Here s is the **semi-perimeter**. Named for Heron of Alexandria; the same result appears in India in Brahmagupta's work.



Split along a diagonal, apply Heron to each triangle, then add the two areas

Fig. 6.2 — Area of a quadrilateral given four sides and one diagonal

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. If any part of this figure is unclear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.*

Quadrilateral by splitting. To find the area of any quadrilateral given its four sides and one diagonal, split it along that diagonal into two triangles and apply Heron's formula to each, then add.

💡 6.2 Ideas that unlock the harder questions

- **Ratio of sides given.** Let the sides be 3x, 5x, 7x, use the perimeter to find x, then apply Heron.
- **Two expressions for the same area.** Area = ½ × base × corresponding height. Finding the area by Heron first, then equating, gives an unknown height in one line.
- **Cost questions.** Area × rate per unit area. Always convert to a single unit (m^2 or cm^2) *before* multiplying.
- **Isosceles triangle.** If sides are a, a, b then the semi-perimeter is (2a + b)/2 and Heron simplifies neatly — but the drop-a-perpendicular method (height = $\sqrt{(a^2 - b^2/4)}$) is usually faster.

⚠ 6.3 Where marks are lost

- Using the **perimeter** in place of the **semi-perimeter** in Heron's formula — the single commonest error in this chapter.
- Mixing units: a side in metres and another in centimetres.
- Writing the area without the unit, or with the wrong unit (cm instead of cm^2).
- Leaving $\sqrt{1800}$ unsimplified instead of $30\sqrt{2}$.

> **Weightage.** Mensuration is Unit V — **14 marks of 80** in the revised scheme (raised from 13). Perimeter and area is the released half; surface areas and volumes will arrive with Part II.

PART C — EXPECTED QUESTIONS (SOLVED)

E1 — 1 Mark

The area of an equilateral triangle of side 4 cm is
(i) $4\sqrt{3}$ cm^2 (ii) $16\sqrt{3}$ cm^2 (iii) $8\sqrt{3}$ cm^2 (iv) $2\sqrt{3}$ cm^2

(i) $4\sqrt{3}$ cm^2

WORKING $(\sqrt{3}/4)(4)^2 = (\sqrt{3}/4)(16) = 4\sqrt{3}$.

E2 — 1 Mark

In Heron's formula, s stands for
(i) the perimeter (ii) half the perimeter (iii) the longest side (iv) the area

(ii) half the perimeter (the semi-perimeter)

E3 — 1 Mark

The area of a rhombus whose diagonals are 10 cm and 24 cm is
(i) 240 cm^2 (ii) 120 cm^2 (iii) 60 cm^2 (iv) 34 cm^2

(ii) 120 cm^2

E4

2 Marks

Find the area of a triangle whose sides are 8 cm, 15 cm and 17 cm.

Step 1 ote $8^2 + 15^2 = 64 + 225 = 289 = 17^2$, so the triangle is right-angled with legs 8 and 15.

Step 2 rea = ½ × 8 × 15 = 60 cm^2

60 cm^2

Heron gives the same answer but takes far longer. Always test for a Pythagorean triple first.

E5

2 Marks

Find the area of an isosceles triangle with equal sides 5 cm and base 8 cm.

Step 1 rop a perpendicular from the apex; it bisects the base, so height = $\sqrt{(5^2 - 4^2)} = \sqrt{9} = 3$ cm.

Step 2 rea = ½ × 8 × 3

12 cm^2

E6

3 Marks

The sides of a triangle are 13 cm, 14 cm and 15 cm. Find its area using Heron's formula.

Step 1 = (13 + 14 + 15)/2 = 42/2 = 21

Step 2 – a = 8, s – b = 7, s – c = 6

Step 3 rea = $\sqrt{(21 \times 8 \times 7 \times 6)} = \sqrt{7056} = 84$

84 cm^2

E7

3 Marks

The perimeter of a triangle is 42 cm and its sides are in the ratio 3 : 4 : 5. Find its area.

Step 1 et the sides be 3x, 4x, 5x. Then 12x = 42 ⇒ x = 3.5, giving sides 10.5, 14, 17.5 cm.

Step 2 he ratio 3 : 4 : 5 is a right-triangle ratio, so the legs are 10.5 and 14.

Step 3 rea = ½ × 10.5 × 14 = 73.5

73.5 cm^2

E8 **3 Marks**

A triangular park has sides 40 m, 24 m and 32 m. Find the cost of levelling it at ₹15 per m^2.

Step 1 $4^2 + 32^2 = 576 + 1024 = 1600 = 40^2$ — right-angled, legs 24 and 32.

Step 2 rea $= \frac{1}{2} \times 24 \times 32 = 384$ m^2

Step 3 ost $= 384 \times 15 =$ ₹5760

₹5,760

E9 **5 Marks**

A quadrilateral field ABCD has AB = 9 m, BC = 12 m, CD = 14 m, DA = 13 m and the diagonal AC = 15 m. Find its area.

KEY IDEA A quadrilateral has no area formula of its own. Split it along the given diagonal into two triangles, find each area, then add.

Step 1 plit along AC into △ABC and △ACD.

Step 2 **ABC:** check for a Pythagorean triple — $9^2 + 12^2 = 81 + 144 = 225 = 15^2$. So it is right-angled at B.

Area $= \frac{1}{2} \times 9 \times 12 =$ **54 m^2**

Step 3 **ACD:** sides 15, 14, 13. No right angle, so use Heron. $s = (15 + 14 + 13)/2 = 21$

Step 4 rea $= \sqrt{[21(21-15)(21-14)(21-13)]} = \sqrt{(21 \times 6 \times 7 \times 8)} = \sqrt{7056} =$ **84 m^2**

Step 5 tal area $= 54 + 84$

Area of the field ABCD = 138 m^2

Always test the triangle inequality first. Before using Heron, check that each of $(s - a)$, $(s - b)$, $(s - c)$ is **positive**. If any one comes out negative, those three lengths **cannot form a triangle** at all, and there is no area to find — write that down, because it is exactly what the examiner is testing. For example 15, 5 and 8 cannot form a triangle, since $5 + 8 = 13$ is less than 15.

E10 5 Marks

The sides of a triangular plot are 51 m, 37 m and 20 m.

(a) Find its area. (3)

(b) Find the length of the perpendicular from the opposite vertex to the longest side. (2)

Step 1 (a) $s = (51 + 37 + 20)/2 = 108/2 = 54$

Step 2 Area $= \sqrt{[54(54-51)(54-37)(54-20)]} = \sqrt{(54 \times 3 \times 17 \times 34)}$

Step 3 $= \sqrt{93636} = \mathbf{306\ m^2}$

Step 4 (b) Taking the longest side 51 m as base: Area = ½ × base × height

Step 5 $306 = \frac{1}{2} \times 51 \times h \Rightarrow h = 612/51 = \mathbf{12\ m}$

(a) 306 m^2 (b) 12 m

E11 5 Marks

Case study. A school lays out a triangular herb garden with sides 12 m, 35 m and 37 m inside a rectangular plot measuring 50 m by 30 m.

(a) Show that the herb garden is right-angled, and find its area. (3)

(b) Find the area of the plot left over. (1)

(c) Paving the remaining plot costs ₹14 per m^2. Find the cost. (1)

Step 1 (a) Test for a Pythagorean triple: $12^2 + 35^2 = 144 + 1225 = 1369$, and $37^2 = 1369$. Equal, so the triangle is **right-angled**, with the 12 m and 35 m sides as the legs.

Step 2 Area $= \frac{1}{2} \times 12 \times 35 = \mathbf{210\ m^2}$

Step 3 (Heron gives the same: $s = 42$, $\sqrt{[42 \times 30 \times 7 \times 5]} = \sqrt{44100} = 210$ — but spotting the triple is far quicker.)

Step 4 (b) Plot area $= 50 \times 30 = 1500\ m^2$. Remaining $= 1500 - 210 = \mathbf{1290\ m^2}$

Step 5 (c) Cost = 1290 × 14 = **₹18,060**

(a) right-angled; 210 m^2 (b) 1290 m^2 (c) ₹18,060

CHALLENGE SET — ABOVE-AVERAGE / HOTS

Read this before you start. Every question below is **3 to 5 marks** and needs **more than one idea** joined together, or reasoning run *backwards* from the answer. None of them can be finished by putting numbers into a single formula. If a question takes you under two minutes, check your working — you have probably missed a condition.

H1

5 Marks

The perimeter of a rhombus is 100 cm and one of its diagonals is 48 cm. Find the other diagonal and the area of the rhombus.

Why this is above average: Three separate properties of a rhombus are needed — equal sides, perpendicular bisection of diagonals, and the half-product area formula.

Step 1 ll four sides of a rhombus are equal, so side = 100 ÷ 4 = 25 cm.

Step 2 he diagonals bisect each other at right angles. Half of the known diagonal = 24 cm.

Step 3 the right triangle formed by the side and the two half-diagonals: half of the other diagonal = $\sqrt{(25^2 - 24^2)} = \sqrt{(625 - 576)} = \sqrt{49} = 7$ cm.

Step 4 the other diagonal = 2 × 7 = **14 cm**.

Step 5 rea = $\frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 48 \times 14 =$ **336 cm^2**

Other diagonal = 14 cm; area = 336 cm^2

H2

5 Marks

A trapezium has parallel sides 25 cm and 13 cm, and both non-parallel sides equal to 10 cm. Find its height and its area.

Why this is above average: No height is given and none of the standard formulas produces it — you have to construct the two right triangles yourself first.

Step 1 rop perpendiculars from the two ends of the shorter parallel side onto the longer one. This leaves a rectangle of width 13 cm in the middle.

Step 2 he remaining length is 25 − 13 = 12 cm, shared equally between the two congruent right triangles at the ends (the trapezium is isosceles, since both slant sides are 10 cm).

Step 3 each right triangle has base 12 ÷ 2 = 6 cm and hypotenuse 10 cm.

Step 4 eight = $\sqrt{(10^2 - 6^2)} = \sqrt{(100 - 36)} = \sqrt{64} =$ **8 cm**

Step 5 rea = $\frac{1}{2} \times (25 + 13) \times 8 = \frac{1}{2} \times 38 \times 8 =$ **152 cm^2**

Height = 8 cm; area = 152 cm^2

H3

5 Marks

The area of a triangle with sides 30 m, 34 m and 16 m is to be found.

(a) Show that the triangle is right-angled and find its area. (2)

(b) Find the area again by Heron’s formula and confirm the two agree. (2)

(c) Find the length of the altitude drawn to the longest side. (1)

> **Why this is above average:** You are asked to reach the same answer by two independent routes and then use it a third way. It tests whether you trust your own method.

Step 1) $30^2 + 16^2 = 900 + 256 = 1156$ and $34^2 = 1156$. Equal, so the triangle is right-angled with legs 30 m and 16 m.

Step 2 rea = $\frac{1}{2} \times 30 \times 16 =$ **$240\ m^2$**

Step 3) $s = (30 + 34 + 16)/2 = 80/2 = 40$. Then $s - a = 10$, $s - b = 6$, $s - c = 24$.

Step 4 rea = $\sqrt{(40 \times 10 \times 6 \times 24)} = \sqrt{57600} =$ **$240\ m^2$** — the two methods agree, as they must.

Step 5) Taking the longest side 34 m as base: $240 = \frac{1}{2} \times 34 \times h \Rightarrow h = 480/34 =$ **14.12 m** (2 d.p.)

(a) right-angled, $240\ m^2$ (b) $240\ m^2$ by Heron (c) ≈ 14.12 m

CHAPTER 7

The Mathematics of Maybe: Introduction to Probability

Concept Notes · Formula Sheet · Expected Questions

PART A — CONCEPT NOTES

🎲 7.1 The basic idea

- A **random experiment** is one whose result cannot be predicted with certainty, though all possible results are known.
- Each performance is a **trial**; each possible result is an **outcome**; a set of outcomes we are interested in is an **event**.
- Class 9 studies **empirical (experimental) probability** — probability worked out from what actually happened in a set of trials.

Empirical probability.

$$P(E) = (\text{number of trials in which E happened}) \div (\text{total number of trials})$$

Properties that every question relies on

- **0 ≤ P(E) ≤ 1** always. A probability can never be negative or more than 1.
- P(E) = 0 means the event is **impossible**; P(E) = 1 means it is **certain**.
- The probabilities of all possible outcomes of an experiment **add up to 1**.
- **P(not E) = 1 − P(E)** — E and "not E" are complementary events.

Empirical vs theoretical. Empirical probability is based on observation and can change from one set of trials to another. Theoretical probability is calculated from the structure of the experiment. As the number of trials grows large, the empirical value tends to settle near the theoretical value — this is why a coin tossed 10,000 times gives a head-fraction very close to 0.5.

⚠ 7.2 Where marks are lost

- Dividing by the number of **outcomes** instead of the number of **trials**.
- Giving an answer greater than 1 — always a sign of an arithmetic slip; check before writing it down.
- Leaving the answer as a raw fraction like 45/150 instead of reducing to 3/10.
- Missing the word "**not**" in the question and forgetting to subtract from 1.

> **Weightage.** Probability now shares Unit VI with Statistics for a combined **10 marks of 80**. It is new to Class 9 in 2026–27 and is highly scoring — the arithmetic is simple and the concepts are few.

PART C — EXPECTED QUESTIONS (SOLVED)

E1 — 1 Mark

The probability of an impossible event is

(i) 1 (ii) 0 (iii) ½ (iv) −1

(ii) 0

E2 — 1 Mark

Which of these can **not** be the probability of an event?

(i) 0.7 (ii) 2/3 (iii) 1.5 (iv) 0

(iii) 1.5

REASON Probability never exceeds 1.

E3 — 1 Mark

If P(E) = 0.35, then P(not E) is

(i) 0.35 (ii) 0.65 (iii) 1.35 (iv) 0

(ii) 0.65

E4 **1 Mark**

Assertion (A): A spinner with four equal sectors is spun 160 times and lands on red 44 times; the empirical probability of red is 0.275.
Reason (R): Empirical probability is the number of favourable trials divided by the total number of trials.
(i) Both true, R explains A (ii) Both true, R does not explain A (iii) A true, R false (iv) A false, R true

(i) Both true, and R is the correct explanation of A

WORKING 44 ÷ 160 = 0.275. Note that the *theoretical* probability here would be 0.25 — empirical values sit near, but rarely exactly on, the theoretical one.

E5 **2 Marks**

A bag of 480 seeds is sown and 336 of them germinate. Find the empirical probability that a seed chosen at random (a) germinates, (b) does not germinate.

Step 1 (germinates) = 336/480. Divide both by 48: = 7/10 = 0.7

Step 2 (does not germinate) = 1 − 0.7 = 0.3 (= 3/10)

(a) 7/10 or 0.7 (b) 3/10 or 0.3

E6 **2 Marks**

In a survey of 500 families, 280 own a two-wheeler. Find the probability that a family chosen at random does not own a two-wheeler.

Step 1 amilies without a two-wheeler = 500 − 280 = 220

Step 2 = 220/500 = 11/25 = 0.44

11/25 (or 0.44)

E7 **3 Marks**

Two coins are tossed 400 times with these results: two heads 96, one head 208, no head 96. Find the probability of each outcome and verify that they add to 1.

Step 1 (two heads) = 96/400 = 6/25 = 0.24

Step 2 (one head) = 208/400 = 13/25 = 0.52

Step 3 (no head) = 96/400 = 6/25 = 0.24; Sum = 0.24 + 0.52 + 0.24 = 1.00 ✓

0.24, 0.52, 0.24 — and they total 1, as the probabilities of all possible outcomes must.

E8 **3 Marks**

The record of 1000 patients at a clinic shows blood groups: A — 340, B — 290, AB — 90, O — 280. Find the probability that a patient chosen at random has (a) blood group AB, (b) blood group A or B, (c) not blood group O.

Step 1) P(AB) = 90/1000 = 9/100 = 0.09

Step 2) A or B = 340 + 290 = 630, so P = 630/1000 = 63/100 = 0.63

Step 3) P(O) = 280/1000 = 0.28, so P(not O) = 1 − 0.28 = 0.72

(a) 0.09 (b) 0.63 (c) 0.72

E9 **5 Marks**

The marks of 60 students in a test were recorded as: 0–20 → 7 students, 20–40 → 13, 40–60 → 20, 60–80 → 14, 80–100 → 6.

Find the probability that a student picked at random scored (a) less than 40, (b) 60 or more, (c) between 20 and 80, (d) full marks.

Step 1 otal trials = 60. Check the frequencies: 7 + 13 + 20 + 14 + 6 = 60 ✓

Step 2) Less than 40 = 7 + 13 = 20, so P = 20/60 = **1/3**

Step 3) 60 or more = 14 + 6 = 20, so P = 20/60 = **1/3**

Step 4) Between 20 and 80 = 13 + 20 + 14 = 47, so P = **47/60**

Step 5) No student is recorded at exactly 100 in this data, so the event did not occur in any trial: P = **0**

(a) 1/3 (b) 1/3 (c) 47/60 (d) 0

E10 5 Marks

Case study. A shopkeeper tracked 800 customers over a month. 340 paid in cash, 300 by UPI, 160 by card.

(a) Find P(a customer pays by UPI). (1)

(b) Find P(a customer does not pay in cash). (2)

(c) If 2,000 customers come next month, estimate how many will pay by card. (2)

Step 1 **(a)** P(UPI) = 300/800 = 3/8 = **0.375**

Step 2 **(b)** P(cash) = 340/800 = 17/40 = 0.425

Step 3 P(not cash) = 1 − 0.425 = **0.575** (= 23/40)

Step 4 **(c)** P(card) = 160/800 = 1/5 = 0.2

Step 5 Expected number = 0.2 × 2000 = **400 customers**

(a) 0.375 (b) 0.575 (c) about 400 customers

Part (c) shows why empirical probability matters in practice: a probability estimated from past trials is used to **predict** a future count. State it as an estimate, not a certainty.

CHALLENGE SET — ABOVE-AVERAGE / HOTS

Read this before you start. Every question below is **3 to 5 marks** and needs **more than one idea** joined together, or reasoning run *backwards* from the answer. None of them can be finished by putting numbers into a single formula. If a question takes you under two minutes, check your working — you have probably missed a condition.

H1

5 Marks

In a quality check of 1200 manufactured pens, 48 had a faulty nib, 30 had a faulty cap and 18 leaked. No pen had more than one fault.

(a) Find the probability that a pen chosen at random has a faulty nib. (1)

(b) Find the probability that a pen has some fault. (2)

(c) Find the probability that a pen is fault-free. (1)

(d) In a consignment of 5000 such pens, estimate how many would be fault-free. (1)

> **Why this is above average:** The sentence "no pen had more than one fault" is the hinge — without it the three counts could not simply be added.

Step 1) P(faulty nib) = 48/1200 = 1/25 = **0.04**

Step 2) Total faulty = 48 + 30 + 18 = 96 (they can be added because no pen has two faults). P(some fault) = 96/1200 = 2/25 = **0.08**

Step 3) P(fault-free) = 1 − 0.08 = **0.92**

Step 4) Expected number = 0.92 × 5000 = **4600 pens**

Step 5 ate it as an estimate — an empirical probability predicts, it does not guarantee.

> **(a) 0.04 (b) 0.08 (c) 0.92 (d) about 4600 pens**

H2

4 Marks

A survey of 250 households recorded the number of two-wheelers owned: 0 → 40 households, 1 → 130, 2 → 60, 3 or more → 20.

(a) Find the probability that a household owns at least one two-wheeler. (2)

(b) Find the probability that a household owns fewer than two. (1)

(c) Verify that the probabilities of all four outcomes add to 1. (1)

> **Why this is above average:** "At least one" and "fewer than two" overlap in a way that trips students up; reading the boundary correctly is the whole test.

Step 1 rst check the data: 40 + 130 + 60 + 20 = 250 ✓

Step 2) At least one = 130 + 60 + 20 = 210. P = 210/250 = 21/25 = **0.84** (or 1 − 40/250).

Step 3) Fewer than two means 0 or 1 = 40 + 130 = 170. P = 170/250 = 17/25 = **0.68**

Step 4) 0.16 + 0.52 + 0.24 + 0.08 = **1.00** ✓

> **(a) 0.84 (b) 0.68 (c) they total 1**

H3

4 Marks

A coin is tossed 500 times and a head appears 260 times. A student argues: “Since a coin is fair, the probability of a head must be exactly 0.5, so the experiment is wrong.” Evaluate this argument.

> **Why this is above average:** No formula is being asked for at all. The marks are for distinguishing two concepts and justifying a judgement in words — the "evaluate" band of the paper.

Step 1 he **empirical** probability here is 260/500 = **0.52**, and that value is correct for this set of trials.

Step 2 he **theoretical** probability of a head for a fair coin is 0.5. The two are different kinds of quantity and need not be equal.

Step 3 mpirical probability is computed from what actually happened, and varies from one set of trials to another; theoretical probability is computed from the structure of the experiment.

Step 4 s the number of trials grows very large, the empirical value tends to settle close to the theoretical one — 0.52 after 500 tosses is exactly the small deviation one expects.

> **The student is wrong. The experiment is not faulty; 0.52 is the correct empirical probability, and it differs from the theoretical 0.5 only because the number of trials is finite.**

CHAPTER 8

Predicting What Comes Next: Sequences and Progressions

Concept Notes · Formula Sheet · Expected Questions

PART A — CONCEPT NOTES

🔢 8.1 Sequences, AP and GP

- A **sequence** is an ordered list of numbers; each number is a **term**, written $a_1, a_2, a_3, \ldots, a_n$.
- An **Arithmetic Progression (AP)** is a sequence in which each term after the first is obtained by **adding a fixed number d**, the **common difference**.
- A **Geometric Progression (GP)** is a sequence in which each term after the first is obtained by **multiplying by a fixed number r**, the **common ratio**.

AP. nth term: $a_n = a + (n - 1)d$ Sum of n terms: $S_n = (n/2)[2a + (n - 1)d]$ or $S_n = (n/2)(a + \ell)$ when the last term ℓ is known.

GP. nth term: $a_n = a \cdot r^{(n-1)}$ Sum of n terms: $S_n = a(r^n - 1)/(r - 1)$ for $r \neq 1$.

How to tell them apart

Test	AP	GP
Take consecutive terms and...	**subtract**: $a_2 - a_1 = a_3 - a_2$	**divide**: $a_2/a_1 = a_3/a_2$
Example	3, 7, 11, 15 (d = 4)	3, 6, 12, 24 (r = 2)
Growth	Straight-line, steady	Rapid — doubling, compounding

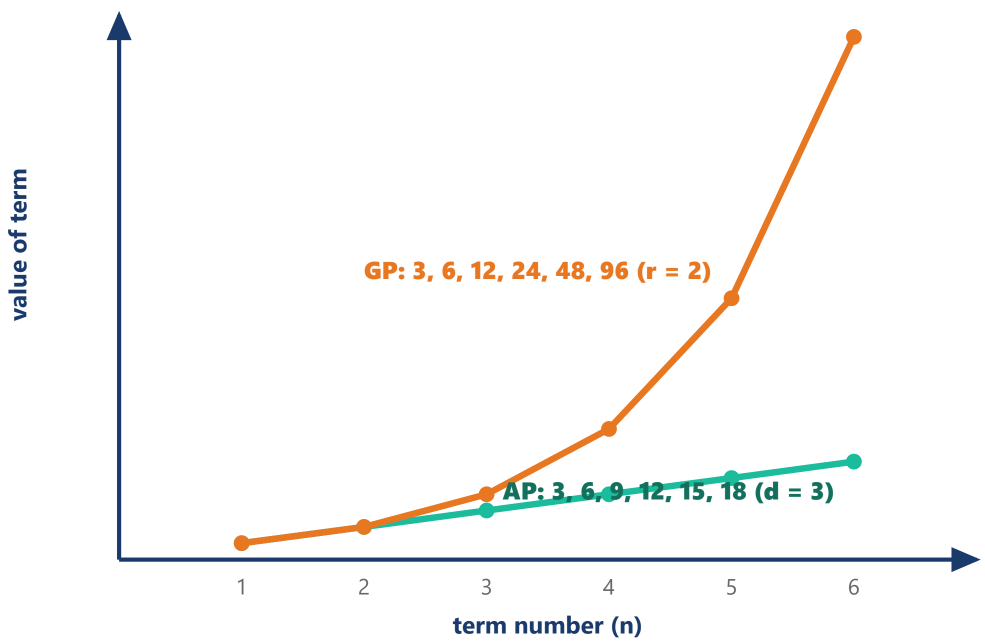


Fig. 8.1 — An AP grows in a straight line; a GP curves upward sharply

*Note: MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. If any part of this figure is unclear, students are kindly requested to refer to the corresponding figure in the NCERT textbook Ganita Manjari, Part I.*

Sums worth memorising. First n natural numbers: $n(n+1)/2$. First n odd numbers: n^2. First n even numbers: $n(n+1)$.

Indian Knowledge Systems link. Pingala's *Chandahshastra* deals with binary-style sequences in Sanskrit prosody, and the sequence usually called Fibonacci appears there as the **maatraameru** — each term the sum of the previous two.

⚠ 8.2 Where marks are lost

- Using n instead of $(n - 1)$ in $a_n = a + (n - 1)d$. The 10th term uses 9d, not 10d.
- Assuming a sequence is an AP without checking that the difference is the **same** for every consecutive pair — check at least two pairs.
- Applying the GP sum formula when $r = 1$ (then every term is a and $S_n = na$).
- Forgetting that d can be **negative** in a decreasing AP.

PART C — EXPECTED QUESTIONS (SOLVED)

E1

1 Mark

The common difference of the AP 10, 7, 4, 1, ... is
(i) 3 (ii) −3 (iii) 7 (iv) −7

(ii) −3

E2

1 Mark

Which of these is a GP?
(i) 2, 4, 6, 8 (ii) 1, 3, 9, 27 (iii) 5, 10, 15, 20 (iv) 1, 4, 9, 16

(ii) 1, 3, 9, 27 — each term is 3 times the previous one

E3 1 Mark

The sum of the first 20 natural numbers is

(i) 200 (ii) 210 (iii) 400 (iv) 420

(ii) 210

WORKING $n(n + 1)/2 = 20 \times 21/2 = 210$.

E4 1 Mark

Assertion (A): The sequence 2, 4, 8, 16 is an AP.

Reason (R): In an AP the difference between consecutive terms is constant.

(i) Both true, R explains A (ii) Both true, R does not explain A (iii) A true, R false (iv) A false, R true

(iv) A is false, R is true

REASON The differences are 2, 4, 8 — not constant. The sequence is a GP ($r = 2$), not an AP.

E5 2 Marks

Find the 18th term of the AP 3, 11, 19, ...

Step 1 $= 3, d = 11 - 3 = 8, n = 18$

Step 2 $_8 = a + (n - 1)d = 3 + 17 \times 8 = 3 + 136$

139

E6 2 Marks

Find the 6th term of the GP 5, 10, 20, ...

Step 1 $= 5, r = 10/5 = 2, n = 6$

Step 2 $= a \cdot r\text{^}(n-1) = 5 \times 2^5 = 5 \times 32$

160

E7 3 Marks

Find the sum of the first 25 terms of the AP 3, 8, 13, ...

Step 1 = 3, d = 5, n = 25

Step 2 = (n/2)[2a + (n − 1)d] = (25/2)[6 + 24 × 5]

Step 3 (25/2)(6 + 120) = (25/2)(126) = 25 × 63

1575

E8 3 Marks

Which term of the AP 6, 13, 20, ... is 104?

Step 1 = 6, d = 13 − 6 = 7. Let the nth term be 104.

Step 2 + (n − 1)7 = 104 ⇒ (n − 1)7 = 98 ⇒ n − 1 = 14

Step 3 = 15

104 is the 15th term.

E9 3 Marks

Find the sum of the first 8 terms of the GP 2, 6, 18, ...

Step 1 = 2, r = 3, n = 8

Step 2 = $a(r^n - 1)/(r - 1) = 2(3^8 - 1)/(3 - 1)$

Step 3 = 6561, so $S_8 = 2(6560)/2 = 6560$

6560

E10 — 5 Marks

The 4th term of an AP is 14 and the 12th term is 46. Find the first term, the common difference and the sum of the first 20 terms.

Step 1 $_4$ = a + 3d = 14 ...(i) a_{12} = a + 11d = 46 ...(ii)

Step 2 ubtract (i) from (ii): 8d = 32 ⇒ **d = 4**

Step 3 ubstitute in (i): a + 12 = 14 ⇒ **a = 2**

Step 4 $_{20}$ = (20/2)[2(2) + 19(4)] = 10(4 + 76)

Step 5 10 × 80 = **800**

a = 2, d = 4, S_{20} = 800

E11 — 5 Marks

Case study. A student saves ₹200 in the first month and increases her saving by ₹50 every month.
(a) Show that the monthly savings form an AP and state a and d. (1)
(b) How much does she save in the 12th month? (2)
(c) What is her total saving in one year? (2)

Step 1) Savings are 200, 250, 300, ... Each term exceeds the previous by the same ₹50, so it is an AP with **a = 200, d = 50**.

Step 2) a_{12} = a + 11d = 200 + 11 × 50 = 200 + 550 = **₹750**

Step 3) Use S_n = (n/2)(a + ℓ) with n = 12, a = 200, ℓ = 750

Step 4 $_2$ = (12/2)(200 + 750) = 6 × 950

Step 5 **₹5,700**

(a) AP with a = 200, d = 50 (b) ₹750 (c) ₹5,700

CHALLENGE SET — ABOVE-AVERAGE / HOTS

Read this before you start. Every question below is **3 to 5 marks** and needs **more than one idea** joined together, or reasoning run *backwards* from the answer. None of them can be finished by putting numbers into a single formula. If a question takes you under two minutes, check your working — you have probably missed a condition.

H1 **5 Marks**

In an AP, the sum of the first 7 terms is 49 and the sum of the first 17 terms is 289. Find the first term, the common difference, and hence the sum of the first n terms.

> **Why this is above average:** Two sum conditions give the two unknowns, and the final part asks for a general expression rather than a number — then rewards you for recognising the result.

Step 1 $_n = (n/2)[2a + (n - 1)d]$. For $n = 7$: $(7/2)(2a + 6d) = 49 \Rightarrow 7(a + 3d) = 49 \Rightarrow a + 3d = 7$ ...(i)

Step 2 or $n = 17$: $(17/2)(2a + 16d) = 289 \Rightarrow 17(a + 8d) = 289 \Rightarrow a + 8d = 17$ ...(ii)

Step 3 ubtract (i) from (ii): $5d = 10 \Rightarrow$ **$d = 2$**. Then from (i), $a = 7 - 6 =$ **1**.

Step 4 $_n = (n/2)[2(1) + (n - 1)2] = (n/2)(2 + 2n - 2) = (n/2)(2n)$

Step 5 o $S_n =$ **n^2** — this AP is 1, 3, 5, 7, ..., the odd numbers, whose sum of n terms is famously n^2.

> **$a = 1, d = 2$ and $S_n = n^2$**

H2 **5 Marks**

The 3rd term of a GP is 20 and the 6th term is 160. Find the first term, the common ratio and the sum of the first 8 terms.

> **Why this is above average:** Dividing one term by another to eliminate a is the technique being tested; solving the two equations any other way is far messier.

Step 1 $_3 = ar^2 = 20$ and $a_6 = ar^5 = 160$.

Step 2 ivide the second by the first: $ar^5/ar^2 = 160/20 \Rightarrow r^3 = 8 \Rightarrow$ **$r = 2$**.

Step 3 ubstitute back: $a(2^2) = 20 \Rightarrow 4a = 20 \Rightarrow$ **$a = 5$**.

Step 4 $_8 = a(r^n - 1)/(r - 1) = 5(2^8 - 1)/(2 - 1) = 5(256 - 1)$

Step 5 $5 \times 255 =$ **1275**

> **$a = 5, r = 2, S_8 = 1275$**

H3 **5 Marks**

An AP consists of 40 terms. Its 5th term is 29 and its last term is 239.

(a) Find the first term and the common difference. (2)

(b) Find the 25th term. (1)

(c) Find the sum of all 40 terms. (2)

Why this is above average: Recognising that "last term of a 40-term AP" means a_{40} is the step that decides the question; the shorter sum formula then saves time.

Step 1) $a_5 = a + 4d = 29$ ...(i); the last term is $a_{40} = a + 39d = 239$ ...(ii)

Step 2 ubtract (i) from (ii): $35d = 210 \Rightarrow$ **$d = 6$**. Then $a = 29 - 24 =$ **5**.

Step 3) $a_{25} = a + 24d = 5 + 144 =$ **149**

Step 4) With the last term known, use $S_n = (n/2)(a + \ell) = (40/2)(5 + 239)$

Step 5 $20 \times 244 =$ **4880**

(a) a = 5, d = 6 (b) 149 (c) 4880

QUICK REVISION

Master Formula Sheet — All 8 Chapters

Print these two pages and keep them inside your notebook

1 & 3 — Coordinates and Numbers

- Distance: $PQ = \sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$ | From origin: $\sqrt{(x^2+y^2)}$
- Midpoint: $((x_1+x_2)/2, (y_1+y_2)/2)$ | Quadrant signs: I (+,+), II (−,+), III (−,−), IV (+,−)
- Collinear if AB + BC = AC | Parallelogram if diagonals share a midpoint
- $\sqrt{a}\cdot\sqrt{b} = \sqrt{(ab)}$ | $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a - b$
- a^m·a^n = a^(m+n) | (a^m)^n = a^(mn) | a^0 = 1 | a^(−n) = 1/a^n | a^(1/n) = $\sqrt[n]{a}$
- Rational ⇔ decimal terminates or recurs. Irrational ⇔ non-terminating, non-recurring.

2 & 4 — Polynomials and Identities

- Zero of ax + b: x = −b/a | Remainder on ÷(x−a) is p(a) | (x−a) is a factor ⇔ p(a) = 0
- Degree of the zero polynomial is **not defined**; a non-zero constant has degree 0
- $(a\pm b)^2 = a^2 \pm 2ab + b^2$ | $a^2 - b^2 = (a+b)(a-b)$
- $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$
- $(a\pm b)^3 = a^3 \pm b^3 \pm 3ab(a\pm b)$
- $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$ — "SOAP"
- $a^3+b^3+c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$; if a+b+c = 0 then $a^3+b^3+c^3 = 3abc$
- $x^2 + 1/x^2 = (x + 1/x)^2 - 2$ | $x^3 + 1/x^3 = (x + 1/x)^3 - 3(x + 1/x)$

5 & 6 — Circles and Mensuration

- Chord: $r^2 = d^2 + \ell^2$ where ℓ is **half** the chord and d the distance from the centre
- Angle at centre = 2 × angle at the remaining arc | Angles in the same segment are equal
- Angle in a semicircle = 90° | Cyclic quadrilateral: opposite angles sum to 180°
- Heron: s = (a+b+c)/2; Area = $\sqrt{[s(s-a)(s-b)(s-c)]}$
- Equilateral: $(\sqrt{3}/4)a^2$ | Rhombus: $½d_1d_2$ | Trapezium: ½(a+b)h | Circle: πr^2, $2\pi r$

7 & 8 — Probability and Progressions

- $P(E)$ = favourable trials / total trials | $0 \le P(E) \le 1$ | $P(\text{not } E) = 1 - P(E)$
- All outcome probabilities of an experiment sum to 1
- AP: $a_n = a + (n-1)d$ | $S_n = (n/2)[2a + (n-1)d] = (n/2)(a + \ell)$
- GP: $a_n = a \cdot r^{(n-1)}$ | $S_n = a(r^n - 1)/(r-1)$, $r \neq 1$
- $1+2+\ldots+n = n(n+1)/2$ | first n odd numbers = n^2 | first n even numbers = $n(n+1)$

PRACTICE

MLI Model Question Paper — Mathematics

Based on the released Ganita Manjari Part I chapters only

MATH LOVE INSTITUTE — CLASS IX MATHEMATICS

Model Paper (Part I chapters) · Session 2026–27 · Time: 3 hours · Maximum Marks: 80

Read before attempting

This paper is set **only on the 8 chapters of Ganita Manjari Part I**. Your school's actual annual paper will also carry Geometry (Lines & Angles, Triangles, Quadrilaterals), Linear Equations in Two Variables, Surface Areas and Volumes, and Statistics from Part II. Treat this as a half-yearly-standard paper, not a full annual model.

General Instructions

1. This question paper has five sections A, B, C, D and E. All questions are compulsory.
2. Section A has 18 MCQs and 2 Assertion–Reason questions of 1 mark each.
3. Section B has 5 questions of 2 marks each. Section C has 6 questions of 3 marks each.
4. Section D has 4 questions of 5 marks each. Section E has 3 case-study questions of 4 marks each.
5. Internal choice is provided in 2 questions of Section B, 2 of Section C, 2 of Section D and in all 3 case studies.
6. Draw neat figures wherever required. Take π = 22/7 unless stated otherwise. Use of a calculator is **not** permitted.

SECTION A — 20 × 1 = 20 marks

1. The point (0, −5) lies on (a) x-axis (b) y-axis (c) quadrant III (d) quadrant IV — 1

2. The distance between (0, 0) and (−9, 12) is (a) 21 (b) 15 (c) 3 (d) 225 — 1

3. The degree of the polynomial 5 is (a) 0 (b) 1 (c) 5 (d) not defined — 1

4. The zero of 7 − 2x is (a) 2/7 (b) 7/2 (c) −7/2 (d) −2/7 — 1

5. The remainder when $x^3 - 1$ is divided by (x − 1) is (a) 1 (b) −1 (c) 0 (d) 2 — 1

6. Which is a rational number? (a) √7 (b) π (c) √81 (d) 0.10110111... — 1

7. $0.\overline{6}$ in p/q form is (a) 6/10 (b) 2/3 (c) 3/5 (d) 6/11 — 1

8. (16)^(3/4) equals (a) 8 (b) 12 (c) 64 (d) 4 — 1

9. $(x + 5)(x - 5)$ equals (a) $x^2 + 25$ (b) $x^2 - 25$ (c) $x^2 - 10x + 25$ (d) $x^2 + 10x + 25$ **1**

10. If $x + y + z = 0$ then $x^3 + y^3 + z^3$ equals (a) 0 (b) xyz (c) $3xyz$ (d) $-3xyz$ **1**

11. A chord 16 cm long lies 6 cm from the centre. The radius is (a) 8 cm (b) 10 cm (c) 12 cm (d) 14 cm **1**

12. In a cyclic quadrilateral PQRS, $\angle Q = 75°$. Then $\angle S =$ (a) 75° (b) 105° (c) 15° (d) 90° **1**

13. An arc subtends 70° at a point on the major arc. The angle at the centre is (a) 35° (b) 70° (c) 140° (d) 110° **1**

14. The semi-perimeter of a triangle with sides 9, 12, 15 cm is (a) 36 cm (b) 18 cm (c) 12 cm (d) 24 cm **1**

15. The area of a rhombus with diagonals 16 cm and 30 cm is (a) 480 cm^2 (b) 240 cm^2 (c) 120 cm^2 (d) 46 cm^2 **1**

16. The probability of a certain event is (a) 0 (b) ½ (c) 1 (d) 100 **1**

17. The 10th term of the AP 4, 9, 14, ... is (a) 45 (b) 49 (c) 54 (d) 50 **1**

18. The sum of the first 15 odd natural numbers is (a) 120 (b) 225 (c) 240 (d) 210 **1**

19. Assertion (A): $\sqrt{2} + \sqrt{3}$ is irrational. **Reason (R):** The sum of two irrational numbers is always irrational. **1**
(a) Both true, R explains A (b) Both true, R does not explain A (c) A true, R false (d) A false, R true

20. Assertion (A): The diagonals of a parallelogram bisect each other. **Reason (R):** The midpoint of both diagonals of a parallelogram is the same point. **1**
(a) Both true, R explains A (b) Both true, R does not explain A (c) A true, R false (d) A false, R true

SECTION B — 5 × 2 = 10 marks

21. Find the distance between A(−6, 2) and B(3, −10). **2**

22. Find the remainder when $2x^3 - 5x^2 + 3x - 4$ is divided by $(x - 2)$. **2**
OR Find the value of k if $(x + 1)$ is a factor of $x^3 + kx^2 - x + 2$.

23. If $x - 1/x = 3$, find $x^2 + 1/x^2$. **2**

24. A chord of a circle of radius 41 cm is 40 cm from the centre. Find its length. **2**
OR An arc subtends 96° at the centre. Find the angle it subtends on the major arc.

25. In a batch of 900 bulbs tested, 837 were found working. Find the empirical probability that a bulb picked at random is defective. **2**

SECTION C — 6 × 3 = 18 marks

26. Show that A(−3, 2), B(1, −2) and C(5, −6) are collinear. **3**

27. Rationalise the denominator of $5/(\sqrt{6} - 1)$ and simplify. **3**

OR Express $1.\overline{27}$ in the form p/q.

28. Factorise: $2x^2 - 7x - 15$ **3**

29. Expand $(3a - 2b + c)^2$. **3**

30. Find the area of a triangle with sides 18 cm, 24 cm and 30 cm. **3**

OR The perimeter of a triangle is 60 cm and its sides are in the ratio 3 : 4 : 5. Find its area.

31. The 5th term of an AP is 19 and the 9th term is 35. Find the first term and the common difference. **3**

SECTION D — 4 × 5 = 20 marks

32. Three vertices of a parallelogram PQRS are P(2, −1), Q(5, 3) and R(7, 6). Find S and verify that PQ = SR. **5**

33. If both $(x - 2)$ and $(x - 3)$ are factors of $p(x) = x^3 + ax^2 + bx - 6$, find a and b, and hence factorise p(x) completely. **5**

OR If $x + 1/x = 6$, find $x^2 + 1/x^2$ and $x^3 + 1/x^3$.

34. Prove that equal chords of a circle are equidistant from the centre. Draw a neat labelled figure. **5**

OR Prove that the angle subtended by an arc at the centre is double the angle it subtends at any point on the remaining part of the circle.

35. A triangular plot has sides 15 m, 41 m and 52 m. Find its area by Heron's formula, and hence the length of the perpendicular from the opposite vertex to the longest side. **5**

SECTION E — 3 × 4 = 12 marks (Case Studies)

36. A park on a grid. A rectangular park is mapped with the gate at O(0, 0). A fountain stands at F(8, 6), a bench at B(0, 6) and a tree at T(8, 0). **4**

(i) In which quadrant does the point (−8, 6) lie? (1)

(ii) Find the distance OF. (1)

(iii) Find the midpoint of BT. (2)

OR (iii) Show that OBFT is a rectangle by comparing OB with TF and OT with BF. (2)

37. A circular garden. A circular garden of radius 17 m has a straight path AB across it, 15 m from the centre O. C is a point on the garden boundary such that AC is a diameter. **4**

(i) Find the length of the path AB. (2)

(ii) State the measure of ∠ABC, with reason. (1)

(iii) If ∠BAC = 40°, find ∠BCA. (1)

38. A savings plan. A shopkeeper deposits ₹1,500 in the first month and increases the deposit by ₹250 each month. 4

(i) State a and d for this AP. (1)

(ii) Find the deposit in the 10th month. (1)

(iii) Find the total deposited in 10 months. (2)

OR (iii) In which month does the monthly deposit first exceed ₹4,000? (2)

Answer Key — Section A

Q	1	2	3	4	5	6	7	8	9	10
Ans	b	b	a	b	c	c	b	a	b	c
Q	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans	b	b	c	b	b	c	b	b	c	a

Notes on the two Assertion–Reason items. Q19: A is true ($\sqrt{2} + \sqrt{3}$ is irrational) but R is false, because $\sqrt{2} + (-\sqrt{2}) = 0$ is rational — so the answer is (c). Q20: both statements are true and R is precisely the coordinate-geometry justification of A — so the answer is (a).

Selected answers, Sections B–E. 21. 15 units · 22. −2 (OR k = −2) · 23. 11 · 24. 18 cm (OR 48°) · 25. 63/900 = 7/100 = 0.07 · 26. AB = BC = $4\sqrt{2}$, AC = $8\sqrt{2}$, so AB + BC = AC · 27. ($\sqrt{6}$ + 1) (OR $1.\overline{27}$ = 14/11 approximately — show the 100x method) · 28. $(2x + 3)(x - 5)$ · 29. $9a^2 + 4b^2 + c^2 - 12ab - 4bc + 6ca$ · 30. 216 cm^2 (OR 150 cm^2) · 31. a = 3, d = 4 · 32. S(4, 2) · 33. a = −6, b = 11, p(x) = $(x-1)(x-2)(x-3)$ (OR 34 and 198) · 35. s = 54; Area = $\sqrt{(54\times39\times13\times2)}$ = 234 m^2; height on the 52 m side = 2×234÷52 = 9 m · 36. (ii) 10 (iii) (4, 3) · 37. (i) 16 m (ii) 90°, angle in a semicircle (iii) 50° · 38. (ii) ₹3,750 (iii) ₹26,250 (OR the 12th month).