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NCERT Solutions — Compiled Volume
Class X Mathematics · CBSE 2026–27

Step-wise solutions with CBSE-pattern mark allocation

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A Note on This Document: Parts of this material have been organised and formatted with the help of AI-based tools, and the solutions have been checked against the NCERT text. Even so, an occasional slip may remain — please bring anything doubtful to your teacher. Mark allocations follow the CBSE pattern and are indicative, not official. **Diagrams are redrawn schematics — correct in construction, but not to scale.**

✓ How to use this document — CBSE Board 2026–27

This volume gives the concept explanation and the **complete NCERT solutions** for the rationalised Class X syllabus. Four points before you start:

- **Nothing from the deleted syllabus is included.** Euclid's Division Lemma and decimal expansions (Real Numbers), the division algorithm for polynomials, the cross-multiplication method and reducible equations (Pair of Linear Equations), area of a triangle from coordinates (Coordinate Geometry), ogives and cumulative-frequency curves (Statistics), the frustum of a cone, and the entire Constructions chapter are **out of syllabus** and are not solved here. Where an old question bank still carries them, ignore it.
- **Keep your NCERT textbook open beside this.** Every figure here is an **MLI-redrawn schematic** — correct in construction but **not to scale**, and not a copy of any textbook figure. Each carries a note saying so. Never measure off these diagrams.
- **Solutions follow the CBSE marking scheme.** Each step is on its own line with its mark allocation, because method and formula marks are awarded independently of the final answer. Write your answer sheet the same way.

- **If anything here disagrees with your printed NCERT book or the official CBSE syllabus document, those win.** Report it to your faculty so this volume can be corrected. Every numerical has been worked through, but an occasional slip may still remain.

Syllabus note. The NCERT Class X Mathematics textbook for **2026–27** has **14 chapters**. The chapter **Constructions** (formerly Chapter 11) has been removed from the syllabus. Exercises belonging to the older, pre-rationalisation edition — such as 3.4–3.7 and 7.3–7.4 — have been **left out of this volume**, because they are no longer part of the course.

✅ **Complete coverage**

All **14 chapters** of the NCERT Class X Mathematics textbook for 2026–27 are included in this volume, with every exercise of the current syllabus solved step by step.

📜 CONTENTS — CHAPTERS INCLUDED

1	**Real Numbers**	Exercises 1.1 – 1.2
2	**Polynomials**	Exercises 2.1 – 2.2
3	**Pair of Linear Equations in Two Variables**	Exercises 3.1 – 3.3
4	**Quadratic Equations**	Exercises 4.1 – 4.3
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6	**Triangles**	Exercises 6.1 – 6.3
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10	**Circles**	Exercises 10.1 – 10.2
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12 **Surface Areas and Volumes** Exercises 12.1 – 12.2

13 **Statistics** Exercises 13.1 – 13.3

14 **Probability** Exercise 14.1

CHAPTER 1

Real Numbers

Exercises 1.1 – 1.2

EXERCISES 1.1 – 1.2

⚠ Important — As Per the Latest CBSE Syllabus (2026–27)

Two topics have been **officially removed** from Chapter 1 and will **not** appear in the CBSE board exam, even though many older question banks still include them:

1. **Euclid's Division Lemma / Division Algorithm** (the a = bq + r method of finding HCF)
2. **Decimal Expansions of Rational Numbers** (terminating vs. non-terminating repeating decimals)

The current NCERT textbook contains only **Exercise 1.1** (Fundamental Theorem of Arithmetic) and **Exercise 1.2** (Revisiting Irrational Numbers). This guide covers exactly this — nothing outdated, nothing extra.

📊 Weightage in CBSE Class 10 Board Exam

Marks Distribution (Out of 80 — Theory Paper)

✅ Verified directly against the official CBSE curriculum document (cbseacademic.nic.in, Mathematics Code 041/241, Class X 2026–27) — not a secondary source. The topic list (Fundamental Theorem of Arithmetic + proofs of irrationality of √2, √3, √5 — with no Euclid's Division Lemma or decimal-expansion item) and the 6-mark Unit I total are quoted verbatim from it. Since Real Numbers is the **only** chapter in Unit I, this 6-mark figure is an exact official number, not an estimate.

Detail	Value
Unit	Number Systems (Unit I)
Chapters in this unit	Real Numbers is the **only** chapter
Marks allotted	**6 marks** out of 80
Percentage of theory paper	**≈ 7.5%**
Typical split	A mix of 1-mark + 2-mark + 3-mark questions totalling ~6 marks (varies slightly by year)

📌 With only 2 exercises now in the syllabus and a small, well-defined question pool, this chapter is one of the most realistic places to target full marks with focused practice.

🎯 Predicted Question Types

Format	Marks	Likely Topic
MCQ / Assertion-Reason	1	HCF×LCM relation, prime factorisation facts, rational/irrational identification
Very Short Answer	1	Express a number as a product of primes; state the Fundamental Theorem of Arithmetic
Short Answer	2	Find HCF/LCM of two numbers by prime factorisation; find LCM given HCF (or vice versa)
Proof (Short/Long)	3	Prove $\sqrt{2}$, $\sqrt{3}$, or $\sqrt{5}$ is irrational — **the single most repeated question type** across years
Proof (Short/Long)	3–5	Prove a combination like $3+2\sqrt{5}$, $5-\sqrt{3}$, $6+\sqrt{2}$, or $7\sqrt{5}$ is irrational
Word Problem	2–3	LCM-based "bells toll together" / "meet again on a track" type questions
Competency-Based Question	4–5 (sub-parts)	Per CBSE's 2026 mandatory 50% CBQ rule — a real-life case study on scheduling, packaging, or number patterns with HCF/LCM sub-questions

📐 All Key Formulas & Definitions

Fundamental Theorem of Arithmetic: Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

HCF: Product of the smallest power of each common prime factor.

LCM: Product of the greatest power of each prime factor involved (common or not).

Key Relation (TWO numbers only): $\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$. This does **NOT** extend to three or more numbers.

Theorem used in irrationality proofs: If p is prime and p divides a^2, then p divides a.

Irrational number: Cannot be written as p/q where p, q are integers and q ≠ 0.

Known irrationals (use directly): √2, √3, √5, √(any prime), π.

Sum/Product Rule: Rational ± Irrational = Irrational. (Non-zero Rational) × Irrational = Irrational.

💡 Shortcuts & Exam-Writing Tips

1. **Memorise the opening line** for every irrationality proof: "Let us assume, to the contrary, that ___ is rational. Then ___ = a/b, where a, b are coprime integers and b ≠ 0." This line alone earns real marks.
2. **Always state "a and b are coprime"** right at the start — the final contradiction depends on it, and examiners check for it explicitly.
3. **For combination proofs** (3+2√5, 6+√2, etc.), isolate the surd algebraically and argue the RHS is rational — contradicting the *known* irrationality of √2/√3/√5. No need to re-derive from scratch.
4. **Factor-tree shortcut:** divide repeatedly by the smallest possible prime (2, then 3, then 5, then 7…) rather than guessing factors.
5. **HCF×LCM shortcut:** if HCF and one number (or LCM) are given, use HCF×LCM = product directly — never re-factorise from scratch.
6. **"Explain why composite"** questions: look for a common factor you can pull out via the distributive law.
7. **"Meet again" word problems are always LCM** — a classic trap is reaching for HCF because the word "common" appears in the question.
8. **Always end proofs with** "Hence, ___ is irrational" — an explicit concluding line is expected by examiners.

📘 NCERT Solutions — Exercise 1.1 (7 Questions)

🔑 Approach for This Exercise

Every question here uses the Fundamental Theorem of Arithmetic. Factorise each number completely into primes first; then HCF = product of the smallest common powers, and LCM = product of the greatest powers of every prime that appears.

Q1. Express each number as a product of its prime factors: (i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429 **5 marks**

GIVEN: *Five composite numbers.*

TO FIND: *Prime factorisation of each.*

SOLUTION:

(i) $140 = 2 \times 70 = 2 \times 2 \times 35 = 2 \times 2 \times 5 \times 7$

$\mathbf{140 = 2^2 \times 5 \times 7}$ **1 mark**

(ii) $156 = 2 \times 78 = 2 \times 2 \times 39 = 2 \times 2 \times 3 \times 13$

$\mathbf{156 = 2^2 \times 3 \times 13}$ **1 mark**

(iii) $3825 = 3 \times 1275 = 3 \times 3 \times 425 = 3 \times 3 \times 5 \times 85 = 3 \times 3 \times 5 \times 5 \times 17$

$\mathbf{3825 = 3^2 \times 5^2 \times 17}$ **1 mark**

(iv) $5005 = 5 \times 1001 = 5 \times 7 \times 143 = 5 \times 7 \times 11 \times 13$

$\mathbf{5005 = 5 \times 7 \times 11 \times 13}$ **1 mark**

(v) $7429 = 17 \times 437 = 17 \times 19 \times 23$

$\mathbf{7429 = 17 \times 19 \times 23}$ **1 mark**

Q2. Find the LCM and HCF of (i) 26, 91 (ii) 510, 92 (iii) 336, 54 and verify LCM × HCF = product of the two numbers **6 marks**

GIVEN: *Three pairs of integers.*

TO FIND: *LCM, HCF of each pair, and verify LCM×HCF = product.*

SOLUTION:

(i) 26 and 91

$26 = 2 \times 13$

$91 = 7 \times 13$

HCF = *13; LCM* = $2 \times 7 \times 13 = 182$ **1 mark**

Verify: $13 \times 182 = 2366$; $26 \times 91 = 2366$ ✓ **½ mark**

(ii) 510 and 92

$510 = 2 \times 3 \times 5 \times 17$

$92 = 2^2 \times 23$

HCF = *2; LCM* = $2^2 \times 3 \times 5 \times 17 \times 23 = 23460$ **1 mark**

Verify: $2 \times 23460 = 46920$; $510 \times 92 = 46920$ ✓ **½ mark**

(iii) 336 and 54

$336 = 2^4 \times 3 \times 7$

$54 = 2 \times 3^3$

HCF = $2 \times 3 = 6$; *LCM* = $2^4 \times 3^3 \times 7 = 3024$ **1 mark**

Verify: $6 \times 3024 = 18144$; $336 \times 54 = 18144$ ✓ **1 mark**

Q3. Find the LCM and HCF of the following integers by applying the prime factorisation method: (i) 12, 15, 21 (ii) 17, 23, 29 (iii) 8, 9, 25 6 marks

GIVEN: *Three sets of three integers each.*

TO FIND: *LCM and HCF of each set.*

SOLUTION:

(i) 12, 15, 21

$12 = 2^2 \times 3;\ \ 15 = 3 \times 5;\ \ 21 = 3 \times 7$

HCF = 3 (only common factor); LCM = $2^2 \times 3 \times 5 \times 7 = 420$ 2 marks

(ii) 17, 23, 29

All three are prime numbers, so they share no common factor.

HCF = 1; LCM = $17 \times 23 \times 29 = 11339$ 2 marks

(iii) 8, 9, 25

$8 = 2^3;\ \ 9 = 3^2;\ \ 25 = 5^2$

No prime is common to all three, so HCF = 1; LCM = $2^3 \times 3^2 \times 5^2 = 1800$ 2 marks

Q4. Given that HCF (306, 657) = 9, find LCM (306, 657) 2 marks

GIVEN: *HCF(306, 657) = 9.*

TO FIND: *LCM(306, 657).*

SOLUTION:

Using $\text{HCF} \times \text{LCM} = 306 \times 657$ ½ mark

$$\text{LCM} = \frac{306 \times 657}{9} = 34 \times 657$$

$\text{LCM} = \mathbf{22338}$ 1½ marks

Q5. Check whether 6^n can end with the digit 0 for any natural number n 2 marks

GIVEN: 6^n, *n a natural number.*

TO FIND: *Whether* 6^n *can end in digit 0.*

SOLUTION:

$6^n = (2 \times 3)^n = 2^n \times 3^n$ ½ mark

For a number to end in the digit 0, it must be divisible by 10 — i.e., its prime factorisation must contain **both** *2 and 5.* 1 mark

The prime factorisation of 6^n *contains only the primes 2 and 3 — the prime 5 never appears.* ½ mark

Therefore, 6^n can never end with the digit 0, for any natural number n.

Q6. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers 2 marks

GIVEN: *Two numeric expressions.*

TO FIND: *Explain why each is composite.*

SOLUTION:

$7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1) = 13 \times (77 + 1) = 13 \times 78$ 1 mark

Since this can be written as a product of two factors (13 and 78), both greater than 1, it is a **composite number**.

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) = 5 \times (1008 + 1) = 5 \times 1009$ 1 mark

Since this can be written as a product of two factors (5 and 1009), both greater than 1, it is also a **composite number**.

Q7. There is a circular path around a sports field. Sonia takes 18 minutes for one round; Ravi takes 12 minutes. If they start together at the same point and go the same direction, after how many minutes will they meet again at the starting point? **2 marks**

GIVEN: *Sonia takes 18 min/round; Ravi takes 12 min/round; both start together, same point, same direction.*

TO FIND: *Time after which they will meet again at the starting point.*

SOLUTION:

They will meet again at the starting point after a time that is a common multiple of 18 and 12 — the earliest such time is their LCM. **½ mark**

$18 = 2 \times 3^2;\ \ 12 = 2^2 \times 3$

$\mathrm{LCM}(18, 12) = 2^2 \times 3^2 = 36$ **1 mark**

Therefore, Sonia and Ravi will meet again at the starting point after 36 minutes. **½ mark**

— End of Exercise 1.1 —

📘 NCERT Solutions — Exercise 1.2 (3 Questions)

🔑 Approach for This Exercise

Assume the number is rational (= a/b, coprime), manipulate algebraically, and reach a contradiction. Q2 and Q3 build directly on the base irrationality of √2 and √5 — no need to re-prove those from scratch each time.

Q1. Prove that √5 is irrational

3 marks

GIVEN: *The number √5.*

TO FIND: *Prove √5 is irrational.*

SOLUTION:

Let us assume, to the contrary, that √5 is rational. Then we can write $\sqrt{5} = \frac{a}{b}$*, where a and b are coprime integers and* $b \neq 0$. 1 mark

Squaring both sides: $5 = \frac{a^2}{b^2} \Rightarrow a^2 = 5b^2$ *...(i)*

This means 5 divides a^2*. Since 5 is prime, 5 must also divide a.* ½ mark

So we can write $a = 5c$ *for some integer c. Substituting in (i):* $(5c)^2 = 5b^2 \Rightarrow 25c^2 = 5b^2 \Rightarrow b^2 = 5c^2$

½ mark

This means 5 divides b^2*, and so 5 divides b as well.* ½ mark

Therefore, a and b have at least 5 as a common factor. But this contradicts the fact that a and b are coprime.

½ mark

This contradiction has arisen because of our incorrect assumption that √5 is rational.

Hence, √5 is irrational.

Q2. Prove that 3 + 2√5 is irrational

3 marks

GIVEN: *The number* $3 + 2\sqrt{5}$.

TO FIND: *Prove* $3 + 2\sqrt{5}$ *is irrational.*

SOLUTION:

Let us assume, to the contrary, that $3 + 2\sqrt{5}$ *is rational. Then we can write* $3 + 2\sqrt{5} = \frac{a}{b}$*, where a and b are coprime integers and* $b \neq 0$. 1 mark

Rearranging: $2\sqrt{5} = \frac{a}{b} - 3 = \frac{a - 3b}{b}$

$\sqrt{5} = \frac{a - 3b}{2b}$ 1 mark

Since a and b are integers, $\frac{a - 3b}{2b}$ *is a rational number, and so √5 would be rational.* ½ mark

But this contradicts the fact that √5 is irrational (proved in Q1). ½ mark

This contradiction has arisen because of our incorrect assumption that $3 + 2\sqrt{5}$ *is rational.*

Hence, $3 + 2\sqrt{5}$ **is irrational.**

Q3. Prove that the following are irrationals: (i) 1/√2 (ii) 7√5 (iii) 6+√2 5 marks

GIVEN: *Three expressions involving surds.*

TO FIND: *Prove each is irrational.*

SOLUTION:

(i) 1/√2:

Let us assume, to the contrary, that $\frac{1}{\sqrt{2}}$ *is rational. Then* $\frac{1}{\sqrt{2}} = \frac{a}{b}$, *where a, b are coprime integers,* $b \neq 0$.

So $\sqrt{2} = \frac{b}{a}$, *which is rational (as a, b are integers).* 1 mark

This contradicts the known fact that √2 is irrational. ½ mark

Hence, 1/√2 is irrational.

(ii) 7√5:

Let us assume, to the contrary, that $7\sqrt{5}$ *is rational. Then* $7\sqrt{5} = \frac{a}{b}$, *where a, b are coprime integers,* $b \neq 0$.

So $\sqrt{5} = \frac{a}{7b}$, *which is rational (as a, b are integers).* 1 mark

This contradicts the fact that √5 is irrational (proved in Q1). ½ mark

Hence, 7√5 is irrational.

(iii) 6+√2:

Let us assume, to the contrary, that $6 + \sqrt{2}$ *is rational. Then* $6 + \sqrt{2} = \frac{a}{b}$, *where a, b are coprime integers,* $b \neq 0$.

So $\sqrt{2} = \frac{a}{b} - 6 = \frac{a - 6b}{b}$, *which is rational (as a, b are integers).* 1 mark

This contradicts the known fact that √2 is irrational. ½ mark

Hence, 6+√2 is irrational.

— End of Exercise 1.2 — Chapter 1 Complete —

📚 **Board questions for this chapter are in the companion volume.** To keep this book to what it is for — concept explanation and the complete NCERT solutions — all previous-year and most-frequently-asked CBSE questions have been moved to *MLI Class 10 Maths — CBSE Most Frequently Asked Questions & PYQ Bank*. Finish the NCERT exercises above first, then work the board bank there.

CHAPTER 2

Polynomials

Exercises 2.1 – 2.2

EXERCISES 2.1 – 2.2

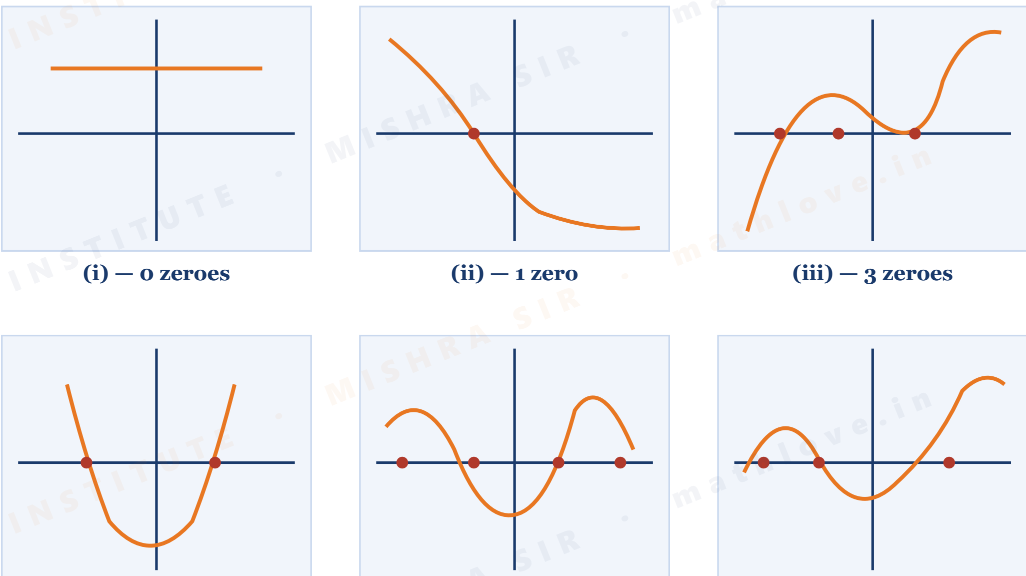
📘 NCERT Solutions — Exercise 2.1 (1 Question, 6 Graphs)

🔑 **Approach for This Exercise**

Only one idea is being tested here: **the zeroes of $p(x)$ are exactly the x-coordinates of the points where the graph of $y = p(x)$ cuts or touches the x-axis**. So the whole question reduces to counting intersection points with the x-axis. Two cautions: a graph that merely *turns* near the axis without touching it contributes no zero, and a graph that only *touches* the axis (without crossing) still counts as one zero.

Q1. The graphs of $y = p(x)$ are given in Fig. 2.10 below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$ in each case. 6 marks

SOLUTION:



(i) — 0 zeroes (ii) — 1 zero (iii) — 3 zeroes

(iv) — 2 zeroes (v) — 4 zeroes (vi) — 3 zeroes

Fig. 2.10 — redrawn schematics of the six textbook graphs. The dark red dots mark the points where each curve meets the x-axis; counting those dots gives the number of zeroes.

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Graph	What the curve does	Number of zeroes
(i)	The curve is a straight line lying entirely *above* the x-axis; it never meets the axis.	**0**
(ii)	The curve cuts the x-axis at exactly one point.	**1**
(iii)	The curve cuts the x-axis at three distinct points.	**3**
(iv)	The curve (a parabola) cuts the x-axis at two points.	**2**
(v)	The curve cuts the x-axis at four distinct points.	**4**
(vi)	The curve cuts the x-axis at three distinct points.	**3**

Answers: (i) 0 (ii) 1 (iii) 3 (iv) 2 (v) 4 (vi) 3 **1 mark each**

📌 **Examiner's tip.** Count *points of contact with the x-axis*, not turning points and not how many times the curve bends. In (i) the line runs parallel to the x-axis above it — since $p(x)$ is never 0, there is no zero at all. Also remember the degree link: a graph meeting the axis 4 times means $p(x)$ has degree at least 4.

— End of Exercise 2.1 —

📘 NCERT Solutions — Exercise 2.2 (2 Questions, 12 Parts)

🔑 Approach for This Exercise

Two standard relations drive everything. For $p(x) = ax^2 + bx + c$ with zeroes α, β: $\alpha + \beta = -\dfrac{b}{a}$ and $\alpha\beta = \dfrac{c}{a}$. In Q1 you find the zeroes by splitting the middle term and then *verify* both relations — the verification carries its own marks, so never skip it. In Q2 you run the same relations backwards: a quadratic with given sum S and product P is $k\left[x^2 - Sx + P\right]$, and taking $k = 1$ (or clearing fractions) gives the simplest answer.

Q1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients: 12 marks

(i) $x^2 - 2x - 8$ **(ii)** $4s^2 - 4s + 1$ **(iii)** $6x^2 - 3 - 7x$ **(iv)** $4u^2 + 8u$ **(v)** $t^2 - 15$ **(vi)** $3x^2 - x - 4$

SOLUTION:

(i) $x^2 - 2x - 8$ here $a = 1,\ b = -2,\ c = -8.$

Split the middle term (product $= 1 \times (-8) = -8$, sum $= -2$): the numbers are -4 and $+2$.

$x^2 - 4x + 2x - 8 = x(x-4) + 2(x-4) = (x-4)(x+2)$

Zeroes: $\alpha = 4,\ \beta = -2$

Verify: $\alpha + \beta = 4 - 2 = 2$ and $-\frac{b}{a} = -\frac{-2}{1} = 2$ ✓

$\alpha\beta = 4 \times (-2) = -8$ and $\frac{c}{a} = \frac{-8}{1} = -8$ ✓ 2 marks

(ii) $4s^2 - 4s + 1$ here $a = 4,\ b = -4,\ c = 1.$

Product $= 4 \times 1 = 4$, sum $= -4$ → the numbers are -2 and -2.

$4s^2 - 2s - 2s + 1 = 2s(2s-1) - 1(2s-1) = (2s-1)(2s-1) = (2s-1)^2$

Zeroes: $\alpha = \beta = \frac{1}{2}$ (a repeated zero)

Verify: $\alpha + \beta = \frac{1}{2} + \frac{1}{2} = 1$ and $-\frac{b}{a} = -\frac{-4}{4} = 1$ ✓

$\alpha\beta = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ and $\frac{c}{a} = \frac{1}{4}$ ✓ 2 marks

(iii) $6x^2 - 3 - 7x$ first write it in standard form: $6x^2 - 7x - 3$, so $a = 6,\ b = -7,\ c = -3.$

Product $= 6 \times (-3) = -18$, sum $= -7$ → the numbers are -9 and $+2$.

$6x^2 - 9x + 2x - 3 = 3x(2x-3) + 1(2x-3) = (3x+1)(2x-3)$

Zeroes: $\alpha = -\frac{1}{3},\ \beta = \frac{3}{2}$

Verify: $\alpha + \beta = -\frac{1}{3} + \frac{3}{2} = \frac{-2+9}{6} = \frac{7}{6}$ and $-\frac{b}{a} = \frac{7}{6}$ ✓

$\alpha\beta = -\frac{1}{3} \times \frac{3}{2} = -\frac{1}{2}$ and $\frac{c}{a} = \frac{-3}{6} = -\frac{1}{2}$ ✓ 2 marks

(iv) $4u^2 + 8u$ here $a = 4,\ b = 8,\ c = 0.$ There is no constant term, so take out the common factor instead of splitting.

$4u^2 + 8u = 4u(u+2)$

Zeroes: $\alpha = 0,\ \beta = -2$

Verify: $\alpha + \beta = 0 - 2 = -2$ and $-\frac{b}{a} = -\frac{8}{4} = -2$ ✓

$\alpha\beta = 0 \times (-2) = 0$ and $\frac{c}{a} = \frac{0}{4} = 0$ ✓ 2 marks

(v) $t^2 - 15$ here $a = 1,\ b = 0,\ c = -15.$ This is a difference of two squares.

$t^2 - 15 = \left(t - \sqrt{15}\right)\left(t + \sqrt{15}\right)$

Zeroes: $\alpha = \sqrt{15},\ \beta = -\sqrt{15}$

Verify: $\alpha + \beta = \sqrt{15} - \sqrt{15} = 0$ and $-\frac{b}{a} = -\frac{0}{1} = 0$ ✓

$\alpha\beta = \left(\sqrt{15}\right)\left(-\sqrt{15}\right) = -15$ and $\dfrac{c}{a} = -15$ ✓ **2 marks**

(vi) $3x^2 - x - 4$ here $a = 3,\ b = -1,\ c = -4.$

Product $= 3 \times (-4) = -12$, sum $= -1$ → the numbers are -4 and $+3$.

$3x^2 - 4x + 3x - 4 = x(3x-4) + 1(3x-4) = (3x-4)(x+1)$

Zeroes: $\alpha = \dfrac{4}{3},\ \beta = -1$

Verify: $\alpha + \beta = \dfrac{4}{3} - 1 = \dfrac{1}{3}$ and $-\dfrac{b}{a} = -\dfrac{-1}{3} = \dfrac{1}{3}$ ✓

$\alpha\beta = \dfrac{4}{3} \times (-1) = -\dfrac{4}{3}$ and $\dfrac{c}{a} = \dfrac{-4}{3}$ ✓ **2 marks**

> 📌 **Where marks are lost.** In (iii) the polynomial is written out of order as $6x^2 - 3 - 7x$; students often read $b = -3$. Always rewrite in the form $ax^2 + bx + c$ *before* reading off the coefficients. And in (ii) the repeated zero $\frac{1}{2}$ must be counted **twice** in the sum, otherwise the verification fails.

Q2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively: **12 marks**

(i) $\dfrac{1}{4},\ -1$ **(ii)** $\sqrt{2},\ \dfrac{1}{3}$ **(iii)** $0,\ \sqrt{5}$ **(iv)** $1,\ 1$ **(v)** $-\dfrac{1}{4},\ \dfrac{1}{4}$ **(vi)** $4,\ 1$

SOLUTION:

Rule used throughout: if the sum of the zeroes is S and their product is P, then a quadratic polynomial with those zeroes is $k\left[x^2 - Sx + P\right]$. Taking $k = 1$ gives the simplest form; where fractions appear, choosing k equal to the common denominator clears them.

(i) $S = \dfrac{1}{4},\ P = -1$

$x^2 - \dfrac{1}{4}x + (-1) = x^2 - \dfrac{1}{4}x - 1$

Multiplying by $k = 4$ to clear the fraction: $\mathbf{4x^2 - x - 4}$ **2 marks**

(ii) $S = \sqrt{2},\ P = \dfrac{1}{3}$

$x^2 - \sqrt{2}x + \dfrac{1}{3}$

Multiplying by $k = 3$: $\mathbf{3x^2 - 3\sqrt{2}x + 1}$ **2 marks**

(iii) $S = 0,\ P = \sqrt{5}$

$x^2 - 0 \cdot x + \sqrt{5} = \mathbf{x^2 + \sqrt{5}}$ *(the middle term vanishes simply because S = 0)* **Note:** with sum 0 and product √5 the discriminant is 0 − 4√5 < 0, so this polynomial has **no real zeroes**. That is perfectly acceptable — the question only asks for a polynomial with the given sum and product, not for one whose zeroes are real. **2 marks**

(iv) $S = 1,\ P = 1$

$\mathbf{x^2 - x + 1}$ 2 marks

(v) $S = -\frac{1}{4},\ P = \frac{1}{4}$

$x^2 - \left(-\frac{1}{4}\right)x + \frac{1}{4} = x^2 + \frac{1}{4}x + \frac{1}{4}$

Multiplying by $k = 4$: $\mathbf{4x^2 + x + 1}$ 2 marks

(vi) $S = 4,\ P = 1$

$\mathbf{x^2 - 4x + 1}$ 2 marks

📌 **Two things the examiner looks for.** First, the **sign of the middle term**: it is $-S$, so a negative sum (as in (v)) produces a *positive* middle term. Second, the answer is not unique — any non-zero multiple k works, so $8x^2 - 2x - 8$ is equally correct in (i). Writing "$k\left[x^2 - Sx + P\right]$, taking $k = 1$" shows you understand this and protects the mark.

— End of Exercise 2.2 — Chapter 2 Complete —

CHAPTER 3

Pair of Linear Equations in Two Variables

Exercises 3.1 – 3.3

EXERCISES 3.1 – 3.3

⚠ Important — As Per the Latest CBSE Syllabus (2026–27)

Two major topics have been **officially removed** from Chapter 3 and will **not** appear in the CBSE board exam, even though many older question banks and guides still include them:

1. **Cross-Multiplication Method** (solving using the $a_1b_2-a_2b_1$ style formula — this was earlier Exercise 3.4 in older editions)
2. **Equations Reducible to a Pair of Linear Equations** (problems needing a 1/x, 1/y type substitution, such as boats/streams with upstream-downstream speeds — this was earlier Exercise 3.5/3.6)

The current NCERT textbook contains only **Exercise 3.1** (Graphical Method), **Exercise 3.2** (Substitution Method) and **Exercise 3.3** (Elimination Method). This guide covers exactly this — nothing outdated, nothing extra. Every word problem below (including the boat-and-stream style question) has been deliberately built so it solves with plain substitution/elimination — no reciprocal substitution needed.

📊 Weightage in CBSE Class 10 Board Exam

Marks Distribution (Out of 80 — Theory Paper)

✅ Verified directly against the official CBSE curriculum document (cbseacademic.nic.in, Mathematics Code 041/241, Class X 2026–27) — not a secondary source. The topic list (graphical method, consistency/inconsistency, substitution, elimination, simple situational problems — with no cross-multiplication or reducible-equations item) and the 20-mark Algebra unit total are quoted verbatim from it. The official document does **not** publish a chapter-by-chapter mark split within that 20; the "typical share" row below is a reasonable estimate based on past years' papers, not an official figure.

Detail	Value
Unit	Algebra (Unit II)
Chapters in this unit	Polynomials, Pair of Linear Equations, Quadratic Equations, Arithmetic Progressions

Marks allotted to Unit II	**20 marks** out of 80 (the single highest-weightage unit)
Typical share for this chapter alone	**≈ 5–6 marks (estimated, not an official figure** — see note above; usually a mix of a 1-mark consistency check, a 2/3-mark solve, and often a full word problem or a CBQ)
Number of exercises to master	Only 3 — Exercise 3.1, 3.2 and 3.3

📌 This chapter is the backbone for word-problem solving skills used again in Quadratic Equations and Arithmetic Progressions — the "form two equations, then solve" habit built here pays off across the whole Algebra unit.

🎯 Predicted Question Types

Format	Marks	Likely Topic
MCQ / Assertion-Reason	1	Consistency condition facts, nature of lines (intersecting/parallel/coincident) from ratios
Very Short Answer	1	Comparing a_1/a_2, b_1/b_2, c_1/c_2 to classify a given pair of equations
Short Answer	2–3	Solve a pair by substitution or elimination method
Short/Long Answer	3	Find the value of k for a pair to have a unique / no / infinitely many solutions
Word Problem	3–5	Ages, two-digit numbers, cost of articles, fixed + variable charges
Competency-Based Question	4–5 (sub-parts)	A real-life scenario (tickets, fare, shopping) with sub-questions on forming equations, solving, and interpreting the graph

📐 All Key Formulas & Definitions

General form: A pair of linear equations in two variables is written as $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

Consistent pair: has at least one solution. **Inconsistent pair:** has no solution. **Dependent pair:** equivalent equations with infinitely many common solutions (always consistent).

Unique solution (intersecting lines): $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ — consistent.

No solution (parallel lines): $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ — inconsistent.

Infinitely many solutions (coincident lines): $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ — dependent and consistent.

Substitution Method (4 steps): (1) Express one variable in terms of the other from either equation. (2) Substitute into the other equation to get a single-variable equation. (3) Solve it. (4) Substitute back to get the other variable.

Elimination Method (4 steps): (1) Multiply one/both equations by suitable constants to make coefficients of one variable numerically equal. (2) Add or subtract to eliminate that variable. (3) Solve the resulting single-variable equation. (4) Substitute back into either original equation.

Special outcomes while solving: If you reach a true statement with no variable (like 18 = 18) → infinitely many solutions. If you reach a false statement (like 0 = 9 or −4 = 0) → no solution.

💡 Shortcuts & Exam-Writing Tips

1. **Always write both equations in the general form** $ax + by + c = 0$ before comparing ratios — a common mistake is comparing ratios directly from an unrearranged equation.
2. **Pick substitution** when a variable already has coefficient 1 or -1 in either equation — it saves a step. Otherwise, prefer elimination.
3. **For "find k" questions,** set up the ratio condition first (unique / no solution / infinite solutions), then solve the resulting equation(s) in k — never guess-and-check.
4. **Word problems:** always state clearly what x and y represent as your very first line — examiners award a mark for this alone.
5. **Digit problems:** a two-digit number with tens digit x and units digit y is always $10x + y$; the number with digits reversed is $10y + x$.
6. **Graphical questions:** plot at least 2–3 points per line using a table, use a ruler, and clearly mark the point of intersection with its coordinates.
7. **Verification is a habit,** not an option — substitute your final answer back into both original equations before writing your final answer.

8. **Never attempt the cross-multiplication formula or 1/x, 1/y substitution** in the 2026–27 exam — these are no longer in the syllabus (see notice above).

📘 NCERT Solutions — Exercise 3.1 (7 Questions, Graphical Method)

🔑 Approach for This Exercise

Form two linear equations from the situation, then either plot both lines to read the solution graphically, or compare a_1/a_2, b_1/b_2, c_1/c_2 to classify the pair without plotting.

Q1. Form the pair of linear equations and find their solutions graphically: (i) 10 students took a quiz; girls are 4 more than boys (ii) 5 pencils + 7 pens cost ₹50; 7 pencils + 5 pens cost ₹46 4 marks

GIVEN: *Two real-life situations.*

TO FIND: *Pair of equations and their graphical solution.*

SOLUTION:

(i) *Let number of boys = x, girls = y.*

$x + y = 10; \quad y = x + 4$ 1 mark

Solving: $x + (x + 4) = 10 \Rightarrow 2x = 6 \Rightarrow x = 3, \ y = 7$. **Boys = 3, Girls = 7** *(plotting both lines, they meet at (3,7))* 1 mark

(ii) *Let cost of a pencil = ₹x, cost of a pen = ₹y.*

$5x + 7y = 50; \quad 7x + 5y = 46$ 1 mark

Point tables for plotting *(take any two convenient points per line):*

5x + 7y = 50	*x = 3*	*x = 10*
y	*5*	*0*
7x + 5y = 46	*x = 3*	*x = −2*
y	*5*	*12*

Both lines pass through (3, 5), so that is the point of intersection. Reading the graph, the lines intersect at $x = 3, y = 5$. **Cost of a pencil = ₹3, cost of a pen = ₹5** 1 mark

Q2. Compare a_1/a_2, b_1/b_2, c_1/c_2 to find whether the lines intersect, are parallel, or coincide: (i) $5x-4y+8=0$; $7x+6y-9=0$ (ii) $9x+3y+12=0$; $18x+6y+24=0$ (iii) $6x-3y+10=0$; $2x-y+9=0$ 3 marks

GIVEN: *Three pairs of linear equations.*

TO FIND: *Classify each pair.*

SOLUTION:

(i) $\frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$. *Since* $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ → **lines intersect at a point** *(unique solution)* 1 mark

(ii) $\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$. *All equal* → **lines are coincident** *(infinitely many solutions)* 1 mark

(iii) $\frac{a_1}{a_2} = \frac{6}{2} = 3, \frac{b_1}{b_2} = \frac{-3}{-1} = 3, \frac{c_1}{c_2} = \frac{10}{9}$. *Since* $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ → **lines are parallel** *(no solution)* 1 mark

Q3. Compare the ratios to find out whether the following pairs are consistent or inconsistent: (i) $3x+2y=5$; $2x-3y=7$ (ii) $2x-3y=8$; $4x-6y=9$ (iii) $3x/2+5y/3=7$; $9x-10y=14$ (iv) $5x-3y=11$; $-10x+6y=-22$ (v) $4x/3+2y=8$; $2x+3y=12$ 5 marks

GIVEN: *Five pairs of linear equations.*

TO FIND: *Consistency of each pair.*

SOLUTION:

(i) $\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{2}{-3}$. *Not equal* → **consistent** *(unique solution)* 1 mark

(ii) *Standard form:* $2x - 3y - 8 = 0,\ 4x - 6y - 9 = 0$. *Ratios:* $\frac{2}{4} = \frac{1}{2}, \frac{-3}{-6} = \frac{1}{2}, \frac{-8}{-9} = \frac{8}{9}$. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ → **inconsistent** *(no solution)* 1 mark

(iii) *Ratios:* $\frac{3/2}{9} = \frac{1}{6}, \frac{5/3}{-10} = \frac{-1}{6}$. *Not equal* → **consistent** *(unique solution)* 1 mark

(iv) *Standard form:* $5x - 3y - 11 = 0,\ -10x + 6y + 22 = 0$. *Ratios:* $\frac{5}{-10} = \frac{-1}{2}, \frac{-3}{6} = \frac{-1}{2}, \frac{-11}{22} = \frac{-1}{2}$. *All equal* → **consistent** *(dependent, infinitely many solutions)* 1 mark

(v) *Standard form:* $\frac{4}{3}x + 2y - 8 = 0,\ 2x + 3y - 12 = 0$. *Ratios:* $\frac{4/3}{2} = \frac{2}{3}, \frac{2}{3}, \frac{-8}{-12} = \frac{2}{3}$. *All equal* → **consistent** *(dependent, infinitely many solutions)* 1 mark

Q4. Which pairs are consistent/inconsistent? If consistent, obtain the solution graphically: (i) x+y=5, 2x+2y=10 (ii) x−y=8, 3x−3y=16 (iii) 2x+y−6=0, 4x−2y−4=0 (iv) 2x−2y−2=0, 4x−4y−5=0 6 marks

GIVEN: *Four pairs of linear equations.*

TO FIND: *Classify each; solve graphically if consistent.*

SOLUTION:

(i) *Ratios:* $\frac{1}{2}, \frac{1}{2}, \frac{-5}{-10} = \frac{1}{2}$. *All equal* → **consistent, dependent** — *infinitely many solutions (e.g. (1,4), (2,3), (3,2) all lie on both lines).* 1½ marks

(ii) *Ratios:* $\frac{1}{3}, \frac{-1}{-3} = \frac{1}{3}, \frac{-8}{-16} = \frac{1}{2}$. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ → **inconsistent**, *no solution.* 1½ marks

(iii) *Ratios:* $\frac{2}{4} = \frac{1}{2}, \frac{1}{-2} = \frac{-1}{2}$. *Not equal* → **consistent**, *unique solution.*

From $2x + y - 6 = 0$: $y = 6 - 2x$. *Sub into* $4x - 2y - 4 = 0$:

$4x - 2(6 - 2x) - 4 = 0 \Rightarrow 4x - 12 + 4x - 4 = 0 \Rightarrow 8x = 16 \Rightarrow x = 2, y = 2$. **Solution: (2, 2)**

1½ marks

(iv) *Simplify (i):* $x - y - 1 = 0$. *Ratios:* $\frac{2}{4} = \frac{1}{2}, \frac{-2}{-4} = \frac{1}{2}, \frac{-2}{-5} = \frac{2}{5}$. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ → **inconsistent**, *no solution.* 1½ marks

Q5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden. 3 marks

GIVEN: *Length = width + 4; half-perimeter = 36 m.*

TO FIND: *Length and width.*

SOLUTION:

Let width = x m, length = y m.

$y = x + 4$ *...(1);* $x + y = 36$ *...(2)* 1 mark

Substituting (1) in (2): $x + (x + 4) = 36 \Rightarrow 2x = 32 \Rightarrow x = 16$ 1 mark

$y = 16 + 4 = 20$. **Width = 16 m, Length = 20 m** 1 mark

Q6. Given 2x + 3y − 8 = 0, write another linear equation such that the pair represents: (i) intersecting lines (ii) parallel lines (iii) coincident lines 3 marks

GIVEN: $2x + 3y - 8 = 0$.

TO FIND: *A second equation for each case.*

SOLUTION:

(i) Intersecting: *choose any equation with a different a/b ratio, e.g.* $\mathbf{3x - 2y - 5 = 0}$ *(here* $\frac{2}{3} \neq \frac{3}{-2}$*)* 1 mark

(ii) Parallel: *keep the same a/b ratio but a different c, e.g.* $\mathbf{4x + 6y - 4 = 0}$ *(ratios* $\frac{2}{4} = \frac{1}{2}$ *match* $\frac{a_1}{a_2}$*, but* $\frac{-8}{-4} = 2 \neq \frac{1}{2}$*)* 1 mark

(iii) Coincident: *multiply the whole equation by a constant, e.g.* $\mathbf{4x + 6y - 16 = 0}$ *(exactly double the original)* 1 mark

Q7. Draw the graphs of $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region. 4 marks

GIVEN: *Two linear equations.*

TO FIND: *Vertices of the triangle formed with the x-axis.*

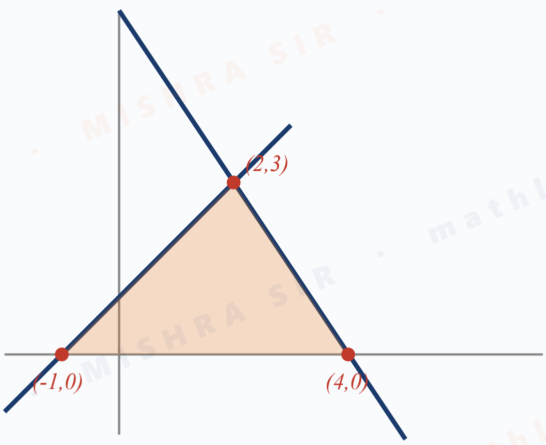
SOLUTION:

Line 1: $x - y + 1 = 0 \Rightarrow y = x + 1$. *x-intercept (y=0):* $x = -1 \Rightarrow (-1, 0)$.

Line 2: $3x + 2y - 12 = 0$. *x-intercept (y=0):* $x = 4 \Rightarrow (4, 0)$.

Intersection of Line 1 & 2: substitute $y = x + 1$ *into Line 2:*

$3x + 2(x + 1) - 12 = 0 \Rightarrow 5x - 10 = 0 \Rightarrow x = 2, y = 3.$ 2 marks



Note: *MLI-redrawn schematic — correct in construction but* **not drawn to scale**, *and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Vertices of the triangle: (−1, 0), (4, 0), (2, 3) — *the shaded region above shows the triangle formed with the x-axis.* 2 marks

— End of Exercise 3.1 —

📘 NCERT Solutions — Exercise 3.2 (3 Questions, Substitution Method)

🔑 Approach for This Exercise

Express one variable in terms of the other from the simpler-looking equation, substitute into the second equation, solve, then back-substitute.

Q1. Solve by substitution: (i) x+y=14, x−y=4 (ii) s−t=3, s/3+t/2=6 (iii) 3x−y=3, 9x−3y=9 (iv) 0.2x+0.3y=1.3, 0.4x+0.5y=2.3 (v) √2x+√3y=0, √3x−√8y=0 (vi) 3x/2−5y/3=−2, x/3+y/2=13/6 12 marks

GIVEN: *Six pairs of equations.*

TO FIND: *Solve each by substitution.*

SOLUTION:

(i) *From eq.1:* $x = 14 - y$. *Sub into eq.2:* $(14 - y) - y = 4 \Rightarrow 14 - 2y = 4 \Rightarrow y = 5,\ x = 9$ 2 marks

(ii) *From eq.1:* $s = t + 3$. *Sub:* $\frac{t+3}{3} + \frac{t}{2} = 6$. *Multiply by 6:*
$2(t + 3) + 3t = 36 \Rightarrow 5t = 30 \Rightarrow t = 6,\ s = 9$ 2 marks

(iii) *From eq.1:* $y = 3x - 3$. *Sub into eq.2:* $9x - 3(3x - 3) = 9 \Rightarrow 9x - 9x + 9 = 9 \Rightarrow 9 = 9$ *(always true)*
→ **infinitely many solutions** *(the equations are dependent)* 2 marks

(iv) *Multiply both by 10:* $2x + 3y = 13,\ 4x + 5y = 23$. *From first:* $x = \frac{13 - 3y}{2}$. *Sub:*
$4 \cdot \frac{13 - 3y}{2} + 5y = 23 \Rightarrow 2(13 - 3y) + 5y = 23 \Rightarrow 26 - y = 23 \Rightarrow y = 3,\ x = 2$ 2 marks

(v) *From eq.1:* $x = \frac{-\sqrt{3}y}{\sqrt{2}}$. *Sub into eq.2:* $\sqrt{3}\left(\frac{-\sqrt{3}y}{\sqrt{2}}\right) - \sqrt{8}y = 0 \Rightarrow \frac{-3y}{\sqrt{2}} - 2\sqrt{2}y = 0$. *Multiplying by* $\sqrt{2}$*:* $-3y - 4y = 0 \Rightarrow -7y = 0 \Rightarrow y = 0,\ x = 0$ 2 marks

(vi) *Multiply eq.1 by 6:* $9x - 10y = -12$. *Multiply eq.2 by 6:* $2x + 3y = 13 \Rightarrow x = \frac{13 - 3y}{2}$. *Sub:*
$9 \cdot \frac{13 - 3y}{2} - 10y = -12 \Rightarrow 9(13 - 3y) - 20y = -24 \Rightarrow 117 - 47y = -24 \Rightarrow y = 3,\ x = 2$ 2 marks

Q2. Solve 2x+3y=11 and 2x−4y=−24 and hence find the value of 'm' for which y = mx+3. 3 marks

GIVEN: $2x + 3y = 11,\ 2x - 4y = -24$*; also* $y = mx + 3$.

TO FIND: *x, y, and m.*

SOLUTION:

Subtracting: $(2x + 3y) - (2x - 4y) = 11 - (-24) \Rightarrow 7y = 35 \Rightarrow y = 5$ 1 mark

From eq.1: $2x + 15 = 11 \Rightarrow x = -2$ 1 mark

Substitute in $y = mx + 3$*:* $5 = m(-2) + 3 \Rightarrow -2m = 2 \Rightarrow \mathbf{m = -1}$ 1 mark

Q3. Form the pair of linear equations and solve by substitution: (i) difference of two numbers is 26, one is 3× the other (ii) larger of two supplementary angles exceeds smaller by 18° (iii) 7 bats+6 balls=₹3800, 3 bats+5 balls=₹1750 (iv) taxi: 10 km costs ₹105, 15 km costs ₹155 (v) fraction becomes 9/11 on adding 2 to both, 5/6 on adding 3 to both (vi) Jacob 5 yrs hence = 3× son's age; 5 yrs ago Jacob = 7× son's age 18 marks

GIVEN: *Six word problems.*

TO FIND: *Form equations and solve each.*

SOLUTION:

(i) *Let numbers be x, y with* $x = 3y$. $x - y = 26 \Rightarrow 3y - y = 26 \Rightarrow y = 13,\ x = 39$ 3 marks

(ii) *Let angles be x, y (x larger).* $x + y = 180,\ x - y = 18$. *Adding:* $2x = 198 \Rightarrow x = 99,\ y = 81$ 3 marks

(iii) *Let bat = ₹x, ball = ₹y.* $7x + 6y = 3800,\ 3x + 5y = 1750$. *From eq.2:* $x = \frac{1750 - 5y}{3}$. *Sub into eq.1:* $7 \cdot \frac{1750 - 5y}{3} + 6y = 3800 \Rightarrow 7(1750 - 5y) + 18y = 11400 \Rightarrow 12250 - 17y = 11400 \Rightarrow y = 50,\ x = 500.$ **Bat = ₹500, Ball = ₹50** 3 marks

(iv) *Let fixed charge = ₹x, rate/km = ₹y.* $x + 10y = 105,\ x + 15y = 155$. *Subtracting:* $5y = 50 \Rightarrow y = 10,\ x = 5$. *For 25 km:* $x + 25y = 5 + 250 = ₹\mathbf{255}$ 3 marks

(v) *Let fraction = x/y.* $\frac{x+2}{y+2} = \frac{9}{11} \Rightarrow 11x - 9y = -4;\ \frac{x+3}{y+3} = \frac{5}{6} \Rightarrow 6x - 5y = -3$. *Solving: multiply first by 5, second by 9:* $55x - 45y = -20,\ 54x - 45y = -27$. *Subtracting:* $x = 7$. *From second:* $42 - 5y = -3 \Rightarrow y = 9$. **Fraction = 7/9** 3 marks

(vi) *Let present ages be J (Jacob), S (son).* $J + 5 = 3(S + 5) \Rightarrow J = 3S + 10;$ $J - 5 = 7(S - 5) \Rightarrow J = 7S - 30$. *Equating:* $3S + 10 = 7S - 30 \Rightarrow S = 10,\ J = 40$. **Jacob = 40 yrs, Son = 10 yrs** 3 marks

— End of Exercise 3.2 —

📘 NCERT Solutions — Exercise 3.3 (2 Questions, Elimination Method)

🔑 Approach for This Exercise

Make the coefficients of one variable numerically equal by multiplying, then add or subtract the equations to eliminate it.

Q1. Solve by elimination and substitution: (i) x+y=5, 2x−3y=4 (ii) 3x+4y=10, 2x−2y=2 (iii) 3x−5y−4=0, 9x=2y+7 (iv) x/2+2y/3=−1, x−y/3=3 12 marks

GIVEN: *Four pairs of equations.*

TO FIND: *Solve each pair.*

SOLUTION:

(i) *Multiply eq.1 by 3:* $3x+3y=15$. *Add to eq.2* ($2x-3y=4$)*:* $5x=19 \Rightarrow x=\frac{19}{5},\ y=5-\frac{19}{5}=\frac{6}{5}$ 3 marks

(ii) *Simplify eq.2:* $x-y=1$. *Multiply eq.2 (original) by 2:* $4x-4y=4$. *Add to eq.1* ($3x+4y=10$)*:* $7x=14 \Rightarrow x=2,\ y=x-1=1$ 3 marks

(iii) *Standard form:* $3x-5y-4=0,\ 9x-2y-7=0$. *Multiply first by 3:* $9x-15y-12=0$. *Subtract second:* $-13y-5=0 \Rightarrow y=\frac{-5}{13}$. *From first:* $3x=4+5\left(\frac{-5}{13}\right)=\frac{27}{13} \Rightarrow x=\frac{9}{13}$ 3 marks

(iv) *Multiply eq.1 by 6:* $3x+4y=-6$. *Multiply eq.2 by 3:* $3x-y=9$. *Subtract:* $5y=-15 \Rightarrow y=-3$. *From* $3x-y=9$*:* $3x+3=9 \Rightarrow x=2$ 3 marks

Q2. Form pairs of linear equations and solve by elimination: (i) fraction reduces to 1 on (+1 num, −1 denom), becomes 1/2 on (+1 denom only) (ii) Nuri was thrice Sonu's age 5 yrs ago; twice 10 yrs later (iii) sum of digits=9, 9×number=2×reversed (iv) Meena withdrew ₹2000 in ₹50 & ₹100 notes, 25 notes total (v) library: fixed charge for 3 days + extra/day; ₹27 for 7 days, ₹21 for 5 days 15 marks

GIVEN: *Five word problems.*

TO FIND: *Form equations and solve by elimination.*

SOLUTION:

(i) *Let fraction = x/y.* $\frac{x+1}{y-1}=1 \Rightarrow x-y=-2;\ \frac{x}{y+1}=\frac{1}{2} \Rightarrow 2x-y=1$. *Subtracting:* $(2x-y)-(x-y)=1-(-2) \Rightarrow x=3$. *Then* $y=5$. **Fraction = 3/5** 3 marks

(ii) *Let present ages: Nuri = N, Sonu = S.* $N-5=3(S-5) \Rightarrow N=3S-10;$ $N+10=2(S+10) \Rightarrow N=2S+10$. *Equating:* $3S-10=2S+10 \Rightarrow S=20,\ N=50$. **Nuri = 50 yrs, Sonu = 20 yrs** 3 marks

(iii) *Let number =* $10x+y$*, digit sum* $x+y=9$. *Also* $9(10x+y)=2(10y+x) \Rightarrow 88x-11y=0 \Rightarrow 8x=y$. *Sub into* $x+y=9$*:* $x+8x=9 \Rightarrow x=1,\ y=8$. **Number = 18** 3 marks

(iv) *Let ₹50 notes = x, ₹100 notes = y.* $x+y=25;\ \ 50x+100y=2000 \Rightarrow x+2y=40$. *Subtracting:* $y=15,\ x=10$. **10 notes of ₹50, 15 notes of ₹100** 3 marks

(v) *Let fixed charge = ₹x, extra charge/day = ₹y.* $x+4y=27$ *(7 days = 3 fixed + 4 extra);* $x+2y=21$ *(5 days = 3 fixed + 2 extra). Subtracting:* $2y=6 \Rightarrow y=3,\ x=15$. **Fixed charge = ₹15, extra charge = ₹3/day** 3 marks

— End of Exercise 3.3 — Chapter 3 Complete —

📚 **Board questions for this chapter are in the companion volume.** All previous-year and most-frequently-asked CBSE questions have been moved to *MLI Class 10 Maths — CBSE Most Frequently Asked Questions & PYQ Bank*, so that this book stays what it is for: concept explanation and the complete NCERT solutions. Finish the exercises above first, then work the board bank there.

CHAPTER 4

Quadratic Equations

Exercises 4.1 – 4.3

EXERCISES 4.1 – 4.3

⚠ Important — As Per the Latest CBSE Syllabus (2026–27)

The current NCERT textbook contains only **three** exercises for this chapter — **Exercise 4.1** (checking and forming quadratic equations), **Exercise 4.2** (solving by factorisation), and **Exercise 4.3** (nature of roots using the discriminant, and the quadratic formula). Older editions and some question banks additionally featured "Solving a Quadratic Equation by Completing the Square" as its own worked-example section — the current chapter does **not** teach or test this as a separate technique; the quadratic formula is given directly and used throughout. You only need two tools for this entire chapter: **factorisation** and the **quadratic formula** (with the discriminant to judge the nature of roots).

📊 Weightage in CBSE Class 10 Board Exam

Marks Distribution (Out of 80 — Theory Paper)

✅ Verified directly against the official CBSE curriculum document (cbseacademic.nic.in, Mathematics Code 041/241, Class X 2026–27) — not a secondary source. The official topic list for this chapter is: "Standard form of a quadratic equation; Solutions of quadratic equations (only real roots) by factorization, and by using quadratic formula; Relationship between discriminant and nature of roots; Situational problems." The 20-mark Algebra unit total is quoted verbatim from it. The official document does **not** publish a chapter-by-chapter mark split within that 20; the "typical share" row below is a reasonable estimate based on past years' papers, not an official figure.

Detail	Value
Unit	Algebra (Unit II)
Chapters in this unit	Polynomials, Pair of Linear Equations, Quadratic Equations, Arithmetic Progressions
Marks allotted to Unit II	**20 marks** out of 80 (the single highest-weightage unit)

Typical share for this chapter alone	**≈ 6–7 marks (estimated, not an official figure** — see note above; this chapter usually carries the heaviest load within Algebra due to its rich word-problem base)
Number of exercises to master	Only 3 — Exercise 4.1, 4.2 and 4.3

📌 This chapter's word problems (ages, speed-distance-time, taps filling a tank, geometry) are among the most reliably repeated patterns across CBSE papers — the same handful of situations reappear year after year with only the numbers changed.

🎯 Predicted Question Types

Format	Marks	Likely Topic
MCQ / Assertion-Reason	1	Identifying whether an equation is quadratic; nature of roots from the discriminant; a root satisfying the equation
Very Short Answer	1	State the discriminant of a given equation and classify the nature of its roots
Short Answer	2	Solve a quadratic equation by factorisation
Short/Long Answer	3	Find the value of k for which an equation has equal roots
Long Answer	3–5	Word problems: ages, consecutive integers, speed-distance-time, taps/pipes, geometry (right triangles, rectangles)
Competency-Based Question	4–5 (sub-parts)	A real-life scenario (ticket pricing, area optimisation, projectile-style) with sub-questions on forming the equation, solving it, and interpreting whether the situation is possible

📐 All Key Formulas & Definitions

Standard form: A quadratic equation in x is $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.

Root of a quadratic equation: A real number α is a root of $ax^2 + bx + c = 0$ if $a\alpha^2 + b\alpha + c = 0$. The zeroes of the polynomial $ax^2 + bx + c$ and the roots of the equation $ax^2 + bx + c = 0$ are the same thing.

Maximum roots: Since a quadratic polynomial has at most 2 zeroes, a quadratic equation has at most 2 roots.

Solving by factorisation: Split the middle term so $ax^2 + bx + c$ factors into two linear factors; set each factor to zero.

Quadratic formula: The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, provided $b^2 - 4ac \geq 0$.

Discriminant: $D = b^2 - 4ac$. It determines the nature of the roots.

Nature of roots: (i) $D > 0 \rightarrow$ two distinct real roots. (ii) $D = 0 \rightarrow$ two equal real roots, both equal to $\dfrac{-b}{2a}$. (iii) $D < 0 \rightarrow$ no real roots.

Word-problem checklist: after solving, always reject roots that are negative (for lengths, ages, speeds, counts) or otherwise physically meaningless, and state clearly which root is the answer.

💡 Shortcuts & Exam-Writing Tips

1. **Before deciding if an equation is quadratic,** fully expand and simplify both sides first — terms often cancel (a cubic can collapse into a quadratic, or a seemingly-quadratic equation can collapse into linear). Never judge from the unsimplified form.
2. **Splitting the middle term fast:** for $ax^2 + bx + c$, find two numbers whose product is $a \times c$ and sum is b.
3. **"Find k for equal roots"** always means set $D = b^2 - 4ac = 0$ and solve for k — this is one of the most repeated question types in the whole syllabus.
4. **"Is it possible to..." word problems** are a discriminant test in disguise — form the equation, compute D, and if $D < 0$ the situation is impossible; state this explicitly rather than just leaving the answer blank.
5. **Always reject the negative/invalid root** in word problems with one line of reasoning ("since x denotes a length, it cannot be negative") — examiners award a mark for this explicitly.
6. **Speed-distance-time and tap/pipe problems** almost always lead to an equation with x and (x±k) in the denominator — clear denominators first by multiplying through before expanding.
7. **Quadratic formula is safest** when factorisation isn't obvious (surds, ugly coefficients) — don't waste time guessing factors past 30–40 seconds.
8. **Always double-check your quadratic form** by substituting a root back into the original (unsimplified) word-problem equation, not just the simplified one — this catches sign errors made while forming the

equation.

📘 NCERT Solutions — Exercise 4.1 (2 Questions)

🔑 **Approach for This Exercise**

Expand and fully simplify every term first. An equation is quadratic only if, after complete simplification, the highest power of x remaining is exactly 2 (with a non-zero coefficient) — higher powers must have cancelled out, and it must not have reduced further to degree 1 or 0.

Q1. Check whether the following are quadratic equations: (i) $(x+1)^2=2(x-3)$ (ii) $x^2-2x=(-2)(3-x)$ (iii) $(x-2)(x+1)=(x-1)(x+3)$ (iv) $(x-3)(2x+1)=x(x+5)$ (v) $(2x-1)(x-3)=(x+5)(x-1)$ (vi) $x^2+3x+1=(x-2)^2$ (vii) $(x+2)^3=2x(x^2-1)$ (viii) $x^3-4x^2-x+1=(x-2)^3$ 8 marks

GIVEN: *Eight equations.*

TO FIND: *Whether each is a quadratic equation.*

SOLUTION:

(i) *LHS* $= (x+1)^2 = x^2+2x+1$. *Equation becomes* $x^2+2x+1 = 2x-6 \Rightarrow x^2+7=0$. *Highest power is 2* $\rightarrow$ **Quadratic equation.** 1 mark

(ii) *RHS* $= -6+2x$. *Equation becomes* $x^2-2x = -6+2x \Rightarrow x^2-4x+6=0$. **Quadratic equation.** 1 mark

(iii) *LHS* $= x^2-x-2$, *RHS* $= x^2+2x-3$. *Equation becomes* $x^2-x-2 = x^2+2x-3 \Rightarrow -3x+1=0$. *The* x^2 *terms cancel* — **Not a quadratic equation** *(it is linear).* 1 mark

(iv) *LHS* $= 2x^2-5x-3$, *RHS* $= x^2+5x$. *Equation becomes* $2x^2-5x-3 = x^2+5x \Rightarrow x^2-10x-3=0$. **Quadratic equation.** 1 mark

(v) *LHS* $= 2x^2-7x+3$, *RHS* $= x^2+4x-5$. *Equation becomes* $x^2-11x+8=0$. **Quadratic equation.** 1 mark

(vi) *RHS* $= x^2-4x+4$. *Equation becomes* $x^2+3x+1 = x^2-4x+4 \Rightarrow 7x-3=0$. *The* x^2 *terms cancel* — **Not a quadratic equation** *(it is linear).* 1 mark

(vii) *LHS* $= (x+2)^3 = x^3+6x^2+12x+8$, *RHS* $= 2x^3-2x$. *Equation becomes* $x^3+6x^2+12x+8 = 2x^3-2x \Rightarrow x^3-6x^2-14x-8=0$. *Highest power is 3* — **Not a quadratic equation** *(it is cubic).* 1 mark

(viii) *RHS* $= (x-2)^3 = x^3-6x^2+12x-8$. *Equation becomes* $x^3-4x^2-x+1 = x^3-6x^2+12x-8 \Rightarrow 2x^2-13x+9=0$. *The* x^3 *terms cancel* — **Quadratic equation.** 1 mark

Q2. Represent the following situations in the form of quadratic equations: (i) area of rectangular plot 528 m², length is one more than twice breadth (ii) product of two consecutive positive integers is 306 (iii) Rohan's mother is 26 years older; product of their ages 3 years from now = 360 (iv) train travels 480 km at uniform speed; 8 km/h less speed takes 3 hours more 8 marks

GIVEN: *Four real-life situations.*

TO FIND: *The quadratic equation representing each.*

SOLUTION:

(i) *Let breadth* $= x$ *m, so length* $= (2x+1)$ *m. Area:* $x(2x+1) = 528 \Rightarrow \mathbf{2x^2 + x - 528 = 0}$ 2 marks

(ii) *Let the integers be* x *and* $x+1$*. Product:* $x(x+1) = 306 \Rightarrow \mathbf{x^2 + x - 306 = 0}$ 2 marks

(iii) *Let Rohan's present age* $= x$ *years; mother's* $= (x+26)$ *years. Three years from now:*

$(x+3)(x+29) = 360 \Rightarrow x^2 + 32x + 87 = 360 \Rightarrow \mathbf{x^2 + 32x - 273 = 0}$ 2 marks

(iv) *Let the speed* $= x$ *km/h. Time taken* $= \dfrac{480}{x}$*; at speed* $(x-8)$*, time taken* $= \dfrac{480}{x-8} = \dfrac{480}{x} + 3$*. Clearing denominators:* $480x - 480(x-8) = 3x(x-8) \Rightarrow 3840 = 3x^2 - 24x \Rightarrow \mathbf{x^2 - 8x - 1280 = 0}$

2 marks

— End of Exercise 4.1 —

📘 NCERT Solutions — Exercise 4.2 (6 Questions, Factorisation Method)

🔑 Approach for This Exercise

Split the middle term (product = a×c, sum = b), factorise into two linear factors, and set each to zero. For word problems: define the variable clearly, form the equation, factorise, then reject any root that doesn't make physical sense.

Q1. Find the roots by factorisation: (i) x²−3x−10=0 (ii) 2x²+x−6=0 (iii) √2x²+7x+5√2=0 (iv) 2x²−x+1/8=0 (v) 100x²−20x+1=0 10 marks

GIVEN: *Five quadratic equations.*

TO FIND: *Roots by factorisation.*

SOLUTION:

(i) $x^2 - 3x - 10 = x^2 - 5x + 2x - 10 = x(x-5) + 2(x-5) = (x-5)(x+2)$. *Roots:* $\mathbf{x = 5, -2}$ 2 marks

(ii) $2x^2 + x - 6 = 2x^2 + 4x - 3x - 6 = 2x(x+2) - 3(x+2) = (2x-3)(x+2)$. *Roots:* $\mathbf{x = \frac{3}{2}, -2}$ 2 marks

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = (\sqrt{2}x + 5)(x + \sqrt{2})$.

Roots: $\mathbf{x = -\frac{5}{\sqrt{2}} = -\frac{5\sqrt{2}}{2}, x = -\sqrt{2}}$ 2 marks

(iv) *Multiply by 8:* $16x^2 - 8x + 1 = 0 = (4x-1)^2$. *Root:* $\mathbf{x = \frac{1}{4}}$ *(repeated)* 2 marks

(v) $100x^2 - 20x + 1 = (10x-1)^2 = 0$. *Root:* $\mathbf{x = \frac{1}{10}}$ *(repeated)* 2 marks

Q2. Solve the problems given in Example 1: (i) John–Jivanti marbles (ii) cottage industry toys 4 marks

GIVEN: $x^2 - 45x + 324 = 0$ *(marbles) and* $x^2 - 55x + 750 = 0$ *(toys), from Example 1.*

TO FIND: *Solve each and interpret.*

SOLUTION:

(i) $x^2 - 45x + 324 = x^2 - 36x - 9x + 324 = x(x-36) - 9(x-36) = (x-36)(x-9)$. *Roots:* $x = 36$ *or* $x = 9$. **John had 36 marbles and Jivanti had 9, or John had 9 and Jivanti had 36** *(both are valid, since neither loses more marbles than they had).* 2 marks

(ii) $x^2 - 55x + 750 = x^2 - 25x - 30x + 750 = x(x-25) - 30(x-25) = (x-25)(x-30)$. *Roots:* $x = 25$ *or* $x = 30$. **The number of toys produced was 25 or 30** *(both are valid positive answers).* 2 marks

Q3. Find two numbers whose sum is 27 and product is 182. 2 marks

GIVEN: *Sum = 27, product = 182.*

SOLUTION:

Let one number be x, the other $(27 - x)$. *Then* $x(27 - x) = 182 \Rightarrow x^2 - 27x + 182 = 0$ 1 mark

$x^2 - 14x - 13x + 182 = x(x-14) - 13(x-14) = (x-14)(x-13) = 0$. **The two numbers are 13 and 14.** 1 mark

Q4. Find two consecutive positive integers, sum of whose squares is 365. 3 marks

GIVEN: *Two consecutive positive integers; sum of squares = 365.*

SOLUTION:

Let the integers be x and $x+1$. $x^2+(x+1)^2=365 \Rightarrow 2x^2+2x+1=365 \Rightarrow x^2+x-182=0$

1 mark

$x^2+14x-13x-182=x(x+14)-13(x+14)=(x+14)(x-13)=0$. *Roots:* $x=13$ *or* $x=-14$. *Since x must be a* **positive** *integer,* $x=-14$ *is rejected.* 1 mark

The integers are 13 and 14. *(Check:* $13^2+14^2=169+196=365$ ✓*)* 1 mark

Q5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides. 3 marks

GIVEN: *Altitude = base − 7; hypotenuse = 13 cm.*

SOLUTION:

Let base = x cm, altitude = $(x-7)$ *cm. By Pythagoras:*

$x^2+(x-7)^2=13^2 \Rightarrow 2x^2-14x+49=169 \Rightarrow x^2-7x-60=0$ 1 mark

$x^2-12x+5x-60=x(x-12)+5(x-12)=(x-12)(x+5)=0$. *Roots:* $x=12$ *or* $x=-5$. *Since base cannot be negative,* $x=-5$ *is rejected.* 1 mark

Base = 12 cm, Altitude = 5 cm. *(Check:* $12^2+5^2=144+25=169=13^2$ ✓*)* 1 mark

Q6. A cottage industry produces pottery articles. The cost of production of each article (in ₹) was 3 more than twice the number of articles produced that day. If total cost was ₹90, find the number of articles and the cost of each. 3 marks

GIVEN: *Cost per article =* $2x+3$*; total cost = ₹90.*

SOLUTION:

Let number of articles = x. Cost per article = $(2x+3)$. *Total:* $x(2x+3)=90 \Rightarrow 2x^2+3x-90=0$

1 mark

$2x^2+15x-12x-90=x(2x+15)-6(2x+15)=(2x+15)(x-6)=0$. *Roots:* $x=6$ *or* $x=-\frac{15}{2}$.

Since the number of articles must be a positive integer, the negative root is rejected. 1 mark

Number of articles = 6, Cost per article = ₹(2×6+3) = ₹15. *(Check:* $6\times 15=90$ ✓*)* 1 mark

— End of Exercise 4.2 —

📘 NCERT Solutions — Exercise 4.3 (5 Questions, Nature of Roots)

🔑 Approach for This Exercise

Compute $D = b^2 - 4ac$ first. If $D \geq 0$, apply the quadratic formula for the roots. For "find k" questions, set $D = 0$ and solve for k. For "is it possible" word problems, form the equation and test its discriminant before attempting to solve.

Q1. Find the nature of the roots; find them if real: (i) $2x^2-3x+5=0$ (ii) $3x^2-4\sqrt{3}x+4=0$ (iii) $2x^2-6x+3=0$ 6 marks

GIVEN: *Three quadratic equations.*

SOLUTION:

(i) $a = 2, b = -3, c = 5$. $D = (-3)^2 - 4(2)(5) = 9 - 40 = -31 < 0$. **No real roots.** 2 marks

(ii) $a = 3, b = -4\sqrt{3}, c = 4$. $D = (-4\sqrt{3})^2 - 4(3)(4) = 48 - 48 = 0$. **Two equal real roots**, *both* $= \frac{-b}{2a} = \frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3}$. 2 marks

(iii) $a = 2, b = -6, c = 3$. $D = 36 - 24 = 12 > 0$. **Two distinct real roots**:

$x = \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$. 2 marks

Q2. Find k for equal roots: (i) $2x^2+kx+3=0$ (ii) $kx(x-2)+6=0$ 4 marks

GIVEN: *Two equations with unknown k; equal roots required.*

SOLUTION:

(i) $a = 2, b = k, c = 3$. *Setting* $D = 0$: $k^2 - 4(2)(3) = 0 \Rightarrow k^2 = 24 \Rightarrow \mathbf{k = \pm 2\sqrt{6}}$ 2 marks

(ii) *Expanding:* $kx^2 - 2kx + 6 = 0$. *For this to be quadratic,* $k \neq 0$. *Setting* $D = 0$:

$(-2k)^2 - 4(k)(6) = 0 \Rightarrow 4k^2 - 24k = 0 \Rightarrow 4k(k-6) = 0 \Rightarrow k = 0$ *or* $k = 6$. *Since* $k = 0$ *is rejected (equation wouldn't be quadratic),* $\mathbf{k = 6}$ 2 marks

Q3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m²? If so, find its length and breadth. 3 marks

GIVEN: *Length = 2 × breadth; area = 800 m².*

SOLUTION:

Let breadth = x m, length = $2x$ *m. Area:* $2x^2 = 800 \Rightarrow x^2 - 400 = 0$ 1 mark

Here $D = 0 - 4(1)(-400) = 1600 > 0$*, so real roots exist —* **yes, it is possible.** $x^2 = 400 \Rightarrow x = \pm 20$*; rejecting the negative root,* $x = 20$. 1 mark

Breadth = 20 m, Length = 40 m. 1 mark

Q4. Is the following situation possible? The sum of the ages of two friends is 20 years. Four years ago, the product of their ages was 48. If so, find their present ages. 3 marks

GIVEN: *Sum of present ages = 20; product of ages 4 years ago = 48.*

SOLUTION:

Let one friend's present age = x, the other's = $(20 - x)$*. Four years ago:*

$(x - 4)(20 - x - 4) = 48 \Rightarrow (x - 4)(16 - x) = 48$ 1 mark

$16x - x^2 - 64 + 4x = 48 \Rightarrow -x^2 + 20x - 112 = 0 \Rightarrow x^2 - 20x + 112 = 0$ 1 mark

$D = (-20)^2 - 4(1)(112) = 400 - 448 = -48 < 0$*. Since D is negative, there is no real solution —* **this situation is not possible.** 1 mark

Q5. Is it possible to design a rectangular park of perimeter 80 m and area 400 m²? If so, find its length and breadth. 3 marks

GIVEN: *Perimeter = 80 m; area = 400 m².*

SOLUTION:

Since perimeter = $2(l + b) = 80$*, we get* $l + b = 40$*. Let length = x, so breadth =* $(40 - x)$*. Area:*

$x(40 - x) = 400 \Rightarrow x^2 - 40x + 400 = 0$ 1 mark

$D = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$*. Since* $D = 0$*, real (equal) roots exist —* **yes, it is possible.**

1 mark

$x = \frac{40}{2} = 20$. **Length = Breadth = 20 m** *(the park is a square).* 1 mark

— End of Exercise 4.3 — Chapter 4 Complete —

📚 **Board questions for this chapter are in the companion volume.** All previous-year and most-frequently-asked CBSE questions have been moved to *MLI Class 10 Maths — CBSE Most Frequently Asked Questions &*

PYQ Bank, so that this book stays what it is for: concept explanation and the complete NCERT solutions. Finish the exercises above first, then work the board bank there.

CHAPTER 5

Arithmetic Progressions

Exercises 5.1 – 5.3 (Ex 5.4 is the optional exercise — not examinable)

EXERCISE 5.1

CBSE Step-wise Marking Scheme Format **Board Exam 2026–27** **Name: _______________**

💡 **CBSE Marking Scheme Guidelines Followed in This Document**

1. Every word problem is structured as **Given** → **To Find** → **Solution**, matching CBSE's official answer-key format.
2. Each step of the solution is written on its own line and tagged with its mark allocation.
3. As per CBSE policy, method/formula marks are earned independently of the final answer — so every step is shown, not just the final result.

EXERCISE 5.1

Q1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why? **4**

(i) The taxi fare after each km, when the fare is ₹15 for the first km and ₹8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs ₹150 for the first metre and rises by ₹50 for each subsequent metre.

(iv) The amount of money in the account every year, when ₹10000 is deposited at compound interest at 8% per annum.

(i) Given: *Fare for 1st km = ₹15; fare for each additional km = ₹8.*

To Find: *Whether the fares for 1, 2, 3, ... km form an AP.*

Solution:

Fare for 1st km = 15

Fare for 2nd km = 15 + 8 = 23

Fare for 3rd km = 23 + 8 = 31

Fare for 4th km = 31 + 8 = 39 **[1 mark]**

So the list of fares is: 15, 23, 31, 39, ...

Check the differences between consecutive terms:

$a_2 - a_1 = 23 - 15 = 8$

$a_3 - a_2 = 31 - 23 = 8$

$a_4 - a_3 = 39 - 31 = 8$ **[1 mark]**

Since the difference between consecutive terms is the same constant value (8), **this list forms an AP**, *with first term 15 and common difference 8.* **[½ mark]**

(ii) Given: Let the initial amount of air in the cylinder = V. Each time, the pump removes $\frac{1}{4}$ of the air remaining, leaving $\frac{3}{4}$ of it.

To Find: Whether the quantities of air remaining after each stroke form an AP.

Solution:

Air remaining after 1st stroke = $\frac{3}{4}V$

Air remaining after 2nd stroke = $\frac{3}{4} \times \frac{3}{4}V = \left(\frac{3}{4}\right)^2 V$

Air remaining after 3rd stroke = $\left(\frac{3}{4}\right)^3 V$ [1 mark]

So the list is: $V,\ \frac{3}{4}V,\ \left(\frac{3}{4}\right)^2 V,\ \left(\frac{3}{4}\right)^3 V, \ldots$

Here, each term is obtained by **multiplying** the previous term by $\frac{3}{4}$ — not by adding a fixed number.

So the difference between consecutive terms is NOT constant. [1 mark]

Therefore, **this list does NOT form an AP**. [½ mark]

(iii) Given: Cost of digging 1st metre = ₹150; cost rises by ₹50 for each subsequent metre.

To Find: Whether the costs of digging 1, 2, 3, ... metres form an AP.

Solution:

Cost for 1st metre = 150

Cost for 2nd metre = 150 + 50 = 200

Cost for 3rd metre = 200 + 50 = 250

Cost for 4th metre = 250 + 50 = 300 [1 mark]

So the list of costs is: 150, 200, 250, 300, ...

Check the differences:

$a_2 - a_1 = 200 - 150 = 50$

$a_3 - a_2 = 250 - 200 = 50$

$a_4 - a_3 = 300 - 250 = 50$ [1 mark]

Since the difference between consecutive terms is constant (50), **this list forms an AP**, with first term 150 and common difference 50. [½ mark]

(iv) Given: Principal = ₹10000, Rate = 8% per annum, compounded annually.

To Find: Whether the amounts at the end of year 1, 2, 3, ... form an AP.

Solution:

Using the compound interest formula, amount after n years is:

$A_n = 10000\left(1 + \frac{8}{100}\right)^n$ [1 mark]

Amount after year 1 = $10000(1.08)^1$

Amount after year 2 = $10000(1.08)^2$

Amount after year 3 = $10000(1.08)^3$ [1 mark]

Here, consecutive terms are related by **multiplication** by a fixed factor (1.08), not by addition of a fixed number.

So the difference between consecutive terms is NOT constant.

Therefore, **this list does NOT form an AP**. [½ mark]

Q2.

Write first four terms of the AP, when the first term a and the common difference d are given as follows:

(i) a = 10, d = 10 (ii) a = −2, d = 0 (iii) a = 4, d = −3 (iv) a = −1, d = $\frac{1}{2}$ (v) a = −1.25, d = −0.25

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Given: First term (a) and common difference (d) for five different APs.

To Find: First four terms of each AP.

Solution: Using the general form of an AP:

$a,\ a + d,\ a + 2d,\ a + 3d$ [½ mark for formula]

(i) a = 10, d = 10:

1st term = 10

2nd term = 10 + 10 = 20

3rd term = 20 + 10 = 30

4th term = 30 + 10 = 40

∴ AP = **10, 20, 30, 40** [1 mark]

(ii) a = −2, d = 0:

Every term = −2 (since d = 0, no change occurs)

∴ AP = **−2, −2, −2, −2** [1 mark]

(iii) a = 4, d = −3:

1st term = 4

2nd term = 4 − 3 = 1

3rd term = 1 − 3 = −2

4th term = −2 − 3 = −5

∴ AP = **4, 1, −2, −5** [1 mark]

(iv) a = −1, d = $\frac{1}{2}$:

1st term = −1

2nd term = $-1 + \frac{1}{2} = -\frac{1}{2}$

3rd term = $-\frac{1}{2} + \frac{1}{2} = 0$

4th term = $0 + \frac{1}{2} = \frac{1}{2}$

∴ AP = **−1,** $-\frac{1}{2}$**, 0,** $\frac{1}{2}$ [1 mark]

(v) a = −1.25, d = −0.25:

1st term = −1.25

2nd term = −1.25 − 0.25 = −1.50

3rd term = −1.50 − 0.25 = −1.75

4th term = −1.75 − 0.25 = −2.00

∴ AP = **−1.25, −1.50, −1.75, −2.00** [1 mark]

Q3.

For the following APs, write the first term and the common difference:

(i) 3, 1, −1, −3, ... (ii) −5, −1, 3, 7, ... (iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots$ (iv) 0.6, 1.7, 2.8, 3.9, ...

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Given: Four different arithmetic progressions.

To Find: First term (a) and common difference (d) of each.

Solution: Using $d = a_2 - a_1$ [½ mark for formula]

(i) First term a = 3

d = $a_2 - a_1 = 1 - 3 = \mathbf{-2}$ [1 mark]

(ii) First term a = −5

d = $a_2 - a_1 = -1 - (-5) = \mathbf{4}$ [1 mark]

(iii) First term a = $\frac{1}{3}$

d = $a_2 - a_1 = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$ ⇒ d = $\frac{4}{3}$ [1 mark]

(iv) First term a = 0.6

d = $a_2 - a_1 = 1.7 - 0.6 = \mathbf{1.1}$ [1 mark]

Q4.

Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

(i) 2, 4, 8, 16, ... (ii) 2, $\frac{5}{2}$, 3, $\frac{7}{2}$, ... (iii) −1.2, −3.2, −5.2, −7.2, ... (iv) −10, −6, −2, 2, ...

(v) 3, 3+√2, 3+2√2, 3+3√2, ... (vi) 0.2, 0.22, 0.222, 0.2222, ... (vii) 0, −4, −8, −12, ...

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$, ... (ix) 1, 3, 9, 27, ... (x) a, 2a, 3a, 4a, ...

(xi) a, a^2, a^3, a^4, ... (xii) √2, √8, √18, √32, ... (xiii) √3, √6, √9, √12, ...

(xiv) 1^2, 3^2, 5^2, 7^2, ... (xv) 1^2, 5^2, 7^2, 73, ...

15

Given: Fifteen different lists of numbers.

To Find: Which lists form an AP; if so, state d and write three more terms.

Solution: Check whether $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$ (i.e., a constant difference) for each list. [½ mark for method]

(i) 2, 4, 8, 16:

Differences: $4 - 2 = 2$; $8 - 4 = 4$; $16 - 8 = 8$

Differences are NOT equal. ⇒ **Not an AP.** [1 mark]

(ii) 2, $\frac{5}{2}$, 3, $\frac{7}{2}$:

Differences: $\frac{5}{2} - 2 = \frac{1}{2}$; $3 - \frac{5}{2} = \frac{1}{2}$; $\frac{7}{2} - 3 = \frac{1}{2}$

Differences are equal. ⇒ **AP, with d = $\frac{1}{2}$**

Next 3 terms: $\frac{7}{2} + \frac{1}{2} = 4$; $4 + \frac{1}{2} = \frac{9}{2}$; $\frac{9}{2} + \frac{1}{2} = 5$

∴ Next three terms: **4, $\frac{9}{2}$, 5** [1 mark]

(iii) −1.2, −3.2, −5.2, −7.2:

Differences: $-3.2 - (-1.2) = -2$; $-5.2 - (-3.2) = -2$; $-7.2 - (-5.2) = -2$

Differences are equal. ⇒ **AP, with d = −2**

Next 3 terms: −9.2, −11.2, −13.2 [1 mark]

(iv) −10, −6, −2, 2:

Differences: $-6-(-10)=4;\ \ -2-(-6)=4;\ \ 2-(-2)=4$

Differences are equal. ⇒ **AP, with d = 4**

Next 3 terms: 6, 10, 14 [1 mark]

(v) 3, 3+√2, 3+2√2, 3+3√2:

Differences: $(3+\sqrt{2})-3=\sqrt{2};\ \ (3+2\sqrt{2})-(3+\sqrt{2})=\sqrt{2};\ \ (3+3\sqrt{2})-(3+2\sqrt{2})=\sqrt{2}$

Differences are equal. ⇒ **AP, with d = √2**

Next 3 terms: $3+4\sqrt{2},\ 3+5\sqrt{2},\ 3+6\sqrt{2}$ [1 mark]

(vi) 0.2, 0.22, 0.222, 0.2222:

Differences: $0.22-0.2=0.02;\ \ 0.222-0.22=0.002;\ \ 0.2222-0.222=0.0002$

Differences are NOT equal. ⇒ **Not an AP.** [1 mark]

(vii) 0, −4, −8, −12:

Differences: $-4-0=-4;\ \ -8-(-4)=-4;\ \ -12-(-8)=-4$

Differences are equal. ⇒ **AP, with d = −4**

Next 3 terms: −16, −20, −24 [1 mark]

(viii) $-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}$:

Differences: all equal to 0

Differences are equal. ⇒ **AP, with d = 0**

Next 3 terms: $-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}$ [1 mark]

(ix) 1, 3, 9, 27:

Differences: $3-1=2;\ \ 9-3=6;\ \ 27-9=18$

Differences are NOT equal. ⇒ **Not an AP.** [1 mark]

(x) a, 2a, 3a, 4a:

Differences: $2a-a=a;\ \ 3a-2a=a;\ \ 4a-3a=a$

Differences are equal. ⇒ **AP, with d = a**

Next 3 terms: 5a, 6a, 7a [1 mark]

(xi) a, a^2, a^3, a^4:

Differences: $a^2 - a$; $a^3 - a^2$; $a^4 - a^3$

These are equal only for special values of a (e.g. a=0 or a=1), not in general.

⇒ **Not an AP** (in general). [1 mark]

(xii) √2, √8, √18, √32:

Simplify each term first: $\sqrt{8} = 2\sqrt{2}$; $\sqrt{18} = 3\sqrt{2}$; $\sqrt{32} = 4\sqrt{2}$

So the list is: $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}$

Differences: $2\sqrt{2} - \sqrt{2} = \sqrt{2}$; $3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$; $4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

Differences are equal. ⇒ **AP, with d = √2**

Next 3 terms: $5\sqrt{2}(= \sqrt{50})$, $6\sqrt{2}(= \sqrt{72})$, $7\sqrt{2}(= \sqrt{98})$ [1 mark]

(xiii) √3, √6, √9, √12:

Simplify: $\sqrt{9} = 3$; $\sqrt{12} = 2\sqrt{3}$

Differences: $\sqrt{6} - \sqrt{3} \approx 0.73$; $3 - \sqrt{6} \approx 0.55$; $2\sqrt{3} - 3 \approx 0.46$

Differences are NOT equal. ⇒ **Not an AP.** [1 mark]

(xiv) 1^2, 3^2, 5^2, 7^2 = 1, 9, 25, 49:

Differences: $9 - 1 = 8$; $25 - 9 = 16$; $49 - 25 = 24$

Differences are NOT equal. ⇒ **Not an AP.** [1 mark]

(xv) 1^2, 5^2, 7^2, 73 = 1, 25, 49, 73:

Differences: $25 - 1 = 24$; $49 - 25 = 24$; $73 - 49 = 24$

Differences are equal. ⇒ **AP, with d = 24**

Next 3 terms: 97, 121, 145 [1 mark]

EXERCISE 5.2

CBSE Step-wise Marking Scheme FormatBoard Exam 2026–27Name: _______________

💡 Key Formula for This Exercise

General (nth) term of an AP: $a_n = a + (n-1)d$ — write this first in every answer to secure the formula mark.

EXERCISE 5.2

Q1.

Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the nth term of the AP.

5

Given: Partial values of a, d, n, a_n for five APs.

To Find: The missing quantity in each row.

Solution: In every part, we use the formula:

$a_n = a + (n-1)d$ [½ mark for formula]

(i) Given: a=7, d=3, n=8. Find a_8.

$a_8 = 7 + (8-1)(3)$

$a_8 = 7 + 21$

$a_8 = \mathbf{28}$ [1 mark]

(ii) Given: a=−18, n=10, a_{10}=0. Find d.

$0 = -18 + (10-1)d$

$0 = -18 + 9d$

$9d = 18$

$d = \mathbf{2}$ [1 mark]

(iii) Given: d=−3, n=18, a_{18}=−5. Find a.

$-5 = a + (18-1)(-3)$

$-5 = a - 51$

$a = -5 + 51$

$a = \mathbf{46}$ [1 mark]

(iv) Given: a=−18.9, d=2.5, a_n=3.6. Find n.

$3.6 = -18.9 + (n-1)(2.5)$

$3.6 + 18.9 = 2.5(n-1)$

$22.5 = 2.5(n-1)$

$n - 1 = 9$

$n = \mathbf{10}$ [1 mark]

(v) Given: a=3.5, d=0, n=105. Find a_{105}.

$a_{105} = 3.5 + (105-1)(0)$

$a_{105} = 3.5 + 0$

$a_{105} = \mathbf{3.5}$ [1 mark]

Q2.

Choose the correct choice and justify:

(i) 30th term of the AP: 10, 7, 4, ..., is (A) 97 (B) 77 (C) −77 (D) −87

(ii) 11th term of the AP: −3, $-\frac{1}{2}$, 2, ..., is (A) 28 (B) 22 (C) −38 (D) $-48\frac{1}{2}$

2

(i) Given: AP is 10, 7, 4, ...; need the 30th term.

To Find: a_{30}, and the correct MCQ option.

Solution:

Here a=10, d = 7−10 = −3, n=30 [½ mark]

$a_{30} = 10 + (30-1)(-3)$

$a_{30} = 10 + 29(-3)$

$a_{30} = 10 - 87$

$a_{30} = \mathbf{-77}$ [½ mark]

∴ **Correct option: (C)**

(ii) Given: AP is −3, $-\frac{1}{2}$, 2, ...; need the 11th term.

To Find: a_{11}, and the correct MCQ option.

Solution:

Here a=−3, $d = -\frac{1}{2} - (-3) = -\frac{1}{2} + 3 = \frac{5}{2}$, n=11 [½ mark]

$a_{11} = -3 + (11-1)\left(\frac{5}{2}\right)$

$a_{11} = -3 + 10\left(\frac{5}{2}\right)$

$a_{11} = -3 + 25$

$a_{11} = \mathbf{22}$ [½ mark]

∴ **Correct option: (B)**

Q3.

In the following APs, find the missing terms in the boxes:

(i) 2, □, 26 (ii) □, 13, □, 3 (iii) 5, □, □, $9\frac{1}{2}$ (iv) −4, □, □, □, □, 6 (v) □, 38, □, □, □, −22

5

Given: Five incomplete APs with some terms missing.

To Find: The missing terms in each.

Solution:

(i) Here $a_1 = 2$, $a_3 = 26$ (this AP has 3 terms).

Since $a_3 = a_1 + 2d$:

$26 = 2 + 2d$

$2d = 24$

$d = 12$

Missing term $a_2 = 2 + 12 = \mathbf{14}$ [1 mark]

(ii) Here $a_2 = 13$, $a_4 = 3$ (4 terms total).

Since $a_4 = a_2 + 2d$:

$3 = 13 + 2d$

$2d = -10$

$d = -5$

$a_1 = a_2 - d = 13 - (-5) = \mathbf{18}$

$a_3 = a_2 + d = 13 + (-5) = \mathbf{8}$

∴ The AP is: 18, 13, 8, 3 [1 mark]

(iii) Here $a_1 = 5$, $a_4 = 9\frac{1}{2} = 9.5$.

Since $a_4 = a_1 + 3d$:

$9.5 = 5 + 3d$

$3d = 4.5$

$d = 1.5$

$a_2 = 5 + 1.5 = \mathbf{6.5}$

$a_3 = 6.5 + 1.5 = \mathbf{8}$

∴ The AP is: 5, 6.5, 8, 9.5 [1 mark]

(iv) Here $a_1 = -4$, $a_6 = 6$.

Since $a_6 = a_1 + 5d$:

$6 = -4 + 5d$

$5d = 10$

$d = 2$

$a_2 = -4 + 2 = \mathbf{-2}$

$a_3 = -2 + 2 = \mathbf{0}$

$a_4 = 0 + 2 = \mathbf{2}$

$a_5 = 2 + 2 = \mathbf{4}$

∴ The AP is: −4, −2, 0, 2, 4, 6 [1 mark]

(v) Here $a_2 = 38$, $a_6 = -22$.

Since $a_6 = a_2 + 4d$:

$-22 = 38 + 4d$

$4d = -60$

$d = -15$

$a_1 = 38 - (-15) = \mathbf{53}$

$a_3 = 38 + (-15) = \mathbf{23}$

$a_4 = 23 - 15 = \mathbf{8}$

$a_5 = 8 - 15 = \mathbf{-7}$

∴ The AP is: 53, 38, 23, 8, −7, −22 [1 mark]

Q4.

Which term of the AP: 3, 8, 13, 18, ..., is 78?

2

Given: AP: 3, 8, 13, 18, ...; a term equal to 78.

To Find: The position (n) of the term 78.

Solution:

Here a=3, d=8−3=5 [½ mark]

Let $a_n = 78$:

$78 = 3 + (n-1)(5)$

$75 = 5(n-1)$

$n - 1 = 15$

$n = \mathbf{16}$ [1 mark]

Therefore, the **16th term** of the AP is 78. [½ mark]

Q5.

Find the number of terms in each of the following APs:

(i) 7, 13, 19, ..., 205 (ii) 18, $15\frac{1}{2}$, 13, ..., −47

3

(i) Given: AP: 7, 13, 19, ..., 205.

To Find: Number of terms n.

Solution:

Here a=7, d=13−7=6, a_n=205 [½ mark]

$205 = 7 + (n-1)(6)$

$198 = 6(n-1)$

$n-1 = 33$

$n = \mathbf{34}$ [1 mark]

(ii) Given: AP: 18, $15\frac{1}{2}$, 13, ..., −47.

To Find: Number of terms n.

Solution:

Here a=18, d=15.5−18=−2.5, a_n=−47 [½ mark]

$-47 = 18 + (n-1)(-2.5)$

$-65 = -2.5(n-1)$

$n-1 = 26$

$n = \mathbf{27}$ [1 mark]

Q6.

Check whether −150 is a term of the AP: 11, 8, 5, 2, ...

2

Given: AP: 11, 8, 5, 2, ...

To Find: Whether −150 is a term of this AP.

Solution:

Here a=11, d=8−11=−3 [½ mark]

Suppose −150 is the nth term:

$-150 = 11 + (n-1)(-3)$

$-161 = -3(n-1)$

$n-1 = \frac{161}{3}$

$n-1 = 53.67$

$n = 54.67$ [1 mark]

Since n is not a positive integer, **−150 is NOT a term** of this AP. [½ mark]

Q7.

Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

3

Given: $a_{11} = 38$, $a_{16} = 73$.

To Find: a_{31}

Solution:

$a + 10d = 38$ ...(1)

$a + 15d = 73$ ...(2) [1 mark]

Subtracting (1) from (2):

$5d = 35$

$d = 7$

Substituting d=7 in (1):

$a + 10(7) = 38$

$a = 38 - 70 = -32$ [1 mark]

Now, $a_{31} = a + 30d$

$a_{31} = -32 + 30(7)$

$a_{31} = -32 + 210$

$a_{31} = \mathbf{178}$ [1 mark]

Q8.

An AP consists of 50 terms of which the 3rd term is 12 and the last term is 106. Find the 29th term.

3

Given: n=50, $a_3 = 12$, $a_{50} = 106$.

To Find: a_{29}

Solution:

$a + 2d = 12$ ...(1)

$a + 49d = 106$ ...(2) [1 mark]

Subtracting (1) from (2):

$47d = 94$

$d = 2$

Substituting d=2 in (1):

$a + 2(2) = 12$

$a = 8$ [1 mark]

Now, $a_{29} = a + 28d$

$a_{29} = 8 + 28(2)$

$a_{29} = 8 + 56$

$a_{29} = \mathbf{64}$ [1 mark]

Q9.

If the 3rd and the 9th terms of an AP are 4 and −8 respectively, which term of this AP is zero?

3

Given: $a_3 = 4$, $a_9 = -8$.

To Find: The value of n for which $a_n = 0$.

Solution:

$a + 2d = 4$ ...(1)

$a + 8d = -8$ ...(2) [1 mark]

Subtracting (1) from (2):

$6d = -12$

$d = -2$

Substituting d=−2 in (1):

$a + 2(-2) = 4$

$a = 8$ [1 mark]

Let $a_n = 0$:

$8 + (n-1)(-2) = 0$

$-2(n-1) = -8$

$n - 1 = 4$

$n = \mathbf{5}$

Therefore, the **5th term** of this AP is zero. [1 mark]

Q10.

The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

2

Given: $a_{17} - a_{10} = 7$

To Find: Common difference d.

Solution:

$[a + 16d] - [a + 9d] = 7$ [1 mark]

$16d - 9d = 7$

$7d = 7$

$d = \mathbf{1}$ [1 mark]

Q11.

Which term of the AP: 3, 15, 27, 39, ... will be 132 more than its 54th term?

3

Given: AP: 3, 15, 27, 39, ...

To Find: n such that $a_n - a_{54} = 132$

Solution:

Here a=3, d=12 [½ mark]

$[a + (n-1)d] - [a + 53d] = 132$

$(n-54)d = 132$

$(n-54)(12) = 132$

$n - 54 = 11$

$n = \mathbf{65}$ [1½ marks]

Q12.

Two APs have the same common difference. The difference between their 100th terms is 100. What is the difference between their 1000th terms?

2

Given: Two APs with the same d; $a_{100} - a'_{100} = 100$.

To Find: $a_{1000} - a'_{1000}$

Solution:

$[a + 99d] - [a' + 99d] = 100$

$a - a' = 100$ ...(*) [1 mark]

Now, $a_{1000} - a'_{1000} = [a + 999d] - [a' + 999d]$

$= a - a'$

From (*), this equals $\mathbf{100}$ [1 mark]

(The difference between corresponding terms of two APs with the same d is always constant.)

Q13.

How many three-digit numbers are divisible by 7?

3

Given: Three-digit numbers divisible by 7.

To Find: Count of such numbers.

Solution:

Smallest 3-digit multiple of 7: $100 \div 7 = 14.28...$, so smallest multiple is $7 \times 15 = 105$

Largest 3-digit multiple of 7: $999 \div 7 = 142.71...$, so largest multiple is $7 \times 142 = 994$ [1 mark]

So the AP is: 105, 112, ..., 994 with a=105, d=7, a_n=994 [½ mark]

$994 = 105 + (n-1)(7)$

$889 = 7(n-1)$

$n - 1 = 127$

$n = \mathbf{128}$ [1½ marks]

Q14.

How many multiples of 4 lie between 10 and 250?

3

Given: Multiples of 4 between 10 and 250.

To Find: Count of such multiples.

Solution:

First multiple of 4 after 10 = 12

Last multiple of 4 before 250 = 248 [1 mark]

So the AP is: 12, 16, ..., 248 with a=12, d=4, a_n=248 [½ mark]

$248 = 12 + (n-1)(4)$

$236 = 4(n-1)$

$n - 1 = 59$

$n = \mathbf{60}$ [1½ marks]

Q15.

For what value of n are the nth terms of two APs: 63, 65, 67, ... and 3, 10, 17, ... equal?

3

Given: Two APs: 63, 65, 67, ... and 3, 10, 17, ...

To Find: n such that the nth terms are equal.

Solution:

For the first AP: a=63, d=2 ⇒ $a_n = 63 + (n-1)(2)$

For the second AP: a=3, d=7 ⇒ $a'_n = 3 + (n-1)(7)$ [1 mark]

Setting the nth terms equal:

$63 + (n-1)(2) = 3 + (n-1)(7)$

$63 - 3 = (n-1)(7-2)$

$60 = 5(n-1)$ [1 mark]

$n - 1 = 12$

$n = \mathbf{13}$ [1 mark]

Q16.

Determine the AP whose 3rd term is 16 and the 7th term exceeds the 5th term by 12.

3

Given: $a_3 = 16$; $a_7 - a_5 = 12$.

To Find: The AP (a and d).

Solution:

$a + 2d = 16$ ...(1) [½ mark]

$[a + 6d] - [a + 4d] = 12$

$2d = 12$

$d = 6$ [1 mark]

Substituting d=6 in (1):

$a + 2(6) = 16$

$a = 4$

Required AP: $a,\ a+d,\ a+2d,\ a+3d,\ \ldots$ = **4, 10, 16, 22, 28, ...** [1½ marks]

Q17.

Find the 20th term from the last term of the AP: 3, 8, 13, ..., 253.

3

Given: AP: 3, 8, 13, ..., 253.

To Find: 20th term from the last term.

Solution:

Here a=3, d=5, l=253. First, find total number of terms n: [½ mark]

$253 = 3 + (n-1)(5)$

$250 = 5(n-1)$

$n - 1 = 50$

$n = 51$ [1 mark]

The 20th term from the last is the $(51 - 20 + 1) = 32$nd term from the start.

$a_{32} = 3 + 31(5)$

$a_{32} = 3 + 155$

$a_{32} = \mathbf{158}$ [1½ marks]

(Alternative method: reverse the AP so a=253, d=−5, then find the 20th term:

$253 + 19(-5) = 253 - 95 = 158$ — *same answer.)*

Q18.

The sum of the 4th and 8th terms of an AP is 24, and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

3

Given: $a_4 + a_8 = 24$; $a_6 + a_{10} = 44$.

To Find: First three terms.

Solution:

$[a + 3d] + [a + 7d] = 24$

$2a + 10d = 24$

$a + 5d = 12$ ...(1)

$[a + 5d] + [a + 9d] = 44$

$2a + 14d = 44$

$a + 7d = 22$ ...(2) [1 mark]

Subtracting (1) from (2):

$2d = 10$

$d = 5$

Substituting d=5 in (1):

$a + 5(5) = 12$

$a = 12 - 25 = -13$ [1 mark]

First three terms: $a,\ a + d,\ a + 2d$ = **−13, −8, −3** [1 mark]

Q19.

Subba Rao started work in 1995 at an annual salary of ₹5000 and received an increment of ₹200 each year. In which year did his income reach ₹7000?

3

Given: Starting salary a=₹5000 (year 1995), annual increment d=₹200.

To Find: The year when income = ₹7000.

Solution:

$7000 = 5000 + (n-1)(200)$ [1 mark]

$2000 = 200(n-1)$

$n - 1 = 10$

$n = 11$ [1 mark]

So his income reached ₹7000 in the 11th year, i.e., in the year:

$1995 + (11-1) = \mathbf{2005}$ [1 mark]

Q20.

Ramkali saved ₹5 in the first week of a year and then increased her weekly savings by ₹1.75. If in the nth week, her weekly savings become ₹20.75, find n.

3

Given: a=₹5, d=₹1.75, a_n=₹20.75.

To Find: n.

Solution:

$20.75 = 5 + (n-1)(1.75)$ [1½ marks]

$15.75 = 1.75(n-1)$

$n - 1 = 9$

$n = \mathbf{10}$ [1½ marks]

EXERCISE 5.3

CBSE Step-wise Marking Scheme FormatBoard Exam 2026–27Name: ______________

💡 Key Formulas for This Exercise

Sum of first n terms: $S_n = \frac{n}{2}[2a + (n-1)d]$ or $S_n = \frac{n}{2}(a+l)$, where l = last term.

Relationship between term and sum: $a_n = S_n - S_{n-1}$. Always write the formula first — it secures the method mark even if a later arithmetic step goes wrong.

EXERCISE 5.3

Q1.

Find the sum of the following APs:

(i) 2, 7, 12, ..., to 10 terms (ii) −37, −33, −29, ..., to 12 terms

(iii) 0.6, 1.7, 2.8, ..., to 100 terms (iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$, ..., to 11 terms

8

Given: Four APs with the number of terms specified.

To Find: Sum of the specified number of terms in each.

Solution: In every part we use:

$S_n = \frac{n}{2}[2a + (n-1)d]$ [½ mark for formula]

(i) Here a=2, d=7−2=5, n=10.

$S_{10} = \frac{10}{2}[2(2) + (10-1)(5)]$

$S_{10} = 5[4 + 45]$

$S_{10} = 5(49)$

$S_{10} = \mathbf{245}$ [1½ marks]

(ii) Here a=−37, d=−33−(−37)=4, n=12.

$S_{12} = \frac{12}{2}[2(-37) + (12-1)(4)]$

$S_{12} = 6[-74 + 44]$

$S_{12} = 6(-30)$

$S_{12} = \mathbf{-180}$ [1½ marks]

(iii) Here a=0.6, d=1.7−0.6=1.1, n=100.

$S_{100} = \frac{100}{2}[2(0.6) + (100-1)(1.1)]$

$S_{100} = 50[1.2 + 108.9]$

$S_{100} = 50(110.1)$

$S_{100} = \mathbf{5505}$ [1½ marks]

(iv) Here a=$\frac{1}{15}$, $d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$, n=11.

$S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\left(\frac{1}{60}\right)\right]$

$S_{11} = \frac{11}{2}\left[\frac{2}{15} + \frac{10}{60}\right]$

$S_{11} = \frac{11}{2}\left[\frac{8}{60} + \frac{10}{60}\right]$

$S_{11} = \frac{11}{2} \times \frac{18}{60}$

$S_{11} = \frac{11}{2} \times \frac{3}{10}$

$S_{11} = \mathbf{1.65}$ [1½ marks]

Q2.

Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 + \ldots + 84$ (ii) 34 + 32 + 30 + ... + 10 (iii) −5 + (−8) + (−11) + ... + (−230)

6

Given: Three AP series with first and last terms specified.

To Find: Sum of each series.

Solution: First find n using $a_n = a + (n-1)d$, then apply $S_n = \frac{n}{2}(a+l)$. [½ mark for method]

(i) Here a=7, d=3.5, l=84.

$84 = 7 + (n-1)(3.5)$

$77 = 3.5(n-1)$

$n - 1 = 22$

$n = 23$ [½ mark]

$S_{23} = \frac{23}{2}(7+84)$

$S_{23} = \frac{23}{2}(91)$

$S_{23} = \mathbf{1046.5}$ [1 mark]

(ii) Here a=34, d=32−34=−2, l=10.

$10 = 34 + (n-1)(-2)$

$-24 = -2(n-1)$

$n - 1 = 12$

$n = 13$ [½ mark]

$S_{13} = \frac{13}{2}(34+10)$

$S_{13} = \frac{13}{2}(44)$

$S_{13} = \mathbf{286}$ [1 mark]

(iii) Here a=−5, d=−8−(−5)=−3, l=−230.

$-230 = -5 + (n-1)(-3)$

$-225 = -3(n-1)$

$n - 1 = 75$

$n = 76$ [½ mark]

$S_{76} = \frac{76}{2}(-5 + (-230))$

$S_{76} = 38(-235)$

$S_{76} = \mathbf{-8930}$ [1 mark]

Q3.

In an AP, find the missing quantities:

(i) a=5, d=3, a_n=50; find n and S_n

(ii) a=7, a_{13}=35; find d and S_{13}

(iii) a_{12}=37, d=3; find a and S_{12}

(iv) a_3=15, S_{10}=125; find d and a_{10}

(v) d=5, S_9=75; find a and a_9

(vi) a=2, d=8, S_n=90; find n and a_n

(vii) a=8, a_n=62, S_n=210; find n and d

(viii) a_n=4, d=2, S_n=−14; find n and a

(ix) a=3, n=8, S=192; find d

(x) l=28, S=144, n=9; find a

20

Given: Ten different sets of AP parameters, two known values each.

To Find: The missing quantities in each part.

Solution:

(i) a=5, d=3, a_n=50.

$a_n = a + (n-1)d$

$50 = 5 + (n-1)(3)$

$45 = 3(n-1)$

$n - 1 = 15$

$n = \mathbf{16}$ [1 mark]

$S_{16} = \frac{16}{2}(5 + 50)$

$S_{16} = 8(55)$

$S_{16} = \mathbf{440}$ [1 mark]

(ii) a=7, a_{13}=35.

$35 = 7 + (13-1)d$

$28 = 12d$

$d = \mathbf{\frac{7}{3}}$ [1 mark]

$S_{13} = \frac{13}{2}(7+35)$

$S_{13} = \frac{13}{2}(42)$

$S_{13} = \mathbf{273}$ [1 mark]

(iii) a_{12}=37, d=3.

$37 = a + (12-1)(3)$

$37 = a + 33$

$a = \mathbf{4}$ [1 mark]

$S_{12} = \frac{12}{2}(4+37)$

$S_{12} = 6(41)$

$S_{12} = \mathbf{246}$ [1 mark]

(iv) a_3=15, S_{10}=125.

$a + 2d = 15$ ...(1)

$\frac{10}{2}[2a+9d] = 125 \Rightarrow 2a + 9d = 25$ ...(2) [1 mark]

From (1): a=15−2d. Substituting in (2):

$2(15-2d) + 9d = 25$

$30 - 4d + 9d = 25$

$30 + 5d = 25$

$5d = -5$

$d = \mathbf{-1}$

Then a=15−2(−1)=17.

$a_{10} = 17 + 9(-1)$

$a_{10} = \mathbf{8}$ [1 mark]

(v) d=5, S_9=75.

$\frac{9}{2}[2a + 8(5)] = 75$

$2a+40=\frac{150}{9}$

$2a+40=\frac{50}{3}$

$2a=\frac{50}{3}-40=\frac{50-120}{3}=\frac{-70}{3}$

$a=-\mathbf{\frac{35}{3}}$ [1 mark]

$a_9=a+8d=-\frac{35}{3}+40=\frac{-35+120}{3}$

$a_9=\mathbf{\frac{85}{3}}$ [1 mark]

(vi) a=2, d=8, S_n=90.

$\frac{n}{2}[2(2)+(n-1)(8)]=90$

$n[4+8n-8]=180$

$n(8n-4)=180$

$8n^2-4n-180=0$

$2n^2-n-45=0$ [1 mark]

Using the quadratic formula:

$n=\frac{1\pm\sqrt{1+360}}{4}=\frac{1\pm 19}{4}$

Taking the positive root: $n=\frac{20}{4}=\mathbf{5}$ (n=−4.5 rejected, as n must be a positive integer)

$a_5=2+4(8)=\mathbf{34}$ [1 mark]

(vii) a=8, a_n=62, S_n=210.

$S_n=\frac{n}{2}(a+a_n)$

$210=\frac{n}{2}(8+62)$

$210=35n$

$n=\mathbf{6}$ [1 mark]

$a_n=a+(n-1)d$

$62=8+5d$

$5d=54$

$d=\mathbf{10.8}$ [1 mark]

(viii) a_n=4, d=2, S_n=−14.

$a_n=a+(n-1)(2)=4$

$a=4-2(n-1)=6-2n$ ...(1)

$S_n=\frac{n}{2}(a+4)=-14$

$n(a+4)=-28$ ...(2) [1 mark]

Substituting (1) into (2):

$n(6-2n+4)=-28$

$n(10-2n)=-28$

$10n-2n^2=-28$

$2n^2-10n-28=0$

$n^2-5n-14=0$

$(n-7)(n+2)=0$

So n=7 or n=−2. Since n must be positive, **n=7**.

From (1): $a=6-2(7)=\mathbf{-8}$ [1 mark]

(ix) a=3, n=8, S=192.

$S_n=\frac{n}{2}[2a+(n-1)d]$

$192=\frac{8}{2}[2(3)+7d]$

$192=4[6+7d]$

$48=6+7d$

$7d=42$

$d=\mathbf{6}$ [2 marks]

(x) l=28, S=144, n=9.

$S=\frac{n}{2}(a+l)$

$144=\frac{9}{2}(a+28)$

$32=a+28$

$a=\mathbf{4}$ [2 marks]

Q4.

How many terms of the AP: 9, 17, 25, ... must be taken to give a sum of 636?

3

Given: AP: 9, 17, 25, ...; S_n=636.

To Find: n.

Solution:

Here a=9, d=8 [½ mark]

$636=\frac{n}{2}[18+8(n-1)]$

$1272=n[18+8n-8]$

$1272=n[10+8n]$

$8n^2+10n-1272=0$

$4n^2+5n-636=0$ [1 mark]

Using the quadratic formula:
$n = \frac{-5 \pm \sqrt{25 + 4(4)(636)}}{8} = \frac{-5 \pm \sqrt{10201}}{8} = \frac{-5 \pm 101}{8}$
Taking the positive root: $n = \frac{96}{8} = \mathbf{12}$ (negative root rejected) [1½ marks]

Q5.

The first term of an AP is 5, the last term is 45, and the sum is 400. Find the number of terms and the common difference.

3

Given: a=5, l=45, S=400.

To Find: n and d.

Solution:

$S_n = \frac{n}{2}(a + l)$

$400 = \frac{n}{2}(5 + 45)$

$400 = \frac{n}{2}(50)$

$400 = 25n$

$n = \mathbf{16}$ [1½ marks]

Using $a_n = a + (n-1)d$:

$45 = 5 + (16 - 1)d$

$40 = 15d$

$d = \mathbf{\frac{8}{3}}$ [1½ marks]

Q6.

The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there, and what is their sum?

3

Given: a=17, l=350, d=9.

To Find: n and S_n.

Solution:

$a_n = a + (n-1)d$

$350 = 17 + (n-1)(9)$

$333 = 9(n-1)$

$n - 1 = 37$

$n = \mathbf{38}$ [1½ marks]

$S_{38} = \frac{38}{2}(17 + 350)$

$S_{38} = 19(367)$

$S_{38} = \mathbf{6973}$ [1½ marks]

Q7.

Find the sum of first 22 terms of an AP in which d = 7 and the 22nd term is 149.

3

Given: d=7, a_{22}=149, n=22.

To Find: S_{22}

Solution:

$a_{22} = a + 21d$

$149 = a + 21(7)$

$149 = a + 147$

$a = 2$ [1½ marks]

$S_{22} = \frac{22}{2}(a + a_{22})$

$S_{22} = 11(2 + 149)$

$S_{22} = 11(151)$

$S_{22} = \mathbf{1661}$ [1½ marks]

Q8.

Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

3

Given: a_2=14, a_3=18.

To Find: S_{51}

Solution:

d = $a_3 - a_2 = 18 - 14 = 4$

$a_2 = a + d = 14$

$a = 14 - 4 = 10$ [1 mark]

$S_{51} = \frac{51}{2}[2a + (51 - 1)d]$

$S_{51} = \frac{51}{2}[2(10) + 50(4)]$

$S_{51} = \frac{51}{2}[20 + 200]$

$S_{51} = \frac{51}{2}(220)$

$S_{51} = \mathbf{5610}$ [2 marks]

Q9.

If the sum of the first 7 terms of an AP is 49, and that of 17 terms is 289, find the sum of the first n terms.

3

Given: S_7=49, S_{17}=289.

To Find: S_n (general formula).

Solution:

$\frac{7}{2}[2a+6d]=49 \Rightarrow 2a+6d=14 \Rightarrow a+3d=7$ ...(1)

$\frac{17}{2}[2a+16d]=289 \Rightarrow 2a+16d=34 \Rightarrow a+8d=17$ ...(2) [1 mark]

Subtracting (1) from (2):

$5d=10$

$d=2$

Substituting d=2 in (1):

$a+3(2)=7$

$a=1$ [1 mark]

$S_n=\frac{n}{2}[2(1)+(n-1)(2)]$

$S_n=\frac{n}{2}[2+2n-2]$

$S_n=\frac{n}{2}(2n)$

$S_n=\mathbf{n^2}$ [1 mark]

(Check: $S_7=7^2=49$ ✓, $S_{17}=17^2=289$ ✓)

Q10.

Show that $a_1, a_2, \ldots, a_n, \ldots$ form an AP where a_n is defined as: (i) $a_n=3+4n$ (ii) $a_n=9-5n$. Also find S_{15} in each case.

4

Given: General term formulas for two sequences.

To Find: Show each is an AP; find S_{15}.

Solution:

(i) $a_n=3+4n$

$a_1=3+4(1)=7$

$a_2=3+4(2)=11$

$a_3=3+4(3)=15$

$a_2-a_1=11-7=4$

$a_3-a_2=15-11=4$ — constant difference ⇒ **it is an AP**, with a=7, d=4. [1 mark]

$S_{15}=\frac{15}{2}[2(7)+(15-1)(4)]$

$S_{15}=\frac{15}{2}[14+56]$

$S_{15}=\frac{15}{2}(70)$

$S_{15}=\mathbf{525}$ [1 mark]

(ii) $a_n=9-5n$

$a_1=9-5(1)=4$

$a_2 = 9 - 5(2) = -1$

$a_3 = 9 - 5(3) = -6$

$a_2 - a_1 = -1 - 4 = -5$

$a_3 - a_2 = -6 - (-1) = -5$ — constant difference ⇒ **it is an AP**, with a=4, d=−5. [1 mark]

$S_{15} = \frac{15}{2}[2(4) + (15-1)(-5)]$

$S_{15} = \frac{15}{2}[8 - 70]$

$S_{15} = \frac{15}{2}(-62)$

$S_{15} = \mathbf{-465}$ [1 mark]

Q11.

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term? Sum of first two terms? Second term? Find the 3rd, 10th, and nth terms.

4

Given: $S_n = 4n - n^2$.

To Find: $a_1, S_2, a_2, a_3, a_{10}, a_n$.

Solution:

First term: $a_1 = S_1 = 4(1) - 1^2 = 4 - 1 = \mathbf{3}$ [½ mark]

Sum of first two terms: $S_2 = 4(2) - 2^2 = 8 - 4 = \mathbf{4}$ [½ mark]

Second term: $a_2 = S_2 - S_1 = 4 - 3 = \mathbf{1}$ [½ mark]

Third term: $S_3 = 4(3) - 3^2 = 12 - 9 = 3$; $a_3 = S_3 - S_2 = 3 - 4 = \mathbf{-1}$ [½ mark]

Tenth term: $S_{10} = 4(10) - 10^2 = 40 - 100 = -60$; $S_9 = 4(9) - 9^2 = 36 - 81 = -45$

$a_{10} = S_{10} - S_9 = -60 - (-45) = \mathbf{-15}$ [1 mark]

nth term:

$a_n = S_n - S_{n-1}$

$a_n = [4n - n^2] - [4(n-1) - (n-1)^2]$

$a_n = 4n - n^2 - 4n + 4 + (n-1)^2$

$a_n = -n^2 + 4 + n^2 - 2n + 1$

$a_n = \mathbf{5 - 2n}$ [1 mark]

(Check: $a_1 = 5 - 2(1) = 3$ ✓, $a_2 = 5 - 4 = 1$ ✓, $a_{10} = 5 - 20 = -15$ ✓)

Q12.

Find the sum of the first 40 positive integers divisible by 6.

2

Given: Multiples of 6; n=40.

To Find: S_{40}

Solution:

Multiples of 6 form an AP: 6, 12, 18, ...; a=6, d=6 [½ mark]

$S_{40} = \frac{40}{2}[2(6) + (40 - 1)(6)]$

$S_{40} = 20[12 + 234]$

$S_{40} = 20(246)$

$S_{40} = \mathbf{4920}$ [1½ marks]

Q13.

Find the sum of the first 15 multiples of 8.

2

Given: Multiples of 8; n=15.

To Find: S_{15}

Solution:

Multiples of 8 form an AP: 8, 16, 24, ...; a=8, d=8 [½ mark]

$S_{15} = \frac{15}{2}[2(8) + (15 - 1)(8)]$

$S_{15} = \frac{15}{2}[16 + 112]$

$S_{15} = \frac{15}{2}(128)$

$S_{15} = 15(64)$

$S_{15} = \mathbf{960}$ [1½ marks]

Q14.

Find the sum of the odd numbers between 0 and 50.

2

Given: Odd numbers between 0 and 50: 1, 3, ..., 49.

To Find: Their sum.

Solution:

Here a=1, d=2, l=49; number of terms n=25 [½ mark]

$S_{25} = \frac{25}{2}(1 + 49)$

$S_{25} = \frac{25}{2}(50)$

$S_{25} = 25(25)$

$S_{25} = \mathbf{625}$ [1½ marks]

Q15.

A contract specifies a penalty for delay: ₹200 for the first day, ₹250 for the second, ₹300 for the third, each day ₹50 more than the previous. Find the total penalty for a 30-day delay.

3

Given: Penalty AP: 200, 250, 300, ...; n=30 days.

To Find: Total penalty S_{30}.

Solution:

Here a=200, d=50 [1 mark]

$S_{30} = \frac{30}{2}[2(200) + (30-1)(50)]$

$S_{30} = 15[400 + 1450]$

$S_{30} = 15(1850)$

$S_{30} = \mathbf{27750}$ [1 mark]

Total penalty = **₹27,750** [1 mark]

Q16.

A sum of ₹700 is used to give 7 cash prizes, each ₹20 less than the preceding one. Find the value of each prize.

3

Given: S_7=700, d=−20, n=7.

To Find: Each prize value.

Solution:

$S_7 = \frac{7}{2}[2a + (7-1)(-20)] = 700$

$2a - 120 = \frac{1400}{7}$

$2a - 120 = 200$

$2a = 320$

$a = 160$ [1½ marks]

Prizes (using $a, a+d, a+2d, \ldots$): **₹160, ₹140, ₹120, ₹100, ₹80, ₹60, ₹40** [1½ marks]

(Check: sum = 160+140+120+100+80+60+40 = 700 ✓)

Q17.

Students of a school plant trees such that each section of each class plants as many trees as the class number (Class I → 1 tree, Class II → 2 trees, ... Class XII → 12 trees), with 3 sections per class. How many trees in total?

3

Given: Trees per section: 1, 2, ..., 12; 3 sections per class.

To Find: Total trees planted.

Solution:

Trees planted by one section of all classes:

$S_{12} = \frac{12}{2}[2(1) + (12-1)(1)]$

$S_{12} = 6[2 + 11]$

$S_{12} = 6(13)$

$S_{12} = 78$ [2 marks]

Since there are 3 sections of each class, total trees:

$= 3 \times 78 = \mathbf{234}$ [1 mark]

Q18.

A spiral is made of 13 successive semicircles with radii 0.5, 1.0, 1.5, 2.0, ... cm. Find the total length of the spiral. (π=22/7)

3

Given: Semicircle radii form AP: a=0.5, d=0.5, n=13.

To Find: Total spiral length.

Solution:

Length of one semicircle of radius r = πr.

Total length = π × (sum of radii) [1 mark]

Sum of radii:

$S_{13} = \frac{13}{2}[2(0.5) + (13 - 1)(0.5)]$

$S_{13} = \frac{13}{2}[1 + 6]$

$S_{13} = \frac{13}{2}(7)$

$S_{13} = 45.5$ cm [1 mark]

Total length $= \frac{22}{7} \times 45.5$

$= 22 \times 6.5$

$= \mathbf{143}\ \mathrm{cm}$ [1 mark]

Q19.

200 logs are stacked: 20 in the bottom row, 19 in the next, 18 next, and so on. In how many rows are the 200 logs placed, and how many logs in the top row?

3

Given: Logs per row: 20, 19, 18, ...; S_n=200.

To Find: n (rows) and a_n (top row logs).

Solution:

Here a=20, d=−1.

$S_n = \frac{n}{2}[2(20) + (n - 1)(-1)] = 200$

$n[40 - (n - 1)] = 400$

$n[41 - n] = 400$

$41n - n^2 = 400$

$n^2 - 41n + 400 = 0$ [1 mark]

Using the quadratic formula:

$n = \frac{41 \pm \sqrt{1681 - 1600}}{2} = \frac{41 \pm \sqrt{81}}{2} = \frac{41 \pm 9}{2}$

So n=25 or n=16 [½ mark]

Checking n=25: $a_{25} = 20 + 24(-1) = -4$ — **not possible** (logs cannot be negative)

Checking n=16: $a_{16} = 20 + 15(-1) = 5$ — **valid**

Therefore, there are **16 rows**, with **5 logs** in the top row. [1½ marks]

Q20.

In a potato race, a bucket is 5 m from the first potato; other potatoes are 3 m apart, 10 potatoes total. A competitor picks up potatoes one at a time, running back to the bucket each time. Find the total distance run.

3

Given: Distance to 1st potato = 5 m; potatoes 3 m apart; 10 potatoes; competitor runs to-and-fro.

To Find: Total distance run.

Solution:

Distance for 1st round trip = $2 \times 5 = 10$ m

Distance for 2nd round trip = $2 \times (5 + 3) = 2 \times 8 = 16$ m

Distance for 3rd round trip = $2 \times (5 + 6) = 2 \times 11 = 22$ m

These distances form an AP: 10, 16, 22, ...; a=10, d=6, n=10 [1½ marks]

$S_{10} = \frac{10}{2}[2(10) + (10 - 1)(6)]$

$S_{10} = 5[20 + 54]$

$S_{10} = 5(74)$

$S_{10} = \mathbf{370}\ \mathrm{m}$ [1½ marks]

Therefore, the total distance the competitor has to run is **370 m**.

EXERCISE 5.4

CBSE Step-wise Marking Scheme FormatBoard Exam 2026–27Name: ______________

ℹ️ Note

These questions are marked "not from the examination point of view" in the NCERT textbook, but are included here in full CBSE marking-scheme format for complete conceptual practice and internal assessment/portfolio use.

EXERCISE 5.4 (Optional)

Q1.

Which term of the AP: 121, 117, 113, ..., is its first negative term?

3

Given: AP: 121, 117, 113, ...

To Find: The position of the first negative term.

Solution:

Here a=121, d=117−121=−4 [½ mark]

We need the smallest n for which $a_n < 0$:

$a_n = 121 + (n-1)(-4) < 0$

$121 - 4n + 4 < 0$

$125 - 4n < 0$

$125 < 4n$

$n > 31.25$ [1½ marks]

Since n must be a whole number, the smallest such n is $n = \mathbf{32}$. [1 mark]

Check: $a_{31} = 121 - 120 = 1$ *(positive);* $a_{32} = 121 - 124 = -3$ *(first negative)* ✓

Q2.

The sum of the third and the seventh terms of an AP is 6, and their product is 8. Find the sum of the first sixteen terms of the AP.

4

Given: $a_3 + a_7 = 6$; $a_3 \times a_7 = 8$.

To Find: S_{16}

Solution:

$[a+2d] + [a+6d] = 6$

$2a + 8d = 6$

$a + 4d = 3$ ...(1) [½ mark]

$[a+2d][a+6d] = 8$ ...(2)

From (1): a=3−4d. Substituting in (2):

$[3 - 4d + 2d][3 - 4d + 6d] = 8$

$[3 - 2d][3 + 2d] = 8$

$9 - 4d^2 = 8$

$4d^2 = 1$

$d^2 = \frac{1}{4}$

$d = \pm\frac{1}{2}$ [1½ marks]

Case 1: d = $\frac{1}{2}$

$a = 3 - 4\left(\frac{1}{2}\right) = 3 - 2 = 1$

$S_{16} = \frac{16}{2}\left[2(1) + (16 - 1)\left(\frac{1}{2}\right)\right]$

$S_{16} = 8[2 + 7.5]$

$S_{16} = 8(9.5)$

$S_{16} = \mathbf{76}$ [1 mark]

Case 2: d = $-\frac{1}{2}$

$a = 3 - 4\left(-\frac{1}{2}\right) = 3 + 2 = 5$

$S_{16} = \frac{16}{2}\left[2(5) + (16 - 1)\left(-\frac{1}{2}\right)\right]$

$S_{16} = 8[10 - 7.5]$

$S_{16} = 8(2.5)$

$S_{16} = \mathbf{20}$ [1 mark]

Therefore, the sum of the first 16 terms is either **76** or **20** (both are valid solutions).

Q3.

A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and bottom rungs are $2\frac{1}{2}$ m apart, what is the length of wood required for the rungs?

3

Given: Rung spacing = 25 cm; rung lengths from 45 cm to 25 cm; total gap = $2\frac{1}{2}$ m = 250 cm.

To Find: Total length of wood for all rungs.

Solution:

Number of gaps between rungs = $\frac{250}{25} = 10$

Number of rungs = $10 + 1 = \mathbf{11}$ [1 mark]

Rung lengths form an AP with a=45, l=25, n=11:

$S_{11} = \frac{11}{2}(a+l)$

$S_{11} = \frac{11}{2}(45+25)$

$S_{11} = \frac{11}{2}(70)$

$S_{11} = \mathbf{385}\ \mathrm{cm}$ [2 marks]

Q4.

The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of house numbers preceding house x equals the sum of house numbers following it. Find this value of x.

4

Given: Houses numbered 1 to 49.

To Find: The value of x satisfying the given condition.

Solution:

Sum of house numbers before house x:

$1 + 2 + \ldots + (x-1) = \frac{(x-1)x}{2}$

Sum of house numbers after house x:

$(x+1) + (x+2) + \ldots + 49$

$= \frac{49 \times 50}{2} - \frac{x(x+1)}{2}$

$= 1225 - \frac{x(x+1)}{2}$ [1½ marks]

Setting the two sums equal:

$\frac{(x-1)x}{2} = 1225 - \frac{x(x+1)}{2}$

Multiplying throughout by 2:

$(x-1)x = 2450 - x(x+1)$

$x^2 - x = 2450 - x^2 - x$

$2x^2 = 2450$

$x^2 = 1225$

$x = \pm 35$ [1½ marks]

Since x represents a house number (must be positive and ≤ 49):

$x = \mathbf{35}$ [1 mark]

Check: Sum of houses 1 to 34 = $\frac{34 \times 35}{2} = 595$*. Sum of houses 36 to 49 =* $1225 - \frac{35 \times 36}{2} = 1225 - 630 = 595$ ✓

Q5.

A small terrace at a football ground comprises 15 steps, each 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m. Calculate the total volume of concrete required to build the terrace.

4

Given: 15 steps, each 50 m long; rise = $\frac{1}{4}$ m; tread = $\frac{1}{2}$ m per step.

To Find: Total volume of concrete.

Solution:

Volume of the 1st step = $\frac{1}{4} \times \frac{1}{2} \times 50$ m^3

Volume of the 2nd step = $2 \times \frac{1}{4} \times \frac{1}{2} \times 50$ m^3 (its height accumulates 2 rises)

In general, volume of the kth step = $k \times \frac{1}{4} \times \frac{1}{2} \times 50$ m^3 [1½ marks]

Total volume = sum of volumes of all 15 steps:

$= \left(\frac{1}{4} \times \frac{1}{2} \times 50\right) \times (1 + 2 + 3 + \ldots + 15)$ [1 mark]

$= 6.25 \times \frac{15 \times 16}{2}$

$= 6.25 \times 120$

$= \mathbf{750\ m^3}$ [1½ marks]

Therefore, the total volume of concrete required is **750 m³**.

🏆 Exam Tip — CBSE Marking Scheme Reminders

1. Always begin word problems with **Given** and **To Find** — examiners specifically look for this structure in board answer sheets.
2. Write the formula before substituting values — this earns the "method mark" independently of the final numeric answer.
3. Show every algebraic step on its own line (do not skip or combine steps) — each correct step earns marks even if a later step has an arithmetic slip.
4. Box or underline the final answer, and include units (₹, cm, m, m^3, etc.) wherever applicable.
5. When a quadratic equation yields two roots, always test both for validity in the context of the problem before rejecting one.

CHAPTER 6

Triangles

Exercises 6.1 – 6.3

EXERCISES 6.1 – 6.3

📖 About the figure numbers in this chapter

Several NCERT questions here begin "In Fig. 6.18…", "In Fig. 6.35…" and so on. Those figures belong to the NCERT textbook and are **not reproduced** here.

- **You can still follow every solution.** Each one opens by stating the configuration in words — which points lie on which sides, which lines are parallel, where the right angles are — so the reasoning is complete on its own.
- **Keep your NCERT book open beside this** while you work through Exercises 6.2 and 6.3, and **copy each figure into your own notebook** before attempting the question. In the board exam a geometry answer without a figure loses a mark, so the habit matters.
- Where a figure carries information that the words alone cannot convey, MLI has redrawn it below the question — see Fig. 6.17, Fig. 6.35 and Fig. 6.38.

📘 NCERT Solutions — Exercise 6.1 (3 Questions)

🔑 Approach for This Exercise

This exercise fixes the **meaning** of similarity before any theorem is used. Keep two facts straight: **congruent** means same shape *and* same size; **similar** means same shape, size may differ. So every congruent pair is similar, but not the reverse. For polygons with the same number of sides, similarity needs **both** conditions together — equal corresponding angles *and* proportional corresponding sides. One alone is not enough, and Q3 is built to expose exactly that.

Q1. Fill in the blanks using the correct word given in brackets: (i) All circles are ___ (congruent, similar) (ii) All squares are ___ (similar, congruent) (iii) All ___ triangles are similar (isosceles, equilateral) (iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are ___ and (b) their corresponding sides are ___ (equal, proportional) 4 marks

SOLUTION:

(i) similar. Every circle has the same shape; only the radius changes. Two circles are congruent only when their radii are equal, so "similar" is the word that is always true. 1 mark

(ii) similar. Every square has four right angles and four equal sides, so the shape never changes — only the side length does. 1 mark

(iii) equilateral. Every equilateral triangle has all three angles 60°, so any two of them are equiangular and hence similar. Isosceles triangles do *not* qualify: one isosceles triangle may have apex angle 40° and another 100°, and those are not similar. 1 mark

(iv) (a) equal (b) proportional. 1 mark

> 📌 **Why (iii) trips students.** "All isosceles triangles are similar" feels right because they look alike, but only two sides are forced to be equal — the angles are still free. Equilateral is the only triangle whose angles are fixed by its name.

Q2. Give two different examples of pair of (i) similar figures (ii) non-similar figures. 2 marks

SOLUTION:

(i) Similar figures — same shape, different size:

• Any two circles (say of radii 3 cm and 5 cm).

• Any two equilateral triangles (say of sides 2 cm and 6 cm).

(Two squares of different sides, or two regular hexagons, are equally acceptable.) 1 mark

(ii) Non-similar figures — shapes that differ:

• A circle and a square.

• A square and a rhombus that is not a square (all four sides equal in both, but the angles differ — so the shape is not the same).

(A rectangle and a square, or a scalene and an equilateral triangle, also work.) 1 mark

> 📌 **Pick examples that prove the point.** The square-and-rhombus pair is the strongest answer for (ii), because it shows that equal sides alone do not give similarity — the angles must match too.

Q3. State whether the following quadrilaterals (Fig. 6.8) are similar or not. 2 marks

SOLUTION:

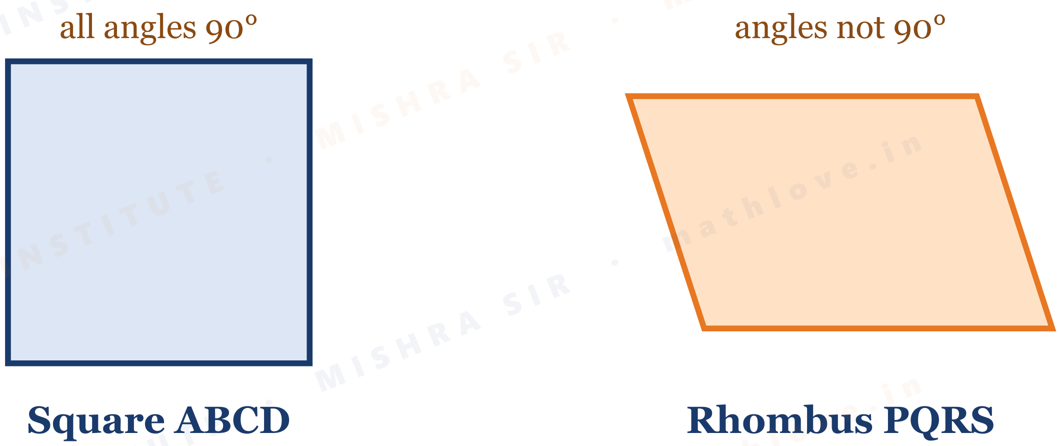


Fig. 6.8 — a square and a rhombus. All eight sides are equal in length, so the sides are in proportion; but the angles do not match.

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

The two quadrilaterals are **NOT similar**. **1 mark**

Reason. For two polygons to be similar, *both* conditions must hold.
• *Sides:* each figure has all four sides equal, so corresponding sides are in the same ratio ✓
• *Angles:* the square has every angle 90°, while the rhombus has two acute and two obtuse angles ✗
Since the corresponding angles are not equal, the second condition fails and the figures are not similar. **1 mark**

📌 **This is the whole lesson of Exercise 6.1.** Proportional sides alone are not enough. The mirror case — equal angles but unequal side ratios — is a square and a rectangle, which is also not similar.

— End of Exercise 6.1 —

📘 NCERT Solutions — Exercise 6.2 (10 Questions)

🔑 Approach for This Exercise

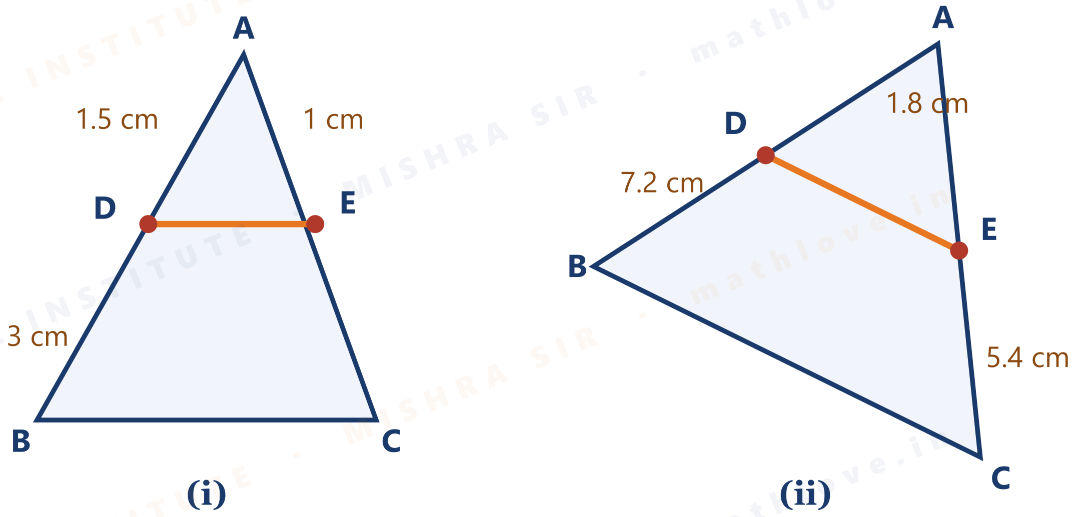
Everything here rests on one theorem and its converse.

Theorem 6.1 (Basic Proportionality Theorem / Thales). If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio: if $DE \parallel BC$ then $\frac{AD}{DB} = \frac{AE}{EC}$.
Theorem 6.2 (Converse). If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Use **6.1** when parallelism is *given* and a length is wanted; use **6.2** when lengths are given and parallelism must be *proved*. In Q3–Q6 the trick is to apply BPT twice in two different triangles and then chain the two results together.

Q1. In Fig. 6.17 (i) and (ii), DE || BC. Find EC in (i) and AD in (ii). **4 marks**

SOLUTION:



***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Fig. 6.17 — redrawn schematic. In both triangles $DE \parallel BC$, so the Basic Proportionality Theorem applies.

(i) To find EC. Given $AD = 1.5$ cm, $DB = 3$ cm, $AE = 1$ cm.

Since $DE \parallel BC$, by BPT: $\frac{AD}{DB} = \frac{AE}{EC}$ **1 mark**

$$\frac{1.5}{3} = \frac{1}{EC}$$

Cross-multiplying: $1.5 \times EC = 3 \times 1 \Rightarrow EC = \frac{3}{1.5}$

$\mathbf{EC = 2}$ cm **1 mark**

(ii) To find AD. Given $DB = 7.2$ cm, $AE = 1.8$ cm, $EC = 5.4$ cm.

By BPT: $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ **1 mark**

$\dfrac{AD}{7.2} = \dfrac{1.8}{5.4} = \dfrac{1}{3}$

$AD = \dfrac{7.2}{3}$

$\mathbf{AD = 2.4}\ \text{cm}$ **1 mark**

> 📌 **Watch which segment is which.** BPT pairs AD with AE (the two pieces next to vertex A) and DB with EC (the two pieces next to the base). Writing $\frac{AD}{AE} = \frac{DB}{EC}$ is also valid, but mixing one from each pairing — such as $\frac{AD}{DB} = \frac{EC}{AE}$ — inverts the ratio and loses the answer.

Q2. E and F are points on the sides PQ and PR respectively of a △PQR. For each of the following cases, state whether EF || QR: (i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm, FR = 2.4 cm (ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm (iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm **6 marks**

SOLUTION:

Here we use the **converse** (Theorem 6.2): compute $\dfrac{PE}{EQ}$ and $\dfrac{PF}{FR}$; if they are equal, then $EF \parallel QR$.

(i) $\dfrac{PE}{EQ} = \dfrac{3.9}{3} = 1.3$ and $\dfrac{PF}{FR} = \dfrac{3.6}{2.4} = 1.5$

Since $1.3 \neq 1.5$, the two ratios are unequal.

⇒ **EF is NOT parallel to QR.** **2 marks**

(ii) $\dfrac{PE}{EQ} = \dfrac{4}{4.5} = \dfrac{8}{9}$ and $\dfrac{PF}{FR} = \dfrac{8}{9}$

The ratios are equal.

⇒ **EF is parallel to QR.** **2 marks**

(iii) Here the *whole* sides are given, so first find the remaining pieces by subtraction:

$EQ = PQ - PE = 1.28 - 0.18 = 1.10$ cm

$FR = PR - PF = 2.56 - 0.36 = 2.20$ cm **1 mark**

$\dfrac{PE}{EQ} = \dfrac{0.18}{1.10} = \dfrac{9}{55}$ and $\dfrac{PF}{FR} = \dfrac{0.36}{2.20} = \dfrac{9}{55}$

The ratios are equal.

⇒ **EF is parallel to QR.** **1 mark**

> 📌 **The trap in (iii).** The question gives PQ and PR, not EQ and FR. Comparing $\frac{PE}{PQ}$ with $\frac{PF}{PR}$ happens to give the same verdict here, but the theorem is stated for the ratio of the *two parts* — do the subtraction and show it, because that step carries a mark.

Q3. In Fig. 6.18, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$. 3 marks

SOLUTION:

The idea is to apply BPT **twice** — once in each triangle — and then equate through the common ratio $\frac{AL}{AC}$.

Step 1 — In △ABC, with $LM \parallel CB$:

By BPT, $\frac{AM}{AB} = \frac{AL}{AC}$ ...(1) 1 mark

Step 2 — In △ACD, with $LN \parallel CD$:

By BPT, $\frac{AN}{AD} = \frac{AL}{AC}$ ...(2) 1 mark

Step 3 — Compare. The right-hand sides of (1) and (2) are identical, so the left-hand sides must be equal:
$\frac{AM}{AB} = \frac{AN}{AD}$ **Hence proved.** 1 mark

> 📌 **The pattern to memorise.** Q3, Q4, Q5 and Q6 are all the same move: two triangles share one segment, BPT is applied in each, and the shared ratio links them. Spot the common segment first — here it is AL/AC.

Q4. In Fig. 6.19, DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$. 3 marks

SOLUTION:

Step 1 — In △ABC, with $DE \parallel AC$:

By BPT, $\frac{BD}{DA} = \frac{BE}{EC}$ ...(1) 1 mark

Step 2 — In △ABE, with $DF \parallel AE$:

By BPT, $\frac{BD}{DA} = \frac{BF}{FE}$ ...(2) 1 mark

Step 3 — Compare (1) and (2). Both equal $\frac{BD}{DA}$, therefore
$\frac{BF}{FE} = \frac{BE}{EC}$ **Hence proved.** 1 mark

> 📌 Here the shared ratio is BD/DA, which sits on side AB — the side common to both triangles ABC and ABE.

Q5. In Fig. 6.20, DE || OQ and DF || OR. Show that EF || QR. 3 marks

SOLUTION:

Step 1 — In △PQO, with $DE \parallel OQ$:

By BPT, $\frac{PE}{EQ} = \frac{PD}{DO}$ ...(1) **1 mark**

Step 2 — In △PRO, with $DF \parallel OR$:

By BPT, $\frac{PF}{FR} = \frac{PD}{DO}$ ...(2) **1 mark**

Step 3 — Combine. From (1) and (2), $\frac{PE}{EQ} = \frac{PF}{FR}$.

Now in △PQR, the line EF divides the two sides PQ and PR in the same ratio. By the **converse of BPT (Theorem 6.2)**,

$EF \parallel QR$ **Hence proved.** **1 mark**

📌 **Note the direction change.** Steps 1 and 2 use BPT (parallel given → ratio). Step 3 runs it backwards with the converse (ratio given → parallel). Naming the converse explicitly is what earns the final mark.

Q6. In Fig. 6.21, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR. **3 marks**

SOLUTION:

Step 1 — In △OPQ, with $AB \parallel PQ$:

By BPT, $\frac{OA}{AP} = \frac{OB}{BQ}$ ...(1) **1 mark**

Step 2 — In △OPR, with $AC \parallel PR$:

By BPT, $\frac{OA}{AP} = \frac{OC}{CR}$ ...(2) **1 mark**

Step 3 — Combine. From (1) and (2), $\frac{OB}{BQ} = \frac{OC}{CR}$.

In △OQR, the line BC divides sides OQ and OR in the same ratio, so by the converse of BPT,

$BC \parallel QR$ **Hence proved.** **1 mark**

Q7. Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. **3 marks**

SOLUTION:

Given: A triangle ABC; D is the mid-point of AB (so $AD = DB$); a line through D parallel to BC meets AC at E.

To prove: E is the mid-point of AC, i.e. $AE = EC$.

Proof. Since $DE \parallel BC$, by Theorem 6.1 (BPT) applied to △ABC:
$\frac{AD}{DB} = \frac{AE}{EC}$ 1 mark

But D is the mid-point of AB, so $AD = DB$, which gives $\frac{AD}{DB} = 1$. 1 mark

Therefore $\frac{AE}{EC} = 1 \Rightarrow AE = EC$.
Hence E is the mid-point of AC — the line bisects the third side. **Hence proved.** 1 mark

Q8. Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. 3 marks

SOLUTION:

Given: A triangle ABC in which D and E are the mid-points of AB and AC respectively.
To prove: $DE \parallel BC$.

Proof. Since D is the mid-point of AB: $AD = DB \Rightarrow \frac{AD}{DB} = 1$ 1 mark
Since E is the mid-point of AC: $AE = EC \Rightarrow \frac{AE}{EC} = 1$ 1 mark

Therefore $\frac{AD}{DB} = \frac{AE}{EC}$, i.e. the line DE divides the sides AB and AC in the same ratio.
By Theorem 6.2 (converse of BPT), $DE \parallel BC$. **Hence proved.** 1 mark

📌 **Q7 and Q8 are mirror images.** Q7 starts from a parallel line and proves a mid-point (uses 6.1); Q8 starts from mid-points and proves parallelism (uses 6.2). The question names which theorem to use — follow it exactly, or the marks go.

Q9. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$. 3 marks

SOLUTION:

Construction. Through O, draw a line $EO \parallel DC$, meeting AD at E. Since $AB \parallel DC$, this line is also parallel to AB. 1 mark

Step 1 — In △ADC, with $EO \parallel DC$:

By BPT, $\frac{AE}{ED} = \frac{AO}{OC}$ ...(1) **1 mark**

Step 2 — In △ABD, with $EO \parallel AB$:
By BPT, $\frac{AE}{ED} = \frac{BO}{OD}$ ...(2)

Step 3 — Combine. From (1) and (2): $\frac{AO}{OC} = \frac{BO}{OD}$.
Rearranging (cross-multiplying and regrouping): $\frac{AO}{BO} = \frac{CO}{DO}$. **Hence proved.** **1 mark**

> 📌 **The construction is the whole question.** Without drawing $EO \parallel DC$ there is no triangle to apply BPT in. State the construction as a separate first line — examiners award a mark for it.

Q10. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium. **3 marks**

SOLUTION:
This is the converse of Q9, so the construction is the same but the logic runs the other way.

Given: $\frac{AO}{BO} = \frac{CO}{DO}$, which rearranges to $\frac{AO}{OC} = \frac{BO}{OD}$ ...(1)

Construction. Through O, draw $EO \parallel AB$, meeting AD at E. **1 mark**

Step 1 — In △ABD, with $EO \parallel AB$:
By BPT, $\frac{AE}{ED} = \frac{BO}{OD}$ ...(2) **1 mark**

Step 2 — Combine with the given. From (1) and (2): $\frac{AE}{ED} = \frac{AO}{OC}$.
Now in △ADC, the line EO divides sides AD and AC in the same ratio, so by the converse of BPT, $EO \parallel DC$
.

Step 3 — Conclude. We constructed $EO \parallel AB$ and have now shown $EO \parallel DC$. Two lines parallel to the same line are parallel to each other, so $AB \parallel DC$.
A quadrilateral with one pair of opposite sides parallel is a trapezium. Hence $ABCD$ is a trapezium. **Hence proved.** **1 mark**

— End of Exercise 6.2 —

📘 NCERT Solutions — Exercise 6.3 (16 Questions)

🔑 Approach for This Exercise

Three criteria decide every question here:

> **AAA (or AA):** if two angles of one triangle equal two angles of another, the triangles are similar.
> **SSS:** if all three pairs of corresponding sides are in the same ratio, the triangles are similar.
> **SAS:** if one pair of angles is equal and the two sides *containing* those angles are in the same ratio, the triangles are similar.

Two habits protect your marks. First, **write the vertices in matching order** — $\triangle ABC \sim \triangle PQR$ says A↔P, B↔Q, C↔R, and a wrong order makes every ratio that follows wrong. Second, in SAS the equal angle must be the *included* angle; if it sits elsewhere, the criterion does not apply.

Q1. State which pairs of triangles in Fig. 6.34 are similar. Write the similarity criterion used, and the pairs in symbolic form. 6 marks

SOLUTION:

(i) Angles $60°, 80°, 40°$ in both triangles — all three pairs of angles equal.
Criterion: **AAA**. $\triangle ABC \sim \triangle PQR$ 1 mark

(ii) Sides $2, 2.5, 3$ and $4, 5, 6$: $\frac{2}{4} = \frac{2.5}{5} = \frac{3}{6} = \frac{1}{2}$ — all three ratios equal.
Criterion: **SSS**. $\triangle ABC \sim \triangle QRP$ 1 mark

(iii) Sides $2.5, 3$ against $5, 6$: $\frac{2.5}{5} = \frac{1}{2}$ but $\frac{3}{6} = \frac{1}{2}$ while the third pair $\frac{2}{4}$ does not correspond to the marked angle.
The proportional sides do not contain the equal angle, so no criterion applies. **Not similar.** 1 mark

(iv) One pair of equal angles ($80°$) with the two sides containing it in the ratio $\frac{2.5}{5} = \frac{3}{6} = \frac{1}{2}$.
Criterion: **SAS**. $\triangle MNL \sim \triangle QPR$ 1 mark

(v) Only one angle ($80°$) and one pair of sides is given; the second side containing that angle is not known.
Not enough information. **Not similar.** 1 mark

(vi) In $\triangle DEF$: $\angle D = 70°,\ \angle E = 80° \Rightarrow \angle F = 30°$.
In $\triangle PQR$: $\angle P = 80°,\ \angle Q = 30° \Rightarrow \angle R = 70°$.
Two angles match, so criterion: **AA**. $\triangle DEF \sim \triangle PQR$ 1 mark

📌 **Always find the third angle first.** In (vi) the given angles look different until you use the angle-sum property — $70^\circ, 80^\circ, 30^\circ$ appear in both. Many students mark this "not similar" because they compare only what is printed.

Q2. In Fig. 6.35, △ODC ~ △OBA, ∠BOC = 125° and ∠CDO = 70°. Find ∠DOC, ∠DCO and ∠OAB. **3 marks**

SOLUTION:

Step 1 — Find ∠DOC. DOB is a straight line, so $\angle DOC$ and $\angle BOC$ form a linear pair:

$\angle DOC = 180^\circ - \angle BOC = 180^\circ - 125^\circ = \mathbf{55^\circ}$ **1 mark**

Step 2 — Find ∠DCO. In △DOC, the angles sum to 180°:

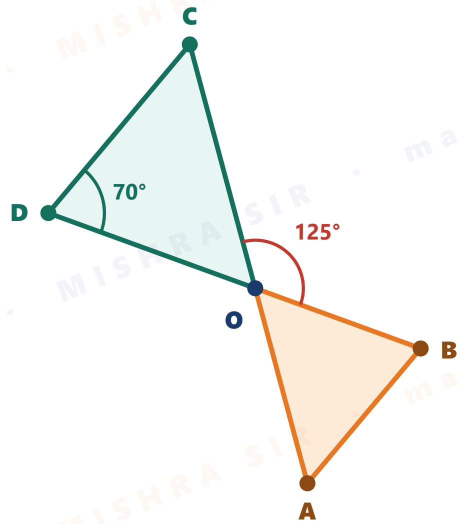
$\angle DCO = 180^\circ - \angle DOC - \angle CDO = 180^\circ - 55^\circ - 70^\circ = \mathbf{55^\circ}$ **1 mark**

Step 3 — Find ∠OAB. Since $\triangle ODC \sim \triangle OBA$, corresponding angles are equal. Matching the order O↔O, D↔B, C↔A:

$\angle OAB = \angle OCD = \mathbf{55^\circ}$ **1 mark**

📌 **Read the similarity statement carefully.** $\triangle ODC \sim \triangle OBA$ pairs C with A, so $\angle OAB$ corresponds to $\angle OCD$, not to $\angle ODC$. Getting this pairing backwards gives 70° — the commonest wrong answer here.

Fig. 6.35 — △ODC and △OBA meet at O; DOB and COA are straight lines



Given ∠CDO = 70° and ∠BOC = 125° (between OB and OC, outside △ODC).
Drawn to scale. △ODC ~ △OBA pairs O↔O, D↔B, C↔A.

Fig. 6.35 — redrawn schematic for Exercise 6.3, Q2

Note: *MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Q3. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion, show that $\frac{OA}{OC} = \frac{OB}{OD}$. 3 marks

SOLUTION:

Step 1 — Compare △AOB and △COD. Since $AB \parallel DC$ and AC is a transversal:

$\angle OAB = \angle OCD$ (alternate interior angles) 1 mark

Since $AB \parallel DC$ and BD is a transversal:

$\angle OBA = \angle ODC$ (alternate interior angles)

Step 2 — Apply AA. Two pairs of angles are equal, therefore

$\triangle AOB \sim \triangle COD$ (AA criterion) 1 mark

Step 3 — Write the ratio of corresponding sides. Matching A↔C, O↔O, B↔D:

$\frac{OA}{OC} = \frac{OB}{OD}$ **Hence proved.** 1 mark

📌 Compare with Exercise 6.2 Q9 — the same result, but proved there by construction and BPT, and here by AA similarity. The question tells you which route to take.

Q4. In Fig. 6.36, $\frac{QR}{QS} = \frac{QT}{PR}$ and ∠1 = ∠2. Show that △PQS ~ △TQR. 3 marks

SOLUTION:

Step 1 — Use ∠1 = ∠2. In △PQR, $\angle 1 = \angle 2$ means the sides opposite them are equal:

$PR = PQ$ (sides opposite equal angles) 1 mark

Step 2 — Rewrite the given ratio. We are given $\frac{QR}{QS} = \frac{QT}{PR}$.

Substituting $PR = PQ$: $\frac{QR}{QS} = \frac{QT}{PQ}$, which rearranges to $\frac{QS}{QR} = \frac{PQ}{QT}$ 1 mark

Step 3 — Apply SAS. In △PQS and △TQR:

$\frac{PQ}{QT} = \frac{QS}{QR}$ (just shown), and $\angle PQS = \angle TQR$ (the same angle Q, included between those sides).

Therefore $\triangle PQS \sim \triangle TQR$ (SAS criterion). **Hence proved.** 1 mark

📌 **Step 1 is the hidden step.** The given ratio contains PR, but the triangles to be compared contain PQ. Converting one into the other via the isosceles property is what makes the proof work.

Q5. S and T are points on sides PR and QR of △PQR such that ∠P = ∠RTS. Show that △RPQ ~ △RTS. 3 marks

SOLUTION:

In △RPQ and △RTS:

$\angle RPQ = \angle RTS$ (given) 1 mark

$\angle PRQ = \angle TRS$ (the same angle at R, common to both triangles) 1 mark

Two pairs of angles are equal, therefore by the **AA criterion**:

$\triangle RPQ \sim \triangle RTS$ **Hence proved.** 1 mark

📌 The common angle at R is free — always check for a shared angle before hunting for anything harder.

Q6. In Fig. 6.37, if △ABE ≅ △ACD, show that △ADE ~ △ABC. 3 marks

SOLUTION:

Step 1 — Extract equal parts from the congruence. Since $\triangle ABE \cong \triangle ACD$, corresponding parts are equal (CPCT):

$AB = AC$ and $AE = AD$ 1 mark

Step 2 — Form the ratio. From $AB = AC$ and $AD = AE$:

$$\frac{AD}{AB} = \frac{AE}{AC}$$ 1 mark

Step 3 — Apply SAS. In △ADE and △ABC:

the sides above are in the same ratio, and $\angle DAE = \angle BAC$ (the same angle at A, included between those sides).

Therefore $\triangle ADE \sim \triangle ABC$ (SAS criterion). **Hence proved.** 1 mark

📌 **Congruence is a source of equal lengths.** The question hands you ≅; your job is to turn it into a ratio and pair it with the common angle.

Q7. In Fig. 6.38, altitudes AD and CE of △ABC intersect each other at the point P. Show that: (i) △AEP ~ △CDP (ii) △ABD ~ △CBE (iii) △AEP ~ △ADB (iv) △PDC ~ △BEC

4 marks

SOLUTION:

Every part uses AA, with one right angle plus one shared or vertically opposite angle.

(i) △AEP and △CDP:

$\angle AEP = \angle CDP = 90^\circ$ (since $CE \perp AB$ and $AD \perp BC$)

$\angle APE = \angle CPD$ (vertically opposite angles)

$\Rightarrow \triangle AEP \sim \triangle CDP$ (AA) 1 mark

(ii) △ABD and △CBE:

$\angle ADB = \angle CEB = 90^\circ$

$\angle ABD = \angle CBE$ (same angle B, common)

$\Rightarrow \triangle ABD \sim \triangle CBE$ (AA) 1 mark

(iii) △AEP and △ADB:

$\angle AEP = \angle ADB = 90^\circ$

$\angle PAE = \angle DAB$ (same angle A, common)

$\Rightarrow \triangle AEP \sim \triangle ADB$ (AA) 1 mark

(iv) △PDC and △BEC:

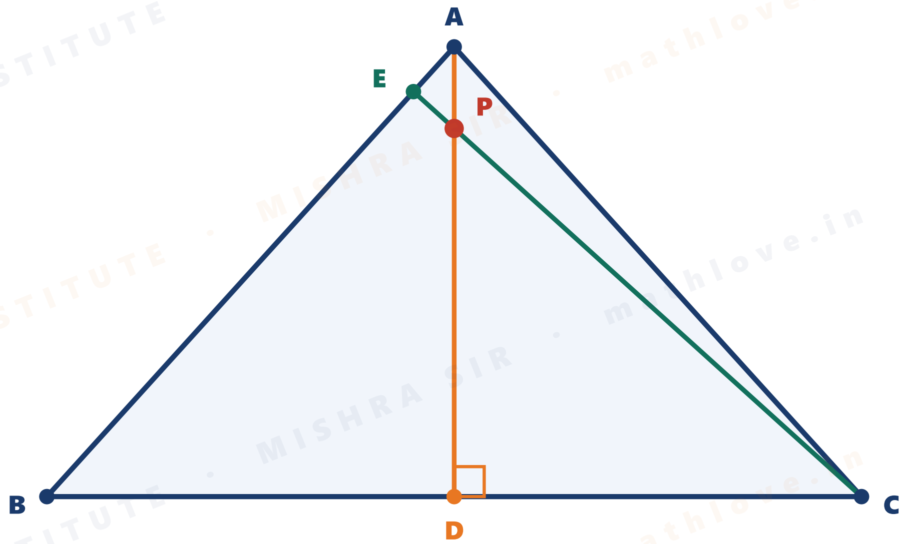
$\angle PDC = \angle BEC = 90^\circ$

$\angle PCD = \angle BCE$ (same angle C, common)

$\Rightarrow \triangle PDC \sim \triangle BEC$ (AA) 1 mark

📌 **A reliable routine for altitude questions.** Write the two right angles first, then look for the second pair: it is either a common angle at a vertex or a vertically opposite pair at the point of intersection. Nothing else is needed.

Fig. 6.38 — altitudes AD and CE of △ABC meet at P



AD ⊥ BC and CE ⊥ AB. Both △AEP and △CDP contain a right angle,
and ∠APE = ∠CPD because they are vertically opposite — that is the AA pair.

Fig. 6.38 — redrawn schematic for Exercise 6.3, Q7

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Q8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that △ABE ~ △CFB. 3 marks

SOLUTION:

In △ABE and △CFB:

First pair. $\angle A = \angle C$ (opposite angles of parallelogram $ABCD$) 1 mark

Second pair. Since $AE \parallel BC$ (as $AD \parallel BC$ and E lies on AD produced), with BE as transversal:
$\angle AEB = \angle CBF$ (alternate interior angles) 1 mark

Two pairs of angles are equal, therefore by **AA**:
$\triangle ABE \sim \triangle CFB$ **Hence proved.** 1 mark

Q9. In Fig. 6.39, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that (i) △ABC ~ △AMP (ii) $\frac{CA}{PA} = \frac{BC}{MP}$ 3 marks

SOLUTION:

(i) In △ABC and △AMP:

$\angle ABC = \angle AMP = 90^\circ$ (given) 1 mark

$\angle BAC = \angle MAP$ (the same angle at A, common to both) 1 mark

By **AA**, $\triangle ABC \sim \triangle AMP$. **Hence proved.**

(ii) Since the triangles are similar, corresponding sides are in the same ratio. Matching A↔A, B↔M, C↔P:

$$\frac{AB}{AM} = \frac{BC}{MP} = \frac{CA}{PA}$$

Taking the last two members: $\frac{CA}{PA} = \frac{BC}{MP}$ **Hence proved.** 1 mark

Q10. CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of △ABC and △EFG. If △ABC ~ △FEG, show that (i) $\frac{CD}{GH} = \frac{AC}{FG}$ (ii) △DCB ~ △HGE (iii) △DCA ~ △HGF 4 marks

SOLUTION:

Setting up. Since $\triangle ABC \sim \triangle FEG$, corresponding angles are equal:

$\angle A = \angle F, \quad \angle B = \angle E, \quad \angle ACB = \angle FGE$ 1 mark

As CD and GH bisect $\angle ACB$ and $\angle FGE$, halving both sides of the last equality gives

$\angle ACD = \angle DCB = \angle FGH = \angle HGE$

(ii) △DCB ~ △HGE:

$\angle DCB = \angle HGE$ (halves of equal angles) and $\angle B = \angle E$.

By **AA**, $\triangle DCB \sim \triangle HGE$. 1 mark

(iii) △DCA ~ △HGF:

$\angle ACD = \angle FGH$ (halves of equal angles) and $\angle A = \angle F$.

By **AA**, $\triangle DCA \sim \triangle HGF$. 1 mark

(i) The ratio: from the similarity in (iii), corresponding sides give

$\frac{CD}{GH} = \frac{AC}{FG}$ **Hence proved.** 1 mark

📌 **Do (ii) and (iii) before (i).** Part (i) is a consequence of part (iii), so proving the similarity first makes the ratio a one-line deduction.

Q11. In Fig. 6.40, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that △ABD ~ △ECF. 3 marks

SOLUTION:

Step 1 — Use the isosceles property. In △ABC, $AB = AC$, so the angles opposite them are equal:

$\angle ABC = \angle ACB$ **1 mark**

Now $\angle ECF$ is vertically opposite to $\angle ACB$ (since E lies on CB produced), so $\angle ECF = \angle ACB = \angle ABD$.

1 mark

Step 2 — Use the perpendiculars.

$\angle ADB = 90^\circ$ (as $AD \perp BC$) and $\angle EFC = 90^\circ$ (as $EF \perp AC$), so $\angle ADB = \angle EFC$.

Step 3 — Apply AA. In △ABD and △ECF, two pairs of angles are equal, therefore

$\triangle ABD \sim \triangle ECF$ **Hence proved.** **1 mark**

Q12. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of △PQR (Fig. 6.41). Show that △ABC ~ △PQR. **4 marks**

SOLUTION:

Given: $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AD}{PM}$, where AD and PM are medians.

Step 1 — Relate the medians to the sides. Since AD is a median, D is the mid-point of BC, so

$BD = \dfrac{1}{2}BC$. Similarly $QM = \dfrac{1}{2}QR$.

Therefore $\dfrac{BD}{QM} = \dfrac{\frac{1}{2}BC}{\frac{1}{2}QR} = \dfrac{BC}{QR}$ **1 mark**

Step 2 — Show △ABD ~ △PQM by SSS. From the given and Step 1:

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

All three pairs of sides are proportional, so $\triangle ABD \sim \triangle PQM$ (SSS). **1 mark**

Step 3 — Extract the equal angle. Corresponding angles of similar triangles are equal, so

$\angle ABD = \angle PQM$, i.e. $\angle ABC = \angle PQR$. **1 mark**

Step 4 — Apply SAS to the full triangles. In △ABC and △PQR:

$\dfrac{AB}{PQ} = \dfrac{BC}{QR}$ (given) and the included angle $\angle ABC = \angle PQR$ (Step 3).

Therefore $\triangle ABC \sim \triangle PQR$ (SAS). **Hence proved.** **1 mark**

📌 **The standard two-stage proof.** A median cannot be used directly in SSS for the whole triangle, so you first prove a smaller similarity (△ABD ~ △PQM), harvest the equal angle from it, and then finish with SAS. Q13–Q16 use the same two-stage idea.

Q13. D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC. Show that $CA^2 = CB \cdot CD$. 3 marks

SOLUTION:

Step 1 — Compare △ADC and △BAC.

$\angle ADC = \angle BAC$ (given) 1 mark

$\angle ACD = \angle BCA$ (the same angle at C, common)

Step 2 — Apply AA. $\triangle ADC \sim \triangle BAC$ 1 mark

Step 3 — Use corresponding sides. Matching A↔B, D↔A, C↔C:

$$\frac{CA}{CB} = \frac{CD}{CA}$$

Cross-multiplying: $CA \times CA = CB \times CD$, that is $\mathbf{CA^2 = CB \cdot CD}$. **Hence proved.** 1 mark

📌 **Whenever a square appears in the result**, expect a similarity where the same side sits in both the numerator and the denominator — that is exactly what produces CA^2.

Q14. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that △ABC ~ △PQR. 4 marks

SOLUTION:

Given: $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$, with AD, PM medians.

Construction. Produce AD to E so that $AD = DE$, and join BE and CE. Similarly produce PM to N so that $PM = MN$, and join QN and RN. 1 mark

Step 1 — The constructions create parallelograms. In $ABEC$, the diagonals BC and AE bisect each other ($BD = DC$ as AD is a median, and $AD = DE$ by construction). So $ABEC$ is a parallelogram, giving $BE = AC$ and $AE = 2AD$.

Likewise $PQNR$ is a parallelogram, giving $QN = PR$ and $PN = 2PM$. 1 mark

Step 2 — Show △ABE ~ △PQN by SSS.

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BE}{QN} \text{ (using } BE = AC,\ QN = PR)$$

$$\frac{AE}{PN} = \frac{2AD}{2PM} = \frac{AD}{PM} = \frac{AB}{PQ}$$

All three ratios are equal, so $\triangle ABE \sim \triangle PQN$ (SSS). **1 mark**

Step 3 — Finish with SAS. From the similarity, $\angle BAE = \angle QPN$. By the same argument applied to $\triangle ACE$ and $\triangle PRN$, $\angle CAE = \angle RPN$. Adding,

$\angle BAC = \angle QPR.$

Now in △ABC and △PQR: $\frac{AB}{PQ} = \frac{AC}{PR}$ with the included angle $\angle BAC = \angle QPR$.

Therefore $\triangle ABC \sim \triangle PQR$ (SAS). **Hence proved.** **1 mark**

> 📌 **Compare with Q12.** There the median went to a side that was *given* proportional, so doubling was unnecessary. Here the two given sides both meet at A, so the median must be doubled into a full parallelogram diagonal before SSS can be applied.

Q15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower. **3 marks**

SOLUTION:

Step 1 — Why the triangles are similar. The pole and the tower are both vertical, so each makes 90° with the ground. The sun's rays arrive at the same angle at the same time, so the angle of elevation is equal for both.

By **AA**, the two triangles (pole–shadow and tower–shadow) are similar. **1 mark**

Step 2 — Form the proportion. Let the height of the tower be h m.

$$\frac{\text{height of pole}}{\text{shadow of pole}} = \frac{\text{height of tower}}{\text{shadow of tower}}$$

$$\frac{6}{4} = \frac{h}{28}$$ **1 mark**

Step 3 — Solve. $h = \frac{6 \times 28}{4} = \frac{168}{4} = 42$

$\text{Height of the tower} = \mathbf{42}\text{ m}$ **1 mark**

> 📌 **Keep the ratio consistent.** Height over shadow on both sides (or pole over tower on both sides) — mixing the two, as in $\frac{6}{4} = \frac{28}{h}$, gives $h = 18.67$ m, a wrong answer that looks plausible.

Q16. If AD and PM are medians of triangles ABC and PQR respectively, where △ABC ~ △PQR, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$. **3 marks**

SOLUTION:

Step 1 — Write what the given similarity provides. Since $\triangle ABC \sim \triangle PQR$:

$\frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle B = \angle Q$ **1 mark**

Step 2 — Halve the sides that carry the medians. As AD and PM are medians, $BD = \frac{1}{2}BC$ and $QM = \frac{1}{2}QR$. Therefore

$\frac{BD}{QM} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{BC}{QR} = \frac{AB}{PQ}$ **1 mark**

Step 3 — Apply SAS to the half-triangles. In △ABD and △PQM:

$\frac{AB}{PQ} = \frac{BD}{QM}$ and the included angle $\angle B = \angle Q$.

Therefore $\triangle ABD \sim \triangle PQM$ (SAS), and so corresponding sides give

$\frac{AB}{PQ} = \frac{AD}{PM}$ **Hence proved.** **1 mark**

📌 **A result worth remembering:** in similar triangles, corresponding medians are in the same ratio as corresponding sides. The same holds for corresponding altitudes and angle bisectors — Q10 proved the bisector version.

— End of Exercise 6.3 — Chapter 6 Complete —

CHAPTER 7

Coordinate Geometry

Exercises 7.1 – 7.2

EXERCISE 7.1

CBSE Step-wise Marking Scheme Format	**Board Exam 2026–27**	**Name: ________________**

💡 **Key Formula for This Exercise**

Distance Formula: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ — Distance from origin: $OP = \sqrt{x^2 + y^2}$

EXERCISE 7.1

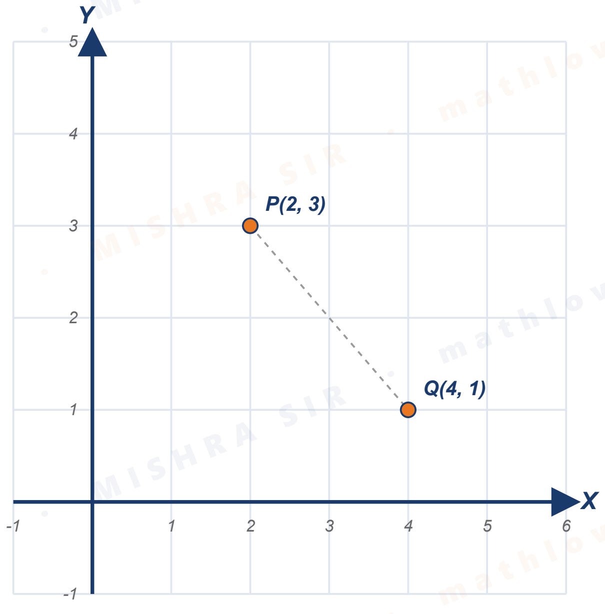
Q1. Find the distance between the following pairs of points: **3**

(i) (2, 3), (4, 1) (ii) (−5, 7), (−1, 3) (iii) (a, b), (−a, −b)

Given: *Three pairs of points.*

To Find: *Distance between each pair.*

Solution: *Using the distance formula:* $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **[½ mark]**



(i) Distance PQ

Note: *MLI-redrawn schematic — correct in construction but* **not drawn to scale**, *and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

(i) *Points: (2,3) and (4,1)*

$d = \sqrt{(4-2)^2 + (1-3)^2}$

$d = \sqrt{2^2 + (-2)^2}$

$d = \sqrt{4+4}$

$d = \sqrt{8}$

$d = \mathbf{2\sqrt{2}}$ units **[½ mark]**

(ii) *Points: (−5,7) and (−1,3)*

$d = \sqrt{(-1-(-5))^2 + (3-7)^2}$

$d = \sqrt{4^2 + (-4)^2}$

$d = \sqrt{16+16}$

$d = \sqrt{32}$

$d = \mathbf{4\sqrt{2}}$ units **[1 mark]**

(iii) *Points: (a,b) and (−a,−b)*

$d = \sqrt{(-a-a)^2 + (-b-b)^2}$

$d = \sqrt{(-2a)^2 + (-2b)^2}$

$d = \sqrt{4a^2 + 4b^2}$

$d = \mathbf{2\sqrt{a^2+b^2}}$ units **[1 mark]**

Q2.

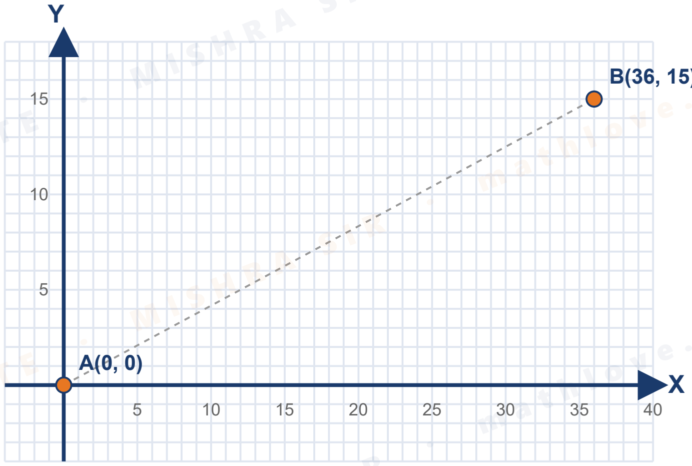
Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2?

2

Given: Points (0,0) and (36,15); Town A at origin, Town B at (36,15) (36 km east, 15 km north of A).

To Find: Distance, and hence distance between towns A and B.

Solution:



Town A to Town B

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

$d = \sqrt{(36-0)^2 + (15-0)^2}$

$d = \sqrt{1296 + 225}$

$d = \sqrt{1521}$

$d = \mathbf{39}\ \text{units}$ [1½ marks]

Since 1 km was taken as one unit on both axes, the distance between towns A and B is also **39 km**. [½ mark]

Q3.

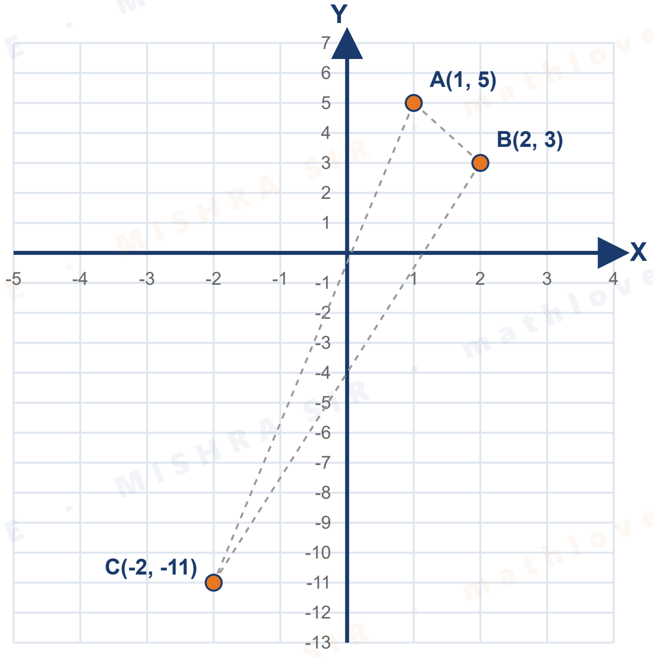
Determine if the points (1, 5), (2, 3) and (−2, −11) are collinear.

3

Given: Points A(1,5), B(2,3), C(−2,−11).

To Find: Whether the points are collinear.

Solution: Points are collinear if the sum of any two distances equals the third. [½ mark]



A, B, C plotted

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

$AB = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{1+4} = \sqrt{5}$

$BC = \sqrt{(-2-2)^2 + (-11-3)^2} = \sqrt{16+196} = \sqrt{212}$

$AC = \sqrt{(-2-1)^2 + (-11-5)^2} = \sqrt{9+256} = \sqrt{265}$ [1½ marks]

Since $AB + BC \neq AC$ (i.e. $\sqrt{5} + \sqrt{212} \neq \sqrt{265}$), the three points do **NOT** satisfy the collinearity condition.

Therefore, the given points are **NOT collinear**. [1 mark]

Q4.

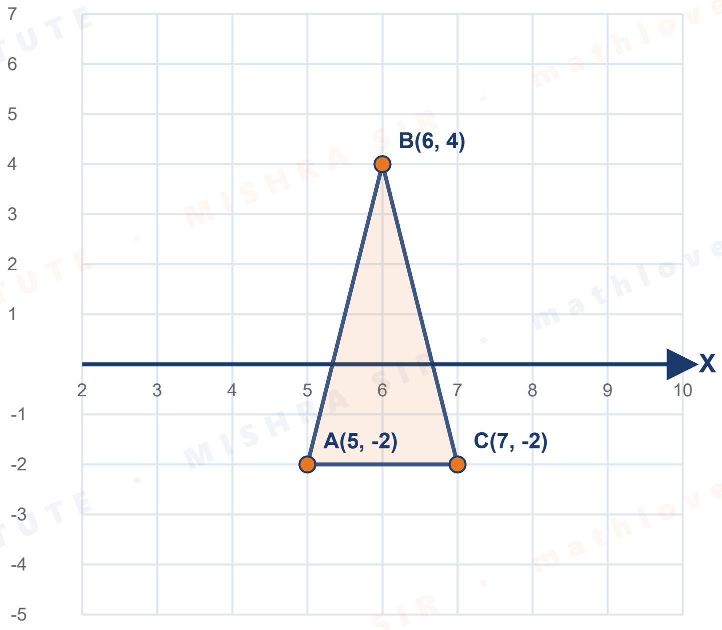
Check whether (5, −2), (6, 4) and (7, −2) are the vertices of an isosceles triangle.

3

Given: Points A(5,−2), B(6,4), C(7,−2).

To Find: Whether ABC is an isosceles triangle.

Solution:



Triangle ABC

Note: *MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

$AB = \sqrt{(6-5)^2 + (4-(-2))^2} = \sqrt{1+36} = \sqrt{37}$

$BC = \sqrt{(7-6)^2 + (-2-4)^2} = \sqrt{1+36} = \sqrt{37}$

$AC = \sqrt{(7-5)^2 + (-2-(-2))^2} = \sqrt{4+0} = 2$ [2 marks]

Since $AB = BC = \sqrt{37}$ (two sides are equal), triangle ABC is an **isosceles triangle**. [1 mark]

Q5.

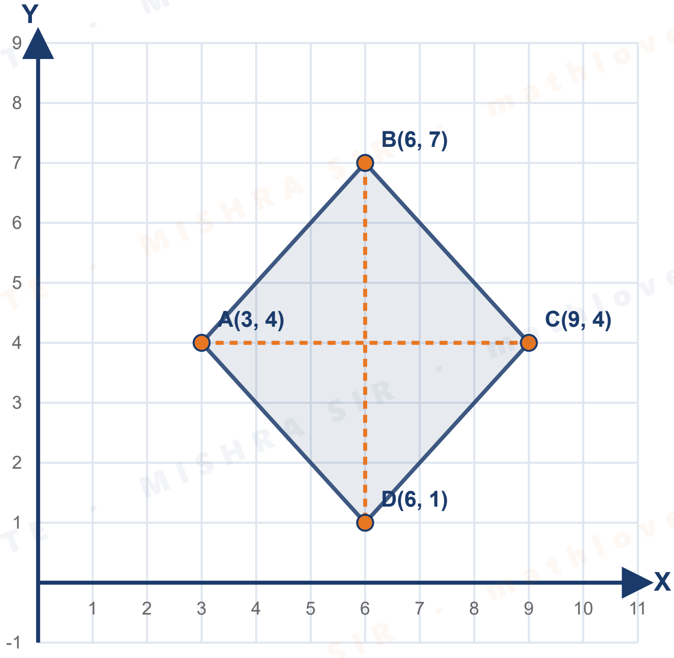
In a classroom, 4 friends are seated at the points A, B, C and D as shown in the figure. Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.

4

Given: From the figure, A(3,4), B(6,7), C(9,4), D(6,1) (columns=x, rows=y).

To Find: Whether ABCD is a square.

Solution:



Classroom seating ABCD

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

For ABCD to be a square, all four sides must be equal AND both diagonals must be equal. [½ mark]

$AB = \sqrt{(6-3)^2 + (7-4)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$BC = \sqrt{(9-6)^2 + (4-7)^2} = \sqrt{9+9} = 3\sqrt{2}$

$CD = \sqrt{(6-9)^2 + (1-4)^2} = \sqrt{9+9} = 3\sqrt{2}$

$DA = \sqrt{(3-6)^2 + (4-1)^2} = \sqrt{9+9} = 3\sqrt{2}$

So AB=BC=CD=DA=$3\sqrt{2}$ (all four sides equal) [1½ marks]

$AC = \sqrt{(9-3)^2 + (4-4)^2} = \sqrt{36} = 6$

$BD = \sqrt{(6-6)^2 + (1-7)^2} = \sqrt{36} = 6$

So AC=BD=6 (both diagonals equal) [1 mark]

Since all sides are equal and both diagonals are equal, **ABCD is a square**. Therefore, **Champa is correct**. [1 mark]

Q6.

Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) (−1, −2), (1, 0), (−1, 2), (−3, 0)

(ii) (−3, 5), (3, 1), (0, 3), (−1, −4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

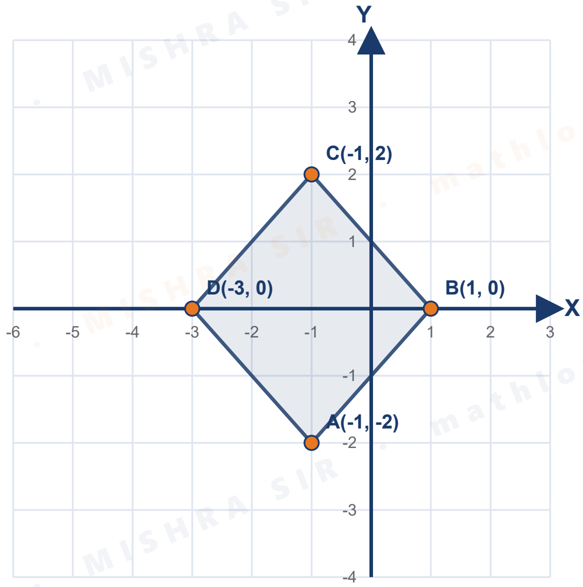
9

Given: Three sets of four points each.

To Find: Type of quadrilateral (if any) formed in each case.

Solution: Compute all four sides and both diagonals; compare using properties of quadrilaterals.

(i) Let A(−1,−2), B(1,0), C(−1,2), D(−3,0).



(i) Square ABCD

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

$AB = \sqrt{(1+1)^2 + (0+2)^2} = \sqrt{4+4} = 2\sqrt{2}$

$BC = \sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$

$CD = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = 2\sqrt{2}$

$DA = \sqrt{(-1+3)^2 + (-2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$

All sides equal. Now check diagonals:

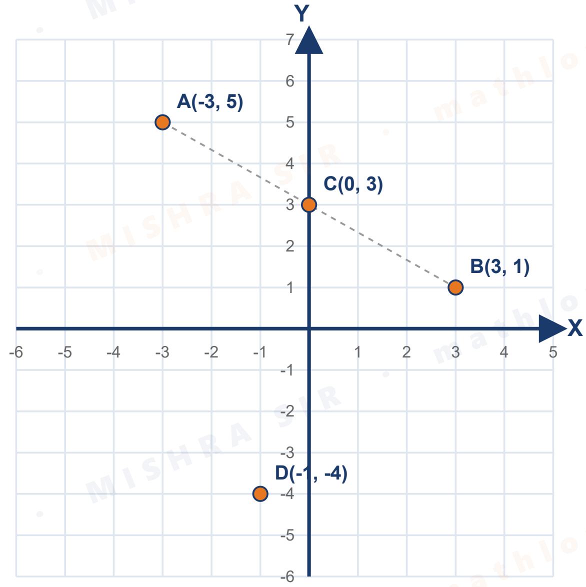
$AC = \sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{0+16} = 4$

$BD = \sqrt{(-3-1)^2 + (0-0)^2} = \sqrt{16+0} = 4$

Diagonals are also equal (AC=BD=4). [2 marks]

Since all sides are equal and both diagonals are equal, ABCD is a **square**. [1 mark]

(ii) Let A(−3,5), B(3,1), C(0,3), D(−1,−4).



(ii) A, B, C are collinear

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

$AC = \sqrt{(0+3)^2 + (3-5)^2} = \sqrt{9+4} = \sqrt{13}$

$BC = \sqrt{(0-3)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$

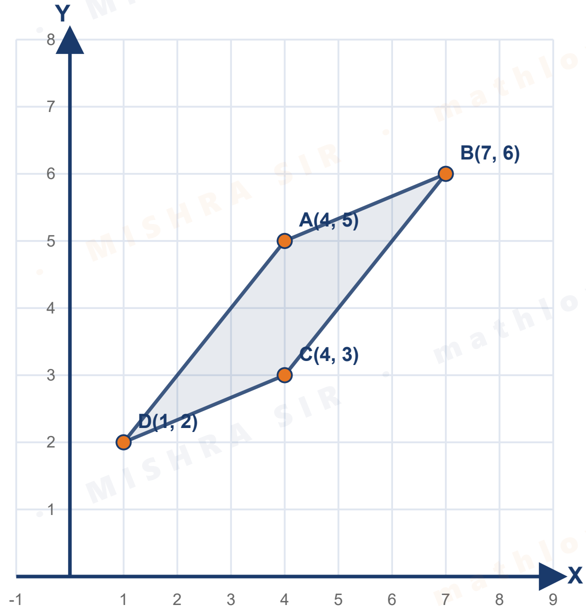
$AB = \sqrt{(3+3)^2 + (1-5)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ [2 marks]

Since $AC + CB = \sqrt{13} + \sqrt{13} = 2\sqrt{13} = AB$, the points A, C, B are **collinear** (C lies between A and B on the same straight line).

Therefore, these four points do **NOT form a quadrilateral** (three of them are collinear).

[1 mark]

(iii) Let A(4,5), B(7,6), C(4,3), D(1,2).



(iii) Parallelogram ABCD

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

$AB = \sqrt{(7-4)^2 + (6-5)^2} = \sqrt{9+1} = \sqrt{10}$

$BC = \sqrt{(4-7)^2 + (3-6)^2} = \sqrt{9+9} = \sqrt{18}$

$CD = \sqrt{(1-4)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$

$DA = \sqrt{(4-1)^2 + (5-2)^2} = \sqrt{9+9} = \sqrt{18}$

So AB=CD and BC=DA (opposite sides equal) [2 marks]

Now check diagonals: $AC = \sqrt{(4-4)^2 + (3-5)^2} = \sqrt{0+4} = 2$

$BD = \sqrt{(1-7)^2 + (2-6)^2} = \sqrt{36+16} = \sqrt{52}$

Since AC ≠ BD (diagonals unequal) but opposite sides are equal, ABCD is a **parallelogram** (not a rectangle). [1 mark]

Q7.

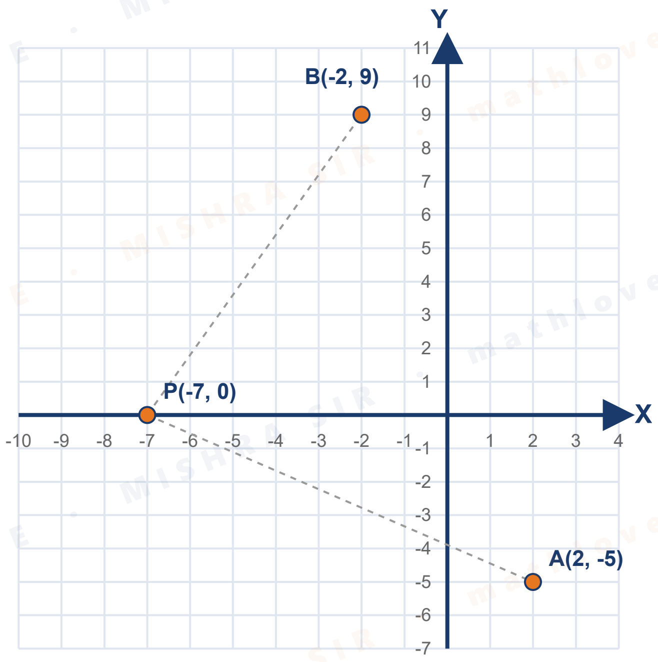
Find the point on the x-axis which is equidistant from (2, −5) and (−2, 9).

3

Given: A(2,−5), B(−2,9); required point lies on the x-axis.

To Find: The point on the x-axis equidistant from A and B.

Solution:



P on x-axis, equidistant from A, B

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

A point on the x-axis has the form P(x, 0). Let P(x,0) be equidistant from A and B.

$PA = PB \Rightarrow PA^2 = PB^2$ [½ mark]

$(x-2)^2 + (0+5)^2 = (x+2)^2 + (0-9)^2$

$x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$

$-4x + 29 = 4x + 85$

$-8x = 56$

$x = \mathbf{-7}$ [1½ marks]

Therefore, the required point is **(−7, 0)**. [1 mark]

Q8.

Find the values of y for which the distance between the points P(2, −3) and Q(10, y) is 10 units.

3

Given: P(2,−3), Q(10,y); PQ = 10 units.

To Find: Value(s) of y.

Solution:

$PQ = \sqrt{(10-2)^2 + (y+3)^2} = 10$ [½ mark]

Squaring both sides:

$64 + (y+3)^2 = 100$

$(y+3)^2 = 36$

$y+3=\pm 6$ [1½ marks]

Case 1: $y+3=6 \Rightarrow y=3$

Case 2: $y+3=-6 \Rightarrow y=-9$

Therefore, $y=\mathbf{3}$ or $y=\mathbf{-9}$ [1 mark]

Q9.

If Q(0, 1) is equidistant from P(5, −3) and R(x, 6), find the values of x. Also find the distances QR and PR.

4

Given: Q(0,1) equidistant from P(5,−3) and R(x,6).

To Find: Value(s) of x, and distances QR, PR.

Solution:

$QP=QR \Rightarrow QP^2=QR^2$ [½ mark]

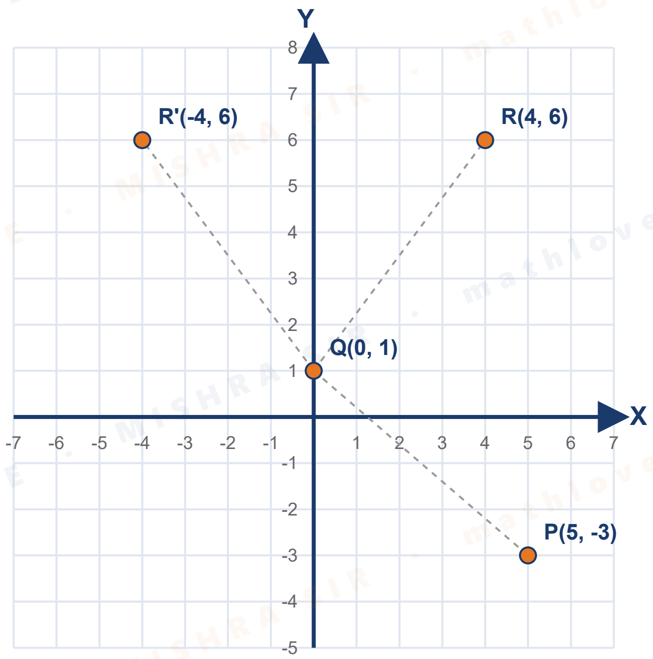
$(5-0)^2+(-3-1)^2=(x-0)^2+(6-1)^2$

$25+16=x^2+25$

$x^2=16$

$x=\pm 4$ [1 mark]

So R is (4, 6) or (−4, 6).



Q equidistant from P and both possible R

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Distance QR (using R(4,6), by symmetry same for R(−4,6)):

$$QR = \sqrt{(4-0)^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{\mathbf{41}}$$

[1 mark]

Distance PR (using R(4,6)):

$$PR = \sqrt{(4-5)^2 + (6-(-3))^2} = \sqrt{1+81} = \sqrt{\mathbf{82}}$$

[1 mark]

Distance PR (using R(−4,6)):

PR = $\sqrt{[(-4-5)^2 + (6-(-3))^2]} = \sqrt{(81+81)} = \sqrt{162} = 9\sqrt{2}$ units [½ mark]

Therefore: x = 4 or x = −4; QR = $\sqrt{41}$ units in both cases; PR = $\sqrt{82}$ units (when R is (4, 6)) or $9\sqrt{2}$ units (when R is (−4, 6)).

Q10.

Find a relation between x and y such that the point (x, y) is equidistant from the points (3, 6) and (−3, 4).

3

Given: P(x,y) equidistant from A(3,6) and B(−3,4).

To Find: The relation between x and y.

Solution:

$PA = PB \Rightarrow PA^2 = PB^2$ [½ mark]

$$(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$

$$x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$$

$$-6x - 12y + 45 = 6x - 8y + 25$$

$$-6x - 6x - 12y + 8y = 25 - 45$$

$$-12x - 4y = -20$$

$$12x + 4y = 20$$

$3x + y = \mathbf{5}$ (dividing throughout by 4) [2½ marks]

This is the required relation, and it represents the equation of the perpendicular bisector of segment AB.

EXERCISE 7.2

CBSE Step-wise Marking Scheme FormatBoard Exam 2026–27Name: ______________

💡 Key Formula for This Exercise

Section Formula: For P dividing AB in ratio $m_1 : m_2$, $P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$ Mid-point Formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ — Area of rhombus $= \frac{1}{2} \times d_1 \times d_2$

EXERCISE 7.2

Q1.

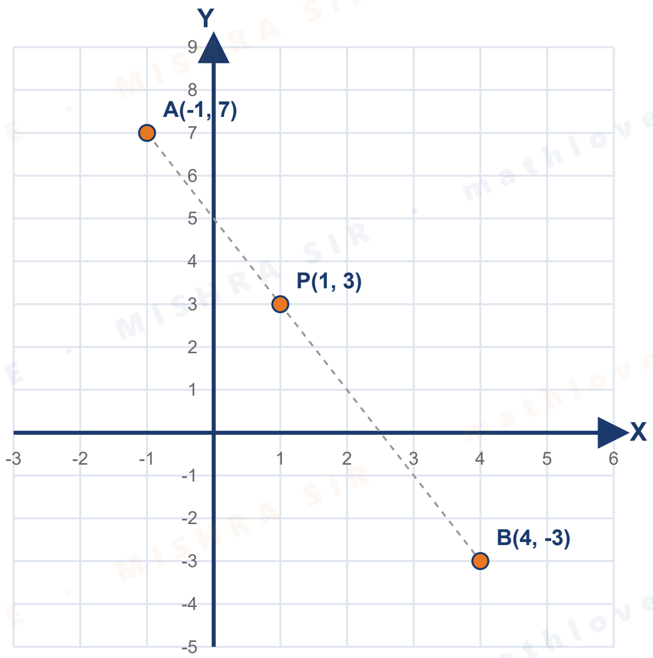
Find the coordinates of the point which divides the join of (−1, 7) and (4, −3) in the ratio 2 : 3.

3

Given: A(−1,7), B(4,−3); ratio $m_1 : m_2 = 2 : 3$.

To Find: Coordinates of point P dividing AB in this ratio.

Solution: Using the section formula: [½ mark]



P divides AB in ratio 2:3

__Note:__ MLI-redrawn schematic — correct in construction but __not drawn to scale__, and not a reproduction of any textbook figure. Never measure a *length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{2(4) + 3(-1)}{2 + 3}$$

$$x = \frac{8 - 3}{5} = \frac{5}{5} = 1 \text{ [1 mark]}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{2(-3) + 3(7)}{2 + 3}$$

$$y = \frac{-6 + 21}{5} = \frac{15}{5} = 3 \text{ [1 mark]}$$

Therefore, the required point is **(1, 3)**. [½ mark]

Q2.

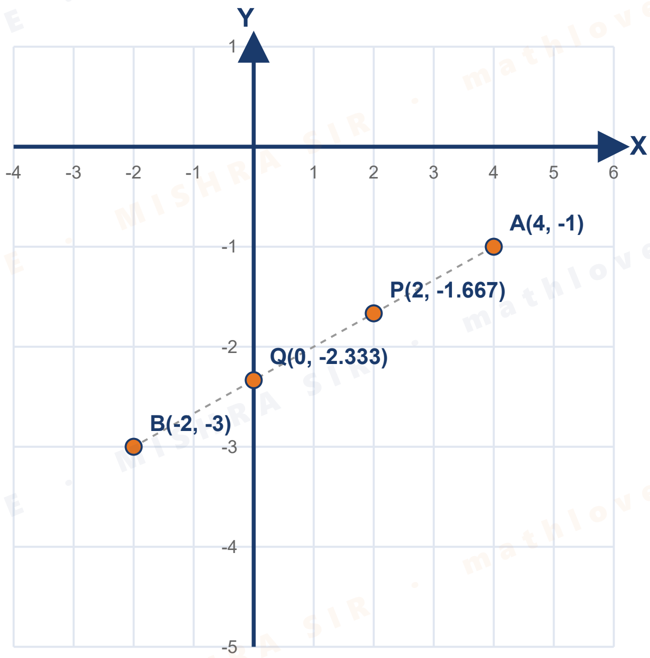
Find the coordinates of the points of trisection of the line segment joining (4, −1) and (−2, −3).

4

Given: A(4,−1), B(−2,−3).

To Find: Coordinates of the two points of trisection P, Q.

Solution: P divides AB in ratio 1:2; Q divides AB in ratio 2:1. [1 mark]



Points of trisection P, Q

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

For P (ratio 1:2):

$$x = \frac{1(-2) + 2(4)}{1 + 2} = \frac{-2 + 8}{3} = \frac{6}{3} = 2$$

$$y = \frac{1(-3) + 2(-1)}{1 + 2} = \frac{-3 - 2}{3} = \frac{-5}{3}$$

∴ P = $\left(2, -\frac{5}{3}\right)$ [1½ marks]

For Q (ratio 2:1):

$$x = \frac{2(-2) + 1(4)}{2 + 1} = \frac{-4 + 4}{3} = 0$$

$$y = \frac{2(-3) + 1(-1)}{2 + 1} = \frac{-6 - 1}{3} = \frac{-7}{3}$$

∴ Q = $\left(0, -\frac{7}{3}\right)$ [1½ marks]

Q3.

To conduct Sports Day activities, in a rectangular school ground ABCD, lines are drawn with chalk powder at a distance of 1 m each. 100 flower pots are placed at a distance of 1 m from each other along AD. Niharika runs $\frac{1}{4}$th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the two flags, where should she post it?

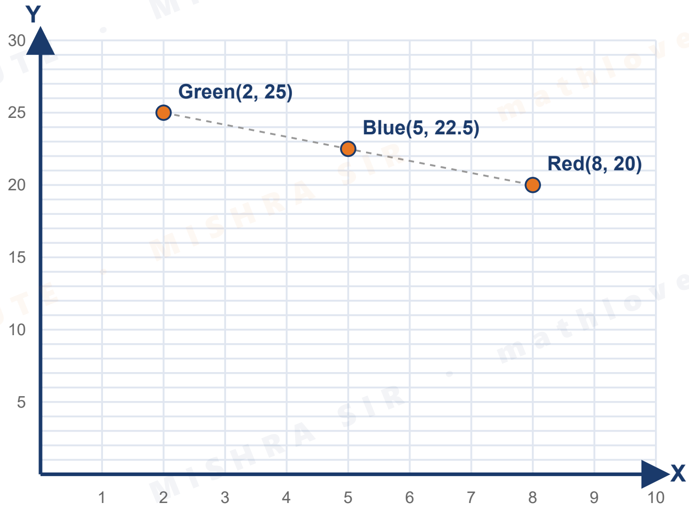
4

Given: Niharika's flag: 2nd line, $\frac{1}{4} \times 100 = 25$ m along AD ⇒ position (2, 25).

Preet's flag: 8th line, $\frac{1}{5} \times 100 = 20$ m along AD ⇒ position (8, 20).

To Find: Distance between the flags; midpoint for the blue flag.

Solution:



Green, Red and Blue flag positions

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Green flag position = (2, 25); Red flag position = (8, 20) [1 mark]

Distance between flags:

$$d = \sqrt{(8-2)^2 + (20-25)^2}$$

$$d = \sqrt{36+25}$$

$$d = \sqrt{\mathbf{61}}\ \mathrm{m}$$ [1½ marks]

For the blue flag, using the mid-point formula:

$$\left(\frac{2+8}{2}, \frac{25+20}{2}\right) = (5, 22.5)$$

Rashmi should post the blue flag on the **5th line, at a distance of 22.5 m along AD**. [1½ marks]

Q4.

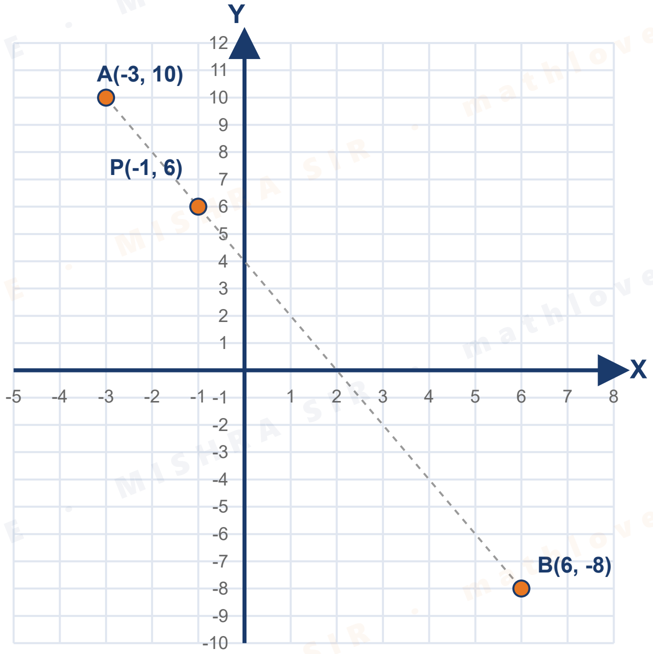
Find the ratio in which the line segment joining the points (−3, 10) and (6, −8) is divided by (−1, 6).

3

Given: A(−3,10), B(6,−8); point P(−1,6) lies on AB.

To Find: The ratio $m_1 : m_2$ in which P divides AB.

Solution: Let P divide AB in the ratio $k : 1$. [½ mark]



P divides AB in ratio 2:7

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Using the x-coordinate of the section formula:

$$-1 = \frac{k(6) + 1(-3)}{k+1}$$

$$-1(k+1) = 6k - 3$$

$$-k - 1 = 6k - 3$$

$$-7k = -2$$

$$k = \frac{2}{7} \text{ [1½ marks]}$$

So the ratio $k : 1 = \mathbf{2 : 7}$ [1 mark]

(You may verify this ratio also satisfies the y-coordinate.)

Q5.

Find the ratio in which the line segment joining A(1, −5) and B(−4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

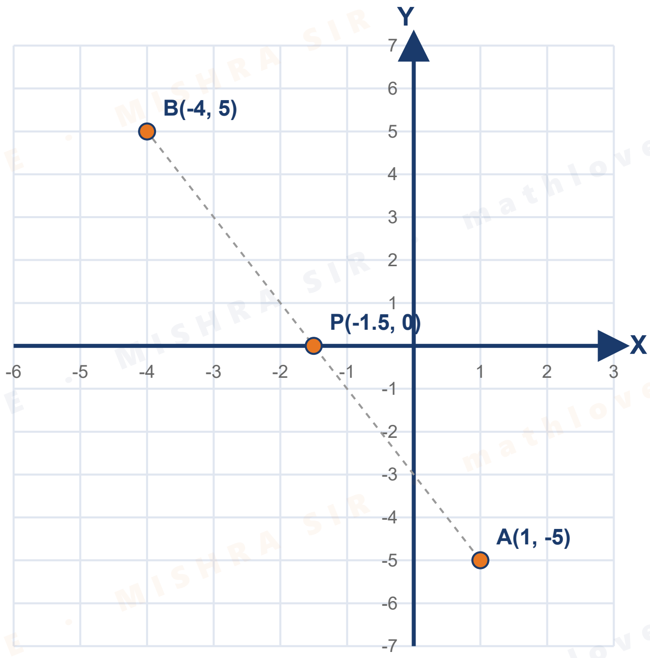
4

Given: A(1,−5), B(−4,5); divided by the x-axis.

To Find: The ratio, and the point of division.

Solution: Let the x-axis divide AB in the ratio $k : 1$ at point P. Since P is on the x-axis, its y-coordinate is 0.

[1 mark]



P: intersection with x-axis

> ***Note:*** *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

$$0 = \frac{k(5) + 1(-5)}{k+1}$$

$$0 = 5k - 5$$

$k = 1$ [1½ marks]

So the ratio is $\mathbf{1 : 1}$ (i.e., the x-axis bisects AB).

x-coordinate of P: $x = \dfrac{1(-4) + 1(1)}{1+1} = \dfrac{-3}{2}$

Therefore, the point of division is $\left(-\dfrac{\mathbf{3}}{\mathbf{2}}, \mathbf{0}\right)$

[1½ marks]

Q6.

If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

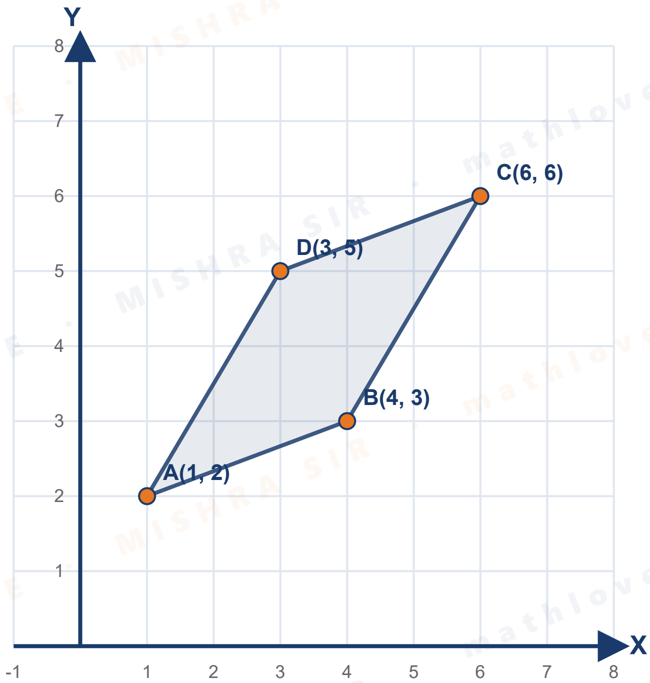
3

Given: A(1,2), B(4,y), C(x,6), D(3,5) are vertices of a parallelogram (in order).

To Find: x and y.

Solution: In a parallelogram, diagonals bisect each other, so the mid-point of AC = mid-point of BD.

[½ mark]



Parallelogram ABCD (x=6, y=3)

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Mid-point of AC = $\left(\frac{1+x}{2}, \frac{2+6}{2}\right)$

Mid-point of BD = $\left(\frac{4+3}{2}, \frac{y+5}{2}\right)$ [½ mark]

Equating x-coordinates:

$\frac{1+x}{2} = \frac{7}{2} \Rightarrow 1+x = 7 \Rightarrow x = \mathbf{6}$ [1 mark]

Equating y-coordinates:

$\frac{8}{2} = \frac{y+5}{2} \Rightarrow 8 = y+5 \Rightarrow y = \mathbf{3}$ [1 mark]

Q7.

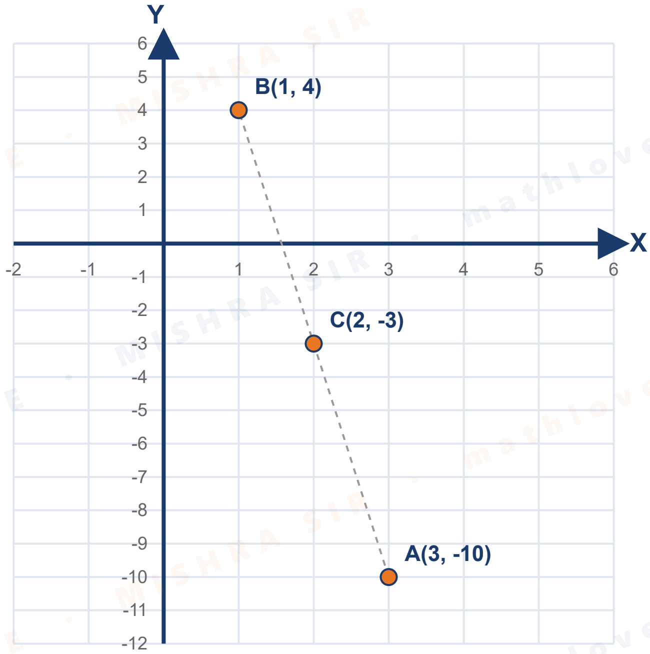
Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, −3) and B is (1, 4).

3

Given: Centre C(2,−3); B(1,4); AB is a diameter.

To Find: Coordinates of A.

Solution: Since AB is a diameter, the centre C is the mid-point of AB. [½ mark]



C is midpoint of diameter AB

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Let A = (x_1, y_1). Using the mid-point formula:

$$2 = \frac{x_1 + 1}{2} \Rightarrow x_1 + 1 = 4 \Rightarrow x_1 = 3$$ [1 mark]

$$-3 = \frac{y_1 + 4}{2} \Rightarrow y_1 + 4 = -6 \Rightarrow y_1 = -10$$ [1 mark]

Therefore, the coordinates of A are **(3, −10)**.

[½ mark]

Q8.

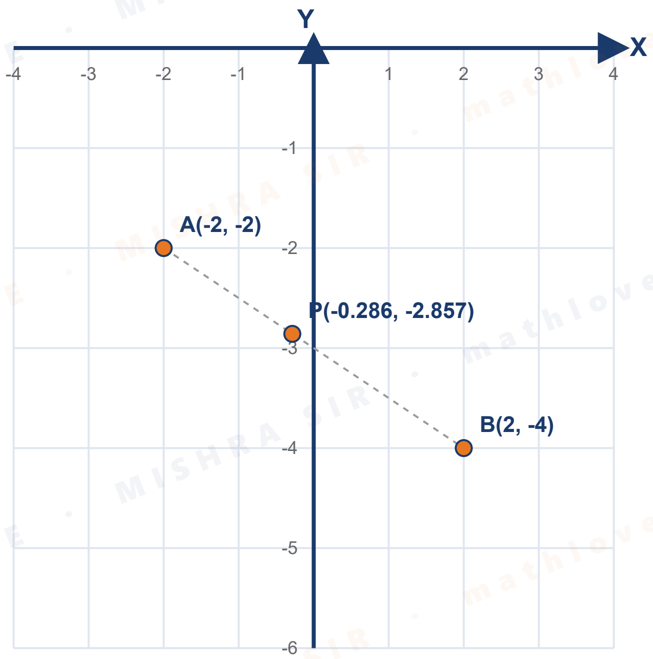
If A and B are (−2, −2) and (2, −4) respectively, find the coordinates of P such that $AP = \frac{3}{7}AB$ and P lies on the line segment AB.

3

Given: A(−2,−2), B(2,−4); $AP = \frac{3}{7}AB$.

To Find: Coordinates of P.

Solution: Since $AP = \frac{3}{7}AB$, we have $PB = AB - AP = \frac{4}{7}AB$. So $AP : PB = 3 : 4$. [1 mark]



P divides AB in ratio 3:4

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Using the section formula with $m_1 : m_2 = 3 : 4$:

$x = \dfrac{3(2) + 4(-2)}{3 + 4} = \dfrac{6 - 8}{7} = -\dfrac{2}{7}$ [1 mark]

$y = \dfrac{3(-4) + 4(-2)}{3 + 4} = \dfrac{-12 - 8}{7} = -\dfrac{20}{7}$ [1 mark]

Therefore, P = $\left(-\mathbf{\dfrac{2}{7}}, -\mathbf{\dfrac{20}{7}}\right)$

Q9.

Find the coordinates of the points which divide the line segment joining A(−2, 2) and B(2, 8) into four equal parts.

4

Given: A(−2,2), B(2,8); divided into 4 equal parts.

To Find: Coordinates of the three division points P, Q, R.

Solution: Q is the mid-point of AB; P is the mid-point of AQ; R is the mid-point of QB. [½ mark]

Four equal parts: P, Q, R

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Q (mid-point of AB, ratio 1:1):

$$Q = \left(\frac{-2+2}{2}, \frac{2+8}{2}\right) = (0,5) \ \ \text{[½ mark]}$$

P (mid-point of AQ, i.e. divides AB in ratio 1:3):

$$P = \left(\frac{1(2)+3(-2)}{1+3}, \frac{1(8)+3(2)}{1+3}\right) = \left(\frac{-4}{4}, \frac{14}{4}\right) = (-1,3.5) \ \ \text{[1½ marks]}$$

R (mid-point of QB, i.e. divides AB in ratio 3:1):

$$R = \left(\frac{3(2)+1(-2)}{3+1}, \frac{3(8)+1(2)}{3+1}\right) = \left(\frac{4}{4}, \frac{26}{4}\right) = (1,6.5) \ \ \text{[1½ marks]}$$

Therefore, the three points are **P(−1, 3.5)**, **Q(0, 5)**, and **R(1, 6.5)**.

Q10.

Find the area of a rhombus if its vertices are (3, 0), (4, 5), (−1, 4) and (−2, −1) taken in order.

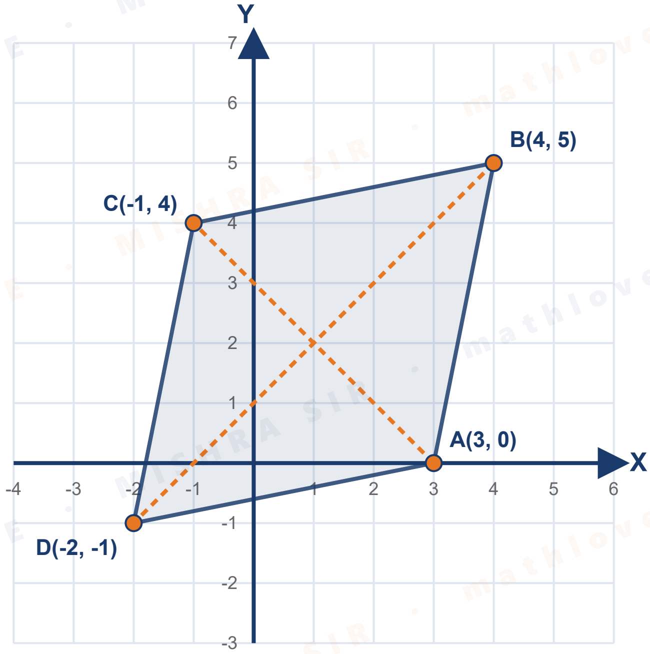
[Hint: Area of rhombus $= \frac{1}{2}$(product of its diagonals)]

4

Given: Rhombus vertices A(3,0), B(4,5), C(−1,4), D(−2,−1).

To Find: Area of the rhombus.

Solution: The diagonals of the rhombus are AC and BD. [½ mark]



Rhombus ABCD with diagonals AC, BD

Note: *MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Diagonal AC:

$$AC = \sqrt{(-1-3)^2 + (4-0)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

[1 mark]

Diagonal BD:

$$BD = \sqrt{(-2-4)^2 + (-1-5)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

[1 mark]

Area of rhombus $= \dfrac{1}{2} \times d_1 \times d_2$:

$$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$$

$$= \frac{1}{2} \times 24 \times 2$$

$= \mathbf{24}$ square units [1½ marks]

🏆 Exam Tip — CBSE Marking Scheme Reminders

1. Always begin word problems with **Given** and **To Find**.
2. Write the formula (distance/section/mid-point) before substituting values.
3. Show every algebraic step on its own line.

4. For quadrilateral-type questions, always check BOTH sides and diagonals before naming the shape (equal sides + equal diagonals = square; equal sides + unequal diagonals = rhombus; opposite sides equal + unequal diagonals = parallelogram).
5. Box or underline the final answer, with units (sq. units, etc.) wherever applicable.

CHAPTER 8

Introduction to Trigonometry

Exercises 8.1 – 8.3

EXERCISES 8.1 – 8.3

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Complete Study Notes + Step-wise NCERT Solutions

Class X Mathematics — Chapter 8: Introduction to Trigonometry (NCERT, Reprint 2026–27) | Session 2026–27

⚠ Please Read First — Scope of This Chapter

1. This is Class X NCERT Chapter 8 — "Introduction to Trigonometry" (Reprint 2026–27). It contains Sections 8.2 (Trigonometric Ratios), 8.3 (Ratios of Specific Angles) and 8.4 (Trigonometric Identities), with **Exercises 8.1, 8.2 and 8.3**.

2. Trigonometric ratios of complementary angles have been REMOVED. Under the rationalised syllabus, the old Section 8.4

on complementary angles (results such as $\sin(90^\circ - A) = \cos A$) and its exercise are **no longer in this textbook**. Do not spend time on that exercise from older editions.

3. Heights and Distances is a separate chapter (Some Applications of Trigonometry) — angles of elevation and depression are **not** part of Chapter 8.

4. Only acute angles. Every ratio in this chapter is defined for $0^\circ \leq A \leq 90^\circ$ only.

📊 Weightage (CBSE Class X Board Exam, 2026–27)

Detail	Value
Unit	Trigonometry (Unit V)
Marks allotted to the whole unit	**12 marks** out of 80 — the joint-highest unit
Chapters in the unit	Ch 8 (Introduction to Trigonometry) + Ch 9 (Some Applications of Trigonometry)
Topics in Chapter 8	Trigonometric ratios; ratios of 0°, 30°, 45°, 60°, 90°; trigonometric identities
Deleted from syllabus	Trigonometric ratios of complementary angles

📌 Chapter 8 is the foundation for Chapter 9. Weak ratios here cost marks twice over — get the table of specific angles memorised in week one.

🎯 Predicted Question Types

Format	Marks	Likely Topic
MCQ / Assertion–	1	Value of a ratio at a specific angle; simplification using an

Reason		identity; true/false on ranges of sin, cos, sec
Very Short / Short Answer I	2	Given one ratio, find another; evaluate an expression at 30°/45°/60°
Short Answer II	3	Proving a trigonometric identity; finding A and B from sin(A−B) and cos(A+B)
Long Answer	5	A harder identity, often needing rationalisation or a conjugate multiplication

📐 The Six Trigonometric Ratios

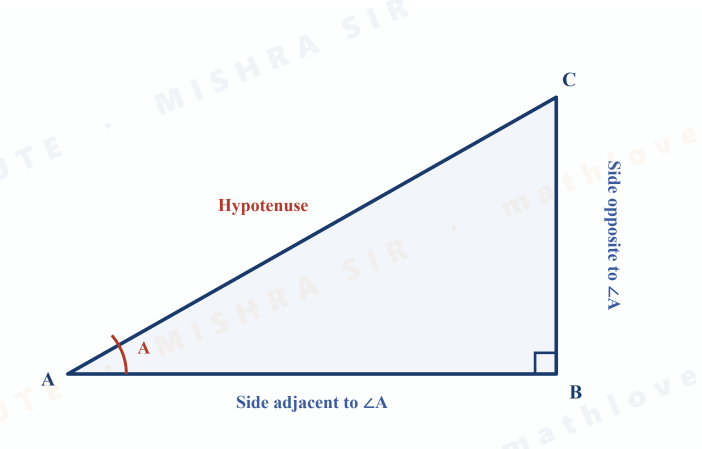

Fig. 8.4 — With respect to ∠A: BC is the side opposite, AB is the side adjacent, AC is the hypotenuse.

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In right $\triangle ABC$, right-angled at B, with respect to $\angle A$:

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BC}{AC} \qquad \cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AB}$$

The three reciprocals:

$$\text{cosec } A = \frac{1}{\sin A} = \frac{AC}{BC}, \quad \sec A = \frac{1}{\cos A} = \frac{AC}{AB}, \quad \cot A = \frac{1}{\tan A} = \frac{AB}{BC}$$

Two quotient relations:

$$\tan A = \frac{\sin A}{\cos A}, \quad \cot A = \frac{\cos A}{\sin A}$$

🧠 **Mnemonic: "Pandit Badri Prasad, Har Har Bole"** — **P**erpendicular/**H**ypotenuse = sin, **B**ase/**H**ypotenuse = cos, **P**erpendicular/**B**ase = tan. (Or the usual **SOH-CAH-TOA**.)

⚠ **Critical:** "opposite" and "adjacent" change places when you switch from ∠A to ∠C. Always re-read *which* angle the question is about before writing a ratio.

📖 Key Facts You Must Be Able to Justify

1. sin A is only a name, not a product. $\sin A$ means "the sine of angle A". "sin" separated from A has no meaning — so $\sin A$ can never be cancelled as $\sin \times A$.

2. Ratios do not depend on the size of the triangle. Any two right triangles with the same acute angle A are similar (AA), so their corresponding sides are proportional and every ratio comes out the same. The ratios depend **only on the angle**.

3. $\sin A \leq 1$ **and** $\cos A \leq 1$ always, because the hypotenuse is the longest side of a right triangle. Consequently $\text{cosec } A \geq 1$ and $\sec A \geq 1$.

4. $\sin^2 A$ **means** $(\sin A)^2$ — but $\sin^{-1} A$ does **not** mean $\text{cosec } A$. $\text{cosec } A = (\sin A)^{-1}$, while $\sin^{-1} A$ is a different idea studied later.

5. If one ratio is known, all six can be found — either by drawing the triangle and using Pythagoras, or by using the identities.

📖 Section 8.3 — Ratios of Specific Angles (with derivations)

🔑 **Never memorise blindly**

Two triangles generate the whole table. If you ever blank out in the exam, redraw these two triangles in ten seconds and read every value straight off them.

A. The 45° triangle (isosceles right triangle)

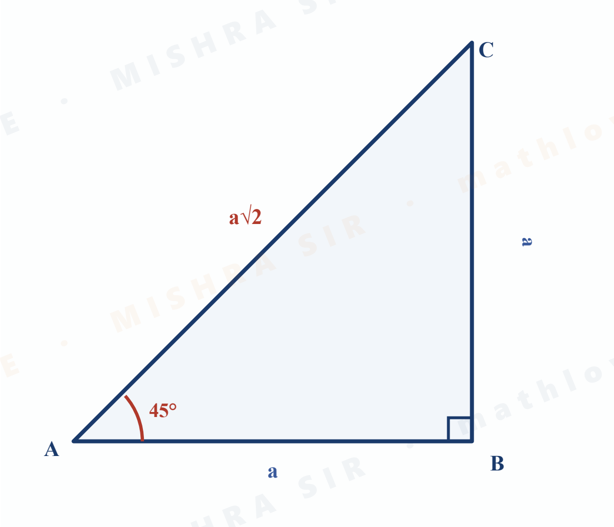

Fig. 8.14 — Right-angled at B with ∠A = ∠C = 45°, so BC = AB = a and AC = a√2.

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If $\angle A = 45^\circ$, then $\angle C = 45^\circ$, so $BC = AB = a$. By Pythagoras $AC^2 = a^2 + a^2 = 2a^2 \Rightarrow AC = a\sqrt{2}$.

$$\sin 45^\circ = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}, \cos 45^\circ = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}},$$

$$\tan 45^\circ = \frac{a}{a} = 1$$

Hence $\operatorname{cosec} 45^\circ = \sqrt{2}$, $\sec 45^\circ = \sqrt{2}$, $\cot 45^\circ = 1$.

B. The 30°–60° triangle (half of an equilateral triangle)

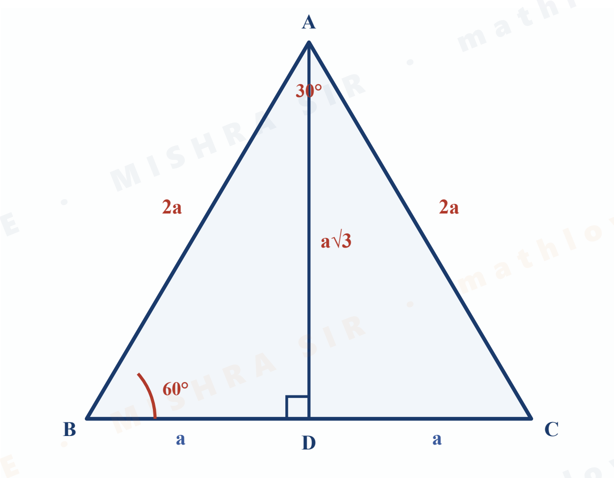


Fig. 8.15 — Equilateral △ABC of side 2a with altitude AD. In △ABD: ∠ABD = 60°, ∠BAD = 30°, BD = a, AD = a√3.

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Take an equilateral triangle ABC with $AB = 2a$; each angle is 60°. Drop the perpendicular AD onto BC. Then $\triangle ABD \cong \triangle ACD$, so $BD = DC = a$ and $\angle BAD = 30^\circ$.

$$AD^2 = AB^2 - BD^2 = (2a)^2 - a^2 = 3a^2 \Rightarrow AD = a\sqrt{3}$$

.

For 30° (angle BAD): $\sin 30^\circ = \dfrac{BD}{AB} = \dfrac{a}{2a} = \dfrac{1}{2}$, $\cos 30^\circ = \dfrac{AD}{AB} = \dfrac{\sqrt{3}}{2}$, $\tan 30^\circ = \dfrac{1}{\sqrt{3}}$

For 60° (angle ABD): $\sin 60^\circ = \dfrac{\sqrt{3}}{2}$, $\cos 60^\circ = \dfrac{1}{2}$, $\tan 60^\circ = \sqrt{3}$

C. The limiting cases 0° and 90°

As $\angle A$ shrinks towards 0°, side BC shrinks towards 0 and AC becomes almost equal to AB. So $\sin A \to 0$ and $\cos A \to 1$. We **define** $\sin 0^\circ = 0$ and $\cos 0^\circ = 1$.

As $\angle A$ grows towards 90°, AB shrinks towards 0 and AC almost coincides with BC. So we define $\sin 90^\circ = 1$ and $\cos 90^\circ = 0$.

> ⚠ **Why some entries are "not defined":** a ratio is undefined exactly when its denominator becomes zero. $\tan 90^\circ = \frac{\sin 90^\circ}{\cos 90^\circ} = \frac{1}{0}$ — undefined. Likewise $\cot 0^\circ$, $\operatorname{cosec} 0^\circ$ and $\sec 90^\circ$.

📋 Table 8.1 — Learn This Cold

∠A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

> 🧠 **Fastest way to reconstruct the sine row:** write $0, 1, 2, 3, 4$; divide each by 4; take the square root. You get $0, \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}, 1$. The cosine row is the same list **reversed**. Then tan = sin / cos, and the last three rows are reciprocals.

📌 **Remark:** as A increases from 0° to 90°, sin A **increases** from 0 to 1 while cos A **decreases** from 1 to 0.

📖 Section 8.4 — The Three Identities

Derivation (all three come from Pythagoras)

$$\text{In right } \triangle ABC \text{ (right angle at B):}$$
$$AB^2 + BC^2 = AC^2 \quad \ldots(1)$$

Dividing (1) by AC^2: $\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = 1$, i.e.

$$\boxed{\sin^2 A + \cos^2 A = 1} \quad \text{true for } 0^\circ \le A \le 90^\circ$$

Dividing (1) by AB^2:

$$\boxed{1 + \tan^2 A = \sec^2 A} \quad \text{true for } 0^\circ \le A < 90^\circ$$

Dividing (1) by BC^2:

$$\boxed{1 + \cot^2 A = \operatorname{cosec}^2 A} \quad \text{true for } 0^\circ < A \le 90^\circ$$

⚠ **The ranges are examinable.** Identity 2 excludes 90° because tan and sec are undefined there; identity 3 excludes 0° because cot and cosec are undefined there.

Useful rearrangements you should recognise instantly:

$\sin^2 A = 1 - \cos^2 A \cdot \cos^2 A = 1 - \sin^2 A \cdot$

$\sec^2 A - \tan^2 A = 1 \cdot \operatorname{cosec}^2 A - \cot^2 A = 1$

Factorised forms: $(\sec A - \tan A)(\sec A + \tan A) = 1$

and $(\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A) = 1$

💡 Shortcuts, Traps & Study Tips

1. Given one ratio, draw the triangle. $\sin A = \frac{3}{4} \rightarrow$ mark opposite 3, hypotenuse 4, find adjacent $= \sqrt{7}$ by Pythagoras.

Faster and safer than juggling identities.

2. Use k, not fixed numbers. $\tan A = \frac{4}{3}$ means $BC = 4k$, $AB = 3k$. The k cancels in every ratio — that *is* the proof that ratios don't depend on size.

3. For identities, convert everything to sin and cos. This single move solves the majority of Exercise 8.3.

4. Work on one side only. Start from the messier side and reduce it to the other. Never move terms across the "=" sign as if it were an equation — that loses marks.

5. Multiply by the conjugate when you see $1 \pm \sin$, $\sec \pm \tan$, or $\operatorname{cosec} \pm \cot$. $\frac{1}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}$ turns the denominator into $\cos^2 A$.

6. $\sin\theta = \frac{4}{3}$ is impossible — sine can never exceed 1. Spotting impossible values is a standard 1-mark question.

7. Rationalise final answers. Write $\frac{\sqrt{3}}{2}$, not $\frac{3}{2\sqrt{3}}$. CBSE expects a rationalised denominator.

8. Equal ratios ⇒ equal angles. If $\sin B = \sin Q$ (both acute), then $\angle B = \angle Q$. The proof uses similar triangles — learn it once, it appears in both Q6 forms.

📘 Solved Textbook Examples (Examples 1–12)

🔑 What each example teaches

Ex 1–5: building all six ratios from one. Ex 6–8: using the table of specific angles. Ex 9–12: identity technique, from easy to hard. Example 12 is the hardest piece of algebra in the chapter — work through it twice.

Example 1. Given tan A = 4/3, find the other trigonometric ratios of angle A. 3 marks

SOLUTION: *Draw right $\triangle ABC$, right-angled at B.*

Step 1. $\tan A = \frac{BC}{AB} = \frac{4}{3}$, *so let $BC = 4k$ and $AB = 3k$ (k a positive number).* 1 mark

Step 2 — Pythagoras:

$$AC^2 = AB^2 + BC^2 = (3k)^2 + (4k)^2 = 25k^2 \Rightarrow AC = 5k$$

1 mark

Step 3 — All six ratios:

$$\sin A = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5},\ \cos A = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$$
$$\cot A = \frac{1}{\tan A} = \frac{3}{4},\ \operatorname{cosec} A = \frac{1}{\sin A} = \frac{5}{4},$$
$$\sec A = \frac{1}{\cos A} = \frac{5}{3}$$ 1 mark

Example 2. If ∠B and ∠Q are acute angles such that sin B = sin Q, prove that ∠B = ∠Q. 3 marks

SOLUTION: *Take right triangles ABC (right angle at C) and PQR (right angle at R).*

Step 1. $\sin B = \frac{AC}{AB}$, $\sin Q = \frac{PR}{PQ}$. *Given equal, so* $\frac{AC}{AB} = \frac{PR}{PQ}$, *i.e.* $\frac{AC}{PR} = \frac{AB}{PQ} = k$ *(say) ...(1)* 1 mark

Step 2 — Pythagoras: $BC = \sqrt{AB^2 - AC^2}$ *and* $QR = \sqrt{PQ^2 - PR^2}$.

$$\frac{BC}{QR} = \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}} = \frac{\sqrt{k^2PQ^2 - k^2PR^2}}{\sqrt{PQ^2 - PR^2}} = \frac{k\sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}}$$ *...(2)* 1½ marks

Step 3. *From (1) and (2):* $\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$. *By SSS similarity,* $\triangle ACB \sim \triangle PRQ$.

∴ *corresponding angles are equal, so* $\angle \mathbf{B} = \angle \mathbf{Q}$. ½ mark

Example 3. In △ACB right-angled at C, AB = 29, BC = 21, ∠ABC = θ. Find (i) $\cos^2\theta + \sin^2\theta$ (ii) $\cos^2\theta - \sin^2\theta$. **3 marks**

SOLUTION:

Step 1.

$$AC = \sqrt{AB^2 - BC^2} = \sqrt{29^2 - 21^2} = \sqrt{(29-21)(29+21)} = \sqrt{8}$$

units **1 mark**

(Using $a^2 - b^2 = (a-b)(a+b)$ avoids squaring 29 and 21 — much faster.)

Step 2. $\sin\theta = \dfrac{AC}{AB} = \dfrac{20}{29}$, $\cos\theta = \dfrac{BC}{AB} = \dfrac{21}{29}$ **½ mark**

(i) $\cos^2\theta + \sin^2\theta = \dfrac{20^2 + 21^2}{29^2} = \dfrac{400 + 441}{841} = \dfrac{841}{841} = \mathbf{1}$

¾ mark

(ii) $\cos^2\theta - \sin^2\theta = \dfrac{21^2 - 20^2}{29^2} = \dfrac{(21+20)(21-20)}{841} = \dfrac{\mathbf{41}}{\mathbf{841}}$

¾ mark

Example 4. In right triangle ABC (right-angled at B), if tan A = 1, verify that 2 sin A cos A = 1. **2 marks**

SOLUTION: $\tan A = \dfrac{BC}{AB} = 1 \Rightarrow BC = AB$. *Let* $AB = BC = k$

. **½ mark**

$$AC = \sqrt{k^2 + k^2} = k\sqrt{2}$$ **½ mark**

$$\sin A = \frac{BC}{AC} = \frac{1}{\sqrt{2}}, \cos A = \frac{AB}{AC} = \frac{1}{\sqrt{2}}$$

$$2\sin A\cos A = 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = 2 \times \frac{1}{2} = \mathbf{1}\ \checkmark$$ **1 mark**

Example 5. In △OPQ right-angled at P, OP = 7 cm and OQ − PQ = 1 cm. Determine sin Q and cos Q. **3 marks**

SOLUTION: $OQ - PQ = 1 \Rightarrow OQ = 1 + PQ.$ **½ mark**

By Pythagoras: $OQ^2 = OP^2 + PQ^2$

$$(1 + PQ)^2 = 7^2 + PQ^2$$

$$1 + PQ^2 + 2PQ = 49 + PQ^2$$ **1 mark**

$2PQ = 48 \Rightarrow PQ = 24$ *cm, so* $OQ = 1 + 24 = 25$ *cm*

1 mark

$$\therefore \sin Q = \frac{OP}{OQ} = \frac{\mathbf{7}}{\mathbf{25}}, \cos Q = \frac{PQ}{OQ} = \frac{\mathbf{24}}{\mathbf{25}}$$ **½ mark**

Example 6. In △ABC right-angled at B, AB = 5 cm and ∠ACB = 30°. Find BC and AC. 2 marks

SOLUTION: *With respect to ∠C, AB is opposite and BC is adjacent.*

$$\tan C = \frac{AB}{BC} \Rightarrow \tan 30^\circ = \frac{5}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{5}{BC}$$

$$\therefore \mathbf{BC = 5\sqrt{3}} \text{ cm}$$ 1 mark

$$\sin C = \frac{AB}{AC} \Rightarrow \sin 30^\circ = \frac{5}{AC} \Rightarrow \frac{1}{2} = \frac{5}{AC} \Rightarrow \mathbf{AC = 10} \text{ cm}$$

1 mark

Check by Pythagoras:

$$AC = \sqrt{5^2 + (5\sqrt{3})^2} = \sqrt{25 + 75} = \sqrt{100} = 10\ cm \checkmark$$

Example 7. In △PQR right-angled at Q, PQ = 3 cm and PR = 6 cm. Determine ∠QPR and ∠PRQ. 2 marks

SOLUTION: *With respect to ∠R, PQ is opposite and PR is the hypotenuse.*

$$\sin R = \frac{PQ}{PR} = \frac{3}{6} = \frac{1}{2}$$ 1 mark

From the table, $\sin 30^\circ = \frac{1}{2}$, *so* $\angle \mathbf{PRQ = 30}^\circ$.

By the angle sum property, $\angle QPR = 180^\circ - 90^\circ - 30^\circ = \mathbf{60}^\circ$

1 mark

Example 8. If sin(A − B) = ½, cos(A + B) = ½, with 0° < A + B ≤ 90° and A > B, find A and B. 3 marks

SOLUTION:

$$\sin(A - B) = \frac{1}{2} = \sin 30^\circ \Rightarrow A - B = 30^\circ \text{ ...(1)}$$ 1 mark

$$\cos(A + B) = \frac{1}{2} = \cos 60^\circ \Rightarrow A + B = 60^\circ \text{ ...(2)}$$ 1 mark

Adding (1) and (2): $2A = 90^\circ \Rightarrow \mathbf{A = 45}^\circ$

Substituting in (2): $B = 60^\circ - 45^\circ = \mathbf{15}^\circ$ 1 mark

Example 9. Express cos A, tan A and sec A in terms of sin A. 3 marks

SOLUTION: *From* $\sin^2 A + \cos^2 A = 1$*:*

$$\cos^2 A = 1 - \sin^2 A \Rightarrow \cos A = \pm\sqrt{1-\sin^2 A}$$

Since A *is acute,* $\cos A$ *is positive, so* $\cos \mathbf{A} = \sqrt{\mathbf{1 - \sin^2 A}}$

1½ marks

$$\tan \mathbf{A} = \frac{\sin \mathbf{A}}{\cos \mathbf{A}} = \frac{\sin \mathbf{A}}{\sqrt{\mathbf{1 - \sin^2 A}}}$$ ¾ mark

$$\sec \mathbf{A} = \frac{\mathbf{1}}{\cos \mathbf{A}} = \frac{\mathbf{1}}{\sqrt{\mathbf{1 - \sin^2 A}}}$$ ¾ mark

Example 10. Prove that sec A (1 − sin A)(sec A + tan A) = 1. 3 marks

SOLUTION: *Convert everything to sin and cos.*

$$\text{LHS} = \frac{1}{\cos A}(1-\sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$$ 1 mark

$$= \frac{1}{\cos A}(1-\sin A)\cdot\frac{1+\sin A}{\cos A} = \frac{(1-\sin A)(1+\sin A)}{\cos^2 A}$$

1 mark

$$= \frac{1-\sin^2 A}{\cos^2 A} = \frac{\cos^2 A}{\cos^2 A} = 1 = \text{RHS} \ \blacksquare$$ 1 mark

Example 11. Prove that (cot A − cos A)/(cot A + cos A) = (cosec A − 1)/(cosec A + 1). 3 marks

SOLUTION: *Write* $\cot A = \dfrac{\cos A}{\sin A}$.

$$\text{LHS} = \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A}$$ 1 mark

Taking $\cos A$ *common from numerator and denominator:*

$$= \frac{\cos A\left(\frac{1}{\sin A} - 1\right)}{\cos A\left(\frac{1}{\sin A} + 1\right)} = \frac{\frac{1}{\sin A} - 1}{\frac{1}{\sin A} + 1}$$ 1 mark

$$= \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} = \text{RHS} \ \blacksquare$$ 1 mark

Example 12. Prove that (sin θ − cos θ + 1)/(sin θ + cos θ − 1) = 1/(sec θ − tan θ), using the identity sec²θ = 1 + tan²θ. **5 marks**

SOLUTION: *Since the RHS involves* $\sec\theta$ *and* $\tan\theta$*, divide the numerator and denominator of the LHS by* $\cos\theta$*.*

Step 1. LHS $= \dfrac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$ **1 mark**

Step 2. *Rewrite as* $\dfrac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1}$ **½ mark**

Step 3. *Multiply numerator and denominator by* $(\tan\theta - \sec\theta)$*:*

$$= \frac{\{(\tan\theta + \sec\theta) - 1\}(\tan\theta - \sec\theta)}{\{(\tan\theta - \sec\theta) + 1\}(\tan\theta - \sec\theta)}$$ **1 mark**

Step 4. *Numerator* $= (\tan^2\theta - \sec^2\theta) - (\tan\theta - \sec\theta)$.

Using $\tan^2\theta - \sec^2\theta = -1$*:*

$$= -1 - \tan\theta + \sec\theta$$ **1 mark**

Step 5. *So*

$$= \frac{-1 - \tan\theta + \sec\theta}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)} = \frac{-(1 + \tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - s}$$

The bracket $(1 + \tan\theta - \sec\theta)$ *cancels with* $(\tan\theta - \sec\theta + 1)$*:*

$$= \frac{-1}{\tan\theta - \sec\theta} = \frac{1}{\sec\theta - \tan\theta} = \text{RHS} \;\blacksquare$$ **1½ marks**

💡 **Much shorter alternative:** *since* $\sec^2\theta - \tan^2\theta = 1$*, we may replace the "1" in the numerator by* $\sec^2\theta - \tan^2\theta$*:* $\dfrac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$ *— multiply top and bottom by* $(\sec\theta - \tan\theta)$ *and simplify. Either route earns full marks.*

— End of Solved Examples —

📘 NCERT Solutions — Exercise 8.1

🔑 Approach for This Exercise

Almost every question follows one routine: from the given ratio, label two sides with a common factor k, find the third by Pythagoras, then read off whatever is asked. Draw the triangle every single time — it takes ten seconds and eliminates the opposite/adjacent confusion that costs most of the lost marks here.

Q1. In $\triangle ABC$ right-angled at B, AB = 24 cm, BC = 7 cm. Determine (i) sin A, cos A (ii) sin C, cos C. 3 marks

SOLUTION:

Step 1 — Third side:

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{24^2 + 7^2} = \sqrt{576 + 49} = \sqrt{625} = 25$$

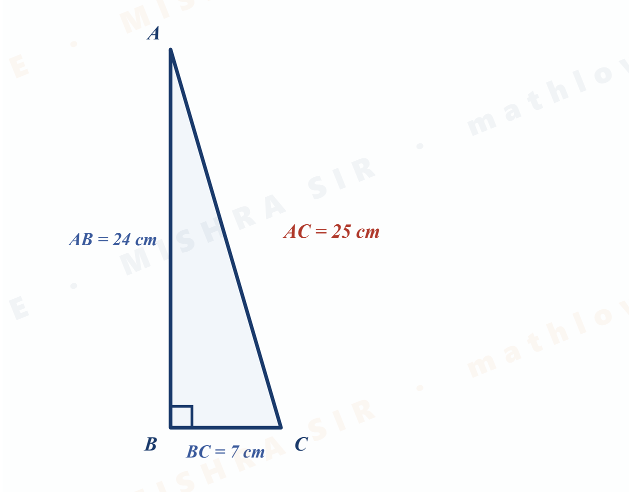
cm 1 mark

(i) For ∠A: *opposite = BC = 7, adjacent = AB = 24, hypotenuse = AC = 25.*

$$\sin A = \frac{BC}{AC} = \frac{\mathbf{7}}{\mathbf{25}}, \cos A = \frac{AB}{AC} = \frac{\mathbf{24}}{\mathbf{25}}$$ 1 mark

(ii) For ∠C: *opposite = AB = 24, adjacent = BC = 7.*

$$\sin C = \frac{AB}{AC} = \frac{\mathbf{24}}{\mathbf{25}}, \cos C = \frac{BC}{AC} = \frac{\mathbf{7}}{\mathbf{25}}$$ 1 mark



Note: *MLI-redrawn schematic — correct in construction but* **not drawn to scale**, *and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Note how "opposite" and "adjacent" swap when we move from ∠A to ∠C — hence $\sin A = \cos C$.

Q2. In Fig. 8.13, find tan P − cot R. 2 marks

SOLUTION: *From Fig. 8.13, △PQR is right-angled at Q, with*

PQ = 12 cm and PR = 13 cm.

Step 1: $QR = \sqrt{PR^2 - PQ^2} = \sqrt{169 - 144} = \sqrt{25} = 5\ cm$

1 mark

Step 2: *For ∠P: opposite = QR = 5, adjacent = PQ = 12 ⇒*

$$\tan P = \frac{5}{12}$$

For ∠R: adjacent = QR = 5, opposite = PQ = 12 ⇒

$$\cot R = \frac{QR}{PQ} = \frac{5}{12}$$

$$\therefore \tan P - \cot R = \frac{5}{12} - \frac{5}{12} = \mathbf{0}$$ 1 mark

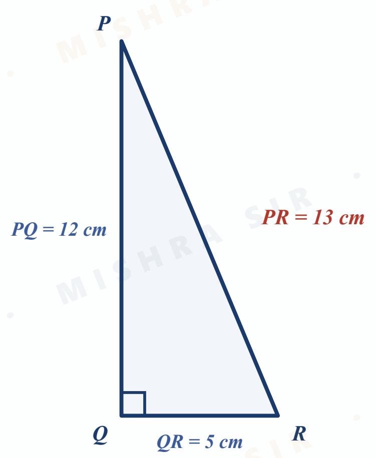


Fig. 8.13 (redrawn) — right-angled at Q, with QR found to be 5 cm.

Note: *MLI-redrawn schematic — correct in construction but* **not drawn to scale***, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Q3. If sin A = 3/4, calculate cos A and tan A. 2 marks

SOLUTION: $\sin A = \frac{BC}{AC} = \frac{3}{4}$, *so let* $BC = 3k$ *and* $AC = 4k$.

½ mark

$$AB = \sqrt{AC^2 - BC^2} = \sqrt{16k^2 - 9k^2} = \sqrt{7}\,k$$ 1 mark

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{7}}{4}, \quad \tan A = \frac{BC}{AB} = \frac{3}{\sqrt{7}}$$ ½ mark

Q4. Given 15 cot A = 8, find sin A and sec A. 2 marks

SOLUTION: $\cot A = \frac{8}{15} = \frac{AB}{BC}$, *so let* $AB = 8k,\ BC = 15k.$

½ mark

$$AC = \sqrt{(8k)^2 + (15k)^2} = \sqrt{64k^2 + 225k^2} = \sqrt{289k^2} = 17k$$

1 mark

$$\sin A = \frac{BC}{AC} = \mathbf{\frac{15}{17}}, \quad \sec A = \frac{AC}{AB} = \mathbf{\frac{17}{8}}$$

½ mark

Q5. Given sec θ = 13/12, calculate all other trigonometric ratios. 3 marks

SOLUTION: $\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{13}{12}$, *so hypotenuse* $= 13k$, *adjacent* $= 12k.$

½ mark

Opposite

$$= \sqrt{(13k)^2 - (12k)^2} = \sqrt{169k^2 - 144k^2} = \sqrt{25k^2} = 5k$$

1 mark

$$\cos\theta = \mathbf{\frac{12}{13}},\ \sin\theta = \mathbf{\frac{5}{13}},\ \tan\theta = \mathbf{\frac{5}{12}}$$
$$\operatorname{cosec}\theta = \mathbf{\frac{13}{5}},\ \cot\theta = \mathbf{\frac{12}{5}}$$

1½ marks

Q6. If ∠A and ∠B are acute angles such that cos A = cos B, show that ∠A = ∠B. 3 marks

SOLUTION: *Take two right triangles:* $\triangle ABC$ *right-angled at C and* $\triangle PQR$ *right-angled at R, with* $\angle A$ *and* $\angle B$ *placed at A and Q respectively.*

Step 1. $\cos A = \frac{AC}{AB}$ *and* $\cos B = \frac{QR}{PQ}$. *Given equal:*

$$\frac{AC}{AB} = \frac{QR}{PQ} \Rightarrow \frac{AC}{QR} = \frac{AB}{PQ} = k \text{ (say) } ...(1)$$

1 mark

Step 2. *By Pythagoras,* $BC = \sqrt{AB^2 - AC^2}$ *and* $PR = \sqrt{PQ^2 - QR^2}.$

$$\frac{BC}{PR} = \frac{\sqrt{k^2PQ^2 - k^2QR^2}}{\sqrt{PQ^2 - QR^2}} = \frac{k\sqrt{PQ^2 - QR^2}}{\sqrt{PQ^2 - QR^2}} = k \ ...(2)$$

1½ marks

Step 3. *From (1) and (2), all three pairs of sides are in the same ratio k, so the triangles are similar (SSS).*

$\therefore$ *corresponding angles are equal, giving* $\angle\mathbf{A} = \angle\mathbf{B}.$

½ mark

Q7. If cot θ = 7/8, evaluate (i) (1 + sin θ)(1 − sin θ) / (1 + cos θ)(1 − cos θ) (ii) cot²θ. 3 marks

SOLUTION — (I): *Use* $(1+x)(1-x) = 1 - x^2$ *first — do* **not** *substitute numbers yet.*

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$$

1½ marks

$$= \left(\frac{7}{8}\right)^2 = \mathbf{\frac{49}{64}}$$ ½ mark

SOLUTION — (II): $\cot^2\theta = \left(\frac{7}{8}\right)^2 = \mathbf{\frac{49}{64}}$ 1 mark

> 💡 *Parts (i) and (ii) give the same answer — that is the point of the question. Recognising the identity turns a page of work into two lines.*

Q8. If 3 cot A = 4, check whether (1 − tan²A)/(1 + tan²A) = cos²A − sin²A or not. 4 marks

SOLUTION: $\cot A = \frac{4}{3} \Rightarrow \tan A = \frac{3}{4}$. *So opposite = 3k, adjacent = 4k, hypotenuse =* $\sqrt{9k^2+16k^2} = 5k$. 1 mark

$$\sin A = \frac{3}{5},\ \cos A = \frac{4}{5}$$ ½ mark

LHS: $\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$ 1¼ marks

RHS: $\cos^2 A - \sin^2 A = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$ 1 mark

Since LHS = RHS, **the given statement is verified.** ¼ mark

Q9. In △ABC right-angled at B, if tan A = 1/√3, find (i) sin A cos C + cos A sin C (ii) cos A cos C − sin A sin C. 3 marks

SOLUTION: $\tan A = \dfrac{1}{\sqrt{3}} = \tan 30^\circ \Rightarrow \angle A = 30^\circ$. *Since* $\angle B = 90^\circ$, $\angle C = 60^\circ$. 1 mark

$\sin A = \dfrac{1}{2}$, $\cos A = \dfrac{\sqrt{3}}{2}$, $\sin C = \dfrac{\sqrt{3}}{2}$, $\cos C = \dfrac{1}{2}$ ½ mark

(i)

$$\sin A \cos C + \cos A \sin C = \frac{1}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \mathbf{1}$$

¾ mark

(ii)

$$\cos A \cos C - \sin A \sin C = \frac{\sqrt{3}}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = \mathbf{0}$$

¾ mark

💡 *These are secretly* $\sin(A + C) = \sin 90^\circ = 1$ *and* $\cos(A + C) = \cos 90^\circ = 0$ — *formulas you meet in Class 11.*

Q10. In $\triangle$PQR right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine sin P, cos P and tan P. 4 marks

SOLUTION: *Let $QR = x$ cm. Then $PR = 25 - x$ cm.*

½ mark

By Pythagoras: $PR^2 = PQ^2 + QR^2$

$(25 - x)^2 = 5^2 + x^2$ 1 mark

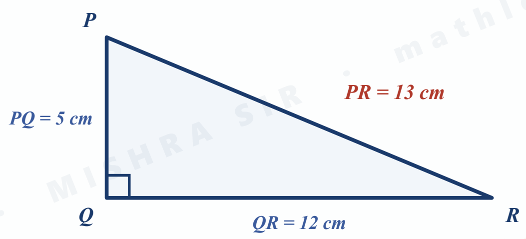
$625 - 50x + x^2 = 25 + x^2$

$-50x = -600 \Rightarrow x = 12$ 1 mark

$\therefore QR = 12$ *cm and* $PR = 25 - 12 = 13$ *cm.* ½ mark

For ∠P: opposite = QR = 12, adjacent = PQ = 5, hypotenuse = PR = 13.

$\sin P = \dfrac{12}{13}, \cos P = \dfrac{5}{13}, \tan P = \dfrac{12}{5}$ 1 mark



The familiar 5–12–13 triple appears again — worth memorising alongside 3–4–5, 7–24–25 and 8–15–17.

Note: *MLI-redrawn schematic — correct in construction but* **not drawn to scale**, *and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Q11. State whether true or false, and justify: (i) tan A is always less than 1 (ii) sec A = 12/5 for some angle A (iii) cos A is the abbreviation for cosecant of A (iv) cot A is the product of cot and A (v) sin θ = 4/3 for some angle θ. 5 marks

SOLUTION:

(i) FALSE. $\tan A$ *can take any non-negative value for an acute angle. For example* $\tan 60^\circ = \sqrt{3} \approx 1.73$, *which is greater than 1.* 1 mark

(ii) TRUE. $\sec A = \dfrac{12}{5} = 2.4$, *and since* $\sec A \geq 1$ *for all acute A, a value of 2.4 is perfectly possible (it corresponds to* $\cos A = \frac{5}{12}$*).* 1 mark

(iii) FALSE. $\cos A$ *is the abbreviation for the* **cosine** *of angle A. The cosecant is written* $\operatorname{cosec} A$. 1 mark

(iv) FALSE. $\cot A$ *is a single symbol meaning "the cotangent of angle A". "cot" separated from A has no meaning, so it is not a product.* 1 mark

(v) FALSE. $\dfrac{4}{3} \approx 1.33$, *but* $\sin\theta$ *can never exceed 1 because the opposite side is never longer than the hypotenuse.* 1 mark

— End of Exercise 8.1 —

📘 NCERT Solutions — Exercise 8.2

🔑 Approach for This Exercise

Substitute the table values first, simplify the numerator and denominator separately, then rationalise. Do not try to simplify symbolically before substituting — for these particular questions, numbers are quicker.

Q1(i). Evaluate sin 60° cos 30° + sin 30° cos 60°. 2 marks

SOLUTION: $= \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{3}}{2} + \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{3}{4} + \dfrac{1}{4} = \mathbf{1}$

2 marks

Q1(ii). Evaluate 2 tan²45° + cos²30° − sin²60°. 2 marks

SOLUTION: $= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2 + \frac{3}{4} - \frac{3}{4} = \mathbf{2}$

2 marks

Spot that $\cos 30^\circ = \sin 60^\circ$*, so the last two terms cancel before any arithmetic.*

Q1(iii). Evaluate cos 45° / (sec 30° + cosec 30°). 3 marks

SOLUTION:

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$ 1 mark

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2(1+\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2}(1+\sqrt{3})}$$ 1 mark

Multiply numerator and denominator by $(\sqrt{3}-1)$*; note* $(1+\sqrt{3})(\sqrt{3}-1) = 2$*:*

$$= \frac{\sqrt{3}(\sqrt{3}-1)}{2\sqrt{2}\times 2} = \frac{3-\sqrt{3}}{4\sqrt{2}} = \frac{(3-\sqrt{3})\sqrt{2}}{8} = \mathbf{\frac{3\sqrt{2}-\sqrt{6}}{8}}$$

1 mark

Q1(iv). Evaluate (sin 30° + tan 45° − cosec 60°) / (sec 30° + cos 60° + cot 45°). 3 marks

SOLUTION:

$$\textbf{Numerator} = \frac{1}{2} + 1 - \frac{2}{\sqrt{3}} = \frac{3}{2} - \frac{2}{\sqrt{3}} = \frac{3\sqrt{3}-4}{2\sqrt{3}}$$

1 mark

$$\textbf{Denominator} = \frac{2}{\sqrt{3}} + \frac{1}{2} + 1 = \frac{2}{\sqrt{3}} + \frac{3}{2} = \frac{4+3\sqrt{3}}{2\sqrt{3}}$$

1 mark

The common factor $2\sqrt{3}$ *cancels:*

$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4}$. *Multiplying top and bottom by* $(3\sqrt{3}-4)$*:*

$$= \frac{(3\sqrt{3}-4)^2}{27-16} = \frac{27-24\sqrt{3}+16}{11} = \mathbf{\frac{43-24\sqrt{3}}{11}}$$ 1 mark

Q1(v). Evaluate (5 cos²60° + 4 sec²30° − tan²45°) / (sin²30° + cos²30°). 3 marks

SOLUTION:

Denominator $= \sin^2 30^\circ + \cos^2 30^\circ = 1$ *(by the identity — no arithmetic needed).* 1 mark

Numerator $= 5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2 = \frac{5}{4} + 4 \times \frac{4}{3} - 1$ 1 mark

$$= \frac{5}{4} + \frac{16}{3} - 1 = \frac{15 + 64 - 12}{12} = \frac{67}{12}$$

$\therefore$ *the value is* $\mathbf{\frac{67}{12}}$ 1 mark

Q2. Choose the correct option and justify: (i) 2tan30°/(1 + tan²30°) (ii) (1 − tan²45°)/(1 + tan²45°) (iii) sin 2A = 2 sin A is true when A = (iv) 2tan30°/(1 − tan²30°) 4 marks

SOLUTION — (I):

$$\frac{2 \times \frac{1}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

∴ (A) sin 60° 1 mark

(II): $\frac{1-1}{1+1} = \frac{0}{2} = 0$ **∴ (D) 0** 1 mark

(III): *Test each option. At* $A = 0^\circ$*:* $LHS = \sin 0^\circ = 0$*,* $RHS = 2\sin 0^\circ = 0$ ✓.

At $A = 30^\circ$*:* $LHS = \sin 60^\circ = \frac{\sqrt{3}}{2}$*,* $RHS = 2 \times \frac{1}{2} = 1$ ✗.

∴ (A) 0° 1 mark

(IV): $\frac{2 \times \frac{1}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \frac{3}{\sqrt{3}} = \sqrt{3} = \tan 60^\circ$

∴ (C) tan 60° 1 mark

> 💡 *Parts (i), (ii) and (iv) are the double-angle formulas* $\sin 2A$, $\cos 2A$ *and* $\tan 2A$ *in disguise, evaluated at* $A = 30^\circ$ *or* 45°.

Q3. If tan(A + B) = $\sqrt{3}$ and tan(A − B) = 1/$\sqrt{3}$, with 0° < A + B ≤ 90° and A > B, find A and B. 3 marks

SOLUTION:

$$\tan(A+B) = \sqrt{3} = \tan 60^\circ \Rightarrow A + B = 60^\circ \textit{ ...(1)}$$ 1 mark

$$\tan(A-B) = \frac{1}{\sqrt{3}} = \tan 30^\circ \Rightarrow A - B = 30^\circ \textit{ ...(2)}$$ 1 mark

$$\textit{Adding: } 2A = 90^\circ \Rightarrow \mathbf{A = 45^\circ}$$

$$\textit{Subtracting: } 2B = 30^\circ \Rightarrow \mathbf{B = 15^\circ}$$ 1 mark

Q4. State whether true or false, with justification: (i) sin(A + B) = sin A + sin B (ii) sin θ increases as θ increases (iii) cos θ increases as θ increases (iv) sin θ = cos θ for all θ (v) cot A is not defined for A = 0°. 5 marks

SOLUTION:

(i) FALSE. *Counter-example with* $A = 30^\circ, B = 60^\circ$*: LHS* $= \sin 90^\circ = 1$*, but RHS* $= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2} \approx 1.37$*. Not equal.*

1 mark

(ii) TRUE. *As θ increases from 0° to 90°,* $\sin\theta$ *increases steadily from 0 to 1 (see the table:* $0, \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}, 1$*).* 1 mark

(iii) FALSE. $\cos\theta$ **decreases** *from 1 to 0 as θ goes from 0° to 90° (table row:* $1, \frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}, 0$*).* 1 mark

(iv) FALSE. *It holds only at* $\theta = 45^\circ$*. For instance at* $\theta = 0^\circ$, $\sin 0^\circ = 0$ *but* $\cos 0^\circ = 1$. 1 mark

(v) TRUE. $\cot A = \dfrac{\cos A}{\sin A}$*, and at* $A = 0^\circ$ *the denominator* $\sin 0^\circ = 0$*. Division by zero is undefined.* 1 mark

— End of Exercise 8.2 —

📘 NCERT Solutions — Exercise 8.3 (Identities)

🔑 **The universal method for Q4**

Take the messier side. Convert to sin and cos. Combine into a single fraction. Use $\sin^2 + \cos^2 = 1$ to simplify. Where a $1 \pm \sin$,

$\sec \pm \tan$ or $\operatorname{cosec} \pm \cot$ appears, multiply by the conjugate. Work on one side only until it becomes the other side.

Q1. Express sin A, sec A and tan A in terms of cot A. **3 marks**

SOLUTION: *Start from* $1 + \cot^2 A = \operatorname{cosec}^2 A.$

$$\operatorname{cosec} A = \sqrt{1 + \cot^2 A} \text{ (positive, as } A \text{ is acute)}$$

$$\therefore \sin \mathbf{A} = \frac{\mathbf{1}}{\sqrt{\mathbf{1} + \cot^{\mathbf{2}} \mathbf{A}}}$$ **1 mark**

$$\tan \mathbf{A} = \frac{\mathbf{1}}{\cot \mathbf{A}}$$ **1 mark**

$$\sec^2 A = 1 + \tan^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\therefore \sec \mathbf{A} = \frac{\sqrt{\mathbf{1} + \cot^{\mathbf{2}} \mathbf{A}}}{\cot \mathbf{A}}$$ **1 mark**

Q2. Write all the other trigonometric ratios of ∠A in terms of sec A. **4 marks**

SOLUTION: *Start from*

$$1 + \tan^2 A = \sec^2 A \Rightarrow \tan A = \sqrt{\sec^2 A - 1}.$$

$$\cos \mathbf{A} = \frac{\mathbf{1}}{\sec \mathbf{A}}$$ **¾ mark**

$$\tan \mathbf{A} = \sqrt{\sec^{\mathbf{2}} \mathbf{A} - \mathbf{1}}$$ **¾ mark**

$$\sin A = \tan A \cos A = \frac{\sqrt{\sec^{\mathbf{2}} \mathbf{A} - \mathbf{1}}}{\sec \mathbf{A}}$$ **¾ mark**

$$\operatorname{cosec} \mathbf{A} = \frac{\sec \mathbf{A}}{\sqrt{\sec^{\mathbf{2}} \mathbf{A} - \mathbf{1}}}$$ **¾ mark**

$$\cot \mathbf{A} = \frac{\mathbf{1}}{\sqrt{\sec^{\mathbf{2}} \mathbf{A} - \mathbf{1}}}$$ **1 mark**

Q3. Choose the correct option and justify: (i) 9sec²A − 9tan²A (ii) (1 + tanθ + secθ)(1 + cotθ − cosecθ) (iii) (secA + tanA)(1 − sinA) (iv) (1 + tan²A)/(1 + cot²A) 4 marks

(I): $9(\sec^2 A - \tan^2 A) = 9 \times 1 = 9 \therefore$ **(B) 9** 1 mark

(II): *Convert to sin and cos:*

$$\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right) = \frac{(\sin\theta + \cos\theta)^2 - 1}{\sin\theta\cos\theta}$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta} = 2 \therefore \textbf{(C) 2}$$ 1 mark

(III):

$$\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A) = \frac{(1 + \sin A)(1 - \sin A)}{\cos A} = \frac{\cos^2 A}{\cos A}$$

$\therefore$ **(D) cos A** 1 mark

(IV): $$\frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A \therefore \textbf{(D) tan²A}$$

1 mark

Q4(i). Prove: (cosec θ − cot θ)² = (1 − cos θ)/(1 + cos θ) 3 marks

SOLUTION:

$$\text{LHS} = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \frac{(1 - \cos\theta)^2}{\sin^2\theta}$$ 1 mark

$$= \frac{(1 - \cos\theta)^2}{1 - \cos^2\theta} = \frac{(1 - \cos\theta)^2}{(1 - \cos\theta)(1 + \cos\theta)}$$ 1 mark

$$= \frac{1 - \cos\theta}{1 + \cos\theta} = \text{RHS} \blacksquare$$ 1 mark

Q4(ii). Prove: cos A/(1 + sin A) + (1 + sin A)/cos A = 2 sec A 3 marks

SOLUTION: *Take the LCM* $(1 + \sin A)\cos A$.

$$\text{LHS} = \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A)\cos A}$$ 1 mark

$$= \frac{\cos^2 A + 1 + 2\sin A + \sin^2 A}{(1 + \sin A)\cos A} = \frac{1 + 1 + 2\sin A}{(1 + \sin A)\cos A}$$

1 mark

$$= \frac{2(1 + \sin A)}{(1 + \sin A)\cos A} = \frac{2}{\cos A} = 2\sec A = \text{RHS} \blacksquare$$ 1 mark

Q4(iii). Prove: tan θ/(1 − cot θ) + cot θ/(1 − tan θ) = 1 + sec θ cosec θ **5 marks**

SOLUTION: *Write* $s = \sin\theta$, $c = \cos\theta$ *for brevity.*

First term: $\dfrac{s/c}{1-\frac{c}{s}} = \dfrac{s/c}{\frac{s-c}{s}} = \dfrac{s^2}{c(s-c)}$ **1 mark**

Second term: $\dfrac{c/s}{1-\frac{s}{c}} = \dfrac{c/s}{\frac{c-s}{c}} = \dfrac{c^2}{s(c-s)} = \dfrac{-c^2}{s(s-c)}$

1 mark

Adding *over the common denominator* $sc(s-c)$*:*

$$= \frac{s^3 - c^3}{sc(s-c)}$$ **1 mark**

Using $s^3 - c^3 = (s-c)(s^2+sc+c^2)$*:*

$$= \frac{(s-c)(s^2+sc+c^2)}{sc(s-c)} = \frac{1+sc}{sc} \quad (\textit{since } s^2+c^2=1)$$

1½ marks

$$= \frac{1}{sc} + 1 = 1 + \sec\theta\,\mathrm{cosec}\,\theta = \mathrm{RHS} \ \blacksquare$$ **½ mark**

Q4(iv). Prove: (1 + sec A)/sec A = sin²A/(1 − cos A) **3 marks**

[Hint: simplify LHS and RHS separately]

SOLUTION:

LHS $= \dfrac{1}{\sec A} + \dfrac{\sec A}{\sec A} = \cos A + 1$ **1½ marks**

RHS

$$= \frac{\sin^2 A}{1-\cos A} = \frac{1-\cos^2 A}{1-\cos A} = \frac{(1-\cos A)(1+\cos A)}{1-\cos A} = 1 + \cos A$$

1½ marks

$\therefore$ *LHS = RHS* $\blacksquare$

Q4(v). Prove: (cos A − sin A + 1)/(cos A + sin A − 1) = cosec A + cot A, using cosec²A = 1 + cot²A 5 marks

SOLUTION: *Divide the numerator and denominator by* $\sin A$*:*

Step 1. $\text{LHS} = \dfrac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$ 1 mark

Step 2. *Group as* $\dfrac{(\cot A + \operatorname{cosec} A) - 1}{(\cot A - \operatorname{cosec} A) + 1}$ ½ mark

Step 3. *Multiply numerator and denominator by*

$(\cot A - \operatorname{cosec} A)$*:*

Numerator

$$= (\cot^2 A - \operatorname{cosec}^2 A) - (\cot A - \operatorname{cosec} A) = -1 - \cot A + \operatorname{cosec} A$$

1½ marks

Step 4.

$$\text{LHS} = \frac{-(1 + \cot A - \operatorname{cosec} A)}{(\cot A - \operatorname{cosec} A + 1)(\cot A - \operatorname{cosec} A)} = \frac{-1}{\cot A - \operatorname{cosec} A}$$

1 mark

Step 5. *Multiply by the conjugate, using* $\operatorname{cosec}^2 A - \cot^2 A = 1$*:*

$$= \frac{\operatorname{cosec} A + \cot A}{(\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A)} = \frac{\operatorname{cosec} A + \cot A}{1} = \text{RHS}$$

▰ 1 mark

Q4(vi). Prove: √((1 + sin A)/(1 − sin A)) = sec A + tan A 3 marks

SOLUTION: *Multiply inside the root by* $\dfrac{1 + \sin A}{1 + \sin A}$*:*

$$\text{LHS} = \sqrt{\frac{(1 + \sin A)^2}{(1 - \sin A)(1 + \sin A)}} = \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}}$$

1½ marks

$$= \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} = \frac{1 + \sin A}{\cos A}$$ 1 mark

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A = \text{RHS}$$ ▰ ½ mark

Q4(vii). Prove: (sin θ − 2 sin³θ)/(2 cos³θ − cos θ) = tan θ **3 marks**

SOLUTION: *Take* $\sin\theta$ *common above and* $\cos\theta$ *common below.*

$$\text{LHS} = \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$$ **1 mark**

$$\textit{Now } 1-2\sin^2\theta = 1-2(1-\cos^2\theta) = 2\cos^2\theta-1$$

1½ marks

So the brackets are identical and cancel:

$$\text{LHS} = \frac{\sin\theta}{\cos\theta} = \tan\theta = \text{RHS} \ \blacksquare$$ **½ mark**

Q4(viii). Prove: (sin A + cosec A)² + (cos A + sec A)² = 7 + tan²A + cot²A **4 marks**

SOLUTION: *Expand both squares.*

$$\text{LHS} = \sin^2 A + 2\sin A\,\text{cosec}\,A + \text{cosec}^2 A + \cos^2 A + 2\cos A\sec A$$

1 mark

Since $\sin A\,\text{cosec}\,A = 1$ *and* $\cos A\sec A = 1$*:*

$$= (\sin^2 A + \cos^2 A) + 2 + 2 + \text{cosec}^2 A + \sec^2 A$$ **1 mark**

$$= 1 + 4 + (1+\cot^2 A) + (1+\tan^2 A)$$ **1½ marks**

$$= 7 + \tan^2 A + \cot^2 A = \text{RHS} \ \blacksquare$$ **½ mark**

Q4(ix). Prove: (cosec A − sin A)(sec A − cos A) = 1/(tan A + cot A) [Hint: simplify LHS and RHS separately] **4 marks**

SOLUTION:

LHS

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right) = \frac{1-\sin^2 A}{\sin A}\cdot\frac{1-\cos^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\sin A}\cdot\frac{\sin^2 A}{\cos A} = \sin A\cos A$$ **2 marks**

RHS

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A\cos A}} = \frac{1}{\frac{1}{\sin A\cos A}} = \sin A\cos$$

2 marks

$\therefore$ *LHS = RHS* $\blacksquare$

Q4(x). Prove: (1 + tan²A)/(1 + cot²A) = ((1 − tan A)/(1 − cot A))² = tan²A 4 marks

SOLUTION — FIRST PART:

$$\frac{1+\tan^2 A}{1+\cot^2 A}=\frac{\sec^2 A}{\mathrm{cosec}^2 A}=\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}=\frac{\sin^2 A}{\cos^2 A}=\tan^2 A$$

2 marks

SECOND PART: *Write* $\cot A=\dfrac{1}{\tan A}$:

$$1-\cot A=1-\frac{1}{\tan A}=\frac{\tan A-1}{\tan A}$$ 1 mark

$$\therefore\ \frac{1-\tan A}{1-\cot A}=\frac{(1-\tan A)\tan A}{\tan A-1}=\frac{-(\tan A-1)\tan A}{\tan A-1}=-\tan$$

Squaring: $\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=(-\tan A)^2=\tan^2 A$ 1 mark

Hence all three expressions are equal. ∎

— End of Exercise 8.3 —

📝 Additional Practice Questions (Newly Created)

🎯 About this set

Freshly written practice in CBSE board style, arranged by mark value. Not reproduced from any past paper — pure extra drill on ratios, the specific-angle table, and identities.

A MCQ / Very Short Answer (6 Questions)

A1. The value of sin 30° + cos 60° is: (a) ½ (b) 1 (c) √3/2 (d) 0 1 mark

SOLUTION: $\frac{1}{2}+\frac{1}{2}=1$. **(b) 1** 1 mark

A2. If cos A = 3/5, then sin A equals: (a) 3/4 (b) 4/5 (c) 5/4 (d) 5/3 1 mark

SOLUTION: $\sin A=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$. **(b) 4/5**

1 mark

A3. The value of $\sec^2\theta - \tan^2\theta$ is: 1 mark

SOLUTION: *By the identity* $1 + \tan^2\theta = \sec^2\theta$*, the value is* **1**. 1 mark

A4. Evaluate tan 30° × tan 60°. 1 mark

SOLUTION: $\frac{1}{\sqrt{3}} \times \sqrt{3} = \mathbf{1}$ 1 mark

A5. If sin θ = cos θ where θ is acute, find θ. 1 mark

SOLUTION: *Dividing by* $\cos\theta$*:*

$\tan\theta = 1 = \tan 45^\circ \Rightarrow \theta = \mathbf{45}^\circ$ 1 mark

A6. Simplify $(1 - \cos^2 A)\operatorname{cosec}^2 A$. 1 mark

SOLUTION: $= \sin^2 A \times \frac{1}{\sin^2 A} = \mathbf{1}$ 1 mark

B Short Answer — 2 Marks (5 Questions)

B1. Evaluate $\sin^2 30^\circ + \cos^2 45^\circ + \tan^2 60^\circ$. 2 marks

SOLUTION:

$$= \left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2 = \tfrac{1}{4} + \tfrac{1}{2} + 3 = \tfrac{\mathbf{15}}{\mathbf{4}}$$

2 marks

B2. If tan θ = 3/4, find the value of sin θ + cos θ. 2 marks

SOLUTION: *Opposite = 3k, adjacent = 4k, hypotenuse = 5k.*

$\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$ $\Rightarrow \sin\theta + \cos\theta = \frac{\mathbf{7}}{\mathbf{5}}$ 2 marks

B3. Prove that (1 + tan²A) cos²A = 1. 2 marks

SOLUTION:

$$\text{LHS} = \sec^2 A \cdot \cos^2 A = \frac{1}{\cos^2 A} \times \cos^2 A = 1 = \text{RHS} \blacksquare$$

2 marks

B4. If 2 sin 2A = √3, find A. 2 marks

SOLUTION:

$$\sin 2A = \frac{\sqrt{3}}{2} = \sin 60^\circ \Rightarrow 2A = 60^\circ \Rightarrow \mathbf{A = 30}^\circ$$

2 marks

B5. Evaluate cos²60° + sin²30° + tan²45°. 2 marks

SOLUTION: $= \frac{1}{4} + \frac{1}{4} + 1 = \frac{3}{2}$ 2 marks

C Short/Long Answer — 3 Marks (5 Questions)

C1. If sec θ + tan θ = p, show that sec θ − tan θ = 1/p, and hence find sin θ in terms of p. 3 marks

SOLUTION: *Since* $\sec^2\theta - \tan^2\theta = 1$, *we have*

$$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1.$$

$$\therefore \sec\theta - \tan\theta = \frac{1}{p}$$ 1 mark

Adding: $2\sec\theta = p + \frac{1}{p} = \frac{p^2+1}{p}$; *Subtracting:*

$$2\tan\theta = p - \frac{1}{p} = \frac{p^2-1}{p}$$ 1 mark

$$\sin\theta = \frac{\tan\theta}{\sec\theta} = \frac{p^2-1}{p^2+1}$$ 1 mark

C2. Prove that (sin A + cos A)² + (sin A − cos A)² = 2. 3 marks

SOLUTION:

$$\text{LHS} = (\sin^2 A + 2\sin A\cos A + \cos^2 A) + (\sin^2 A - 2\sin A$$

1½ marks

The middle terms cancel:

$$= 2(\sin^2 A + \cos^2 A) = 2 \times 1 = 2 = \text{RHS} \;\blacksquare$$

1½ marks

C3. If 5 tan θ = 4, find the value of (5 sin θ − 3 cos θ)/(5 sin θ + 2 cos θ). 3 marks

SOLUTION: *Divide numerator and denominator by* $\cos\theta$*:*

$$= \frac{5\tan\theta - 3}{5\tan\theta + 2}$$ 1½ marks

Given $5\tan\theta = 4$*:*

$$= \frac{4-3}{4+2} = \frac{\mathbf{1}}{\mathbf{6}}$$ 1½ marks

💡 *Dividing throughout by* $\cos\theta$ *is the standard trick whenever every term is degree-one in sin and cos.*

C4. Prove that tan A/(1 + tan²A)² + cot A/(1 + cot²A)² = sin A cos A. 3 marks

SOLUTION: *Use* $1 + \tan^2 A = \sec^2 A$ *and*

$$1 + \cot^2 A = \operatorname{cosec}^2 A.$$

$$\text{LHS} = \frac{\tan A}{\sec^4 A} + \frac{\cot A}{\operatorname{cosec}^4 A}$$ 1 mark

$$= \frac{\sin A}{\cos A}\cos^4 A + \frac{\cos A}{\sin A}\sin^4 A = \sin A\cos^3 A + \cos A\sin^3 A$$

1 mark

$$= \sin A\cos A(\cos^2 A + \sin^2 A) = \sin A\cos A = \text{RHS} \;\blacksquare$$

1 mark

C5. If cos θ + sin θ = √2 cos θ, prove that cos θ − sin θ = √2 sin θ. 3 marks

SOLUTION: *From the given equation:*

$$\sin\theta = \sqrt{2}\cos\theta - \cos\theta = (\sqrt{2}-1)\cos\theta \text{ ...(1)}$$

1 mark

Now consider $\cos\theta - \sin\theta$ *and substitute from (1):*

$$= \cos\theta - (\sqrt{2}-1)\cos\theta = (2-\sqrt{2})\cos\theta$$ 1 mark

$$= \sqrt{2}(\sqrt{2}-1)\cos\theta = \sqrt{2}\sin\theta \text{ (using (1) again)}$$

$$\therefore \cos\theta - \sin\theta = \sqrt{2}\sin\theta \;\blacksquare$$ 1 mark

D Application / Case-Study (2 Questions)

D1. A ladder rests against a vertical wall, making an angle of 60° with the level ground. The foot of the ladder is 2.5 m from the base of the wall. (i) Find the length of the ladder. (ii) Find the height on the wall it reaches. 4 marks

SOLUTION: *A right triangle is formed with the 60° angle at the foot of the ladder; the 2.5 m is the side* **adjacent** *and the ladder is the* **hypotenuse**.

(i) $\cos 60^\circ = \dfrac{2.5}{L} \Rightarrow \dfrac{1}{2} = \dfrac{2.5}{L} \Rightarrow \mathbf{L = 5}\text{ m}$ 2 marks

(ii) $\sin 60^\circ = \dfrac{h}{5} \Rightarrow h = 5 \times \dfrac{\sqrt{3}}{2} = \dfrac{\mathbf{5\sqrt{3}}}{\mathbf{2}} \approx \mathbf{4.33}\text{ m}$

2 marks

D2. A wheelchair ramp is 3 m long along its slope and rises 1.5 m vertically. (i) Find the angle the ramp makes with the ground. (ii) Find the horizontal distance it covers. 4 marks

SOLUTION: *The 1.5 m rise is the side opposite the angle; the 3 m slope is the hypotenuse.*

(i) $\sin\theta = \frac{1.5}{3} = \frac{1}{2} = \sin 30^\circ \Rightarrow \theta = \mathbf{30^\circ}$ 2 marks

(ii) *Horizontal distance*

$= 3\cos 30^\circ = 3 \times \frac{\sqrt{3}}{2} = \frac{\mathbf{3\sqrt{3}}}{\mathbf{2}} \approx \mathbf{2.60}$ m 2 marks

— End of Additional Practice Questions —

📋 Chapter Summary — Revise in 60 Seconds

#	Result
1	$\sin A = \frac{\text{opp}}{\text{hyp}}$, $\cos A = \frac{\text{adj}}{\text{hyp}}$, $\tan A = \frac{\text{opp}}{\text{adj}}$
2	$\text{cosec } A = \frac{1}{\sin A}$, $\sec A = \frac{1}{\cos A}$, $\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A}$, $\tan A = \frac{\sin A}{\cos A}$
3	If one ratio of an acute angle is known, all the others can be determined.
4	Values of all six ratios at 0°, 30°, 45°, 60°, 90° (Table 8.1).
5	$\sin A \le 1$ and $\cos A \le 1$; $\sec A \ge 1$ $(0^\circ \le A < 90^\circ)$ and $\text{cosec } A \ge 1$ $(0^\circ < A \le 90^\circ)$.
6	$\sin^2 A + \cos^2 A = 1$; $\sec^2 A - \tan^2 A = 1$ for $0^\circ \le A < 90^\circ$; $\text{cosec}^2 A = 1 + \cot^2 A$ for $0^\circ < A \le 90^\circ$.

CHAPTER 9

Some Applications of Trigonometry

Exercise 9.1

EXERCISE 9.1

📘 NCERT Solutions — Exercise 9.1 (15 Questions)

🔑 Approach for This Exercise

Every question in this chapter becomes a right-angled triangle once you draw it. A fixed routine handles all fifteen: **(1)** draw the vertical object and the horizontal ground; **(2)** mark the angle *at the observer's eye*, between the line of sight and the horizontal; **(3)** decide which ratio links what you know to what you want — use **tan** when the two legs (height and horizontal distance) are involved, and **sin** or **cos** when the hypotenuse (a rope, string, slide or line of sight) is involved; **(4)** substitute the standard value and solve.

Angle	sin	cos	tan
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

One rule that saves marks: the *angle of depression* from a top point equals the *angle of elevation* from the bottom point, because the horizontal at the top and the ground are parallel (alternate angles). Always convert depression into elevation before writing any ratio.

Q1. A circus artist is climbing a 20 m long rope, tightly stretched and tied 2 marks from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground is 30°.

SOLUTION:

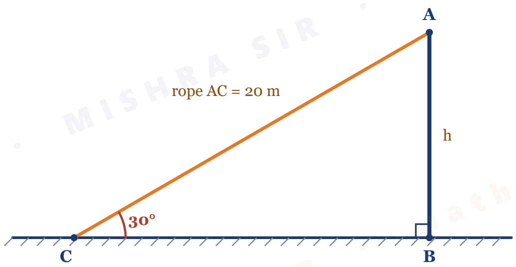


Fig. Q1 — The rope AC is the hypotenuse; the pole AB is opposite the 30° angle.

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Given: $AC = 20$ m (rope), $\angle ACB = 30^\circ$. **To find:** $AB = h$, the height of the pole.

The rope is the **hypotenuse** and the pole is the side **opposite** the angle, so use $\sin$:

$$\sin 30^\circ = \frac{AB}{AC} = \frac{h}{20}$$ **1 mark**

$$\frac{1}{2} = \frac{h}{20}$$

$$h = \frac{20}{2} = \mathbf{10}\ \mathrm{m}$$ **1 mark**

📌 The rope's length is *not* the horizontal distance — using tan here is the commonest error. Whenever a rope, string, ladder or slide is mentioned, that is the hypotenuse.

Q2. A tree breaks due to a storm and the broken part bends so that the top **3 marks**

touches the ground making an angle of 30° with it. The distance from the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

SOLUTION:

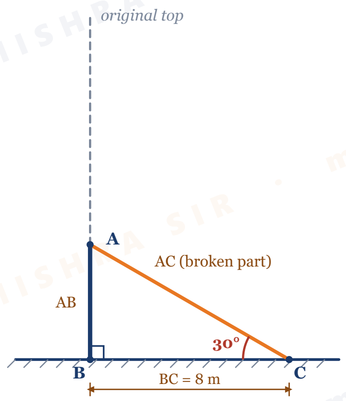


Fig. Q2 — Tree broken at A: standing part AB, fallen part AC; the full height is AB + AC.

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Given: $BC = 8$ m, $\angle ACB = 30^\circ$. **To find:** total height $= AB + AC$.

Step 1 — Find AB (the standing part). It is opposite the angle, with BC adjacent, so use $\tan$:

$$\tan 30^\circ = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{8}$$

$$AB = \frac{8}{\sqrt{3}} \text{ m}$$ **1 mark**

Step 2 — Find AC (the broken part). It is the hypotenuse, with BC adjacent, so use $\cos$:

$$\cos 30^\circ = \frac{BC}{AC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{AC}$$

$$AC = \frac{16}{\sqrt{3}} \text{ m}$$ **1 mark**

Step 3 — Add. The original tree was the standing part plus the broken part:

$$h = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} = \frac{24\sqrt{3}}{3} = 8\sqrt{3}$$

Height of the tree $= \mathbf{8\sqrt{3}}$ m $\approx$ **13.86** m **1 mark**

📌 **The broken part is not lost.** Students often answer $8/\sqrt{3}$ m and stop. The tree's original height includes the bent portion, so both pieces must be found and added.

Q3. A contractor plans two slides. For children below 5 years: top at height 1.5 m, inclined at 30°. For elder children: height 3 m, inclined at 60°. What should be the length of each slide? **3 marks**

SOLUTION:

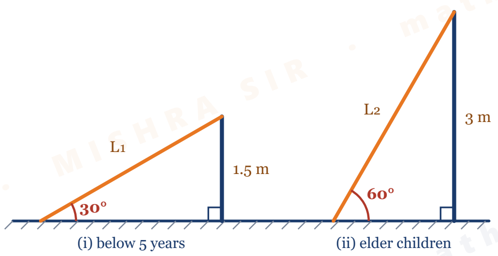


Fig. Q3 — Each slide is the hypotenuse; the height of its top is opposite the angle.

Note: *MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

In both cases the slide is the **hypotenuse** and the height is **opposite** the angle, so $\sin$ is the right ratio.

(i) Slide for younger children: height $= 1.5$ m, angle $= 30°$.

$$\sin 30^\circ = \frac{1.5}{L} \Rightarrow \frac{1}{2} = \frac{1.5}{L}$$

$$L = 1.5 \times 2 = \mathbf{3}\ \text{m}$$ **1½ marks**

(ii) Slide for elder children: height $= 3$ m, angle $= 60^\circ$.

$$\sin 60^\circ = \frac{3}{L} \Rightarrow \frac{\sqrt{3}}{2} = \frac{3}{L}$$

$$L = \frac{3 \times 2}{\sqrt{3}} = \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

$$\mathbf{L = 2\sqrt{3}\ \text{m} \approx 3.46\ \text{m}}$$ **1½ marks**

📌 **A sanity check worth doing.** The second slide is twice as tall but only a little longer — that is exactly what "steeper" means. If your steep slide came out much longer, you have used the wrong ratio.

Q4. The angle of elevation of the top of a tower from a point on the ground, 30 m away from the foot of the tower, is 30°. Find the height of the tower. **2 marks**

SOLUTION:

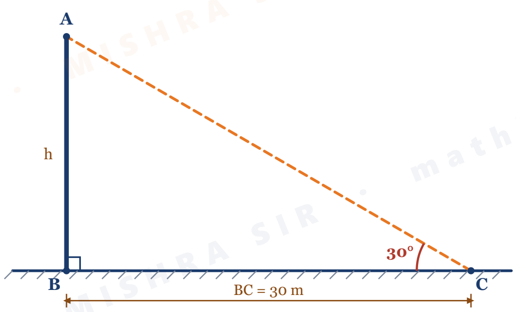


Fig. Q4 — Line of sight CA; the height AB is opposite the 30° angle and BC is adjacent.

__Note:__ MLI-redrawn schematic — correct in construction but __not drawn to scale__, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.

Given: distance $BC = 30$ m, $\angle ACB = 30^\circ$. **To find:** height $AB = h$.

Both the height (opposite) and the distance (adjacent) are involved, so use $\tan$:

$$\tan 30^\circ = \frac{h}{30} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{30}$$ **1 mark**

$$h = \frac{30}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}$$

$$\text{Height} = \mathbf{10\sqrt{3}}\ \text{m} \approx \mathbf{17.32}\ \text{m}$$ **1 mark**

📌 **Always rationalise.** $\frac{30}{\sqrt{3}}$ is correct but $10\sqrt{3}$ is the expected form; leaving a surd in the denominator can cost the final half mark.

Q5. A kite is flying at a height of 60 m above the ground. The string is tied to a point on the ground and makes an angle of 60° with it. Find the length of the string, assuming no slack. **2 marks**

SOLUTION:

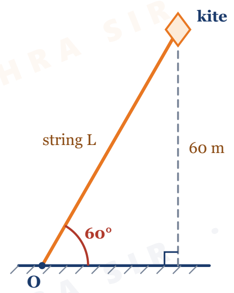


Fig. Q5 — The string is the hypotenuse; the kite's height of 60 m is opposite the 60° angle.

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Given: height $= 60$ m, angle $= 60^\circ$. **To find:** length of string L (the hypotenuse).

Height is opposite, string is hypotenuse → use sin:

$$\sin 60^\circ = \frac{60}{L} \Rightarrow \frac{\sqrt{3}}{2} = \frac{60}{L}$$ **1 mark**

$$L = \frac{60 \times 2}{\sqrt{3}} = \frac{120}{\sqrt{3}} = \frac{120\sqrt{3}}{3} = 40\sqrt{3}$$

$$\text{Length of string} = \mathbf{40\sqrt{3}}\ \text{m} \approx \mathbf{69.28}\ \text{m}$$ **1 mark**

📌 "No slack" is the phrase that makes the string a straight line — without it the string would be a curve and no triangle would exist.

Q6. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked. **3 marks**

SOLUTION:

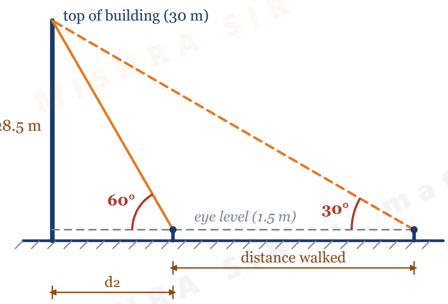


Fig. Q6 — Measure from the boy's eyes: the height above eye level is 30 − 1.5 = 28.5 m.

Note: *MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Step 1 — Subtract the boy's height. The angles are measured from his *eyes*, not the ground, so the effective height of the building above eye level is

$$30 - 1.5 = 28.5 \text{ m} = \frac{57}{2} \text{ m}$$ **1 mark**

Step 2 — Far position (30°). Let the horizontal distance be d_1:

$$\tan 30^\circ = \frac{28.5}{d_1} \Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{d_1} \Rightarrow d_1 = 28.5\sqrt{3} \text{ m}$$

½ mark

Step 3 — Near position (60°). Let the distance be d_2:

$$\tan 60^\circ = \frac{28.5}{d_2} \Rightarrow \sqrt{3} = \frac{28.5}{d_2} \Rightarrow d_2 = \frac{28.5}{\sqrt{3}} = 9.5\sqrt{3} \text{ m}$$

½ mark

Step 4 — The distance walked is the difference.

$$d_1 - d_2 = 28.5\sqrt{3} - 9.5\sqrt{3} = 19\sqrt{3}$$

$$\text{Distance walked} = \mathbf{19\sqrt{3}} \text{ m} \approx \mathbf{32.91} \text{ m}$$ **1 mark**

> 📌 **The 1.5 m is the whole point of this question.**
> Forgetting to subtract it gives $20\sqrt{3}$ m — a near-miss answer that earns almost nothing, because the error is conceptual rather than arithmetic.

Q7. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower. **3 marks**

SOLUTION:

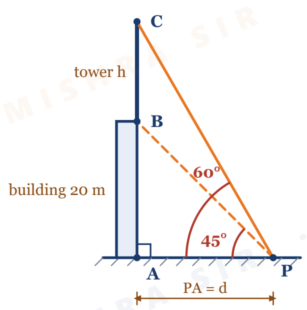


Fig. Q7 — Both angles are at P: 45° to the top of the building B, 60° to the top of the tower C.

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Let the point be P, the building be $AB = 20$ m, and the tower $BC = h$ sit on top of it. Let $PA = d$.

Step 1 — Use the 45° angle (to the bottom of the tower, i.e. top of the building).

$$\tan 45^\circ = \frac{20}{d} \Rightarrow 1 = \frac{20}{d} \Rightarrow d = 20 \text{ m}$$ **1 mark**

Step 2 — Use the 60° angle (to the top of the tower).

The total height is $20 + h$:

$$\tan 60^\circ = \frac{20+h}{d} = \frac{20+h}{20}$$

$$\sqrt{3} = \frac{20+h}{20} \Rightarrow 20 + h = 20\sqrt{3}$$ **1 mark**

Step 3 — Solve for h.

$$h = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1)$$

$$\text{Height of tower} = \mathbf{20(\sqrt{3} - 1)}\text{ m} \approx \mathbf{14.64}\text{ m}$$ **1 mark**

📌 **Start with the 45° angle whenever it appears —** $\tan 45^\circ = 1$ gives the distance in one line, and everything else follows from it.

Q8. A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and of the top of the pedestal is 45°. Find the height of the pedestal. **3 marks**

SOLUTION:

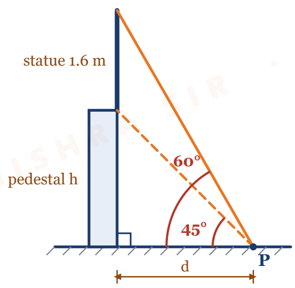

Fig. Q8 — Top of pedestal at 45°, top of statue at 60°, both seen from P.

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Let the pedestal height be h m and the horizontal distance be d m.

Step 1 — The 45° angle (top of pedestal).

$$\tan 45^\circ = \frac{h}{d} \Rightarrow 1 = \frac{h}{d} \Rightarrow d = h$$ **1 mark**

Step 2 — The 60° angle (top of statue, height $h + 1.6$).

$$\tan 60^\circ = \frac{h+1.6}{d} \Rightarrow \sqrt{3} = \frac{h+1.6}{h} \quad \text{(substituting } d = h\text{)}$$ **1 mark**

Step 3 — Solve.

$$\sqrt{3}h = h + 1.6 \Rightarrow h(\sqrt{3}-1) = 1.6 \Rightarrow h = \frac{1.6}{\sqrt{3}-1}$$

Rationalising by multiplying numerator and denominator by $(\sqrt{3}+1)$:

$$h = \frac{1.6(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{1.6(\sqrt{3}+1)}{3-1} = \frac{1.6(\sqrt{3}+1)}{2} = 0.8($$

$$\text{Height of pedestal} = \mathbf{0.8(\sqrt{3}+1)}\ \text{m} \approx \mathbf{2.19}\ \text{m}$$ **1 mark**

📌 **Rationalising a two-term denominator** uses the conjugate: $(\sqrt{3}-1)(\sqrt{3}+1) = 3 - 1 = 2$. This step appears again in Q11 and Q13.

Q9. The angle of elevation of the top of a building from the foot of a tower is 30°, and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building. 3 marks

SOLUTION:

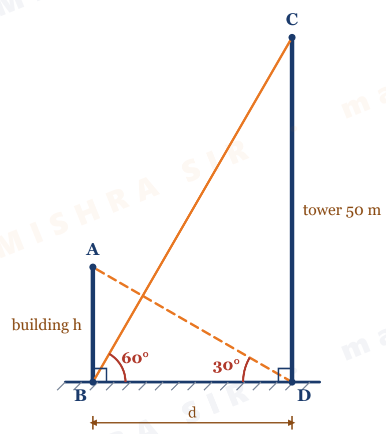

Fig. Q9 — The feet B and D are the observation points; the distance d is shared by both triangles.

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Let the building be $AB = h$, the tower be $CD = 50$ m, and the distance between their feet be d.

Step 1 — Find d using the tower. From the foot of the building, the tower's top has elevation 60°:

$$\tan 60^\circ = \frac{50}{d} \Rightarrow \sqrt{3} = \frac{50}{d} \Rightarrow d = \frac{50}{\sqrt{3}} \text{ m}$$ 1 mark

Step 2 — Use d with the building. From the foot of the tower, the building's top has elevation 30°:

$$\tan 30^\circ = \frac{h}{d} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{50/\sqrt{3}}$$ 1 mark

Step 3 — Solve.

$$h = \frac{1}{\sqrt{3}} \times \frac{50}{\sqrt{3}} = \frac{50}{3}$$

$$\text{Height of building} = \frac{50}{3}\ \text{m} \approx \mathbf{16.67}\ \text{m}$$ **1 mark**

📌 **Both triangles share the same base d** — that shared distance is the bridge between them. Find it from the triangle where a length is known, then carry it across.

Q10. Two poles of equal height stand opposite each other on either side of an 80 m wide road. From a point between them on the road, the angles of elevation of their tops are 60° and 30°. Find the height of the poles and the distances of the point from them. **4 marks**

SOLUTION:

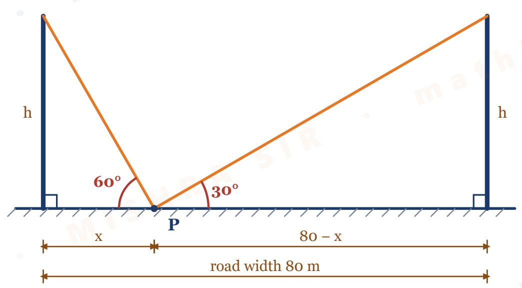

Fig. Q10 — P lies between the poles; the two distances add up to the 80 m road.

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Let the height of each pole be h m and let P be at distance x m from the pole seen at 60°. Then the other pole is $(80 - x)$ m away.

Step 1 — The 60° triangle. $\tan 60^\circ = \dfrac{h}{x} \Rightarrow h = x\sqrt{3}$ ... (1) **1 mark**

Step 2 — The 30° triangle.

$$\tan 30^\circ = \frac{h}{80 - x} \Rightarrow h = \frac{80 - x}{\sqrt{3}} \quad \text{...(2)}$$ **1 mark**

Step 3 — Equate (1) and (2).

$$x\sqrt{3} = \frac{80 - x}{\sqrt{3}} \Rightarrow 3x = 80 - x \Rightarrow 4x = 80 \Rightarrow x = 20 \text{ m}$$

1 mark

Step 4 — Find the height and the other distance.

$h = 20\sqrt{3}$ m; the other distance $= 80 - 20 = 60$ m.

Height of each pole $= 20\sqrt{3}$ **m** ≈ 34.64 **m; the point is 20 m from one pole and 60 m from the other.**

1 mark

Check: from 60 m, $\tan 30^\circ = \frac{20\sqrt{3}}{60} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$ ✓

> 📌 **Using one unknown x instead of two** is what makes this quick — the road width lets you write the second distance as $80 - x$ rather than introducing a new letter.

Q11. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top is 60°. From another point 20 m away from this point on the line joining it to the foot of the tower, the elevation is 30°. Find the height of the tower and the width of the canal. **4 marks**

SOLUTION:

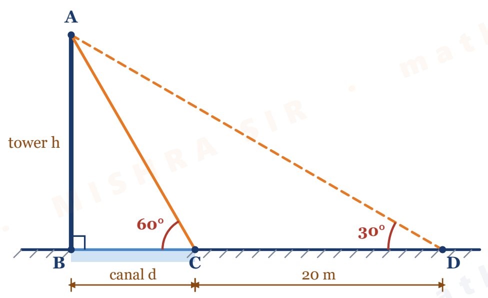

Fig. Q11 — C is directly opposite the tower across the canal; D is 20 m further back.

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Let the tower be $AB = h$, the canal width be $BC = d$ (C is the point directly opposite), and D be 20 m beyond C, so $BD = d + 20$.

Step 1 — From C (60°). $\tan 60^\circ = \dfrac{h}{d} \Rightarrow h = d\sqrt{3}$ ...(1)

1 mark

Step 2 — From D (30°).

$\tan 30^\circ = \dfrac{h}{d+20} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{d+20} \Rightarrow h\sqrt{3} = d + 20$ ...(2)

1 mark

Step 3 — Substitute (1) into (2).

$(d\sqrt{3})\sqrt{3} = d + 20 \Rightarrow 3d = d + 20 \Rightarrow 2d = 20 \Rightarrow d = 10$ m

1 mark

Step 4 — Find the height. From (1), $h = 10\sqrt{3}$ m.

Width of the canal $= 10$ **m; height of the tower** $= 10\sqrt{3}\ \text{m} \approx 17.32$ **m.** **1 mark**

📌 **The further point always has the smaller angle.** If your working gives the 30° point closer than the 60° point, the two triangles have been swapped — check before solving.

Q12. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower. **3 marks**

SOLUTION:

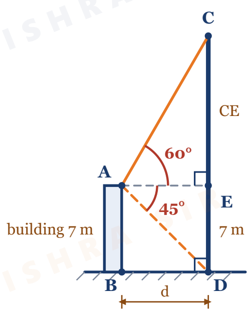

Fig. Q12 — The horizontal AE splits the tower: DE equals the building (7 m), CE is the part above.

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Let the building be $AB = 7$ m and the tower be CD, with the horizontal distance between them $= d$.
Draw a horizontal line from the top of the building A meeting the tower at E. This splits the tower into $DE = 7$ m (level with the building) and CE (the part above).

Step 1 — Use the depression of 45° to find d. The angle of depression from A to the foot D equals the angle of elevation from D to A (alternate angles), so

$$\tan 45^\circ = \frac{7}{d} \Rightarrow 1 = \frac{7}{d} \Rightarrow d = 7 \text{ m}$$ **1 mark**

Step 2 — Use the elevation of 60° to find CE.

$$\tan 60^\circ = \frac{CE}{d} = \frac{CE}{7} \Rightarrow CE = 7\sqrt{3} \text{ m}$$ **1 mark**

Step 3 — Total height of the tower.

$$CD = CE + DE = 7\sqrt{3} + 7 = 7(\sqrt{3} + 1)$$

Height of tower $= \mathbf{7(\sqrt{3}+1)}$ m $\approx \mathbf{19.12}$ m **1 mark**

📌 **The horizontal line from the observer is essential.** It creates the rectangle that makes DE equal to the building's height — without drawing it, the lower part of the tower has no length and the answer comes out as just $7\sqrt{3}$.

Q13. As observed from the top of a 75 m high lighthouse from sea level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. **3 marks**

SOLUTION:

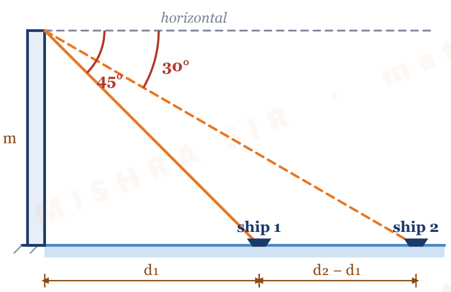

Fig. Q13 — Angles of depression are measured below the horizontal at the top of the lighthouse.

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

The angles of depression equal the angles of elevation from the ships. The ship at 45° is nearer; the one at 30° is further.

Step 1 — Nearer ship (45°).

$$\tan 45^\circ = \frac{75}{d_1} \Rightarrow 1 = \frac{75}{d_1} \Rightarrow d_1 = 75 \text{ m}$$ **1 mark**

Step 2 — Further ship (30°).

$$\tan 30^\circ = \frac{75}{d_2} \Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{d_2} \Rightarrow d_2 = 75\sqrt{3} \text{ m}$$ **1 mark**

Step 3 — Distance between the ships.

$$d_2 - d_1 = 75\sqrt{3} - 75 = 75(\sqrt{3} - 1)$$

$$\text{Distance} = \mathbf{75(\sqrt{3} - 1)} \text{ m} \approx \mathbf{54.9} \text{ m}$$ **1 mark**

📌 **"Exactly behind the other on the same side"** means you subtract the distances. If the ships were on opposite sides of the lighthouse, you would add them instead — read this phrase carefully.

Q14. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from her eyes is 60°; after some time it reduces to 30°. Find the distance travelled by the balloon during the interval. **3 marks**

SOLUTION:

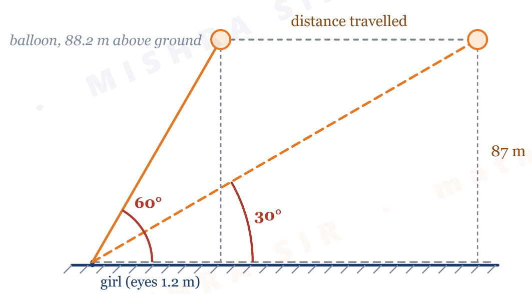

Fig. Q14 — The balloon moves along a horizontal line 87 m above the girl's eye level.

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram;*

work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.

Step 1 — Height above eye level. The angles are measured from the girl's eyes:

$88.2 - 1.2 = 87$ m **1 mark**

Step 2 — First position (60°).

$$\tan 60^\circ = \frac{87}{d_1} \Rightarrow \sqrt{3} = \frac{87}{d_1} \Rightarrow d_1 = \frac{87}{\sqrt{3}} = 29\sqrt{3}\text{ m}$$

½ mark

Step 3 — Second position (30°).

$$\tan 30^\circ = \frac{87}{d_2} \Rightarrow \frac{1}{\sqrt{3}} = \frac{87}{d_2} \Rightarrow d_2 = 87\sqrt{3}\text{ m}$$ **½ mark**

Step 4 — Distance travelled. The balloon moves horizontally, so the distance is the difference:

$$d_2 - d_1 = 87\sqrt{3} - 29\sqrt{3} = 58\sqrt{3}$$

$$\text{Distance travelled} = \mathbf{58\sqrt{3}}\ \text{m} \approx \mathbf{100.46}\ \text{m}$$ **1 mark**

📌 **The 1.2 m matters here as in Q6.** Note that $87 = 29 \times 3$, which is why $\frac{87}{\sqrt{3}}$ simplifies neatly to $29\sqrt{3}$ — the numbers are chosen to work out cleanly, so an ugly answer signals an error.

Q15. A straight highway leads to the foot of a tower. A man at the top of the tower observes a car at an angle of depression of 30°, approaching the foot with uniform speed. Six seconds later the angle of depression is 60°. Find the time taken by the car to reach the foot of the tower from this point. **4 marks**

SOLUTION:

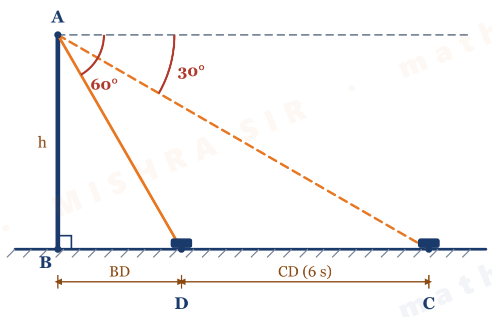

Fig. Q15 — Car at C (depression 30°), six seconds later at D (depression 60°), heading for the foot B.

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Let the tower height be h. Let the car first be at C (30°) and six seconds later at D (60°), with the foot at B.

Step 1 — Distance at 60° (point D).

$$\tan 60^\circ = \frac{h}{BD} \Rightarrow \sqrt{3} = \frac{h}{BD} \Rightarrow BD = \frac{h}{\sqrt{3}}$$ **1 mark**

Step 2 — Distance at 30° (point C).

$$\tan 30^\circ = \frac{h}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{BC} \Rightarrow BC = h\sqrt{3}$$ **1 mark**

Step 3 — Distance covered in 6 seconds.

$$CD = BC - BD = h\sqrt{3} - \frac{h}{\sqrt{3}} = \frac{3h - h}{\sqrt{3}} = \frac{2h}{\sqrt{3}}$$

1 mark

Step 4 — Compare with the remaining distance. The speed is uniform, so time is proportional to distance:

$$\frac{\text{time for } BD}{\text{time for } CD} = \frac{BD}{CD} = \frac{h/\sqrt{3}}{2h/\sqrt{3}} = \frac{1}{2}$$

So the time for BD is half the time for CD:

$$\text{time} = \frac{1}{2} \times 6 = \mathbf{3} \text{ seconds}$$ **1 mark**

📌 **The height h never needs to be found.** It cancels in the ratio, which is why the answer is a clean 3 seconds regardless of how tall the tower is. Setting up the ratio rather than hunting for h is the intended method.

— End of Exercise 9.1 — Chapter 9 Complete —

CHAPTER 10

Circles

Exercises 10.1 – 10.2

EXERCISES 10.1 – 10.2

📘 NCERT Solutions — Exercise 10.1 (4 Questions)

🔑 Approach for This Exercise

This exercise settles the vocabulary before any proof begins. Keep three words apart: a **secant** cuts the circle at two points; a **tangent** touches it at exactly one point (the **point of contact**); and a line missing the circle touches it nowhere. The single fact that powers the whole chapter is:

> **Theorem 10.1.** The tangent at any point of a circle is **perpendicular** to the radius through the point of contact.

So the moment a tangent and a radius meet, you have a right angle — and therefore Pythagoras.

Q1. How many tangents can a circle have? 1 mark

SOLUTION:

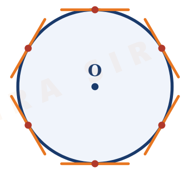

Fig. 10.1–Q1 — A tangent can be drawn at every point of the circle, so there are infinitely many.

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

A circle has **infinitely many tangents**. 1 mark

Reason. There are infinitely many points on a circle, and at *each* such point exactly one tangent can be drawn. Since the points are infinite in number, so are the tangents.

📌 Do not confuse this with the number of tangents *from a given external point*, which is exactly 2. This question asks about the circle as a whole.

Q2. Fill in the blanks: (i) A tangent to a circle intersects it in ___ point(s). (ii) A line intersecting a circle in two points is called a ___. (iii) A circle can have ___ parallel tangents at the most. (iv) The common point of a tangent to a circle and the circle is called ___. 4 marks

SOLUTION:

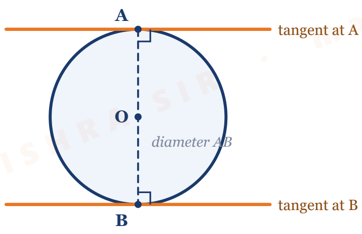

Fig. 10.1–Q2 — The only pair of parallel tangents touches the circle at the two ends of a diameter.

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

(i) one. This is the definition of a tangent — it touches the circle at a single point. **1 mark**

(ii) secant. A line cutting the circle at two distinct points is a secant. **1 mark**

(iii) two. Parallel tangents occur only at the two ends of a diameter; any direction gives exactly one such pair. **1 mark**

(iv) point of contact. **1 mark**

Q3. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is: (A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ cm **2 marks**

SOLUTION:

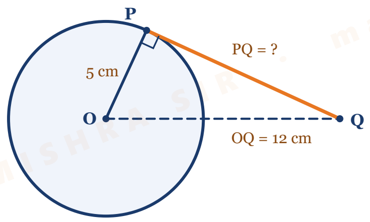

Fig. 10.1–Q3 — OP ⊥ PQ at the point of contact, so OQ (12 cm) is the hypotenuse.

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Given: $OP = 5$ cm (radius), $OQ = 12$ cm. **To find:** PQ.

Step 1. By Theorem 10.1, the tangent is perpendicular to the radius at the point of contact, so $\angle OPQ = 90^\circ$ and $\triangle OPQ$ is right-angled at P. **1 mark**

Step 2 — Apply Pythagoras (note OQ is the hypotenuse, being opposite the right angle):

$$OQ^2 = OP^2 + PQ^2$$
$$12^2 = 5^2 + PQ^2$$
$$144 = 25 + PQ^2 \Rightarrow PQ^2 = 119$$
$$PQ = \sqrt{119} \text{ cm}$$

Answer: option (D) $\sqrt{119}$ cm. **1 mark**

📌 **Option (B) is the trap.** 5-12-13 is a familiar triple, so students answer 13 out of habit — but that would make 12 a *leg*. Here 12 is the hypotenuse, so the answer must be *less* than 12, and $\sqrt{119} \approx 10.9$ fits.

Q4. Draw a circle and two lines parallel to a given line such that one is a **2 marks**

tangent and the other a secant to the circle.

SOLUTION:

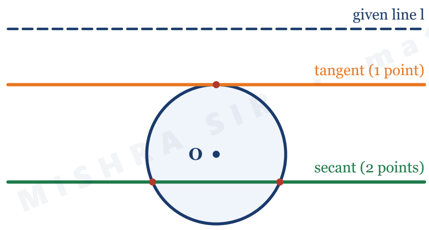

Fig. 10.1–Q4 — Both lines are parallel to ℓ: the tangent touches at one point, the secant cuts at two.

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Steps of construction:

1. Draw the given line ℓ.

2. Draw a circle with centre O of any convenient radius, placed to one side of ℓ. **1 mark**

3. Draw a line parallel to ℓ that touches the circle at exactly one point — this is the required **tangent**.

4. Draw another line parallel to ℓ that cuts across the circle, meeting it at two points — this is the required **secant**. **1 mark**

📌 The tangent is the line whose distance from O equals the radius; the secant is any parallel line whose distance from O is *less* than the radius. Saying this in one line earns the reasoning mark.

— End of Exercise 10.1 —

📘 NCERT Solutions — Exercise 10.2 (13 Questions)

🔑 Approach for This Exercise

Two theorems decide almost every question:

> **Theorem 10.1.** The tangent at a point is perpendicular to the radius through that point → gives a right angle, hence Pythagoras.
> **Theorem 10.2.** The lengths of the two tangents drawn from an external point to a circle are **equal** → gives pairs of equal segments.

For the numerical questions (Q1, Q6, Q7) use Theorem 10.1 with Pythagoras. For questions about a figure circumscribing a circle (Q8, Q11, Q12) use Theorem 10.2 to name equal tangent lengths — label them x, y, z, w and the algebra does the rest. For angle questions (Q2, Q3, Q10) remember that $OPTQ$ forms a quadrilateral with two right angles, so the remaining two angles are supplementary.

Q1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is (A) 7 cm (B) 12 cm (C) 15 cm (D) 24.5 cm **2 marks**

SOLUTION:

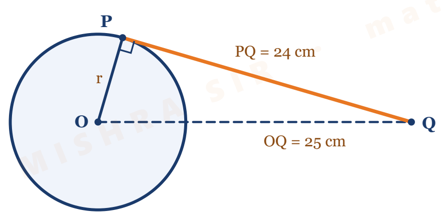

Fig. 10.2–Q1 — Radius OP ⊥ tangent PQ; the distance from the centre, 25 cm, is the hypotenuse.

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Let P be the point of contact. Then $PQ = 24$ cm (tangent), $OQ = 25$ cm, and $OP = r$ is the radius.

By Theorem 10.1, $\angle OPQ = 90°$, so by Pythagoras:

1 mark

$$OQ^2 = OP^2 + PQ^2$$
$$25^2 = r^2 + 24^2$$
$$625 = r^2 + 576 \Rightarrow r^2 = 49 \Rightarrow r = 7$$

Answer: option (A) 7 cm. 1 mark

📌 Here 7-24-25 is the Pythagorean triple at work. Recognising 25 as the hypotenuse (it is the distance from the *centre*, always the longest) makes this a five-second question.

Q2. In Fig. 10.11, if TP and TQ are two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ equals (A) 60° (B) 70° (C) 80° (D) 90° 2 marks

SOLUTION:

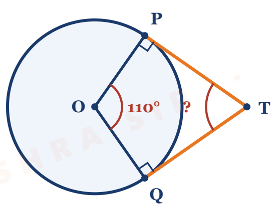

Fig. 10.2–Q2 — OPTQ has right angles at P and Q, so ∠POQ + ∠PTQ = 180°.

__Note:__ MLI-redrawn schematic — correct in construction but __not drawn to scale__, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.

Step 1 — Mark the right angles. By Theorem 10.1, $OP \perp TP$ and $OQ \perp TQ$, so

$\angle OPT = \angle OQT = 90^\circ$ **1 mark**

Step 2 — Use the angle sum of quadrilateral OPTQ (which is 360°):

$$\angle POQ + \angle OPT + \angle PTQ + \angle OQT = 360^\circ$$

$$110^\circ + 90^\circ + \angle PTQ + 90^\circ = 360^\circ$$

$$\angle PTQ = 360^\circ - 290^\circ = 70^\circ$$

Answer: option (B) 70°. **1 mark**

📌 **The shortcut:** since two angles are 90° each, $\angle POQ$ and $\angle PTQ$ always add to 180° — they are supplementary. That single fact answers Q2, Q3 and Q10.

Q3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°, then ∠POA equals (A) 50° (B) 60° (C) 70° (D) 80° **2 marks**

SOLUTION:

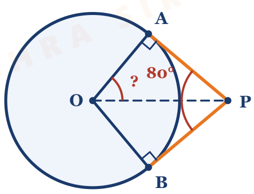

Fig. 10.2–Q3 — OP bisects ∠AOB; ∠AOB = 180° − 80° = 100°, so ∠POA = 50°.

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Step 1 — Use the supplementary relation. From the quadrilateral $OAPB$ with right angles at A and B:

$$\angle AOB + \angle APB = 180^\circ$$
$$\angle AOB = 180^\circ - 80^\circ = 100^\circ$$ **1 mark**

Step 2 — Use symmetry. $\triangle OAP \cong \triangle OBP$ (since $OA = OB$ as radii, $PA = PB$ by Theorem 10.2, and OP is common, by SSS). So OP bisects $\angle AOB$:

$$\angle POA = \frac{100^\circ}{2} = 50^\circ$$

Answer: option (A) 50°. **1 mark**

📌 **The line from the centre to the external point is an axis of symmetry.** It bisects both the angle at the centre and the angle between the tangents — worth remembering for the whole exercise.

Q4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel. **3 marks**

SOLUTION:

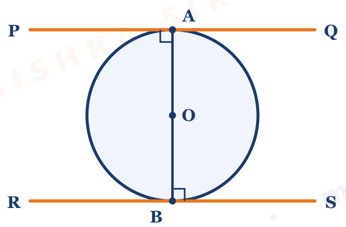

Fig. 10.2–Q4 — ∠OAP and ∠OBS are both 90° and are alternate angles on the transversal AB.

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Given: A circle with centre O and diameter AB. Tangents PQ at A and RS at B.

To prove: $PQ \parallel RS$.

Proof. By Theorem 10.1, the tangent at a point is perpendicular to the radius there:

$$AB \perp PQ \Rightarrow \angle OAP = 90^\circ$$ **1 mark**

$$AB \perp RS \Rightarrow \angle OBS = 90^\circ$$ **1 mark**

Now AB is a transversal cutting the two lines PQ and RS, and

$$\angle OAP = \angle OBS = 90^\circ$$

These are alternate interior angles, and they are equal. Therefore

$PQ \parallel RS$ **Hence proved.** **1 mark**

📌 Any two lines perpendicular to the *same* line are parallel — the diameter is that common perpendicular here.

Q5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre. **3 marks**

SOLUTION:

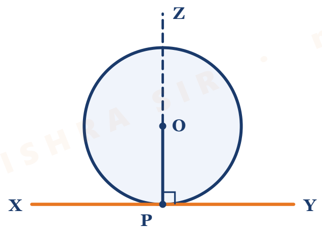

Fig. 10.2–Q5 — The perpendicular PZ to the tangent XY at P coincides with the radius PO, so it passes through O.

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

This is proved by **contradiction**.

Given: A circle with centre O, a tangent XY touching it at P, and a line $PZ \perp XY$ at P.
To prove: PZ passes through O.

Proof. Suppose, to the contrary, that PZ does *not* pass through O. Draw OP, the radius to the point of contact.
1 mark

By Theorem 10.1, the tangent is perpendicular to the radius at the point of contact, so

$$\angle OPY = 90^\circ$$

Also, by our construction, $\angle ZPY = 90^\circ$. **1 mark**

Therefore $\angle OPY = \angle ZPY$. But this is possible only if the rays PO and PZ coincide — two distinct lines through P cannot both make 90° with XY on the same side.

This contradicts our supposition. Hence PZ must pass through the centre O. **Hence proved.** **1 mark**

📌 **This is the converse of Theorem 10.1.** Whenever a question says "prove ... passes through the centre", the contradiction method is the expected route.

Q6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle. **2 marks**

SOLUTION:

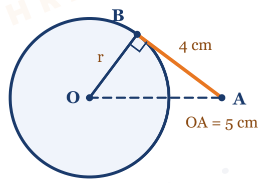

Fig. 10.2–Q6 — OB ⊥ AB at the point of contact B, so $OA^2 = OB^2 + AB^2$.

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Let O be the centre and B the point of contact. Then $OA = 5$ cm, $AB = 4$ cm and $OB = r$.

By Theorem 10.1, $\angle OBA = 90^\circ$, so by Pythagoras: **1 mark**

$$OA^2 = OB^2 + AB^2$$
$$5^2 = r^2 + 4^2$$
$$25 = r^2 + 16 \Rightarrow r^2 = 9 \Rightarrow r = 3$$

$$\text{Radius} = \mathbf{3}\ \text{cm}$$ **1 mark**

📌 The 3-4-5 triple again. The distance from the centre is always the hypotenuse, so it must be the largest of the three lengths — a quick check that you have set the equation up correctly.

Q7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle. **3 marks**

SOLUTION:

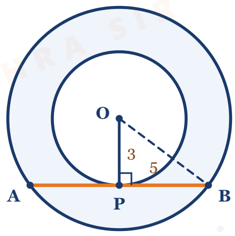

Fig. 10.2–Q7 — AB is a chord of the larger circle touching the smaller circle at P, so OP ⊥ AB and AP = PB.

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Step 1 — Set up. Let AB be the chord of the larger circle touching the smaller circle at P. Since AB is tangent to the smaller circle at P, by Theorem 10.1: $OP \perp AB$, with $OP = 3$ cm. **1 mark**

Step 2 — Use the chord property. The perpendicular from the centre to a chord bisects it, so $AP = PB$ and $AB = 2 \times AP$. **1 mark**

Step 3 — Apply Pythagoras in $\triangle$OPA (right-angled at P, with $OA = 5$ cm the radius of the larger circle):

$$OA^2 = OP^2 + AP^2$$

$$5^2 = 3^2 + AP^2 \Rightarrow 25 = 9 + AP^2 \Rightarrow AP^2 = 16 \Rightarrow AP = 4$$ cm

$$\text{Therefore } AB = 2 \times 4 = 8$$

$$\text{Length of the chord} = \mathbf{8}\text{ cm}$$ **1 mark**

📌 **Two facts combine here** — tangent ⊥ radius, and the perpendicular from the centre bisects the chord. Forgetting to double AP gives 4 cm, the commonest wrong answer.

Q8. A quadrilateral ABCD is drawn to circumscribe a circle (Fig. 10.12). Prove that AB + CD = AD + BC. **3 marks**

SOLUTION:

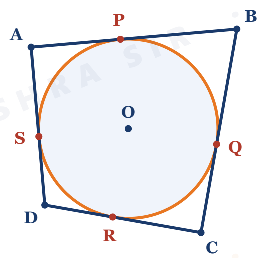

Fig. 10.2–Q8 — From each vertex the two tangents are equal: AP = AS, BP = BQ, CR = CQ, DR = DS.

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Given: Quadrilateral $ABCD$ circumscribing a circle, touching AB, BC, CD, DA at P, Q, R, S respectively.

To prove: $AB + CD = AD + BC$.

Proof. By Theorem 10.2, the two tangents from an external point are equal. Applying this at each vertex:

1 mark

$AP = AS$ ...(1) (from A)

$BP = BQ$ ...(2) (from B)

$$CR = CQ \text{ ...(3) (from } C)$$
$$DR = DS \text{ ...(4) (from } D)$$

Adding (1), (2), (3) and (4): **1 mark**

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

Regrouping the left side as $(AP + BP) + (CR + DR)$ and the right as $(AS + DS) + (BQ + CQ)$:

$AB + CD = AD + BC$ **Hence proved.** **1 mark**

> 📌 **The regrouping is the whole trick.** $AP + BP$ makes up the side AB, and $AS + DS$ makes up AD — write that step explicitly rather than jumping to the conclusion.

Q9. In Fig. 10.13, XY and X′Y′ are two parallel tangents to a circle with centre O, and another tangent AB with point of contact C intersects XY at A and X′Y′ at B. Prove that ∠AOB = 90°. **3 marks**

SOLUTION:

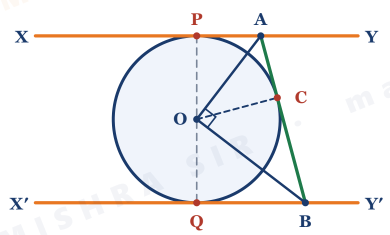

Fig. 10.2–Q9 — XY ∥ X′Y′ touch at P and Q (ends of a diameter); the third tangent touches at C. To prove: ∠AOB = 90°.

> ***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Let the tangent XY touch the circle at P and $X'Y'$ touch it at Q.

Step 1 — OA bisects ∠POC. From the external point A, two tangents AP and AC are drawn. Joining OA, we get $\triangle OPA \cong \triangle OCA$ (SSS: $OP = OC$ radii, $AP = AC$ by Theorem 10.2, OA common).
Therefore $\angle POA = \angle COA$, i.e. $\angle POC = 2\angle COA$ ...(1)

1 mark

Step 2 — OB bisects ∠QOC. Similarly, from B: $\triangle OQB \cong \triangle OCB$, so
$\angle QOB = \angle COB$, i.e. $\angle QOC = 2\angle COB$ ...(2) **1 mark**

Step 3 — Use the straight line PQ. Since $XY \parallel X'Y'$ and both are tangents, P, O, Q are collinear (PQ is a diameter). So

$$\angle POC + \angle QOC = 180^\circ$$

Substituting from (1) and (2):

$$2\angle COA + 2\angle COB = 180^\circ$$

$$\angle COA + \angle COB = 90^\circ$$

That is, $\angle AOB = 90^\circ$ **Hence proved.** **1 mark**

📌 **Why P, O, Q are collinear:** parallel tangents touch at the ends of a diameter (Exercise 10.1 Q2 part iii). Stating this justifies the 180° in Step 3.

Q10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre. **3 marks**

SOLUTION:

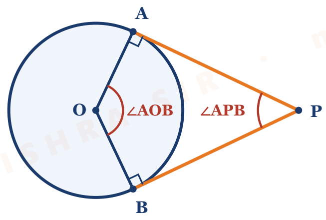

Fig. 10.2–Q10 — In quadrilateral OAPB the angles at A and B are 90°, so ∠APB + ∠AOB = 180°.

***Note:** MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Given: A circle with centre O; tangents PA and PB from an external point P, touching at A and B.

To prove: $\angle APB + \angle AOB = 180°$.

Proof. By Theorem 10.1, the tangent is perpendicular to the radius at the point of contact: **1 mark**

$$\angle OAP = 90° \text{ and } \angle OBP = 90°$$

Now consider the quadrilateral $OAPB$. The sum of its angles is $360°$: **1 mark**

$$\angle OAP + \angle APB + \angle OBP + \angle AOB = 360°$$
$$90° + \angle APB + 90° + \angle AOB = 360°$$
$$\angle APB + \angle AOB = 360° - 180° = 180°$$

Hence the two angles are supplementary. **Hence proved.** **1 mark**

📌 **This is the general proof behind Q2 and Q3.** Once established, those two MCQs become one-line substitutions.

Q11. Prove that the parallelogram circumscribing a circle is a rhombus. **3 marks**

SOLUTION:

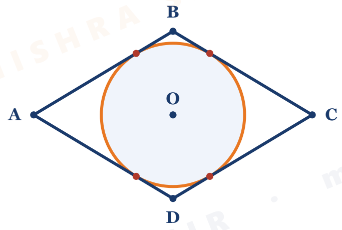

Fig. 10.2–Q11 — A parallelogram that circumscribes a circle must have all four sides equal — a rhombus.

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Given: A parallelogram $ABCD$ circumscribing a circle.

To prove: $ABCD$ is a rhombus, i.e. all four sides are equal.

Step 1 — Apply the result of Q8. For any quadrilateral circumscribing a circle,

$AB + CD = AD + BC$ ...(1) **1 mark**

Step 2 — Use the parallelogram property. Opposite sides of a parallelogram are equal:

$AB = CD$ and $AD = BC$ ...(2) **1 mark**

Step 3 — Substitute (2) into (1).

$$AB + AB = AD + AD$$

$$2AB = 2AD \Rightarrow AB = AD$$

Combining with (2): $AB = BC = CD = DA$.

A parallelogram with all four sides equal is a **rhombus**.

Hence proved. **1 mark**

📌 **Q8 does the heavy lifting.** Quote it as a known result rather than re-deriving the four tangent equalities — that is what the examiner expects here.

Q12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC, into which BC is divided by the point of contact D, are of lengths 8 cm and 6 cm respectively (Fig. 10.14). Find the sides AB and AC. **4 marks**

SOLUTION:

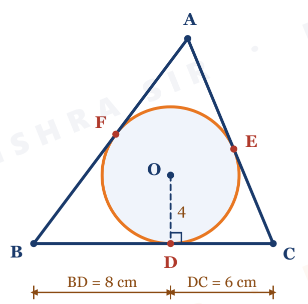

Fig. 10.2–Q12 — The incircle touches BC at D, CA at E and AB at F; tangents from each vertex are equal.

Note: *MLI-redrawn schematic — correct in construction but* ***not drawn to scale****, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Let the circle touch AB at F and AC at E, with centre O and radius 4 cm.

Step 1 — Name the equal tangent lengths. By Theorem 10.2:

$BD = BF = 8$ cm, $\quad CD = CE = 6$ cm, $\quad AF = AE = x$ (say) **1 mark**

So $AB = x + 8$, $AC = x + 6$, $BC = 8 + 6 = 14$ cm.

Step 2 — Find the area two ways. First by Heron's formula. The semi-perimeter is

$$s = \frac{(x+8)+(x+6)+14}{2} = \frac{2x+28}{2} = x+14$$

$$s-a = (x+14)-14 = x, \ \ s-b = (x+14)-(x+6) = 8,$$

$$s-c = (x+14)-(x+8) = 6$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{(x+14)\cdot x\cdot 8\cdot 6} = \sqrt{48x$$

1 mark

Step 3 — Area using the incircle. For a triangle with an inscribed circle of radius r, Area $= r \times s$:

$$\text{Area} = 4(x+14)$$ 1 mark

Step 4 — Equate and solve.

$$\sqrt{48x(x+14)} = 4(x+14)$$

Squaring: $48x(x+14) = 16(x+14)^2$

Dividing both sides by $16(x+14)$ (non-zero): $3x = x+14$

$$2x = 14 \Rightarrow x = 7$$

Therefore $AB = 7+8 = \mathbf{15}$ cm and $AC = 7+6 = \mathbf{13}$ cm.

1 mark

Check: sides 15, 13, 14 give $s = 21$; Area $= \sqrt{21\cdot 6\cdot 8\cdot 7} = \sqrt{7056} = 84$; and $r\cdot s = 4\times 21 = 84$ ✓

📌 **Area = r × s is the key formula** for any triangle with an inscribed circle. Equating it with Heron's formula turns a geometry problem into a single equation in x.

Q13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle. 4 marks

SOLUTION:

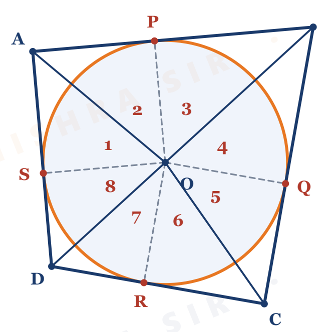

Fig. 10.2–Q13 — Angles at O numbered 1–8 in order S, A, P, B, Q, C, R, D. ∠2 + ∠3 = ∠AOB and ∠6 + ∠7 = ∠COD.

Note: *MLI-redrawn schematic — correct in construction but **not drawn to scale**, and not a reproduction of any textbook figure. Never measure a length or an angle off this diagram; work only from the given data. If anything is unclear, refer to the corresponding figure in your NCERT textbook.*

Given: Quadrilateral $ABCD$ circumscribing a circle with centre O, touching AB, BC, CD, DA at P, Q, R, S.

To prove: $\angle AOB + \angle COD = 180^\circ$ and

$\angle BOC + \angle AOD = 180^\circ$.

Step 1 — Congruent triangle pairs. From the external point A, tangents AP and AS are equal, and $OP = OS$ (radii), with OA common. So

$\triangle OPA \cong \triangle OSA$ (SSS) $\Rightarrow \angle 1 = \angle 2$

Similarly at B, C and D:

$$\angle 3 = \angle 4,\ \ \angle 5 = \angle 6,\ \ \angle 7 = \angle 8$$

(numbering the eight angles at O in order around the centre) **1 mark**

Step 2 — All eight angles fill a full turn.

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$$

1 mark

Step 3 — Pair them using Step 1.

$$2(\angle 2 + \angle 3) + 2(\angle 6 + \angle 7) = 360^\circ$$

$$(\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180^\circ$$ **1 mark**

Step 4 — Identify the angles. $\angle 2 + \angle 3 = \angle AOB$ and

$\angle 6 + \angle 7 = \angle COD$. Therefore

$$\angle AOB + \angle COD = 180^\circ$$

The remaining four angles give

$$\angle BOC + \angle AOD = 360^\circ - 180^\circ = 180^\circ.$$

Hence opposite sides subtend supplementary angles at the centre. **Hence proved.** 1 mark

📌 **Label the eight angles at O before writing anything.** The proof is just "equal in pairs, and the whole is 360°" — but without clear labelling it becomes impossible to write down.

— End of Exercise 10.2 — Chapter 10 Complete —

CHAPTER 11

Areas Related to Circles

Exercise 11.1

EXERCISE 11.1

CBSE Step-wise Marking Scheme Format	**Board Exam 2026–27**	**Name:** ____________

💡 **CBSE Marking Scheme Guidelines Followed in This Document**

1. Every question is structured as **Given** → **To Find** → **Solution**, matching CBSE's official answer-key format.
2. Each step is written on its own line and tagged with its mark allocation.
3. A **labelled rough figure** is drawn wherever the question involves a sector, segment, chord or grazing region — CBSE awards the first ½ mark for a correct figure.
4. Method/formula marks are earned independently of the final answer, so every substitution is shown, not just the result.
5. Unless the question says otherwise, $\pi = \frac{22}{7}$. Where the question states $\pi = 3.14$ or $\sqrt{3} = 1.73$, those values are used exactly as given.

📝 Formulae Used in This Exercise

Quantity	Formula	Remark
Area of circle	πr^2	Full disc, $\theta = 360^\circ$
Circumference	$2\pi r$	Used to find r when perimeter is given
Length of arc of sector of angle θ	$\dfrac{\theta}{360} \times 2\pi r$	Answer in cm / m (length)
Area of sector of angle θ	$\dfrac{\theta}{360} \times \pi r^2$	Answer in cm² / m²
Area of minor segment	Area of sector − Area of △ OAB	Chord cuts off the segment
Area of major sector	πr^2 − area of minor sector, or $\dfrac{360-\theta}{360} \times \pi r^2$	Both methods accepted
Area of major segment	πr^2 − area of minor segment	—
Area of △ OAB (two radii r, included angle θ)	$\dfrac{1}{2} r^2 \sin\theta$	$\theta = 60^\circ \Rightarrow \dfrac{\sqrt{3}}{4} r^2$ (equilateral); $\theta = 90^\circ \Rightarrow \dfrac{1}{2} r^2$; $\theta = 120^\circ \Rightarrow \dfrac{\sqrt{3}}{4} r^2$

Key idea: Every question in this exercise reduces to **sector area**, and a **segment** is always *sector minus triangle*. Identify r and θ first — the rest is substitution.

EXERCISE 11.1 **Total: 34 Marks**

Q1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°. **2**

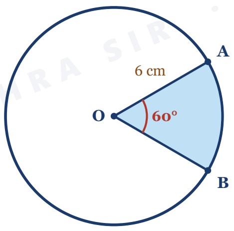

Fig. Q1 — Shaded: sector OAB of angle 60° and radius 6 cm.

Note: *MLI-redrawn figure, constructed from the given data — angles true and lengths to scale. Work only from the given data, never by measuring the figure.*

Given: *Radius* $r = 6$ *cm, angle of sector* $\theta = 60^\circ$, $\pi = \frac{22}{7}$.

To Find: *Area of the sector.*

Solution:

Step 1 — Write the formula:

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2$$ *[½ mark]*

Step 2 — Substitute the values:

$$= \frac{60}{360} \times \frac{22}{7} \times 6 \times 6$$ *[½ mark]*

Step 3 — Simplify:

$$= \frac{1}{6} \times \frac{22}{7} \times 36 = \frac{22 \times 6}{7} = \frac{132}{7}$$ *[1 mark]*

∴ Area of the sector

$$= \frac{132}{7}\ \text{cm}^2 = 18\frac{6}{7}\ \text{cm}^2 \approx 18.86\ \text{cm}^2.$$

Q2. Find the area of a quadrant of a circle whose circumference is 22 cm. **2**

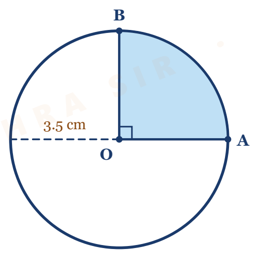

Fig. Q2 — Shaded: quadrant OAB (a 90° sector). The radius comes from $2\pi r = 22$.

Note: *MLI-redrawn figure, constructed from the given data — angles true and lengths to scale. Work only from the given data, never by measuring the figure.*

Given: *Circumference of the circle* $= 22$ *cm*, $\pi = \dfrac{22}{7}$.

To Find: *Area of a quadrant (a sector of angle* $90°$*).*

Solution:

Step 1 — Find the radius from the circumference:

$$2\pi r = 22 \Rightarrow 2 \times \frac{22}{7} \times r = 22$$

$$\frac{44r}{7} = 22 \Rightarrow r = \frac{22 \times 7}{44} = \frac{7}{2} = 3.5 \text{ cm}$$ ***[1 mark]***

Step 2 — A quadrant means $\theta = 90°$:

$$Area = \frac{90}{360} \times \pi r^2 = \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$ ***[½ mark]***

Step 3 — Simplify:

$$= \frac{1}{4} \times \frac{22}{7} \times \frac{49}{4} = \frac{22 \times 49}{112} = \frac{1078}{112} = \frac{77}{8}$$

[½ mark]

∴ Area of the quadrant $= \dfrac{77}{8}$ cm^2 $= 9.625$ cm^2.

Q3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes. **2**

Fig. Q3 — From 12 to 1 the minute hand turns 30° (5 minutes × 6° per minute).

Note: *MLI-redrawn figure, constructed from the given data — angles true and lengths to scale. Work only from the given data, never by measuring the figure.*

Given: *Length of minute hand* $=$ *radius* $r = 14$ *cm; time* $= 5$ *minutes.*

To Find: *Area swept by the minute hand.*

Solution:

Step 1 — Find the angle swept. *The minute hand turns* 360° *in 60 minutes.*

$$\therefore \text{Angle in 1 minute} = \frac{360^\circ}{60} = 6^\circ$$

$$\therefore \text{Angle in 5 minutes} = 5 \times 6^\circ = 30^\circ$$ **[1 mark]**

Step 2 — The area swept is a sector of angle 30°:

$$\text{Area} = \frac{\theta}{360} \times \pi r^2 = \frac{30}{360} \times \frac{22}{7} \times 14 \times 14$$ **[½ mark]**

Step 3 — Simplify:

$$= \frac{1}{12} \times 22 \times 2 \times 14 = \frac{616}{12} = \frac{154}{3}$$ **[½ mark]**

∴ Area swept in 5 minutes

$$= \frac{154}{3}\ \text{cm}^2 = 51\frac{1}{3}\ \text{cm}^2 \approx 51.33\ \text{cm}^2.$$

Q4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding: (i) minor segment (ii) major sector. (Use $\pi = 3.14$) **3**

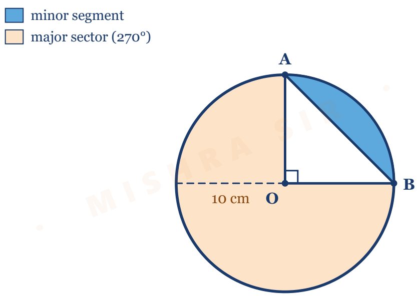

Fig. Q4 — Chord AB subtends 90° at O. Dark: minor segment. Warm: major sector.

Note: *MLI-redrawn figure, constructed from the given data — angles true and lengths to scale. Work only from the given data, never by measuring the figure.*

Given: *$r = 10$ cm, chord AB subtends $\angle AOB = 90^\circ$ at the centre, $\pi = 3.14$.*

To Find: *(i) Area of minor segment (ii) Area of major sector.*

Solution:

Step 1 — Area of the minor sector OAB:

$$= \frac{90}{360} \times 3.14 \times 10 \times 10 = \frac{1}{4} \times 314 = 78.5 \text{ cm}^2$$

[1 mark]

Step 2 — Area of $\triangle$OAB. *Since $\angle AOB = 90^\circ$, OA and OB are the base and height:*

$$= \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$$ ***[½ mark]***

Step 3 — (i) Area of minor segment = sector − triangle:

$$= 78.5 - 50 = 28.5 \text{ cm}^2$$ ***[½ mark]***

Step 4 — (ii) Area of major sector. *Angle of major sector $= 360^\circ - 90^\circ = 270^\circ$:*

$$= \frac{270}{360} \times 3.14 \times 100 = \frac{3}{4} \times 314 = 235.5 \text{ cm}^2$$

[1 mark]

Alternatively: $\pi r^2 -$ minor sector

$$= 314 - 78.5 = 235.5 \text{ cm}^2.$$

$\therefore$ **(i) Minor segment** $= 28.5 \text{ cm}^2$ **(ii) Major sector** $= 235.5 \text{ cm}^2$**.**

Q5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find: (i) the length of the arc (ii) area of the sector formed by the arc (iii) area of the segment formed by the corresponding chord. 3

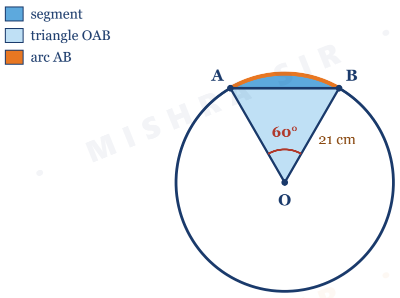

Fig. Q5 — Arc AB (orange), sector OAB = triangle OAB + segment (dark).

Note: *MLI-redrawn figure, constructed from the given data — angles true and lengths to scale. Work only from the given data, never by measuring the figure.*

Given: $r = 21$ *cm*, $\theta = 60^\circ$, $\pi = \dfrac{22}{7}$.

To Find: *(i) arc length (ii) sector area (iii) segment area.*

Solution:

Step 1 — (i) Length of arc AB:

$$= \frac{\theta}{360} \times 2\pi r = \frac{60}{360} \times 2 \times \frac{22}{7} \times 21$$

$$= \frac{1}{6} \times 132 = 22 \text{ cm}$$ *[1 mark]*

Step 2 — (ii) Area of sector OAB:

$$= \frac{60}{360} \times \frac{22}{7} \times 21 \times 21 = \frac{1}{6} \times 22 \times 3 \times 21 = \frac{1}{6} \times 1386 = 231 \text{ cm}^2$$ *[1 mark]*

Step 3 — Area of $\triangle$OAB. *Here $OA = OB = 21$ cm and $\angle AOB = 60^\circ$, so $\triangle OAB$ is* **equilateral** *of side 21 cm:*

$$= \frac{\sqrt{3}}{4} \times (21)^2 = \frac{441\sqrt{3}}{4} \text{ cm}^2$$ *[½ mark]*

Step 4 — (iii) Area of segment = sector − triangle:

$$= \left(231 - \frac{441\sqrt{3}}{4}\right) \text{ cm}^2$$ *[½ mark]*

Taking $\sqrt{3} = 1.73$: $\dfrac{441 \times 1.73}{4} = \dfrac{762.93}{4} = 190.73$

$$\Rightarrow 231 - 190.73 = 40.27 \text{ cm}^2$$

$\therefore$ **(i) 22 cm (ii) 231 cm² (iii)** $\left(231 - \dfrac{441\sqrt{3}}{4}\right) \text{cm}^2 \approx 40.27 \text{ cm}^2$.

Q6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$) **3**

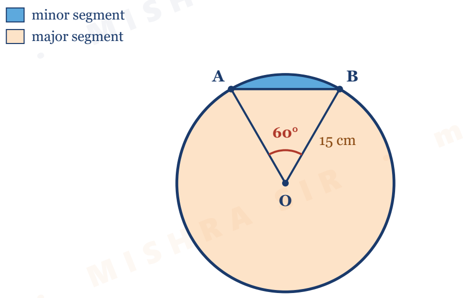

Fig. Q6 — Chord AB subtends 60° at O. Dark: minor segment; warm: major segment.

Note: *MLI-redrawn figure, constructed from the given data — angles true and lengths to scale. Work only from the given data, never by measuring the figure.*

Given: $r = 15$ *cm*, $\theta = 60^\circ$, $\pi = 3.14$, $\sqrt{3} = 1.73$.

To Find: *Areas of the minor and major segments.*

Solution:

Step 1 — Area of the minor sector OAB:

$$= \frac{60}{360} \times 3.14 \times 15 \times 15 = \frac{1}{6} \times 3.14 \times 225 = \frac{706.5}{6} = 117.75 \text{ cm}^2$$

[1 mark]

Step 2 — Area of △OAB. *$OA = OB = 15$ cm with 60° between them ⇒ equilateral triangle of side 15 cm:*

$$= \frac{\sqrt{3}}{4} \times (15)^2 = \frac{1.73 \times 225}{4} = \frac{389.25}{4} = 97.3125 \text{ cm}^2$$

[½ mark]

Step 3 — Area of minor segment:

$$= 117.75 - 97.3125 = 20.4375 \approx 20.44 \text{ cm}^2$$

[½ mark]

Step 4 — Area of the whole circle:

$$= \pi r^2 = 3.14 \times 225 = 706.5 \text{ cm}^2$$ ***[½ mark]***

Step 5 — Area of major segment = circle − minor segment:

$= 706.5 - 20.4375 = 686.0625 \approx 686.06 \text{ cm}^2$

[½ mark]

∴ **Minor segment** $\approx 20.44 \text{ cm}^2$; **Major segment** $\approx 686.06 \text{ cm}^2$.

Q7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$) **3**

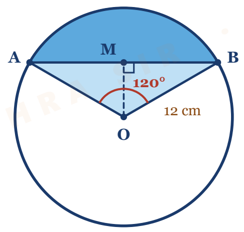

Fig. Q7 — Chord AB subtends 120°; OM ⊥ AB splits it into two 60° angles. Shaded dark: the segment.

Note: *MLI-redrawn figure, constructed from the given data — angles true and lengths to scale. Work only from the given data, never by measuring the figure.*

Given: $r = 12$ *cm*, $\theta = 120^\circ$, $\pi = 3.14$, $\sqrt{3} = 1.73$.

To Find: *Area of the corresponding (minor) segment.*

Solution:

Step 1 — Area of sector OAB:

$$= \frac{120}{360} \times 3.14 \times 12 \times 12 = \frac{1}{3} \times 3.14 \times 144 = \frac{452.16}{3} = 150.72 \text{ cm}^2$$

[1 mark]

Step 2 — Area of △OAB. *Draw OM ⊥ AB. Then*

$$\angle \text{AOM} = \angle \text{BOM} = 60^\circ, \textit{ so}$$

$$\text{OM} = \text{OA} \cos 60^\circ = 12 \times \frac{1}{2} = 6 \text{ cm}$$

$$\text{AM} = \text{OA} \sin 60^\circ = 12 \times \frac{\sqrt{3}}{2} = 6\sqrt{3} \text{ cm} \Rightarrow \text{AB} = 12\sqrt{3} \text{ cm}$$

[1 mark]

$$\text{ar}(\triangle \text{OAB}) = \frac{1}{2} \times \text{AB} \times \text{OM} = \frac{1}{2} \times 12\sqrt{3} \times 6 = 36\sqrt{3}$$

$$= 36 \times 1.73 = 62.28 \text{ cm}^2$$ [½ mark]

Step 3 — Area of segment = sector − triangle:

$$= 150.72 - 62.28 = 88.44 \text{ cm}^2$$ [½ mark]

∴ **Area of the corresponding segment** $= 88.44 \text{ cm}^2$.

Shortcut:

$$\text{ar}(\triangle\text{OAB}) = \frac{1}{2}r^2 \sin\theta = \frac{1}{2} \times 144 \times \sin 120^\circ = 36\sqrt{3}$$

— the same result in one line.

Q8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find (i) the area of that part of the field in which the horse can graze. (ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use $\pi = 3.14$) **3**

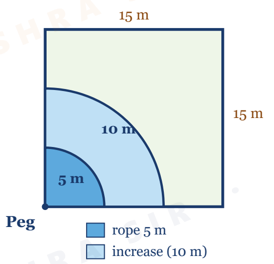

Fig. Q8 — At a corner the horse grazes a quadrant; the lighter band is the increase with the 10 m rope.

Note: *MLI-redrawn figure, constructed from the given data — angles true and lengths to scale. Work only from the given data, never by measuring the figure.*

Given: *Square field of side 15 m; horse tied at a corner; rope lengths 5 m and 10 m;* $\pi = 3.14$.

To Find: *(i) grazing area with 5 m rope (ii) increase in grazing area with 10 m rope.*

Solution:

Step 1 — Identify the shape. *At a corner of a square the two sides meet at* 90°*, so the horse can graze over a* **quadrant** *(sector of angle* 90°*) of radius = rope length. Since* $5 < 15$ *and* $10 < 15$*, the quadrant stays inside the field in both cases.* **[½ mark]**

Step 2 — (i) Grazing area with $r_1 = 5$ m:

$$A_1 = \frac{90}{360} \times 3.14 \times 5 \times 5 = \frac{1}{4} \times 3.14 \times 25 = \frac{78.5}{4} = 19.625 \text{ m}^2$$

[1 mark]

Step 3 — Grazing area with $r_2 = 10$ m:

$$A_2 = \frac{1}{4} \times 3.14 \times 10 \times 10 = \frac{314}{4} = 78.5 \text{ m}^2$$

[1 mark]

Step 4 — (ii) Increase in grazing area:

$= A_2 - A_1 = 78.5 - 19.625 = 58.875 \text{ m}^2$ *[½ mark]*

$\therefore$ **(i)** 19.625 m^2 **(ii) Increase** $= 58.875 \text{ m}^2$**.**

Q9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors. Find: (i) the total length of the silver wire required. (ii) the area of each sector of the brooch. **3**

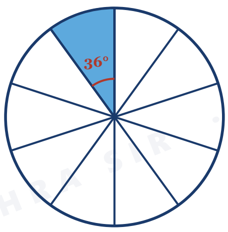

Fig. Q9 — Five diameters make 10 equal sectors of 36° each; one is shaded.

Note: *MLI-redrawn figure, constructed from the given data — angles true and lengths to scale. Work only from the given data, never by measuring the figure.*

Given: *Diameter of circle* $= 35$ *mm* $\Rightarrow r = \frac{35}{2} = 17.5$ *mm; 5 diameters divide the circle into 10 equal sectors;* $\pi = \frac{22}{7}$.

To Find: *(i) total length of silver wire (ii) area of each sector.*

Solution:

Step 1 — (i) Length of the circular wire (circumference):

$$= 2\pi r = 2 \times \frac{22}{7} \times \frac{35}{2} = \frac{22 \times 35}{7} = 110 \text{ mm}$$

[½ mark]

Step 2 — Length of wire in 5 diameters:

$= 5 \times 35 = 175$ mm *[½ mark]*

Step 3 — Total length of silver wire:

$= 110 + 175 = 285$ mm *[1 mark]*

Step 4 — (ii) Angle of each sector. *The 10 sectors are equal, so*

$$\theta = \frac{360^\circ}{10} = 36^\circ$$ *[½ mark]*

Step 5 — Area of each sector:

$$= \frac{36}{360} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} = \frac{1}{10} \times \frac{22}{7} \times \frac{1225}{4}$$

$$= \frac{1}{10} \times \frac{26950}{28} = \frac{1}{10} \times 962.5 = 96.25 \text{ mm}^2 = \frac{385}{4} \text{ mm}^2$$

[½ mark]

∴ (i) Total wire $= 285$ mm (ii) Area of each sector $= \frac{385}{4}$ mm^2 $= 96.25$ mm^2.

Check: $10 \times 96.25 = 962.5 = \pi r^2$ ✓

Q10. An umbrella has 8 ribs which are equally spaced. Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs. **2**

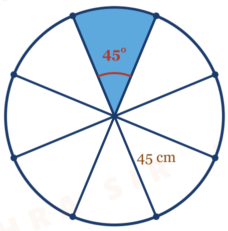

Fig. Q10 — 8 equally spaced ribs make 8 sectors of 45°; the area between two ribs is shaded.

Note: *MLI-redrawn figure, constructed from the given data — angles true and lengths to scale. Work only from the given data, never by measuring the figure.*

Given: *Umbrella = flat circle of radius $r = 45$ cm with 8 equally spaced ribs;* $\pi = \frac{22}{7}$.

To Find: *Area between two consecutive ribs.*

Solution:

Step 1 — Find the angle between two consecutive ribs.

The 8 ribs divide the circle into 8 equal sectors:

$$\theta = \frac{360^\circ}{8} = 45^\circ$$ *[½ mark]*

Step 2 — Area of one such sector:

$$= \frac{45}{360} \times \frac{22}{7} \times 45 \times 45 = \frac{1}{8} \times \frac{22}{7} \times 2025$$ *[½ mark]*

Step 3 — Simplify:

$$= \frac{22 \times 2025}{56} = \frac{44550}{56} = \frac{22275}{28} \text{ cm}^2$$ *[1 mark]*

∴ Area between two consecutive ribs

$$= \frac{22275}{28} \text{ cm}^2 \approx 795.54 \text{ cm}^2.$$

Q11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades. **2**

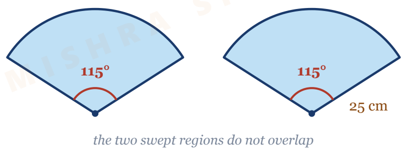

Fig. Q11 — Each blade sweeps a 115° sector of radius 25 cm; the two sectors do not overlap.

Note: *MLI-redrawn figure, constructed from the given data — angles true and lengths to scale. Work only from the given data, never by measuring the figure.*

Given: *Two wipers, each blade length (radius)* $r = 25$ *cm, sweep angle* $\theta = 115^\circ$*, blades do* **not** *overlap;* $\pi = \frac{22}{7}$.

To Find: *Total area cleaned at each sweep.*

Solution:

Step 1 — Area cleaned by one wiper (a sector):

$$= \frac{115}{360} \times \frac{22}{7} \times 25 \times 25 = \frac{23}{72} \times \frac{22}{7} \times 625$$ *[½ mark]*

$$= \frac{23 \times 22 \times 625}{504} = \frac{316250}{504} = \frac{158125}{252} \text{ cm}^2$$

[1 mark]

Step 2 — Since the two sweeps do not overlap, total area = 2× one sector:

$$= 2 \times \frac{158125}{252} = \frac{158125}{126} \text{ cm}^2$$ *[½ mark]*

∴ Total area cleaned $= \frac{158125}{126} \text{ cm}^2 \approx 1254.96 \text{ cm}^2$.

Note: The words "do not overlap" are the reason we may simply **double** *the single-sector area — if they* *overlapped, the common region would have to be subtracted.*

Q12. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use $\pi = 3.14$) **2**

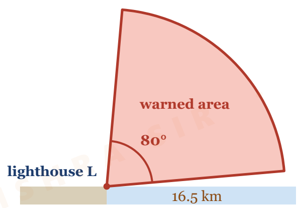

Fig. Q12 — The red light covers a sector of angle 80° and radius 16.5 km.

Note: *MLI-redrawn figure, constructed from the given data — angles true and lengths to scale. Work only from the given data, never by measuring the figure.*

Given: *Sector angle* $\theta = 80^\circ$, *radius (distance of light)* $r = 16.5$ *km*, $\pi = 3.14$.

To Find: *Area of the sea over which ships are warned.*

Solution:

Step 1 — The warned region is a sector:

$$Area = \frac{\theta}{360} \times \pi r^2 = \frac{80}{360} \times 3.14 \times 16.5 \times 16.5$$

[½ mark]

Step 2 — Simplify:

$$(16.5)^2 = 272.25$$

$$= \frac{2}{9} \times 3.14 \times 272.25 = \frac{2}{9} \times 854.865$$ *[1 mark]*

$$= \frac{1709.73}{9} = 189.97$$ *[½ mark]*

$\therefore$ **Area of the sea warned** $= 189.97\ \text{km}^2$**.**

Q13. A round table cover has six equal designs as shown in the figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹0.35 per cm². (Use $\sqrt{3} = 1.7$) **3**

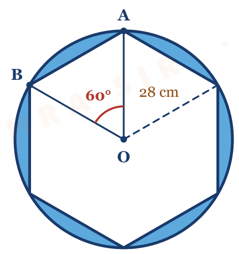

Fig. Q13 — The six designs (shaded) are the segments outside the regular hexagon; each chord subtends 60°.

Note: *MLI-redrawn figure, constructed from the given data — angles true and lengths to scale. Work only from the given data, never by measuring the figure.*

Given: *Radius of cover* $r = 28$ *cm; six equal designs (the six segments outside a regular hexagon inscribed in the circle); rate =₹0.35 per cm²;* $\pi = \frac{22}{7}$, $\sqrt{3} = 1.7$.

To Find: *Total cost of making the designs.*

Solution:

Step 1 — Find the angle of one design. *The six designs are equal, so each chord of the hexagon subtends* $\theta = \frac{360^\circ}{6} = 60^\circ$ *at the centre. Each design is therefore a* **segment** *of angle* 60°. **[½ mark]**

Step 2 — Area of one sector OAB:

$$= \frac{60}{360} \times \frac{22}{7} \times 28 \times 28 = \frac{1}{6} \times 22 \times 4 \times 28 = \frac{2464}{6} = 410.67 \text{ cm}^2$$

[1 mark]

Step 3 — Area of △OAB. $OA = OB = 28$ *cm with* $\angle AOB = 60^\circ \Rightarrow$ *equilateral triangle of side 28 cm:*

$$= \frac{\sqrt{3}}{4} \times (28)^2 = \frac{1.7 \times 784}{4} = 1.7 \times 196 = 333.2 \text{ cm}^2$$

[½ mark]

Step 4 — Area of one design (segment):

$$= 410.67 - 333.2 = 77.47 \text{ cm}^2$$

Area of six designs $= 6 \times 77.47 = 464.8 \text{ cm}^2$

[½ mark]

Step 5 — Cost at ₹0.35 per cm²:

$= 464.8 \times 0.35 = 162.68$ **[½ mark]**

∴ Cost of making the designs =₹162.68 (approximately).

Q14. Tick the correct answer in the following: Area of a sector of angle p (in degrees) of a circle with radius R is **1**

(A) $\frac{p}{180} \times 2\pi R$ (B) $\frac{p}{180} \times \pi R^2$ (C) $\frac{p}{360} \times 2\pi R$ (D) $\frac{p}{720} \times 2\pi R^2$

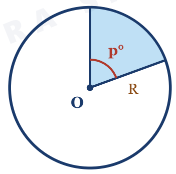

Fig. Q14 — A sector of angle p° in a circle of radius R.

Note: *MLI-redrawn figure, constructed from the given data — angles true and lengths to scale. Work only from the given data, never by measuring the figure.*

Given: *A sector of angle p degrees in a circle of radius R.*

To Find: *The correct expression for its area.*

Solution:

Step 1 — Standard formula: *Area of sector*

$$= \frac{p}{360} \times \pi R^2.$$

Step 2 — Match with the options. *Multiply numerator and denominator by 2:*

$$\frac{p}{360} \times \pi R^2 = \frac{p}{720} \times 2\pi R^2,$$ *which is option (D).*

[1 mark]

Why the others are wrong: *(A) and (C) have $2\pi R$, which is a* **length** *(arc/circumference), not an area. (B) uses 180 in the denominator, which would give twice the correct area.*

$\therefore$ **Correct option: (D)** $\frac{p}{720} \times 2\pi R^2$**.**

✅ Quick Answer Key — Exercise 11.1

Q.No.	Answer	Key Concept
1	$\frac{132}{7}$ cm² ≈ 18.86 cm²	Sector area
2	$\frac{77}{8}$ cm² = 9.625 cm²	Circumference → r; quadrant
3	$\frac{154}{3}$ cm² ≈ 51.33 cm²	6° per minute
4	(i) 28.5 cm² (ii) 235.5 cm²	Segment; major sector
5	(i) 22 cm (ii) 231 cm² (iii) $\left(231 - \frac{441\sqrt{3}}{4}\right)$ cm²	Arc, sector, segment
6	Minor ≈ 20.44 cm²; Major ≈ 686.06 cm²	Equilateral triangle
7	88.44 cm²	$\theta = 120^\circ$, OM ⊥ AB
8	(i) 19.625 m² (ii) 58.875 m²	Quadrant at corner
9	(i) 285 mm (ii) 96.25 mm²	Wire = circumference + 5 diameters
10	$\frac{22275}{28}$ cm² ≈ 795.54 cm²	$\theta = 45^\circ$
11	$\frac{158125}{126}$ cm² ≈ 1254.96 cm²	2 × sector (no overlap)
12	189.97 km²	Sector, $\pi = 3.14$
13	₹162.68	6 segments × rate
14	(D)	$\frac{p}{360}\pi R^2 = \frac{p}{720} \times 2\pi R^2$

🎯 Special Angles — Triangle Area at a Glance (memorise this)

$\angle$ AOB	Nature of $\triangle$ OAB	Area $= \frac{1}{2}r^2 \sin\theta$	Appears in
60°	Equilateral (side $= r$)	$\frac{\sqrt{3}}{4}r^2$	Q5, Q6, Q13
90°	Right isosceles	$\frac{1}{2}r^2$	Q4
120°	Isosceles obtuse	$\frac{\sqrt{3}}{4}r^2$	Q7

Observe: 60° and 120° give the *same* triangle area because $\sin 60^\circ = \sin 120^\circ$ — but the sectors are different, so the segments differ.

⚠ Common Mistakes to Avoid (Board Exam)

1. Using the **diameter** in place of the radius — see Q9 (35 mm is the diameter, so $r = 17.5$ mm).
2. Forgetting to **subtract the triangle** when a *segment* is asked; students often stop at the sector area.
3. Mixing up **minor** and **major**: the major sector angle is $360^\circ - \theta$, and the major segment = circle − minor segment.
4. Switching the value of π. Use $\pi = 3.14$ only where the question says so (Q4, Q6, Q7, Q8, Q12); elsewhere $\pi = \frac{22}{7}$.
5. Using $\sqrt{3} = 1.73$ in Q13 — that question specifies $\sqrt{3} = 1.7$, and the answer changes if you use the wrong value.
6. Writing area units as cm instead of cm^2, or omitting units altogether (Q5 mixes a length answer with area answers).
7. Doubling the wiper area in Q11 *after* already using 230° — do one or the other, not both.
8. Not drawing the figure. In segment questions CBSE reserves a mark for a correct labelled diagram.

CHAPTER 12

Surface Areas and Volumes

Exercises 12.1 – 12.2

EXERCISE 12.1

CBSE Step-wise Marking Scheme Format	**Board Exam 2026–27**	**Name:** ________________

💡 **CBSE Marking Scheme Guidelines Followed in This Document**

1. Every question is structured as **Given** → **To Find** → **Solution**, matching CBSE's official answer-key format.
2. Each step of the solution is written on its own line and tagged with its mark allocation.
3. As per CBSE policy, method/formula marks are earned independently of the final answer — so every step is shown, not just the final result.

📝 Key Idea & Formulae Used in This Exercise
(Surface Area of a Combination of Solids)

Solid / Concept	Formula
Golden Rule	When two solids are joined, the surface where they touch **disappears**. TSA of the combined solid = sum of the **curved/visible** surfaces only — NOT the sum of the individual total surface areas.
Cube (edge a)	$\text{TSA} = 6a^2$
Cuboid (l, b, h)	$\text{TSA} = 2(lb + bh + hl)$
Cylinder (radius r, height h)	$\text{CSA} = 2\pi rh$; Area of one circular base $= \pi r^2$
Cone (radius r, slant height l)	$\text{CSA} = \pi rl$; $l = \sqrt{r^2 + h^2}$
Sphere (radius r)	Surface area $= 4\pi r^2$
Hemisphere (radius r)	$\text{CSA} = 2\pi r^2$; TSA (with flat base) $= 3\pi r^2$

Take $\pi = \dfrac{22}{7}$ throughout this exercise, unless stated otherwise.

EXERCISE 12.1 — **Total: 28 Marks**

Q1. 2 cubes each of volume 64 cm³ are joined end to end. Find the surface area of the resulting cuboid. **[2]**

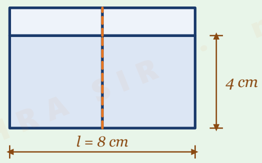

Fig. 12.1–Q1 — Two 4 cm cubes joined end to end: the joining faces (orange, dashed) are hidden inside.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Given: *Two cubes, each of volume 64 cm³, joined end to end.*

To Find: *The surface area of the resulting cuboid.*

Solution:

Step 1 — Find the edge of each cube.

$$\text{Volume of cube} = a^3 = 64$$

$$a = \sqrt[3]{64} = 4 \text{ cm}$$ **[½ mark]**

Step 2 — Determine the dimensions of the resulting cuboid.

When the two cubes are joined end to end, the length doubles while breadth and height stay the same as the cube's edge:

$$\text{Length } l = 2a = 2 \times 4 = 8 \text{ cm}$$

$$\text{Breadth } b = a = 4 \text{ cm}$$

$$\text{Height } h = a = 4 \text{ cm}$$ **[½ mark]**

Step 3 — Apply the surface area formula for a cuboid:

$$\text{TSA} = 2(lb + bh + hl)$$

$$\text{TSA} = 2(8 \times 4 + 4 \times 4 + 4 \times 8)$$

$$\text{TSA} = 2(32 + 16 + 32)$$

$$\text{TSA} = 2 \times 80$$

$$\text{TSA} = \mathbf{160} \text{ cm}^2$$ **[1 mark]**

∴ Surface area of the resulting cuboid = 160 cm².

Q2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. **3**

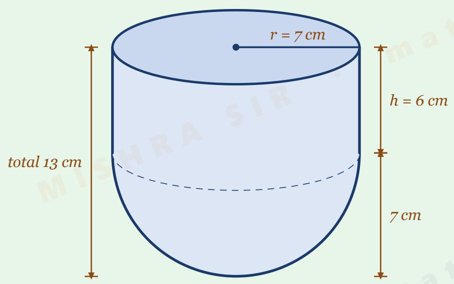

Fig. 12.1–Q2 — Hollow cylinder on a hollow hemisphere of the same radius; the hemisphere takes 7 cm of the 13 cm.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Given: *A hollow hemisphere mounted by a hollow cylinder; diameter of hemisphere = 14 cm; total height of the vessel = 13 cm.*

To Find: *The inner surface area of the vessel.*

Solution:

Step 1 — Find the common radius and the height of the cylindrical part.

$$\textit{Radius } r = \frac{14}{2} = 7 \textit{ cm}$$

Since the hemisphere occupies a height equal to its radius (7 cm) at the bottom, height of the cylindrical part:

$$h = 13 - 7 = 6 \textit{ cm}$$ **[1 mark]**

Step 2 — Identify the visible inner surfaces.

The inner surface area consists of the CSA of the cylinder **plus** *the CSA of the hemisphere (the flat circular top of the vessel and the joining circle are open/not part of the surface):*

$$\text{Inner S.A.} = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$$ **[½ mark]**

Step 3 — Substitute the values:

$$\text{Inner S.A.} = 2 \times \frac{22}{7} \times 7 \times (6 + 7)$$

$$\text{Inner S.A.} = 2 \times 22 \times 13$$

$$\text{Inner S.A.} = \mathbf{572} \textit{ cm}^2$$ **[1½ marks]**

∴ Inner surface area of the vessel = 572 cm².

Q3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy. 3

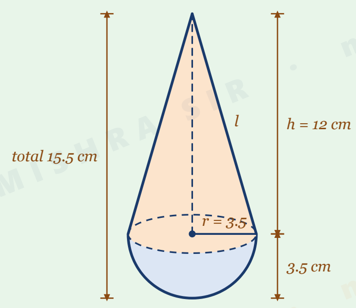

Fig. 12.1–Q3 — Cone on a hemisphere of radius 3.5 cm. The joining circle (dashed) is inside, not on the surface.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Given: *A toy = cone mounted on a hemisphere of the same radius; radius* $r = 3.5$ *cm; total height of toy = 15.5 cm.*

To Find: *The total surface area of the toy.*

Solution:

Step 1 — Find the height of the cone.

Height of cone h = *total height − radius of hemisphere*

$h = 15.5 - 3.5 = 12$ *cm* **[½ mark]**

Step 2 — Find the slant height of the cone using $l = \sqrt{r^2 + h^2}$**:**

$$l = \sqrt{(3.5)^2 + (12)^2}$$

$$l = \sqrt{12.25 + 144}$$

$$l = \sqrt{156.25}$$

$l = 12.5$ *cm* **[1 mark]**

Step 3 — Apply the total surface area formula:

$$\text{TSA} = \text{CSA of cone} + \text{CSA of hemisphere}$$

$$\text{TSA} = \pi r l + 2\pi r^2 = \pi r(l + 2r)$$

$$\text{TSA} = \frac{22}{7} \times 3.5 \times (12.5 + 7)$$

$$\text{TSA} = 11 \times 19.5$$

$\text{TSA} = \mathbf{214.5}$ *cm²* **[1½ marks]**

∴ Total surface area of the toy = 214.5 cm².

Q4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid. **3**

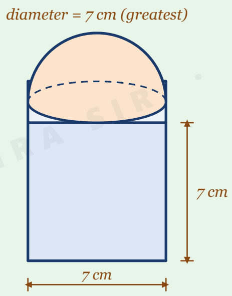

Fig. 12.1–Q4 — The hemisphere's base fits the top face exactly, so the greatest diameter is 7 cm.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Given: *A cubical block of side 7 cm, with a hemisphere placed on top.*

To Find: *(a) The greatest possible diameter of the hemisphere, (b) the surface area of the resulting solid.*

Solution:

Step 1 — Find the greatest diameter of the hemisphere.

For the hemisphere to fit on the cube's face without overhanging, its diameter cannot exceed the side of the cube.

$\therefore$ *Greatest diameter* $=$ **7** *cm, so radius* $r = 3.5$ *cm*

[1 mark]

Step 2 — Identify the visible surfaces of the combined solid.

When the hemisphere sits on the cube, its circular base (πr^2) covers part of the cube's top face and is not part of the outer surface; the hemisphere contributes only its curved surface ($2\pi r^2$):

$$\text{S.A.} = \text{TSA of cube} - \pi r^2 + 2\pi r^2 = \text{TSA of cube} + \pi r^2$$

[½ mark]

Step 3 — Substitute the values:

TSA of cube $= 6a^2 = 6 \times 7 \times 7 = 294$ *cm²*

$$\pi r^2 = \frac{22}{7} \times 3.5 \times 3.5 = 38.5\ cm^2$$

$$\text{S.A.} = 294 + 38.5$$

$$\text{S.A.} = \mathbf{332.5}\ cm^2$$ **[1½ marks]**

∴ Greatest diameter = 7 cm; Surface area of the solid = 332.5 cm².

Q5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid. **3**

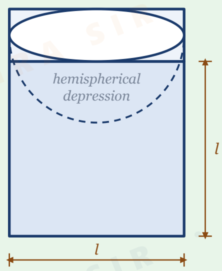

Fig. 12.1–Q5 — A hemisphere of diameter l (dashed) is scooped out of the top face of the cube.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Given: *A cube of edge l, with a hemispherical* **depression (cavity)** *cut from one face; diameter of hemisphere* $= l$ *(edge of cube).*

To Find: *The surface area of the remaining solid, in terms of l.*

Solution:

Step 1 — Find the radius of the hemisphere.

$$r = \frac{l}{2}$$ **[½ mark]**

Step 2 — Identify the visible surfaces.

Since the hemisphere is **scooped out** *(a cavity, not a bump), the flat circular area* (πr^2) *that would have been on the cube's face is removed, but the* **curved inner surface** *of the cavity* $(2\pi r^2)$ *becomes newly visible:*

$$\text{S.A.} = \text{TSA of cube} - \pi r^2 + 2\pi r^2 = \text{TSA of cube} + \pi r^2$$

[1 mark]

Step 3 — Substitute $r = \dfrac{l}{2}$:

$$\text{S.A.} = 6l^2 + \pi\left(\frac{l}{2}\right)^2$$

$$\text{S.A.} = 6l^2 + \frac{\pi l^2}{4}$$

$$\text{S.A.} = l^2\left(6 + \frac{\pi}{4}\right)$$

Using $\pi = \dfrac{22}{7}$:

$$6 + \frac{22}{7 \times 4} = 6 + \frac{22}{28} = 6 + \frac{11}{14} = \frac{84 + 11}{14} = \frac{95}{14}$$

$$\text{S.A.} = \frac{\mathbf{95}}{\mathbf{14}}\mathbf{l^2} \text{ sq. units}$$ *[1½ marks]*

∴ Surface area of the remaining solid

$$= \left(6l^2 + \frac{\pi l^2}{4}\right) = \frac{95}{14}l^2 \textbf{ sq. units.}$$

Q6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area. **3**

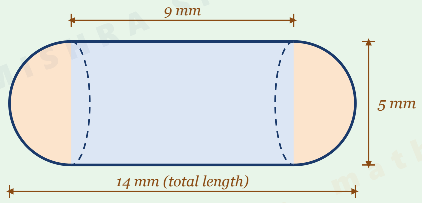

Fig. 12.1–Q6 — A cylinder with a hemisphere at each end: radius 2.5 mm, cylindrical part 9 mm.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Given: *A capsule = cylinder with hemispheres stuck at both ends; total length = 14 mm; diameter = 5 mm.*

To Find: *The surface area of the capsule.*

Solution:

Step 1 — Find the radius and the height of the cylindrical part.

Radius $r = \dfrac{5}{2} = 2.5\ mm$

Since a hemisphere (of height = r) is attached at each end, height of cylindrical part:

$$h = 14 - 2r = 14 - 2(2.5) = 14 - 5 = 9\ mm$$

[1 mark]

Step 2 — Identify the visible surfaces.

Both flat circular ends of the cylinder are covered by the hemispheres, so only the curved surfaces are visible:

$$\text{S.A.} = \text{CSA of cylinder} + 2 \times \text{CSA of hemisphere} = 2\pi rh + 2(2\pi r^2) = 2\pi r(h + 2r)$$

[½ mark]

Step 3 — Substitute the values:

$$\text{S.A.} = 2 \times \frac{22}{7} \times 2.5 \times (9 + 5)$$

$$\text{S.A.} = 2 \times \frac{22}{7} \times 2.5 \times 14$$

$$\text{S.A.} = 2 \times 22 \times 2.5 \times 2$$

$\text{S.A.} = \mathbf{220}\ mm^2$ *[1½ marks]*

∴ Surface area of the capsule = 220 mm².

Q7. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹500 per m². (Note that the base of the tent will not be covered with canvas.) 4

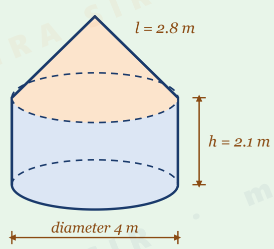

Fig. 12.1–Q7 — Canvas = CSA of cylinder + CSA of cone; the base (bottom circle) is not covered.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Given: *A tent = cylinder + cone (conical top); height of cylinder $h = 2.1$ m; diameter $= 4$ m; slant height of cone $l = 2.8$ m; base is not covered with canvas.*

To Find: *(a) the area of canvas used, (b) the cost of the canvas at ₹500/m².*

Solution:

Step 1 — Find the common radius.

$$r = \frac{4}{2} = 2\ m \quad \text{[½ mark]}$$

Step 2 — Identify the surfaces that need canvas.

Since the base is open (not covered), canvas is required only for the CSA of the cylinder and the CSA of the cone:

$$\text{Area} = 2\pi rh + \pi rl = \pi r(2h + l) \quad \text{[½ mark]}$$

Step 3 — Substitute the values:

$$\text{Area} = \frac{22}{7} \times 2 \times (2 \times 2.1 + 2.8)$$

$$\text{Area} = \frac{22}{7} \times 2 \times (4.2 + 2.8)$$

$$\text{Area} = \frac{22}{7} \times 2 \times 7$$

$$\text{Area} = 22 \times 2$$

$$\text{Area} = \mathbf{44}\ m^2 \quad \text{[1 mark]}$$

Step 4 — Find the cost of the canvas:

$$\text{Cost} = \text{Area} \times \text{Rate} = 44 \times 500$$

Cost = ₹22,000 [2 marks]

∴ Area of canvas used = 44 m²; Cost of the canvas = ₹22,000.

Q8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm². 4

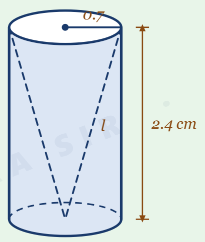

Fig. 12.1–Q8 — A cone of the same height and base (dashed) is hollowed out from the top face.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Given: *A solid cylinder of height 2.4 cm, diameter 1.4 cm; a conical cavity of the* **same height and same diameter** *is hollowed out from one flat face.*

To Find: *The total surface area of the remaining solid, to the nearest cm².*

Solution:

Step 1 — Find the radius and the slant height of the cone.

Radius $r = \frac{1.4}{2} = 0.7$ *cm; Height of cone (= height of cylinder)* $h = 2.4$ *cm*

$l = \sqrt{r^2 + h^2} = \sqrt{(0.7)^2 + (2.4)^2} = \sqrt{0.49 + 5.76} = \sqrt{6.25} = 2.5$ *cm* *[1 mark]*

Step 2 — Identify the visible surfaces of the remaining solid.

Since the cavity has the **same diameter** *as the cylinder, the entire top circular face is replaced by the cone's curved (lateral) surface. The bottom circular face of the cylinder remains untouched, and the cylinder's own curved surface remains visible:*

$$\text{TSA} = \text{Area of bottom circle} + \text{CSA of cylinder} + \text{CSA of cone (cavity)}$$

$$\text{TSA} = \pi r^2 + 2\pi rh + \pi rl$$ *[1 mark]*

Step 3 — Substitute the values:

$$\pi r^2 = \frac{22}{7} \times 0.7 \times 0.7 = 1.54 \ cm^2$$

$$2\pi rh = 2 \times \frac{22}{7} \times 0.7 \times 2.4 = 10.56 \ cm^2$$

$$\pi rl = \frac{22}{7} \times 0.7 \times 2.5 = 5.50 \ cm^2$$

$$\text{TSA} = 1.54 + 10.56 + 5.50$$

$$\text{TSA} = 17.60 \ cm^2$$ *[1 mark]*

Step 4 — Round to the nearest cm²:

$$\text{TSA} \approx \mathbf{18} \ cm^2$$ *[1 mark]*

∴ Total surface area of the remaining solid ≈ 18 cm² (to the nearest cm²).

Q9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article. **3**

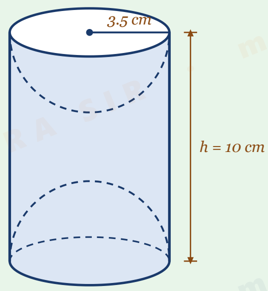

Fig. 12.1–Q9 — A hemisphere (dashed) is scooped out of each end; each flat end is replaced by a curved hollow.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Given: *A solid cylinder of height 10 cm and radius 3.5 cm, with a hemisphere scooped out from* **each** *flat end (radius of hemisphere = radius of cylinder).*

To Find: *The total surface area of the wooden article.*

Solution:

Step 1 — Identify the visible surfaces.

Since the hemispheres scooped out have the **same radius** *as the cylinder, both flat circular ends disappear entirely and are replaced by the curved (concave) surfaces of the two hemispherical cavities:*

$$\text{TSA} = \text{CSA of cylinder} + 2 \times \text{CSA of hemisphere} = 2\pi rh + 2(2\pi r^2) = 2\pi r(h + 2r)$$

[1 mark]

Step 2 — Substitute the values *($r = 3.5$ cm, $h = 10$ cm):*

$$\text{TSA} = 2 \times \frac{22}{7} \times 3.5 \times (10 + 7)$$

$$\text{TSA} = 2 \times 11 \times 17$$

$\text{TSA} = \mathbf{374}$ *cm²* **[2 marks]**

∴ Total surface area of the article = 374 cm².

✅ Quick Answer Key — Exercise 12.1

Q.No.	Final Answer
1	160 cm^2
2	572 cm^2
3	214.5 cm^2
4	Diameter = 7 cm; Surface area = 332.5 cm^2
5	$\left(6+\frac{\pi}{4}\right)l^2 = \frac{95}{14}l^2$ sq. units
6	220 mm^2
7	Area = 44 m^2; Cost = ₹22,000
8	≈ 18 cm^2
9	374 cm^2

⚠ Common Mistakes to Avoid (Board Exam)

1. Adding the **total** surface areas of the two constituent solids. Always identify which surfaces actually remain **visible** after joining — the joint face is never part of the answer.
2. Forgetting that a **depression/cavity** (Q5, Q8, Q9) behaves oppositely to a solid stuck on top: a flat area is **removed** and a curved area of the **same or larger** size takes its place.
3. Using the wrong height for the cylindrical part — always subtract the hemisphere's radius (not diameter) from the total height when a hemisphere sits at one or both ends (Q2, Q3, Q6, Q9).
4. Forgetting to compute the slant height $l = \sqrt{r^2 + h^2}$ before finding CSA of a cone (Q3, Q8).
5. In Q7, including the base area of the tent in the canvas calculation — the question explicitly states the base is not covered.
6. Not rounding to the specified precision when the question demands it (Q8 asks for the nearest cm^2).

EXERCISE 12.2

CBSE Step-wise Marking Scheme Format	**Board Exam 2026–27**	**Name:** ________________

💡 **CBSE Marking Scheme Guidelines Followed in This Document**

1. Every question is structured as **Given → To Find → Solution**, matching CBSE's official answer-key format.
2. Each step of the solution is written on its own line and tagged with its mark allocation.
3. As per CBSE policy, method/formula marks are earned independently of the final answer — so every step is shown, not just the final result.

📝 Key Idea & Formulae Used in This Exercise (Volume of a Combination of Solids)

Solid / Concept	Formula
Golden Rule	Unlike surface area, the volume of a combined solid is simply the **sum (or difference)** of the volumes of its parts — no volume is "lost" at the joint.
Cuboid (l, b, h)	$\text{Volume} = l \times b \times h$
Cylinder (radius r, height h)	$\text{Volume} = \pi r^2 h$
Cone (radius r, height h)	$\text{Volume} = \frac{1}{3}\pi r^2 h$
Sphere (radius r)	$\text{Volume} = \frac{4}{3}\pi r^3$
Hemisphere (radius r)	$\text{Volume} = \frac{2}{3}\pi r^3$

Take $\pi = \frac{22}{7}$ unless a different value of π is specified in the question.

EXERCISE 12.2 — Total: 28 Marks

Q1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π. **[3]**

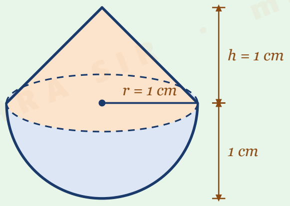

Fig. 12.2–Q1 — Cone (height = radius = 1 cm) standing on a hemisphere of radius 1 cm.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Given: *A solid = cone standing on a hemisphere; radius of both $r = 1$ cm; height of cone h = radius = 1 cm.*

To Find: *The volume of the solid, in terms of π.*

Solution:

Step 1 — Write the volume as the sum of the two parts:

$$\text{Volume} = \text{Volume of cone} + \text{Volume of hemisphere}$$

$$\text{Volume} = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$ **[1 mark]**

Step 2 — Substitute $r = 1$ cm, $h = 1$ cm:

$$\text{Volume} = \frac{1}{3}\pi(1)^2(1) + \frac{2}{3}\pi(1)^3$$

$$\text{Volume} = \frac{1}{3}\pi + \frac{2}{3}\pi$$ **[1 mark]**

Step 3 — Add the fractions:

$$\text{Volume} = \left(\frac{1}{3} + \frac{2}{3}\right)\pi = \frac{3}{3}\pi$$

$$\text{Volume} = \pi \ cm^3$$ **[1 mark]**

∴ Volume of the solid = π cm³.

Q2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.) **3**

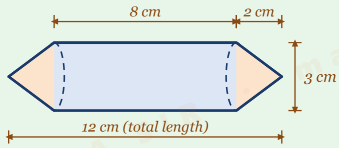

Fig. 12.2–Q2 — A cylinder of length 8 cm with a cone of height 2 cm at each end; diameter 3 cm.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Given: *A model = cylinder with a cone attached at each end; diameter = 3 cm; total length = 12 cm; height of each cone = 2 cm.*

To Find: *The volume of air contained in the model.*

Solution:

Step 1 — Find the common radius and the length of the cylindrical part.

$$\text{Radius } r = \frac{3}{2} = 1.5 \text{ cm}$$

Length of cylindrical part = total length − heights of both cones

$$h = 12 - 2(2) = 12 - 4 = 8 \text{ cm}$$ **[1 mark]**

Step 2 — Write the total volume as the sum of the three parts:

$$\text{Volume} = \text{Volume of cylinder} + 2 \times \text{Volume of one cone}$$

$$\text{Volume} = \pi r^2 h + 2\left(\frac{1}{3}\pi r^2 h_{\text{cone}}\right)$$ **[½ mark]**

Step 3 — Substitute the values ($r = 1.5$, $h = 8$, $h_{\text{cone}} = 2$):

Volume of cylinder

$$= \frac{22}{7} \times (1.5)^2 \times 8 = \frac{22}{7} \times 2.25 \times 8 = \frac{22}{7} \times 18 = \frac{396}{7} \text{ cm}^3$$

Volume of 2 cones

$$= 2 \times \frac{1}{3} \times \frac{22}{7} \times (1.5)^2 \times 2 = 2 \times \frac{1}{3} \times \frac{22}{7} \times 2.25 \times 2 = \frac{66}{7} \text{ cm}^3$$

Step 4 — Add the two volumes:

$$\text{Volume} = \frac{396}{7} + \frac{66}{7} = \frac{462}{7}$$

$$\text{Volume} = \mathbf{66} \text{ cm}^3$$ **[1½ marks]**

∴ Volume of air contained in the model = 66 cm³.

Q3. A *gulab jamun* contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 *gulab jamuns*, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm.

4

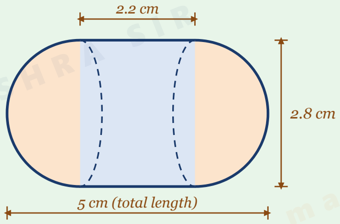

Fig. 12.2–Q3 — A cylinder with a hemisphere at each end: radius 1.4 cm, cylindrical part 2.2 cm.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Given: *45 gulab jamuns, each = cylinder with a hemisphere at each end; total length = 5 cm; diameter = 2.8 cm; syrup = 30% of volume.*

To Find: *The approximate volume of syrup in 45 gulab jamuns.*

Solution:

Step 1 — Find the radius and the length of the cylindrical part (of one gulab jamun).

$$\text{Radius } r = \frac{2.8}{2} = 1.4 \text{ cm}$$

Length of cylindrical part

$h = 5 - 2r = 5 - 2(1.4) = 5 - 2.8 = 2.2$ *cm* **[1 mark]**

Step 2 — Write the volume of one gulab jamun *(two hemispheres together make one full sphere):*

$$\text{Volume} = \pi r^2 h + \frac{4}{3}\pi r^3$$ **[½ mark]**

Step 3 — Substitute the values:

$$\pi r^2 h = \frac{22}{7} \times (1.4)^2 \times 2.2 = \frac{22}{7} \times 1.96 \times 2.2 = 6.16 \times 2.2 = 13.552 \text{ cm}^3$$

$$\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (1.4)^3 = \frac{4}{3} \times \frac{22}{7} \times 2.744 = \frac{4}{3} \times 8.624 = 11.499 \text{ cm}^3$$

$$\text{Volume (one)} = 13.552 + 11.499 = 25.05 \text{ cm}^3 \text{ (approx.)}$$

[1½ marks]

Step 4 — Find the volume of 45 gulab jamuns:

Volume $(45) = 45 \times 25.05 = 1127.25$ *cm³ (approx.)*

Step 5 — Find 30% of this volume (the syrup):

$$\text{Syrup} = \frac{30}{100} \times 1127.25$$

Syrup $=$ **338.17** *cm³ (approx.)* **[1 mark]**

∴ Approximate volume of syrup in 45 gulab jamuns ≈ 338.17 cm³.

Q4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand. **3**

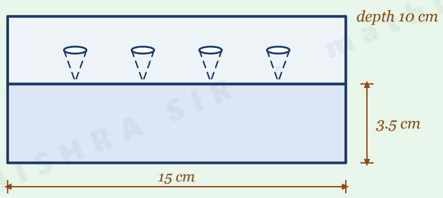

Fig. 12.2–Q4 — Four conical holes (radius 0.5 cm, depth 1.4 cm, dashed) in a 15 × 10 × 3.5 cm block.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Given: *A wooden cuboid pen stand of dimensions* 15 cm × 10 cm × 3.5 cm, *with 4 conical depressions of radius 0.5 cm and depth 1.4 cm each.*

To Find: *The volume of wood in the entire stand.*

Solution:

Step 1 — Find the volume of the cuboid:

Volume of cuboid $= l \times b \times h = 15 \times 10 \times 3.5 = 525$ *cm³* **[1 mark]**

Step 2 — Find the volume of one conical depression:

$$\text{Volume of one cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (0.5)^2 \times 1.4$$

$$= \frac{1}{3} \times \frac{22}{7} \times 0.25 \times 1.4 = \frac{1}{3} \times 1.1 = 0.3667 \text{ cm}^3$$

[1 mark]

Step 3 — Find the volume of 4 depressions:

Volume (4 cones) $= 4 \times 0.3667 = 1.467$ *cm³*

Step 4 — Subtract from the volume of the cuboid:

$$\text{Volume of wood} = 525 - 1.467$$

$$\text{Volume of wood} = \mathbf{523.53}\ cm^3\ (approx.)$$ **[1 mark]**

$\therefore$ **Volume of wood in the entire stand ≈ 523.53 cm^3.**

Q5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel. **4**

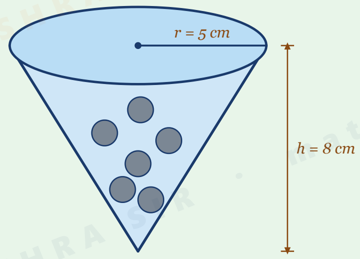

Fig. 12.2–Q5 — Inverted cone full of water; lead shots of radius 0.5 cm push one-fourth of the water out.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Given: *An inverted conical vessel, height = 8 cm, radius of open top = 5 cm, filled to the brim with water. Lead shots (spheres of radius 0.5 cm) are dropped in, causing $\frac{1}{4}$ of the water to overflow.*

To Find: *The number of lead shots dropped into the vessel.*

Solution:

Step 1 — Find the total volume of water (= volume of the cone).

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 8$$

$$= \frac{1}{3} \times \frac{22}{7} \times 25 \times 8 = \frac{4400}{21}\ cm^3$$ **[1 mark]**

Step 2 — Find the volume of water that overflows.

Since the volume of water displaced by the shots equals the volume that overflows:

Volume overflowed $= \frac{1}{4} \times$ Volume of cone $= \frac{1}{4} \times \frac{4400}{21} = \frac{1100}{21}$ cm^3 **[1 mark]**

Step 3 — Find the volume of one lead shot (sphere of radius 0.5 cm):

Volume of one shot $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (0.5)^3 = \frac{4}{3} \times \frac{22}{7} \times 0.125 = \frac{11}{21}$ cm^3 **[1 mark]**

Step 4 — Find the number of lead shots.

$$\text{Number of shots} = \frac{\text{Volume overflowed}}{\text{Volume of one shot}} = \frac{\frac{1100}{21}}{\frac{11}{21}} = \frac{1100}{11}$$

Number of shots $=$ **100** **[1 mark]**

∴ Number of lead shots dropped in the vessel = 100.

Q6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm³ of iron has approximately 8g mass. (Use $\pi = 3.14$) **4**

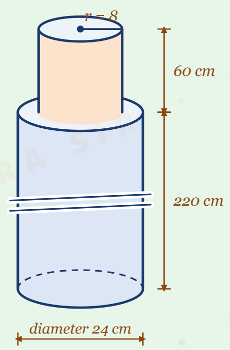

Fig. 12.2–Q6 — Heights are compressed (break marks) — the 220 cm part is drawn shorter so the figure fits.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Given: *A solid iron pole = large cylinder (height 220 cm, diameter 24 cm) topped by a smaller cylinder (height 60 cm, radius 8 cm); mass of 1 cm³ iron ≈ 8 g;* $\pi = 3.14$.

To Find: *The mass of the pole.*

Solution:

Step 1 — Find the radius of the larger cylinder.

$$r_1 = \frac{24}{2} = 12\ cm; \quad h_1 = 220\ cm$$

$$r_2 = 8\ cm\ (given); \quad h_2 = 60\ cm$$ *[½ mark]*

Step 2 — Write the total volume as the sum of the two cylinders:

$$\text{Volume} = \pi r_1^2 h_1 + \pi r_2^2 h_2 = \pi(r_1^2 h_1 + r_2^2 h_2)$$

[½ mark]

Step 3 — Substitute the values:

$$r_1^2 h_1 = (12)^2 \times 220 = 144 \times 220 = 31680$$

$$r_2^2 h_2 = (8)^2 \times 60 = 64 \times 60 = 3840$$

$$\text{Volume} = 3.14 \times (31680 + 3840) = 3.14 \times 35520$$

$$\text{Volume} = 111532.8\ cm^3$$ *[2 marks]*

Step 4 — Find the mass of the pole:

$$\text{Mass} = \text{Volume} \times 8\,\text{g/cm}^3 = 111532.8 \times 8$$

$$\text{Mass} = 892262.4\ g = \mathbf{892.2624}\ kg$$ *[1 mark]*

∴ Mass of the pole ≈ 892.26 kg.

Q7. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm. **4**

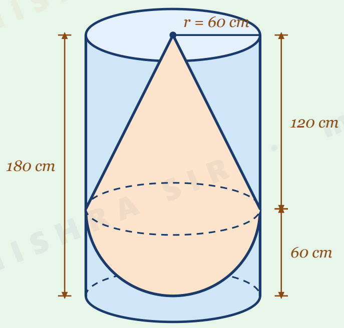

Fig. 12.2–Q7 — Cone + hemisphere (orange) standing in a cylinder of water of the same radius; water fills the rest.

> **Note:** *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Given: *A solid = cone (height 120 cm, radius 60 cm) on a hemisphere (radius 60 cm), placed upright inside a* *cylinder (radius 60 cm, height 180 cm) full of water, touching the bottom.*

To Find: *The volume of water left in the cylinder.*

Solution:

Step 1 — Find the volume of the solid (cone + hemisphere).

Volume of solid $= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$ *[½ mark]*

Substitute $r = 60$ cm, $h = 120$ cm:

$$\frac{1}{3}\pi(60)^2(120) = \frac{1}{3}\pi \times 3600 \times 120 = 144000\pi$$

$$\frac{2}{3}\pi(60)^3 = \frac{2}{3}\pi \times 216000 = 144000\pi$$

Volume of solid $= 144000\pi + 144000\pi = 288000\pi$ *cm³*

[1 mark]

Step 2 — Find the volume of the cylinder:

Volume of cylinder $= \pi r^2 H = \pi(60)^2(180) = \pi \times 3600 \times 180 = 648000\pi$ *cm³* *[1 mark]*

Step 3 — Find the volume of water left (cylinder − solid):

Volume left $= 648000\pi - 288000\pi = 360000\pi$

Using $\pi = \frac{22}{7}$:

Volume left $= 360000 \times \frac{22}{7} = \frac{7920000}{7}$

Volume left $\approx$ **1131428.57** *cm³* *[1½ marks]*

∴ Volume of water left in the cylinder ≈ 1131428.57 cm³ (≈ 1.131 m³).

Q8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm³. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$. 3

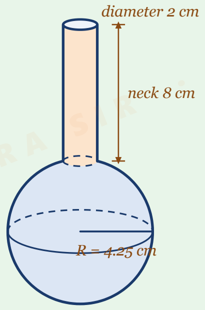

Fig. 12.2–Q8 — Spherical part of diameter 8.5 cm with a cylindrical neck 8 cm long and 2 cm across.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Given: *A spherical glass vessel with a cylindrical neck of length 8 cm, diameter 2 cm; diameter of the spherical part* $= 8.5$ *cm; child's measured volume* $= 345$ *cm³;* $\pi = 3.14$.

To Find: *Whether the child's measurement is correct, by computing the actual volume.*

Solution:

Step 1 — Find the radii of the two parts.

Radius of spherical part $R = \frac{8.5}{2} = 4.25$ *cm*

Radius of cylindrical neck $r = \frac{2}{2} = 1$ *cm; length of neck* $h = 8$ *cm* **[½ mark]**

Step 2 — Write the total volume as the sum of the sphere and the cylinder:

$$\text{Volume} = \frac{4}{3}\pi R^3 + \pi r^2 h$$ **[½ mark]**

Step 3 — Substitute the values:

$$(4.25)^3 = 76.765625$$

$$\frac{4}{3}\pi R^3 = \frac{4}{3} \times 3.14 \times 76.765625 = 321.39 \text{ cm}^3 \text{ (approx.)}$$

$$\pi r^2 h = 3.14 \times (1)^2 \times 8 = 25.12 \text{ cm}^3$$ **[1 mark]**

Step 4 — Add the two volumes:

$$\text{Volume} = 321.39 + 25.12$$

$$\text{Volume} = \mathbf{346.51} \text{ cm}^3 \text{ (approx.)}$$ **[1 mark]**

Step 5 — Compare with the child's answer.

Actual volume ≈ 346.51 *cm³, but the child measured 345 cm³.*

Since $346.51 \neq 345$, **the child's measurement is not correct** *— the actual capacity of the vessel is about*

346.51 cm³, which is about 1.51 cm³ more than what she found.

✅ Quick Answer Key — Exercise 12.2

Q.No.	Final Answer
1	π cm³
2	66 cm³
3	≈ 338.17 cm³ syrup
4	≈ 523.53 cm³
5	100 lead shots
6	≈ 892.26 kg
7	≈ 1131428.57 cm³
8	Actual volume ≈ 346.51 cm³; child's figure (345 cm³) is NOT correct

⚠ Common Mistakes to Avoid (Board Exam)

1. Unlike surface area, for **volume** problems the parts are simply **added** (or **subtracted** for cavities) — no adjustment is needed for the joint.
2. Forgetting to subtract the heights/lengths of attached solids to get the length of the middle cylindrical part (Q2, Q3).
3. In "shots dropped in water" type problems (Q5), remember that the volume of water that **overflows equals** the total volume of the shots submerged — not the full volume of the cone.
4. Missing the unit conversion or leaving mass in grams when kilograms are more natural (Q6).
5. In "check whether correct" type problems (Q8), always state clearly whether the given value is correct or not, along with the actual computed value — this conclusion carries its own mark.
6. Not simplifying $\frac{1100}{21} \div \frac{11}{21}$ correctly in Q5 — the 21 cancels out neatly if π is kept as a fraction throughout rather than converted to a decimal early.

CHAPTER 13

Statistics

Exercises 13.1 – 13.3

EXERCISE 13.1

CBSE Step-wise Marking Scheme Format	**Board Exam 2026–27**	**Name:** ________________

💡 **CBSE Marking Scheme Guidelines Followed in This Document**

1. Every question is structured as **Given → To Find → Solution**, matching CBSE's official answer-key format.
2. Each step of the solution is written on its own line and tagged with its mark allocation.
3. As per CBSE policy, method/formula marks are earned independently of the final answer — so every step is shown, not just the final result.
4. All working tables are drawn **vertically** (class intervals down the rows) — this is the layout CBSE examiners expect in the answer sheet.

📝 Formulae Used in This Exercise (Mean of Grouped Data)

Method	Formula	When to use it
1. Direct Method	$\bar{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$	When x_i and f_i are small numbers, or when class sizes are unequal.
2. Assumed Mean Method	$\bar{x} = a + \dfrac{\Sigma f_i d_i}{\Sigma f_i}$, where $d_i = x_i - a$	When x_i are large but have no common factor.
3. Step-deviation Method	$\bar{x} = a + h\left(\dfrac{\Sigma f_i u_i}{\Sigma f_i}\right)$, where $u_i = \dfrac{x_i - a}{h}$	When x_i and f_i are large and all d_i have a common factor h.
Class mark	$x_i = \dfrac{\text{Upper limit} + \text{Lower limit}}{2}$	Always the first step for grouped data.

Note: a = assumed mean (choose a middle class mark), h = class size. All three methods give the **same** answer.

EXERCISE 13.1 — Total: 27 Marks

Q1. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house. **3**

Number of plants	0–2	2–4	4–6	6–8	8–10	10–12	12–14
Number of houses	1	2	1	5	6	2	3

Which method did you use for finding the mean, and why?

Given: *A grouped frequency distribution of the number of plants in 20 houses, with class size* $h = 2$.

To Find: *The mean number of plants per house, and the method used with a reason.*

Solution:

Step 1 — Choose the method. *Here the class marks* x_i *(1, 3, 5, ...) and the frequencies* f_i *are all* **small numbers**, *so multiplying* $f_i x_i$ *is easy. Hence the* **Direct Method** *is the most appropriate choice.* ***[½ mark]***

Step 2 — Find the class mark of each class *using* $x_i = \dfrac{\text{Upper limit} + \text{Lower limit}}{2}$, *then find* $f_i x_i$:

Number of plants (Class interval)	Number of houses (f_i)	Class mark x_i	$f_i x_i$
0 – 2	1	1	1
2 – 4	2	3	6
4 – 6	1	5	5
6 – 8	5	7	35
8 – 10	6	9	54
10 – 12	2	11	22
12 – 14	3	13	39
Total	$\Sigma f_i = 20$	—	$\Sigma f_i x_i = 162$

[1½ marks for correct table]

Step 3 — Apply the direct-method formula:

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\bar{x} = \frac{162}{20}$$

$$\bar{x} = 8.1$$ *[1 mark]*

∴ **Mean number of plants per house = 8.1 plants.**

Reason for the method: *The Direct Method was used because the values of x_i and f_i are small, so the products $f_i x_i$ can be calculated quickly without simplification.*

Q2. Consider the following distribution of daily wages of 50 workers of a factory. 3

Daily wages (in ₹)	500–520	520–540	540–560	560–580	580–600
Number of workers	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

Given: *Distribution of daily wages of 50 workers; all class sizes are equal, $h = 20$.*

To Find: *The mean daily wage of the workers.*

Solution:

Step 1 — Choose the method. *The class marks (510, 530, 550, ...) are* **large numbers** *and the deviations have a common factor 20. So the* **Step-deviation Method** *is the appropriate choice.*

Take assumed mean $a = 550$ *(the middle class mark) and class size* $h = 20$. ***[½ mark]***

Step 2 — Prepare the working table *with* x_i, $d_i = x_i - a$, $u_i = \frac{d_i}{h}$ *and* f_iu_i:

Daily wages (in ₹)	Number of workers (f_i)	Class mark x_i	$d_i = x_i - 550$	$u_i = \frac{d_i}{20}$	f_iu_i
500 – 520	12	510	–40	–2	–24
520 – 540	14	530	–20	–1	–14
540 – 560	8	550 = a	0	0	0
560 – 580	6	570	20	1	6
580 – 600	10	590	40	2	20
Total	$\Sigma f_i = 50$	—	—	—	$\Sigma f_iu_i = -12$

[1½ marks for correct table]

Step 3 — Apply the step-deviation formula:

$$\bar{x} = a + h\left(\frac{\Sigma f_iu_i}{\Sigma f_i}\right)$$

$$\bar{x} = 550 + 20\left(\frac{-12}{50}\right)$$

$$\bar{x} = 550 + 20 \times (-0.24)$$

$$\bar{x} = 550 - 4.8$$

$$\bar{x} = 545.20$$ ***[1 mark]***

∴ Mean daily wage of the workers = ₹545.20.

Q3. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ₹18. Find the missing frequency f. **3**

Daily pocket allowance (in ₹)	11–13	13–15	15–17	17–19	19–21	21–23	23–25
Number of children	7	6	9	13	f	5	4

Given: *A frequency distribution of daily pocket allowance with one missing frequency f, and mean $\bar{x} = 18$.*

To Find: *The value of the missing frequency f.*

Solution:

Step 1 — Choose the method. *The class marks are small (12, 14, 16, ...), so the* **Direct Method** *is used. The unknown f is carried through the table as an algebraic term.* *[½ mark]*

Step 2 — Prepare the working table:

Daily pocket allowance (in ₹)	Number of children (f_i)	Class mark x_i	$f_i x_i$
11 – 13	7	12	84
13 – 15	6	14	84
15 – 17	9	16	144
17 – 19	13	18	234
19 – 21	f	20	$20f$
21 – 23	5	22	110
23 – 25	4	24	96
Total	$\Sigma f_i = 44 + f$	—	$\Sigma f_i x_i = 752 + 20f$

[1 mark for correct table]

Step 3 — Substitute in the mean formula:

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$18 = \frac{752 + 20f}{44 + f}$$ [½ mark]

Step 4 — Solve the equation:

Cross-multiplying: $18(44 + f) = 752 + 20f$

$$792 + 18f = 752 + 20f$$

$$792 - 752 = 20f - 18f$$

$$40 = 2f$$

$$f = 20$$ [1 mark]

∴ The missing frequency $f = 20$.

Verification: $\Sigma f_i = 44 + 20 = 64$ *and*

$\Sigma f_i x_i = 752 + 20(20) = 1152$; $\frac{1152}{64} = 18$ ✓ *(matches the given mean).*

Q4. Thirty women were examined in a hospital by a doctor and the number of heartbeats per minute were recorded and summarised as follows. Find the mean heartbeats per minute for these women, choosing a suitable method. **3**

Number of heartbeats per minute	65–68	68–71	71–74	74–77	77–80	80–83	83–86
Number of women	2	4	3	8	7	4	2

Given: *Distribution of heartbeats per minute of 30 women; class size $h = 3$.*

To Find: *The mean number of heartbeats per minute.*

Solution:

Step 1 — Choose the method. *The class marks are fairly large and every deviation is a multiple of 3, so the* **Step-deviation Method** *is suitable.*

Take $a = 75.5$ (the middle class mark) and $h = 3$.

[½ mark]

Step 2 — Prepare the working table:

Number of heartbeats per minute	Number of women (f_i)	Class mark x_i	$d_i = x_i - 75.5$	$u_i = \frac{d_i}{3}$	$f_i u_i$
65 – 68	2	66.5	–9	–3	–6
68 – 71	4	69.5	–6	–2	–8
71 – 74	3	72.5	–3	–1	–3
74 – 77	8	75.5 $= a$	0	0	0
77 – 80	7	78.5	3	1	7
80 – 83	4	81.5	6	2	8
83 – 86	2	84.5	9	3	6
Total	$\Sigma f_i = 30$	—	—	—	$\Sigma f_i u_i = 4$

[1½ marks for correct table]

Step 3 — Apply the step-deviation formula:

$$\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$\bar{x} = 75.5 + 3\left(\frac{4}{30}\right)$$

$$\bar{x} = 75.5 + \frac{12}{30}$$

$$\bar{x} = 75.5 + 0.4$$

$$\bar{x} = 75.9$$ *[1 mark]*

∴ Mean number of heartbeats = 75.9 beats per minute.

Q5. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes. **3**

Number of mangoes	50–52	53–55	56–58	59–61	62–64
Number of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Given: *Distribution of mangoes per box for 400 boxes. The classes are* **discontinuous** *(inclusive form): 50–52, 53–55, ...*

To Find: *The mean number of mangoes per box, and the method chosen.*

Solution:

Step 1 — Make the classes continuous. *The gap between the upper limit of one class and the lower limit of the next is* $53 - 52 = 1$.

$$\text{Adjustment factor} = \frac{1}{2} = 0.5$$

Subtract 0.5 from every lower limit and add 0.5 to every upper limit:

50–52 → 49.5–52.5, 53–55 → 52.5–55.5, 56–58 → 55.5–58.5, 59–61 → 58.5–61.5, 62–64 → 61.5–64.5

[½ mark]

Note: *The class marks are unchanged by this adjustment, so the mean is unaffected — but the corrected form is required for full marks.*

Step 2 — Choose the method. *Here the frequencies* f_i *are* **very large** *(110, 135, 115), so the* **Step-deviation Method** *is chosen to keep the multiplication simple. Take* $a = 57$ *and* $h = 3$.

Step 3 — Prepare the working table:

Number of mangoes (continuous classes)	Number of boxes (f_i)	Class mark x_i	$d_i = x_i - 57$	$u_i = \frac{d_i}{3}$	$f_i u_i$
49.5 – 52.5	15	51	–6	–2	–30
52.5 – 55.5	110	54	–3	–1	–110
55.5 – 58.5	135	57 = a	0	0	0
58.5 – 61.5	115	60	3	1	115
61.5 – 64.5	25	63	6	2	50
Total	$\Sigma f_i = 400$	—	—	—	$\Sigma f_i u_i = 25$

[1½ marks for correct table]

Step 4 — Apply the step-deviation formula:

$$\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$\bar{x} = 57 + 3\left(\frac{25}{400}\right)$$

$$\bar{x} = 57 + \frac{75}{400}$$

$$\bar{x} = 57 + 0.1875$$

$$\bar{x} = 57.1875 \approx 57.19$$ **[1 mark]**

∴ Mean number of mangoes per box ≈ 57.19 (about 57 mangoes).

Method chosen: *Step-deviation Method — because the frequencies are large numbers and all the deviations d_i share the common factor 3.*

Q6. The table below shows the daily expenditure on food of 25 households in a locality. **3**

Daily expenditure (in ₹)	100–150	150–200	200–250	250–300	300–350
Number of households	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

Given: *Distribution of daily expenditure on food of 25 households; class size $h = 50$.*

To Find: *The mean daily expenditure on food.*

Solution:

Step 1 — Choose the method. *The class marks (125, 175, 225, …) are large and all deviations are multiples of 50, so use the* **Step-deviation Method** *with $a = 225$ and $h = 50$.* **[½ mark]**

Step 2 — Prepare the working table:

Daily expenditure (in ₹)	Number of households (f_i)	Class mark x_i	$d_i = x_i - 225$	$u_i = \frac{d_i}{50}$	$f_i u_i$
100 – 150	4	125	–100	–2	–8
150 – 200	5	175	–50	–1	–5
200 – 250	12	225 = a	0	0	0
250 – 300	2	275	50	1	2
300 – 350	2	325	100	2	4
Total	$\Sigma f_i = 25$	—	—	—	$\Sigma f_i u_i = -7$

[1½ marks for correct table]

Step 3 — Apply the step-deviation formula:

$$\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$\bar{x} = 225 + 50\left(\frac{-7}{25}\right)$$

$$\bar{x} = 225 + (2 \times (-7))$$

$$\bar{x} = 225 - 14$$

$$\bar{x} = 211$$ *[1 mark]*

∴ **Mean daily expenditure on food = ₹211.**

Q7. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below: **3**

Concentration of SO_2 (in ppm)	0.00–0.04	0.04–0.08	0.08–0.12	0.12–0.16	0.16–0.20	0.20–0.24
Frequency	4	9	9	2	4	2

Find the mean concentration of SO_2 in the air.

Given: *Distribution of SO_2 concentration in 30 localities; class size $h = 0.04$.*

To Find: *The mean concentration of SO_2 in the air.*

Solution:

Step 1 — Choose the method. *The class marks are small decimals which are awkward to multiply, and all deviations are multiples of 0.04. So the* **Step-deviation Method** *is used with $a = 0.14$ and $h = 0.04$.* *[½ mark]*

Step 2 — Prepare the working table:

Concentration of SO_2 (in ppm)	Frequency (f_i)	Class mark x_i	$d_i = x_i - 0.14$	$u_i = \frac{d_i}{0.04}$	$f_i u_i$
0.00 – 0.04	4	0.02	–0.12	–3	–12
0.04 – 0.08	9	0.06	–0.08	–2	–18
0.08 – 0.12	9	0.10	–0.04	–1	–9
0.12 – 0.16	2	0.14 $= a$	0	0	0
0.16 – 0.20	4	0.18	0.04	1	4
0.20 – 0.24	2	0.22	0.08	2	4
Total	$\Sigma f_i = 30$	—	—	—	$\Sigma f_i u_i = -31$

[1½ marks for correct table]

Step 3 — Apply the step-deviation formula:

$$\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$\bar{x} = 0.14 + 0.04\left(\frac{-31}{30}\right)$$

$$\bar{x} = 0.14 - \frac{1.24}{30}$$

$$\bar{x} = 0.14 - 0.0413$$

$$\bar{x} = 0.0987 \approx 0.099$$ *[1 mark]*

∴ **Mean concentration of SO_2 in the air ≈ 0.099 ppm.**

Q8. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent. **3**

Number of days	**0–6**	**6–10**	**10–14**	**14–20**	**20–28**	**28–38**	**38–40**
Number of students	11	10	7	4	4	3	1

Given: *Absentee record of 40 students. Note that the class sizes are* **unequal** *(6, 4, 4, 6, 8, 10, 2).*

To Find: *The mean number of days a student was absent.*

Solution:

Step 1 — Choose the method. *Since the class sizes are unequal, there is no single common factor h for the step-*

deviation method, and the values of x_i and f_i are small. Hence the **Direct Method** *is used.* ***[½ mark]***

Step 2 — Prepare the working table:

Number of days (Class interval)	Number of students (f_i)	Class mark x_i	$f_i x_i$
0 – 6	11	3	33
6 – 10	10	8	80
10 – 14	7	12	84
14 – 20	4	17	68
20 – 28	4	24	96
28 – 38	3	33	99
38 – 40	1	39	39
Total	$\Sigma f_i = 40$	—	$\Sigma f_i x_i = 499$

[1½ marks for correct table]

Step 3 — Apply the direct-method formula:

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\bar{x} = \frac{499}{40}$$

$$\bar{x} = 12.475 \approx 12.48$$ ***[1 mark]***

∴ Mean number of days a student was absent ≈ 12.48 days.

Q9. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate. **3**

Literacy rate (in %)	45–55	55–65	65–75	75–85	85–95
Number of cities	3	10	11	8	3

Given: *Distribution of literacy rates of 35 cities; class size $h = 10$.*

To Find: *The mean literacy rate.*

Solution:

Step 1 — Choose the method. *The class marks (50, 60,* *70, ...) are large and all deviations are multiples of 10, so use the* **Step-deviation Method** *with $a = 70$ and $h = 10$.*

[½ mark]

Step 2 — Prepare the working table:

Literacy rate (in %)	Number of cities (f_i)	Class mark x_i	$d_i = x_i - 70$	$u_i = \frac{d_i}{10}$	$f_i u_i$
45 – 55	3	50	–20	–2	–6
55 – 65	10	60	–10	–1	–10
65 – 75	11	70 = a	0	0	0
75 – 85	8	80	10	1	8
85 – 95	3	90	20	2	6
Total	$\Sigma f_i = 35$	—	—	—	$\Sigma f_i u_i = -2$

[1½ marks for correct table]

Step 3 — Apply the step-deviation formula:

$$\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$\bar{x} = 70 + 10\left(\frac{-2}{35}\right)$$

$$\bar{x} = 70 - \frac{20}{35}$$

$$\bar{x} = 70 - 0.5714$$

$$\bar{x} = 69.4286 \approx 69.43$$ *[1 mark]*

∴ **Mean literacy rate = 69.43%.**

Quick Answer Key — Exercise 13.1

Q.No.	Answer	Method Used
1	8.1 plants per house	Direct Method
2	₹545.20	Step-deviation Method
3	$f = 20$	Direct Method
4	75.9 beats per minute	Step-deviation Method
5	≈ 57.19 mangoes	Step-deviation Method
6	₹211	Step-deviation Method
7	≈ 0.099 ppm	Step-deviation Method
8	≈ 12.48 days	Direct Method (unequal classes)
9	69.43 %	Step-deviation Method

⚠ Common Mistakes to Avoid (Board Exam)

1. Forgetting to convert **inclusive (discontinuous) classes** into continuous form — see Q5. Marks are deducted even though the mean is unchanged.
2. Using the step-deviation method when class sizes are **unequal** without a common divisor — see Q8, where the direct method is safer.
3. Writing only $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$ as a final answer without showing the working table. The table itself carries method marks.
4. Not writing the units (₹, ppm, %, days, beats/minute) with the final answer.

EXERCISE 13.2

CBSE Step-wise Marking Scheme Format	Board Exam 2026–27	Name: ________________

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4. All working tables are drawn **vertically** (class intervals down the rows) — this is the layout CBSE examiners expect in the answer sheet.

📝 Formulae Used in This Exercise (Mode of Grouped Data)

Item	Formula / Meaning
Modal class	The class interval having the **maximum frequency**.
Mode	$\text{Mode} = l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$
l	Lower limit of the modal class
h	Size of the class interval (all class sizes assumed equal)
f_1	Frequency of the modal class
f_0	Frequency of the class **preceding** the modal class
f_2	Frequency of the class **succeeding** the modal class
Mean (for comparison)	$\bar{x} = a + h\left(\dfrac{\Sigma f_i u_i}{\Sigma f_i}\right)$, where $u_i = \dfrac{x_i - a}{h}$

Interpretation: The **mode** tells the value that occurs **most often**; the **mean** tells the **average** of all observations.

EXERCISE 13.2 Total: 24 Marks

Q1. The following table shows the ages of the patients admitted in a hospital during a year: 5

Age (in years)	5–15	15–25	25–35	35–45	45–55	55–65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Given: *Distribution of ages of patients admitted in a hospital; class size* $h = 10$*, total* $\Sigma f_i = 80$*.*

To Find: *(a) the mode, (b) the mean, and (c) a comparison and interpretation of the two.*

PART (a) — Calculation of MODE

Step 1 — Identify the modal class.

Maximum frequency = 23, which belongs to the class 35 – 45.

∴ **Modal class = 35 – 45** *[½ mark]*

Step 2 — Write down the values needed:

Symbol	Meaning	Value
l	Lower limit of modal class	35
h	Class size	10
f_1	Frequency of modal class (35–45)	23
f_0	Frequency of preceding class (25–35)	21
f_2	Frequency of succeeding class (45–55)	14

[½ mark]

Step 3 — Apply the mode formula:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\text{Mode} = 35 + \left(\frac{23 - 21}{2(23) - 21 - 14}\right) \times 10$$

$$\text{Mode} = 35 + \left(\frac{2}{46 - 35}\right) \times 10$$

$$\text{Mode} = 35 + \frac{2}{11} \times 10$$

$$\text{Mode} = 35 + \frac{20}{11} = 35 + 1.818$$

$$\text{Mode} = 36.818 \approx \mathbf{36.8}\ \textit{years}$$ *[1½ marks]*

PART (b) — Calculation of MEAN

Step 4 — Use the step-deviation method *with* $a = 30$ *and* $h = 10$*:*

Age (in years)	Number of patients (f_i)	Class mark x_i	$d_i = x_i - 30$	$u_i = \frac{d_i}{10}$	$f_i u_i$
5 – 15	6	10	–20	–2	–12
15 – 25	11	20	–10	–1	–11
25 – 35	21	30 = a	0	0	0
35 – 45	23	40	10	1	23
45 – 55	14	50	20	2	28
55 – 65	5	60	30	3	15
Total	$\Sigma f_i = 80$	—	—	—	$\Sigma f_i u_i = 43$

[1 mark for correct table]

Step 5 — Apply the mean formula:

$$\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$\bar{x} = 30 + 10\left(\frac{43}{80}\right)$$

$$\bar{x} = 30 + \frac{430}{80}$$

$$\bar{x} = 30 + 5.375$$

$$\bar{x} = 35.375 \approx \mathbf{35.37}\ \textit{years}$$ *[1 mark]*

PART (c) — Comparison and Interpretation

Mode = 36.8 years and Mean = 35.37 years, so **Mode > Mean** *here.*

• *The* **mode** *means that the* **maximum number of patients** *admitted to the hospital were of age about* **36.8 years**.

• *The* **mean** *means that on an* **average**, *the age of a patient admitted to the hospital was about* **35.37 years**.

[½ mark]

Q2. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components: **3**

Lifetimes (in hours)	0–20	20–40	40–60	60–80	80–100	100–120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

Given: *Distribution of observed lifetimes of 225 electrical components; class size* $h = 20$.

To Find: *The modal lifetime of the components.*

Solution:

Step 1 — Identify the modal class.

Maximum frequency = 61, which belongs to the class 60 – 80.

∴ Modal class = 60 – 80 *[½ mark]*

Step 2 — Write down the values needed:

Symbol	Meaning	Value
l	Lower limit of modal class	60
h	Class size	20
f_1	Frequency of modal class (60–80)	61
f_0	Frequency of preceding class (40–60)	52
f_2	Frequency of succeeding class (80–100)	38

[½ mark]

Step 3 — Apply the mode formula:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\text{Mode} = 60 + \left(\frac{61 - 52}{2(61) - 52 - 38}\right) \times 20$$

$$\text{Mode} = 60 + \left(\frac{9}{122 - 90}\right) \times 20$$

$$\text{Mode} = 60 + \frac{9}{32} \times 20$$

$$\text{Mode} = 60 + \frac{180}{32}$$

$$\text{Mode} = 60 + 5.625$$

$$\text{Mode} = \mathbf{65.625}\ \textit{hours}$$ *[2 marks]*

∴ Modal lifetime of the components = 65.625 hours.

Interpretation: *The maximum number of components had a lifetime of about 65.6 hours.*

Q3. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure: **5**

Expenditure (in ₹)	1000–1500	1500–2000	2000–2500	2500–3000	3000–3500	3500–4000	4000–4500	4500–5000
Number of families	24	40	33	28	30	22	16	7

Given: *Distribution of monthly household expenditure of 200 families; class size $h = 500$.*

To Find: *(a) the modal monthly expenditure, (b) the mean monthly expenditure.*

PART (a) — Calculation of MODE

Step 1 — Identify the modal class.

Maximum frequency = 40, which belongs to the class 1500 – 2000.

∴ Modal class = 1500 – 2000 *[½ mark]*

Step 2 — Write down the values needed:

Symbol	Meaning	Value
l	Lower limit of modal class	1500
h	Class size	500
f_1	Frequency of modal class (1500–2000)	40
f_0	Frequency of preceding class (1000–1500)	24
f_2	Frequency of succeeding class (2000–2500)	33

Step 3 — Apply the mode formula:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\text{Mode} = 1500 + \left(\frac{40 - 24}{2(40) - 24 - 33}\right) \times 500$$

$$\text{Mode} = 1500 + \left(\frac{16}{80 - 57}\right) \times 500$$

$$\text{Mode} = 1500 + \frac{16}{23} \times 500$$

$$\text{Mode} = 1500 + \frac{8000}{23}$$

$$\text{Mode} = 1500 + 347.83$$

$$\text{Mode} = \mathbf{1847.83}$$ *[2 marks]*

∴ Modal monthly expenditure = ₹1847.83 (approximately ₹1847.83).

PART (b) — Calculation of MEAN

Step 4 — Use the step-deviation method *with* $a = 2750$ *and* $h = 500$*:*

Expenditure (in ₹)	Number of families (f_i)	Class mark x_i	$d_i = x_i - 2750$	$u_i = \frac{d_i}{500}$	$f_i u_i$
1000 – 1500	24	1250	–1500	–3	–72
1500 – 2000	40	1750	–1000	–2	–80
2000 – 2500	33	2250	–500	–1	–33
2500 – 3000	28	2750 $= a$	0	0	0
3000 – 3500	30	3250	500	1	30
3500 – 4000	22	3750	1000	2	44
4000 – 4500	16	4250	1500	3	48
4500 – 5000	7	4750	2000	4	28
Total	$\Sigma f_i = 200$	—	—	—	$\Sigma f_i u_i = -35$

[1½ marks for correct table]

Step 5 — Apply the mean formula:

$$\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$\bar{x} = 2750 + 500\left(\frac{-35}{200}\right)$$

$$\bar{x} = 2750 + \frac{-17500}{200}$$

$$\bar{x} = 2750 - 87.5$$

$$\bar{x} = \mathbf{2662.50}$$ *[1 mark]*

∴ Mean monthly expenditure = ₹2662.50.

Interpretation: *The largest number of families spend about ₹1847.83 per month, while on an average a family spends ₹2662.50 per month.*

Q4. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures. **5**

Number of students per teacher	15–20	20–25	25–30	30–35	35–40	40–45	45–50	50–55
Number of states / U.T.	3	8	9	10	3	0	0	2

Given: *State-wise distribution of number of students per teacher for 35 states/U.T.; class size* $h = 5$.

To Find: *(a) the mode, (b) the mean, and (c) interpretation of both.*

PART (a) — Calculation of MODE

Step 1 — Identify the modal class.

Maximum frequency = 10, which belongs to the class 30 – 35.

∴ Modal class = 30 – 35 *[½ mark]*

Step 2 — Write down the values needed:

Symbol	Meaning	Value
l	Lower limit of modal class	30
h	Class size	5
f_1	Frequency of modal class (30–35)	10
f_0	Frequency of preceding class (25–30)	9
f_2	Frequency of succeeding class (35–40)	3

Step 3 — Apply the mode formula:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\text{Mode} = 30 + \left(\frac{10 - 9}{2(10) - 9 - 3}\right) \times 5$$

$$\text{Mode} = 30 + \left(\frac{1}{20 - 12}\right) \times 5$$

$$\text{Mode} = 30 + \frac{5}{8}$$

$$\text{Mode} = 30 + 0.625$$

$$\text{Mode} = \mathbf{30.6} \text{ (approximately)}$$ *[1½ marks]*

PART (b) — Calculation of MEAN

Step 4 — Use the step-deviation method *with* $a = 32.5$ *and* $h = 5$*:*

Number of students per teacher	Number of states/U.T. (f_i)	Class mark x_i	$d_i = x_i - 32.5$	$u_i = \frac{d_i}{5}$	$f_i u_i$
15 – 20	3	17.5	–15	–3	–9
20 – 25	8	22.5	–10	–2	–16
25 – 30	9	27.5	–5	–1	–9
30 – 35	10	32.5 $= a$	0	0	0
35 – 40	3	37.5	5	1	3
40 – 45	0	42.5	10	2	0
45 – 50	0	47.5	15	3	0
50 – 55	2	52.5	20	4	8
Total	$\Sigma f_i = 35$	—	—	—	$\Sigma f_i u_i = -23$

[1½ marks for correct table]

Step 5 — Apply the mean formula:

$$\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$\bar{x} = 32.5 + 5\left(\frac{-23}{35}\right)$$

$$\bar{x} = 32.5 - \frac{115}{35}$$

$$\bar{x} = 32.5 - 3.2857$$

$$\bar{x} = 29.2143 \approx \mathbf{29.2}$$ *[1 mark]*

PART (c) — Interpretation

- **Mode = 30.6:** *Most of the states/U.T. have a teacher-student ratio of about* **30.6 students per teacher**.
- **Mean = 29.2:** *On an* **average**, *a state/U.T. has about* **29.2 students per teacher**. *[½ mark]*

Q5. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches. **3**

Runs scored	3000–4000	4000–5000	5000–6000	6000–7000	7000–8000	8000–9000	9000–10000	10000–11000
Number of batsmen	4	18	9	7	6	3	1	1

Find the mode of the data.

Given: *Distribution of runs scored by 49 top batsmen; class size* $h = 1000$.

To Find: *The mode of the data.*

Solution:

Step 1 — Identify the modal class.

Maximum frequency = 18, which belongs to the class 4000 – 5000.

∴ Modal class = 4000 – 5000 *[½ mark]*

Step 2 — Write down the values needed:

Symbol	Meaning	Value
l	Lower limit of modal class	4000
h	Class size	1000
f_1	Frequency of modal class (4000–5000)	18
f_0	Frequency of preceding class (3000–4000)	4
f_2	Frequency of succeeding class (5000–6000)	9

[½ mark]

Step 3 — Apply the mode formula:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\text{Mode} = 4000 + \left(\frac{18 - 4}{2(18) - 4 - 9}\right) \times 1000$$

$$\text{Mode} = 4000 + \left(\frac{14}{36 - 13}\right) \times 1000$$

$$\text{Mode} = 4000 + \frac{14}{23} \times 1000$$

$$\text{Mode} = 4000 + \frac{14000}{23}$$

$$\text{Mode} = 4000 + 608.695$$

$$\text{Mode} = 4608.695 \approx \mathbf{4608.7} \textit{ runs}$$ *[2 marks]*

∴ **Mode of the data ≈ 4608.7 runs.**

Interpretation: *The largest number of batsmen scored about 4608.7 runs.*

Q6. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data: 3

Number of cars	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	7	14	13	12	20	11	15	8

Given: *Distribution of the number of cars passing a spot in 100 periods of 3 minutes each; class size* $h = 10$.

To Find: *The mode of the data.*

Solution:

Step 1 — Identify the modal class.

Maximum frequency = 20, which belongs to the class 40 – 50.

∴ **Modal class = 40 – 50** *[½ mark]*

Step 2 — Write down the values needed:

Symbol	Meaning	Value
l	Lower limit of modal class	40
h	Class size	10
f_1	Frequency of modal class (40–50)	20
f_0	Frequency of preceding class (30–40)	12
f_2	Frequency of succeeding class (50–60)	11

[½ mark]

Step 3 — Apply the mode formula:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\text{Mode} = 40 + \left(\frac{20 - 12}{2(20) - 12 - 11}\right) \times 10$$

$$\text{Mode} = 40 + \left(\frac{8}{40 - 23}\right) \times 10$$

$$\text{Mode} = 40 + \frac{8}{17} \times 10$$

$$\text{Mode} = 40 + \frac{80}{17}$$

$$\text{Mode} = 40 + 4.7058$$

$$\text{Mode} = 44.7058 \approx \mathbf{44.7} \textit{ cars}$$ **[2 marks]**

∴ **Mode of the data ≈ 44.7 cars.**

Interpretation: *In the largest number of 3-minute periods, about 44.7 (i.e. about 45) cars passed through that spot.*

✅ Quick Answer Key — Exercise 13.2

Q.No.	Modal Class	Mode	Mean
1	35 – 45	36.8 years	35.37 years
2	60 – 80	65.625 hours	—
3	1500 – 2000	₹1847.83	₹2662.50
4	30 – 35	30.6	29.2
5	4000 – 5000	4608.7 runs	—
6	40 – 50	44.7 cars	—

⚠ Common Mistakes to Avoid (Board Exam)

1. Taking f_0 and f_2 in the wrong order — f_0 is **above** the modal class and f_2 is **below** it in the table.
2. Writing the denominator as $2f_1 - f_0 + f_2$. The correct denominator is $2f_1 - f_0 - f_2$ (both are subtracted).
3. Using the **upper** limit of the modal class as l. Always use the **lower** limit.
4. Forgetting that mode requires **continuous** class intervals — convert inclusive classes first if needed.
5. Not writing the interpretation sentence when the question says "compare and interpret" — that sentence carries its own mark.

EXERCISE 13.3

CBSE Step-wise Marking Scheme Format	**Board Exam 2026–27**	**Name:** ________________

💡 **CBSE Marking Scheme Guidelines Followed in This Document**

1. Every question is structured as **Given → To Find → Solution**, matching CBSE's official answer-key format.
2. Each step of the solution is written on its own line and tagged with its mark allocation.
3. As per CBSE policy, method/formula marks are earned independently of the final answer — so every step is shown, not just the final result.
4. All working tables are drawn **vertically** (class intervals down the rows) — this is the layout CBSE examiners expect in the answer sheet.

📝 Formulae Used in This Exercise (Median of Grouped Data)

Item	Formula / Meaning
Cumulative frequency (cf)	The frequency of a class **plus** the frequencies of all classes preceding it.
Median class	The class whose cumulative frequency is **greater than and nearest to** $\frac{n}{2}$.
Median	$\text{Median} = l + \left(\frac{\frac{n}{2} - \text{cf}}{f}\right) \times h$
l	Lower limit of the median class
n	Total number of observations $= \Sigma f_i$
cf	Cumulative frequency of the class **preceding** the median class
f	Frequency of the median class
h	Class size
Empirical relation	**3 Median = Mode + 2 Mean**

Important: For the median formula, class intervals **must be continuous**. Convert inclusive classes first.

EXERCISE 13.3 — Total: 30 Marks

Q1. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them. **6**

Monthly consumption (in units)	65–85	85–105	105–125	125–145	145–165	165–185	185–205
Number of consumers	4	5	13	20	14	8	4

Given: *Distribution of monthly electricity consumption of 68 consumers; class size* $h = 20$, $n = 68$.

To Find: *(a) median, (b) mean, (c) mode, and (d) a comparison of the three.*

PART (a) — Calculation of MEDIAN

Step 1 — Prepare the cumulative frequency table:

Monthly consumption (in units)	Number of consumers (f_i)	Cumulative frequency (cf)
65 – 85	4	4
85 – 105	5	4 + 5 = 9
105 – 125	13	9 + 13 = 22
125 – 145	**20**	22 + 20 = **42**
145 – 165	14	42 + 14 = 56
165 – 185	8	56 + 8 = 64
185 – 205	4	64 + 4 = 68
Total	$n = 68$	—

[1 mark]

Step 2 — Locate the median class.

$$\frac{n}{2} = \frac{68}{2} = 34$$

The cumulative frequency just greater than 34 is **42**, *which belongs to the class 125 – 145.*

∴ **Median class = 125 – 145**, *with* $l = 125$, $cf = 22$,

$f = 20,\ h = 20$ **[½ mark]**

Step 3 — Apply the median formula:

$$\text{Median} = l + \left(\frac{\frac{n}{2} - \text{cf}}{f}\right) \times h$$

$$\text{Median} = 125 + \left(\frac{34 - 22}{20}\right) \times 20$$

$$\text{Median} = 125 + \frac{12}{20} \times 20$$

$$\text{Median} = 125 + 12$$

$$\text{Median} = \mathbf{137}\ \textit{units}$$ **[1 mark]**

PART (b) — Calculation of MEAN

Step 4 — Use the step-deviation method *with* $a = 135$ *and* $h = 20$*:*

Monthly consumption (in units)	Number of consumers (f_i)	Class mark x_i	$d_i = x_i - 135$	$u_i = \frac{d_i}{20}$	$f_i u_i$
65 – 85	4	75	–60	–3	–12
85 – 105	5	95	–40	–2	–10
105 – 125	13	115	–20	–1	–13
125 – 145	20	135 = a	0	0	0
145 – 165	14	155	20	1	14
165 – 185	8	175	40	2	16
185 – 205	4	195	60	3	12
Total	$\Sigma f_i = 68$	—	—	—	$\Sigma f_i u_i = 7$

[1 mark]

Step 5 — Apply the mean formula:

$$\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$\bar{x} = 135 + 20\left(\frac{7}{68}\right)$$

$$\bar{x} = 135 + \frac{140}{68}$$

$$\bar{x} = 135 + 2.0588$$

$$\bar{x} = 137.0588 \approx \mathbf{137.06}\ \textit{units}$$ **[1 mark]**

PART (c) — Calculation of MODE

Step 6 — Identify the modal class. *Maximum frequency = 20* ⇒ **Modal class = 125 – 145**

Here $l = 125,\ h = 20,\ f_1 = 20,\ f_0 = 13,\ f_2 = 14.$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\text{Mode} = 125 + \left(\frac{20 - 13}{2(20) - 13 - 14}\right) \times 20$$

$$\text{Mode} = 125 + \left(\frac{7}{40 - 27}\right) \times 20$$

$$\text{Mode} = 125 + \frac{7}{13} \times 20 = 125 + \frac{140}{13}$$

$$\text{Mode} = 125 + 10.769$$

$\text{Mode} = 135.769 \approx \mathbf{135.77}$ *units* **[1 mark]**

PART (d) — Comparison

Measure	Value (units)
Median	137
Mean	137.06
Mode	135.77

The three measures of central tendency are **approximately equal** *(about 137 units), which shows that the distribution is nearly symmetrical.*

Check with the empirical relation:

$$3\,\text{Median} = \text{Mode} + 2\,\text{Mean}$$

LHS $= 3 \times 137 = 411$; *RHS = 135.77 + 2(137.06) = 409.89 — close to 411, as expected, since the empirical relation holds only approximately (the small gap comes from rounding and from the data not being perfectly symmetrical).* **[½ mark]**

Q2. If the median of the distribution given below is 28.5, find the values of x and y. **4**

Class interval	0–10	10–20	20–30	30–40	40–50	50–60	Total
Frequency	5	x	20	15	y	5	60

Given: *A frequency distribution with two unknown frequencies* x *and* y*; total frequency* $n = 60$*; median = 28.5; class size* $h = 10$.

To Find: *The values of* x *and* y.

Solution:

Step 1 — Prepare the cumulative frequency table (carrying x and y as algebraic terms):

Class interval	Frequency (f_i)	Cumulative frequency (cf)
0 – 10	5	5
10 – 20	x	$5+x$
20 – 30	20	$25+x$
30 – 40	15	$40+x$
40 – 50	y	$40+x+y$
50 – 60	5	$45+x+y$
Total	$n=60$	—

[1 mark]

Step 2 — Form the first equation from the total frequency:

$$45+x+y=60$$

$$x+y=15 \quad ...(1)$$ *[½ mark]*

Step 3 — Locate the median class.

Median = 28.5, which lies in the class **20 – 30**.

∴ *Median class = 20 – 30, with* $l=20,\ f=20,$ *cf* $=5+x,\ h=10,\ \frac{n}{2}=\frac{60}{2}=30$ *[½ mark]*

Step 4 — Substitute in the median formula:

$$\text{Median}=l+\left(\frac{\frac{n}{2}-\text{cf}}{f}\right)\times h$$

$$28.5=20+\left(\frac{30-(5+x)}{20}\right)\times 10$$

$$28.5=20+\left(\frac{25-x}{20}\right)\times 10$$

$$28.5=20+\frac{25-x}{2}$$

$$28.5-20=\frac{25-x}{2}$$

$$8.5=\frac{25-x}{2}$$

$$17=25-x$$

$$x=25-17$$

$$x=\mathbf{8}$$ *[1½ marks]*

Step 5 — Find y from equation (1):

$$x+y=15$$

$$8+y=15$$

$$y=\mathbf{7}$$ *[½ mark]*

∴ $x=8$ and $y=7$.

Q3. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years. **4**

Age (in years)	Below 20	Below 25	Below 30	Below 35	Below 40	Below 45	Below 50	Below 55	Below 60
Number of policy holders	2	6	24	45	78	89	92	98	100

Given: *A "less than type" cumulative frequency distribution of ages of 100 policy holders. Policies are issued only from age 18 up to (but not including) 60 years.*

To Find: *The median age of the policy holders.*

Solution:

Step 1 — Convert the "less than" table into ordinary class intervals.

Since policies begin at age 18, the first class is **18 – 20**. *The frequency of each class is found by* **subtracting** *the previous cumulative frequency:*

Age (in years)	Frequency (f_i) = difference of cf	Cumulative frequency (cf)
18 – 20	2	2
20 – 25	6 − 2 = 4	6
25 – 30	24 − 6 = 18	24
30 – 35	45 − 24 = 21	45
35 – 40	**78 − 45 = 33**	**78**
40 – 45	89 − 78 = 11	89
45 – 50	92 − 89 = 3	92
50 – 55	98 − 92 = 6	98
55 – 60	100 − 98 = 2	100
Total	$n = 100$	—

[2 marks for correct table]

Step 2 — Locate the median class.

$$\frac{n}{2} = \frac{100}{2} = 50$$

The cumulative frequency just greater than 50 is **78**, *which belongs to the class 35 – 40.*

∴ **Median class = 35 – 40**, *with* $l = 35$, $cf = 45$, $f = 33$, $h = 5$ **[½ mark]**

Step 3 — Apply the median formula:

$$\text{Median} = l + \left(\frac{\frac{n}{2} - \text{cf}}{f}\right) \times h$$

$$\text{Median} = 35 + \left(\frac{50 - 45}{33}\right) \times 5$$

$$\text{Median} = 35 + \frac{5}{33} \times 5$$

$$\text{Median} = 35 + \frac{25}{33}$$

$$\text{Median} = 35 + 0.7576$$

$$\text{Median} = 35.7576 \approx \mathbf{35.76}\ \textit{years}$$ **[1½ marks]**

∴ **Median age of the policy holders ≈ 35.76 years.**

Interpretation: *About half of the policy holders are younger than 35.76 years and about half are older.*

Q4. The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table: **4**

Length (in mm)	118–126	127–135	136–144	145–153	154–162	163–171	172–180
Number of leaves	3	5	9	12	5	4	2

Find the median length of the leaves.

(**Hint:** The data needs to be converted to continuous classes for finding the median, since the formula assumes continuous classes.)

Given: *Lengths of 40 leaves in* **inclusive (discontinuous) classes** *118–126, 127–135, ...*

To Find: *The median length of the leaves.*

Solution:

Step 1 — Convert the classes into continuous form.

Gap between one upper limit and the next lower limit

$$= 127 - 126 = 1$$

$$\textit{Adjustment factor} = \frac{1}{2} = 0.5$$

Subtract 0.5 from each lower limit and add 0.5 to each *upper limit. The class size becomes $h = 9$.* ***[1 mark]***

Step 2 — Prepare the cumulative frequency table:

Length (in mm) (continuous classes)	Number of leaves (f_i)	Cumulative frequency (cf)
117.5 – 126.5	3	3
126.5 – 135.5	5	8
135.5 – 144.5	9	17
144.5 – 153.5	**12**	**29**
153.5 – 162.5	5	34
162.5 – 171.5	4	38
171.5 – 180.5	2	40
Total	$n = 40$	—

[1 mark]

Step 3 — Locate the median class.

$$\frac{n}{2} = \frac{40}{2} = 20$$

The cumulative frequency just greater than 20 is **29***,*

which belongs to the class 144.5 – 153.5.

∴ **Median class = 144.5 – 153.5**, *with* $l = 144.5$, $cf = 17$,

$f = 12$, $h = 9$ ***[½ mark]***

Step 4 — Apply the median formula:

$$\text{Median} = l + \left(\frac{\frac{n}{2} - \text{cf}}{f} \right) \times h$$

$$\text{Median} = 144.5 + \left(\frac{20 - 17}{12} \right) \times 9$$

$$\text{Median} = 144.5 + \frac{3}{12} \times 9$$

$$\text{Median} = 144.5 + \frac{27}{12}$$

$$\text{Median} = 144.5 + 2.25$$

$$\text{Median} = \mathbf{146.75}\ mm$$ ***[1½ marks]***

∴ **Median length of the leaves = 146.75 mm.**

Q5. The following table gives the distribution of the life time of 400 neon lamps: **3**

Life time (in hours)	1500–2000	2000–2500	2500–3000	3000–3500	3500–4000	4000–4500	4500–5000
Number of lamps	14	56	60	86	74	62	48

Find the median life time of a lamp.

Given: *Distribution of the life time of 400 neon lamps; class size* $h = 500$, $n = 400$.

To Find: *The median life time of a lamp.*

Solution:

Step 1 — Prepare the cumulative frequency table:

Life time (in hours)	Number of lamps (f_i)	Cumulative frequency (cf)
1500 – 2000	14	14
2000 – 2500	56	70
2500 – 3000	60	130
3000 – 3500	**86**	**216**
3500 – 4000	74	290
4000 – 4500	62	352
4500 – 5000	48	400
Total	$n = 400$	—

[1 mark]

Step 2 — Locate the median class.

$$\frac{n}{2} = \frac{400}{2} = 200$$

The cumulative frequency just greater than 200 is **216**, *which belongs to the class 3000 – 3500.*

∴ **Median class = 3000 – 3500**, *with* $l = 3000$, $cf = 130$,

$f = 86$, $h = 500$ *[½ mark]*

Step 3 — Apply the median formula:

$$\text{Median} = l + \left(\frac{\frac{n}{2} - \text{cf}}{f}\right) \times h$$

$$\text{Median} = 3000 + \left(\frac{200 - 130}{86}\right) \times 500$$

$$\text{Median} = 3000 + \frac{70}{86} \times 500$$

$$\text{Median} = 3000 + \frac{35000}{86}$$

$$\text{Median} = 3000 + 406.98$$

$\text{Median} = 3406.98 \approx \mathbf{3406.98}$ *hours* *[1½ marks]*

∴ **Median life time of a lamp ≈ 3406.98 hours.**

Q6. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows: 6

Number of letters	1–4	4–7	7–10	10–13	13–16	16–19
Number of surnames	6	30	40	16	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames. Also, find the modal size of the surnames.

Given: *Distribution of the number of letters in 100 surnames; class size* $h = 3$, $n = 100$.

To Find: *(a) the median, (b) the mean, and (c) the mode (modal size).*

PART (a) — Calculation of MEDIAN

Step 1 — Prepare the cumulative frequency table:

Number of letters	Number of surnames (f_i)	Cumulative frequency (cf)
1 – 4	6	6
4 – 7	30	36
7 – 10	**40**	**76**
10 – 13	16	92
13 – 16	4	96
16 – 19	4	100
Total	$n = 100$	—

[1 mark]

Step 2 — Locate the median class.

$$\frac{n}{2} = \frac{100}{2} = 50$$

The cumulative frequency just greater than 50 is **76**, *which belongs to the class 7 – 10.*

∴ **Median class = 7 – 10**, *with* $l = 7$, $cf = 36$, $f = 40$, $h = 3$

Step 3 — Apply the median formula:

$$\text{Median} = l + \left(\frac{\frac{n}{2} - \text{cf}}{f}\right) \times h$$

$$\text{Median} = 7 + \left(\frac{50 - 36}{40}\right) \times 3$$

$$\text{Median} = 7 + \frac{14}{40} \times 3$$

$$\text{Median} = 7 + \frac{42}{40}$$

$$\text{Median} = 7 + 1.05$$

$\text{Median} = \mathbf{8.05}$ *letters* *[1½ marks]*

PART (b) — Calculation of MEAN

Step 4 — Use the step-deviation method *with* $a = 11.5$ *and* $h = 3$*:*

Number of letters	Number of surnames (f_i)	Class mark x_i	$d_i = x_i - 11.5$	$u_i = \frac{d_i}{3}$	$f_i u_i$
1 – 4	6	2.5	–9	–3	–18
4 – 7	30	5.5	–6	–2	–60
7 – 10	40	8.5	–3	–1	–40
10 – 13	16	11.5 $= a$	0	0	0
13 – 16	4	14.5	3	1	4
16 – 19	4	17.5	6	2	8
Total	$\Sigma f_i = 100$	—	—	—	$\Sigma f_i u_i = -106$

[1 mark]

Step 5 — Apply the mean formula:

$$\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$\bar{x} = 11.5 + 3\left(\frac{-106}{100}\right)$$

$$\bar{x} = 11.5 - \frac{318}{100}$$

$$\bar{x} = 11.5 - 3.18$$

$$\bar{x} = \mathbf{8.32}\ \textit{letters}$$ *[1 mark]*

PART (c) — Calculation of MODE (modal size)

Step 6 — Identify the modal class. *Maximum frequency = 40* ⇒ **Modal class = 7 – 10**

Here $l = 7,\ h = 3,\ f_1 = 40,\ f_0 = 30,\ f_2 = 16.$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\text{Mode} = 7 + \left(\frac{40 - 30}{2(40) - 30 - 16}\right) \times 3$$

$$\text{Mode} = 7 + \left(\frac{10}{80 - 46}\right) \times 3$$

$$\text{Mode} = 7 + \frac{10}{34} \times 3 = 7 + \frac{30}{34}$$

$$\text{Mode} = 7 + 0.882$$

$$\text{Mode} = 7.882 \approx \mathbf{7.88}\ \textit{letters}$$ *[1½ marks]*

∴ Median = 8.05 letters, Mean = 8.32 letters, Mode (modal size) ≈ 7.88 letters.

Q7. The distribution below gives the weights of 30 students of a class. Find the median weight of the students. **3**

Weight (in kg)	40–45	45–50	50–55	55–60	60–65	65–70	70–75
Number of students	2	3	8	6	6	3	2

Given: *Distribution of the weights of 30 students; class size $h = 5$, $n = 30$.*

To Find: *The median weight of the students.*

Solution:

Step 1 — Prepare the cumulative frequency table:

Weight (in kg)	Number of students (f_i)	Cumulative frequency (cf)
40 – 45	2	2
45 – 50	3	5
50 – 55	8	13
55 – 60	**6**	**19**
60 – 65	6	25
65 – 70	3	28
70 – 75	2	30
Total	$n = 30$	—

[1 mark]

Step 2 — Locate the median class.

$$\frac{n}{2} = \frac{30}{2} = 15$$

The cumulative frequency just greater than 15 is **19**, *which belongs to the class 55 – 60.*

∴ **Median class = 55 – 60**, *with* $l = 55$, $cf = 13$, $f = 6$, $h = 5$ *[½ mark]*

Step 3 — Apply the median formula:

$$\text{Median} = l + \left(\frac{\frac{n}{2} - \text{cf}}{f}\right) \times h$$

$$\text{Median} = 55 + \left(\frac{15 - 13}{6}\right) \times 5$$

$$\text{Median} = 55 + \frac{2}{6} \times 5$$

$$\text{Median} = 55 + \frac{10}{6}$$

$$\text{Median} = 55 + 1.67$$

$$\text{Median} = \mathbf{56.67}\ kg$$ *[1½ marks]*

∴ Median weight of the students ≈ 56.67 kg.

✅ **Quick Answer Key — Exercise 13.3**

Q.No.	Median Class	Median	Mean	Mode
1	125 – 145	137 units	137.06 units	135.77 units
2	20 – 30	$x = 8$, $y = 7$		
3	35 – 40	35.76 years	—	—
4	144.5 – 153.5	146.75 mm	—	—
5	3000 – 3500	3406.98 hours	—	—
6	7 – 10	8.05 letters	8.32 letters	7.88 letters
7	55 – 60	56.67 kg	—	—

📈 Which Measure Should Be Used? (Frequently Asked in Board Exams)

Measure	Best used when…
Mean	All observations must be taken into account, and there are no extreme values. It allows two distributions to be compared.
Median	Individual observations are not important and extreme values are present — e.g. typical productivity rate of workers, average wage in a country.
Mode	The most frequent or most popular item is needed — e.g. most-watched TV programme, most demanded consumer item, most common vehicle colour.

⚠ Common Mistakes to Avoid (Board Exam)

1. Using $\frac{n+1}{2}$ instead of $\frac{n}{2}$. For **grouped** data the median class is located using $\frac{n}{2}$ only.
2. Taking cf of the **median class** instead of the class **preceding** it.
3. Not converting inclusive classes to continuous form — see Q4, where 118–126 must become 117.5–126.5.
4. In "less than type" tables (Q3), forgetting to subtract successive cumulative frequencies to get the actual frequencies.
5. Forgetting the first class 18–20 in Q3 — the question states policies begin at age 18.
6. Leaving the answer as an unsimplified fraction. Always convert to a decimal and write the unit.

CHAPTER 14

Probability

Exercise 14.1

EXERCISE 14.1

📘 NCERT Solutions — Exercise 14.1 (25 Questions)

🔑 Approach for This Exercise

One formula runs the whole chapter:

> $P(E) = \dfrac{\text{Number of outcomes favourable to } E}{\text{Number of all possible outcomes}}$, valid only when all outcomes are **equally likely**.
> Also: $0 \le P(E) \le 1$; $P(\bar{E}) = 1 - P(E)$; impossible event $\to 0$; sure event $\to 1$.

The work is almost always in **counting correctly**, not in dividing. Three habits protect marks: write the **total number of outcomes** first; then count the favourable ones *explicitly* (list them for dice and cards); and **always reduce the fraction** to lowest terms. For "not E" questions the complement rule is faster and safer than recounting. Standard totals worth memorising: a deck has **52** cards (26 red, 26 black, 4 suits of 13, **12 face cards** — kings, queens, jacks); two dice give **36** ordered outcomes; three coins give **8**.

Q1. Complete the following statements: (i) Probability of an event E + Probability of the event 'not E' = ___ (ii) The probability of an event that cannot happen is ___; such an event is called ___ (iii) The probability of an event that is certain to happen is ___; 5 marks such an event is called ___ (iv) The sum of the probabilities of all the elementary events of an experiment is ___ (v) The probability of an event is greater than or equal to ___ and less than or equal to ___

SOLUTION:

(i) 1 — because E and 'not E' are complementary: together they cover every possible outcome. **1 mark**

(ii) 0; impossible event. No outcome is favourable, so the numerator is 0. **1 mark**

(iii) 1; sure (or certain) event. Every outcome is favourable, so numerator = denominator. **1 mark**

(iv) 1 — the elementary events together make up the whole sample space. **1 mark**

(v) 0 and 1, i.e. $0 \leq P(E) \leq 1$. **1 mark**

> 📌 These five facts are the whole theory of the chapter. A probability can never be negative and never exceed 1 — that single check catches most arithmetic slips.

Q2. Which of the following experiments have equally likely outcomes? Explain. (i) A driver attempts to start a car: it starts or does not start. (ii) A player attempts to shoot a basketball: she/he shoots or misses. (iii) A trial is made to answer a true-false question: the answer is right or wrong. (iv) A baby is born: it is a boy or a girl. **4 marks**

SOLUTION:

(i) Not equally likely. Whether the car starts depends on its condition — fuel, battery, servicing. A well-maintained car is far more likely to start than not. **1 mark**

(ii) Not equally likely. The result depends on the player's skill and practice. A trained player is more likely to score than to miss. **1 mark**

(iii) Equally likely. If the answer is guessed at random, there are exactly two options and no reason to favour either, so each has probability $\frac{1}{2}$. **1 mark**

(iv) Equally likely. A newborn is equally likely to be a boy or a girl. **1 mark**

📌 **The test to apply:** is there any reason for one outcome to occur more often than the other? If skill, condition or bias enters the picture (i and ii), the outcomes are not equally likely and the classical formula cannot be used.

Q3. Why is tossing a coin considered a fair way of deciding which team should get the ball at the beginning of a football game? **1 mark**

SOLUTION:

Because a coin toss has only two possible outcomes — head and tail — and for an **unbiased** coin tossed at random these are **equally likely**, each with probability $\frac{1}{2}$. **1 mark**

Since neither team has any advantage over the other, and the result cannot be influenced by either side, the method is fair.

📌 The words "unbiased" and "at random" are what make it fair — a weighted coin, or one placed rather than tossed, would not be.

Q4. Which of the following cannot be the probability of an event? (A) 2/3 (B) −1.5 (C) 15% (D) 0.7 **1 mark**

SOLUTION:

Since $0 \leq P(E) \leq 1$, check each option:

(A) $\frac{2}{3} \approx 0.67$ — lies between 0 and 1 ✓

(B) -1.5 — **negative**, so impossible ✗

(C) $15\% = 0.15$ — lies between 0 and 1 ✓

(D) 0.7 — lies between 0 and 1 ✓

Answer: option (B) −1.5. **1 mark**

📌 Convert percentages to decimals before comparing. A value like 115% would also be invalid, for exceeding 1.

Q5. If P(E) = 0.05, what is the probability of 'not E'? **1 mark**

SOLUTION:

By the complement rule, $P(\bar{E}) = 1 - P(E)$:

$P(\bar{E}) = 1 - 0.05 = \mathbf{0.95}$ **1 mark**

Q6. A bag contains lemon flavoured candies only. Malini takes out one candy without looking. What is the probability that she takes out (i) an orange flavoured candy (ii) a lemon flavoured candy? **2 marks**

SOLUTION:

(i) Orange flavoured candy. The bag contains no orange candies, so no outcome is favourable. This is an **impossible event**:

$$P(\text{orange}) = \mathbf{0}$$ **1 mark**

(ii) Lemon flavoured candy. Every candy in the bag is lemon flavoured, so every outcome is favourable. This is a **sure event**:

$$P(\text{lemon}) = \mathbf{1}$$ **1 mark**

> 📌 This question is the practical illustration of Q1 parts (ii) and (iii). Note also that the two answers add to 1, as complementary events must.

Q7. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday? **1 mark**

SOLUTION:

The two events — "same birthday" and "not the same birthday" — are complementary.

$$P(\text{same birthday}) = 1 - P(\text{not same birthday})$$

$$= 1 - 0.992 = \mathbf{0.008}$$ **1 mark**

Q8. A bag contains 3 red balls and 5 black balls. A ball is drawn at random. What is the probability that the ball drawn is (i) red (ii) not red? **2 marks**

SOLUTION:

Total number of balls $= 3 + 5 = 8$, so there are 8 equally likely outcomes.

(i) Red ball. Favourable outcomes $= 3$

$$P(\text{red}) = \frac{3}{8}$$ **1 mark**

(ii) Not red. Using the complement rule:

$$P(\text{not red}) = 1 - \frac{3}{8} = \frac{5}{8}$$ **1 mark**

(Check by direct counting: the 5 black balls are the favourable ones, giving $\frac{5}{8}$ ✓)

Q9. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out at random. What is the probability that the marble taken out will be (i) red (ii) white (iii) not green? **3 marks**

SOLUTION:

Total marbles $= 5 + 8 + 4 = 17$.

(i) $P(\text{red}) = \frac{5}{17}$ **1 mark**

(ii) $P(\text{white}) = \frac{8}{17}$ **1 mark**

(iii) $P(\text{green}) = \frac{4}{17}$, so

$$P(\text{not green}) = 1 - \frac{4}{17} = \frac{13}{17}$$ **1 mark**

📌 Note that $\frac{5}{17} + \frac{8}{17} + \frac{4}{17} = 1$ — the three elementary colour events exhaust the sample space, exactly as Q1(iv) states.

Q10. A piggy bank contains hundred 50p coins, fifty ₹1 coins, twenty ₹2 coins and ten ₹5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50p coin (ii) will not be a ₹5 coin? **2 marks**

SOLUTION:

Total number of coins $= 100 + 50 + 20 + 10 = 180.$

(i) A 50p coin. Favourable $= 100$

$$P = \frac{100}{180} = \frac{5}{9}$$ 1 mark

(ii) Not a ₹5 coin. P(₹5 coin) = 10/180 = 1/18, so

P(not a ₹5 coin) = 1 − 1/18 = 17/18 1 mark

📌 **Count coins, not rupees.** The values 50p, ₹1, ₹2, ₹5 are distractions — each individual coin is one equally likely outcome, whatever it is worth.

Q11. Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish. What is the probability that the fish taken out is a male fish? 1 mark

SOLUTION:

Total fish $= 5 + 8 = 13$; favourable (male) $= 5.$

$$P(\text{male fish}) = \frac{\mathbf{5}}{\mathbf{13}}$$ 1 mark

Q12. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, and these are equally likely outcomes. What is the probability that it will point at (i) 8 (ii) an odd number (iii) a number greater than 2 (iv) a number less than 9? 4 marks

SOLUTION:

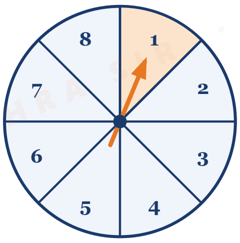

Fig. 14–Q12 — Eight equal sectors; the arrow is shown resting in sector 1. Each number has probability 1/8.

Note: *MLI-redrawn figure, drawn from the given data. Work only from the given data.*

Total outcomes $= 8$ (the numbers 1 to 8).

(i) Pointing at 8. Favourable $= 1$

$$P = \frac{1}{8}$$ **1 mark**

(ii) An odd number. Odd numbers are 1, 3, 5, 7 → favourable $= 4$

$$P = \frac{4}{8} = \frac{1}{2}$$ **1 mark**

(iii) A number greater than 2. These are 3, 4, 5, 6, 7, 8 → favourable $= 6$

$$P = \frac{6}{8} = \frac{3}{4}$$ **1 mark**

(iv) A number less than 9. All of 1 to 8 are less than 9 → favourable $= 8$

$P = \frac{8}{8} = 1$ (a **sure event**) **1 mark**

📌 In (iii), "greater than 2" does **not** include 2 itself. Had it said "2 or more", the count would be 7.

Q13. A die is thrown once. Find the probability of getting (i) a prime **3 marks**

number (ii) a number lying between 2 and 6 (iii) an odd number.

SOLUTION:

Total outcomes $= 6$: the numbers 1, 2, 3, 4, 5, 6.

(i) A prime number. Primes on a die are 2, 3, 5 → favourable $= 3$

$$P = \frac{3}{6} = \frac{1}{2}$$ **1 mark**

(ii) A number lying between 2 and 6. These are 3, 4, 5 (excluding both endpoints) → favourable $= 3$

$$P = \frac{3}{6} = \frac{1}{2}$$ **1 mark**

(iii) An odd number. These are 1, 3, 5 → favourable $= 3$

$$P = \frac{3}{6} = \frac{1}{2}$$ **1 mark**

📌 **1 is not a prime number** — it has only one factor. And in (ii), "between 2 and 6" excludes 2 and 6 themselves; listing the numbers explicitly avoids both traps.

Q14. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) a king of red colour (ii) a face card (iii) a red face card (iv) the jack of hearts (v) a spade (vi) the queen of diamonds. **6 marks**

SOLUTION:

Total outcomes $= 52$. Recall the structure: 4 suits of 13 each — spades ♠ and clubs ♣ (black), hearts ♥ and diamonds ♦ (red). Face cards are kings, queens and jacks, so there are $3 \times 4 = 12$ face cards in all.

(i) A king of red colour. Red kings: king of hearts, king of diamonds → 2

$$P = \frac{2}{52} = \frac{1}{26}$$ **1 mark**

(ii) A face card. Favourable $= 12$

$$P = \frac{12}{52} = \frac{3}{13}$$ **1 mark**

(iii) A red face card. Hearts and diamonds give 3 face cards each → $2 \times 3 = 6$

$$P = \frac{6}{52} = \frac{3}{26}$$ **1 mark**

(iv) The jack of hearts. There is exactly one such card → 1

$$P = \frac{1}{52}$$ **1 mark**

(v) A spade. Favourable $= 13$

$$P = \frac{13}{52} = \frac{1}{4}$$ **1 mark**

(vi) The queen of diamonds. Exactly one such card → 1

$$P = \frac{1}{52}$$ **1 mark**

📌 **Aces are not face cards** in the NCERT convention — only kings, queens and jacks. Counting aces in would give 16 instead of 12 and spoil parts (ii) and (iii).

Q15. Five cards — the ten, jack, queen, king and ace of diamonds — are well-shuffled with their faces downwards. One card is picked up at random. (i) What is the probability that the card is the queen? (ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace (b) a queen? **3 marks**

SOLUTION:

(i) Total cards $= 5$; favourable (the queen) $= 1$

$$P(\text{queen}) = \frac{1}{5}$$ **1 mark**

(ii) The queen has been removed and **not replaced**, so only 4 cards remain: ten, jack, king, ace.

(a) An ace. Favourable $= 1$ out of 4

$$P(\text{ace}) = \frac{1}{4}$$ **1 mark**

(b) A queen. There is no queen left among the remaining cards, so this is an **impossible event**

$$P(\text{queen}) = \frac{0}{4} = 0$$ **1 mark**

📌 **"Put aside" means the total changes.** The denominator drops from 5 to 4 — the single most important idea in without-replacement questions.

Q16. 12 defective pens are accidentally mixed with 132 good ones. One pen is taken out at random. Determine the probability that the pen taken out is a good one. **1 mark**

SOLUTION:

Total pens $= 132 + 12 = 144$; favourable (good) $= 132$.

$$P(\text{good pen}) = \frac{132}{144} = \frac{11}{12}$$ **1 mark**

(dividing numerator and denominator by 12)

Q17. (i) A lot of 20 bulbs contains 4 defective ones. One bulb is drawn at random. What is the probability that this bulb is defective? (ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective? **2 marks**

SOLUTION:

(i) Total bulbs $= 20$; defective $= 4$

$$P(\text{defective}) = \frac{4}{20} = \frac{1}{5}$$ **1 mark**

(ii) One **non-defective** bulb has been drawn and not replaced. So now:

total bulbs $= 20 - 1 = 19$, and non-defective bulbs $= 16 - 1 = 15$.

$$P(\text{not defective}) = \frac{15}{19}$$ **1 mark**

📌 **Both numbers change.** Since the bulb removed was a good one, the good count drops from 16 to 15 *and* the total from 20 to 19. Reducing only the total gives $\frac{16}{19}$, the usual wrong answer.

Q18. A box contains 90 discs numbered from 1 to 90. If one disc is drawn at random, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5. **3 marks**

SOLUTION:

Total outcomes $= 90$.

(i) A two-digit number. These run from 10 to 90 → count $= 90 - 10 + 1 = 81$

$$P = \frac{81}{90} = \frac{9}{10}$$ **1 mark**

(ii) A perfect square. Squares up to 90:

$1, 4, 9, 16, 25, 36, 49, 64, 81$ → count $= 9$

$$P = \frac{9}{90} = \frac{1}{10}$$ **1 mark**

(iii) A number divisible by 5. These are $5, 10, 15, \ldots, 90$ → count $= \frac{90}{5} = 18$

$$P = \frac{18}{90} = \frac{1}{5}$$ **1 mark**

📌 In (i), the single-digit numbers 1 to 9 are the nine excluded ones, so $90 - 9 = 81$ — a quicker route than the range formula. In (ii), stop at $9^2 = 81$ since $10^2 = 100 > 90$.

Q19. A child has a die whose six faces show the letters A, B, C, D, E, A. The die is thrown once. What is the probability of getting (i) A (ii) D? 2 marks

SOLUTION:

Total faces $= 6$, showing A, B, C, D, E, A.

(i) Getting A. The letter A appears on **two** faces → favourable $= 2$

$$P(A) = \frac{2}{6} = \frac{1}{3}$$ 1 mark

(ii) Getting D. The letter D appears on one face → favourable $= 1$

$$P(D) = \frac{1}{6}$$ 1 mark

📌 **Count faces, not distinct letters.** There are five different letters but six faces, and A occupies two of them — that repetition is the whole point of the question.

Q20. Suppose you drop a die at random on a rectangular region of dimensions 3 m × 2 m. What is the probability that it will land inside the circle of diameter 1 m drawn in it? 2 marks

SOLUTION:

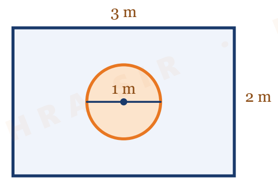

Fig. 14–Q20 — Drawn to scale: a circle of diameter 1 m inside a 3 m × 2 m rectangle.

Note: *MLI-redrawn figure, drawn from the given data. Work only from the given data.*

Here outcomes are measured by **area** rather than by counting.

Step 1 — Total area. $3 \times 2 = 6\ \text{m}^2$

Step 2 — Favourable area. The circle has diameter 1 m, so radius $r = \frac{1}{2}$ m:

$$\text{Area} = \pi r^2 = \pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}\ \text{m}^2$$ **1 mark**

Step 3 — Probability.

$$P = \frac{\text{area of circle}}{\text{area of rectangle}} = \frac{\pi/4}{6} = \frac{\pi}{24}$$

$$\mathbf{P = \frac{\pi}{24} \approx 0.131}$$ **1 mark**

📌 **Halve the diameter first.** Using $r = 1$ gives $\frac{\pi}{6}$, which is wrong by a factor of four.

Q21. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy it if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that (i) she will buy it (ii) she will not buy it?

2 marks

SOLUTION:

Total pens $= 144$; defective $= 20$; good $= 144 - 20 = 124$.

(i) She will buy it — this happens exactly when the pen is good:

$$P(\text{buy}) = \frac{124}{144} = \frac{31}{36}$$ 1 mark

(dividing both by 4)

(ii) She will not buy it — the pen is defective:

$$P(\text{not buy}) = 1 - \frac{31}{36} = \frac{5}{36}$$ 1 mark

(Check directly: $\frac{20}{144} = \frac{5}{36}$ ✓*)*

Q22. Refer to Example 13. (i) Complete the table of probabilities for the sum on 2 dice. (ii) A student argues that there are 11 possible outcomes 2, 3, ..., 12, therefore each has probability 1/11. Do you agree? Justify.

4 marks

SOLUTION:

Two dice give $6 \times 6 = 36$ equally likely ordered outcomes. Counting the ways to make each sum:

Sum	2	3	4	5	6	7	8	9	10
Ways	1	2	3	4	5	6	5	4	3
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$

2 marks

For example, the sum 7 arises from $(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)$ — six ways — while the sum 2 arises only from $(1,1)$.

(ii) No, the student is not correct. 1 mark

Justification. The eleven sums are indeed the possible outcomes, but they are **not equally likely**. As the table shows, the sum 7 occurs in 6 of the 36 ways while the sum 2 occurs in only 1. The formula $P(E) = \frac{\text{favourable}}{\text{total}}$ applies only to equally likely outcomes, so assigning $\frac{1}{11}$ to each is wrong. 1 mark

Check: the eleven probabilities add to $\frac{1+2+3+4+5+6+5+4+3+2+1}{36} = \frac{36}{36} = 1 \checkmark$

📌 **This is the most important question in the chapter.** It shows why "equally likely" is part of the definition and not a decoration. Note also that $(2,5)$ and $(5,2)$ are different outcomes — that is why the total is 36, not 21.

Q23. A game consists of tossing a one-rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result — three heads or three tails — and loses otherwise. Calculate the probability that Hanif will lose the game. 3 marks

SOLUTION:

Step 1 — List the sample space. Three tosses give $2^3 = 8$ equally likely outcomes:
HHH, HHT, HTH, HTT, THH, THT, TTH, TTT 1 mark

Step 2 — Count the winning outcomes. Hanif wins only with all three alike: HHH and TTT → 2 outcomes.

$$P(\text{win}) = \frac{2}{8} = \frac{1}{4}$$ **1 mark**

Step 3 — Use the complement.

$$P(\text{lose}) = 1 - \frac{1}{4} = \frac{3}{4}$$ **1 mark**

(Check: the six losing outcomes are HHT, HTH, HTT, THH, THT, TTH, giving $\frac{6}{8} = \frac{3}{4}$ ✓)

> 📌 **Write out all eight outcomes.** The listing itself carries a mark, and it makes both the winning and losing counts obvious.

Q24. A die is thrown twice. What is the probability that (i) 5 will not come up either time (ii) 5 will come up at least once? **3 marks**

SOLUTION:

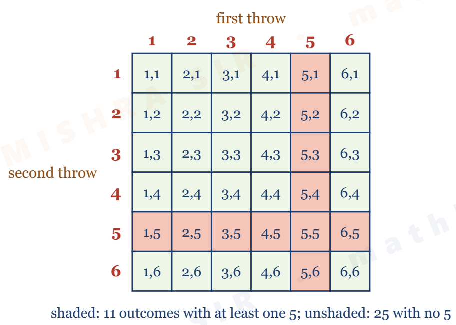

Fig. 14–Q24 — The 36 equally likely outcomes of two throws; the 11 shaded ones contain at least one 5.

> ***Note:** MLI-redrawn figure, drawn from the given data. Work only from the given data.*

Throwing a die twice gives $6 \times 6 = 36$ equally likely ordered outcomes.

(i) 5 does not come up either time. For each throw there are 5 acceptable faces (1, 2, 3, 4, 6), so

$$\text{favourable outcomes} = 5 \times 5 = 25$$ **1 mark**

$$P(\text{no } 5) = \frac{25}{36}$$ **1 mark**

(ii) 5 comes up at least once. This is the complement of (i):

$$P(\text{at least one } 5) = 1 - \frac{25}{36} = \frac{11}{36}$$ **1 mark**

Check by direct counting: 5 on the first throw gives 6 outcomes, 5 on the second gives 6, and $(5, 5)$ is counted twice — so $6 + 6 - 1 = 11$ ✓

> 📌 **"At least once" almost always means use the complement.** Counting directly needs the overlap adjustment shown above; the complement route avoids that entirely.

Q25. Which of the following arguments are correct and which are not? Give reasons. (i) If two coins are tossed simultaneously there are three possible outcomes — two heads, two tails or one of each — therefore each has probability 1/3. (ii) If a die is thrown there are two possible outcomes — an odd number or an even number — therefore the probability of getting an odd number is 1/2. **4 marks**

SOLUTION:

(i) Incorrect. **1 mark**

Tossing two coins gives **four** equally likely outcomes, not three: HH, HT, TH, TT.

• Two heads: HH → 1 way → $P = \frac{1}{4}$

• Two tails: TT → 1 way → $P = \frac{1}{4}$

• One of each: HT and TH → **2** ways → $P = \frac{2}{4} = \frac{1}{2}$

So the three described events are **not equally likely**, and assigning $\frac{1}{3}$ to each is wrong. **1 mark**

(ii) Correct. 1 mark

A die has six equally likely faces. The odd numbers are 1, 3, 5 (three of them) and the even numbers are 2, 4, 6 (also three). Since both events contain the same number of outcomes, they *are* equally likely, and

$$P(\text{odd}) = \frac{3}{6} = \frac{1}{2} \checkmark$$ 1 mark

> 📌 **Compare the two parts carefully.** Both arguments have the same shape — "there are n outcomes, so each has probability $\frac{1}{n}$" — yet one is wrong and one is right. The difference is whether the outcomes are genuinely equally likely, which you can only check by going back to the full sample space.

— End of Exercise 14.1 — Chapter 14 Complete —

End of Compiled Volume

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