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Class 10 — Mathematics
CBSE Most Frequently Asked Questions & PYQ Bank

Board-favourite question types from past CBSE papers, each fully solved with the marking scheme
Rationalised syllabus, Subject Code 041 / 241 · All 14 chapters

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✓ How to use this document

This is the **practice half** of the MLI Class 10 set. The companion volume carries the concept explanation and the complete NCERT solutions; this one carries the question types CBSE keeps coming back to. Four points:

- **Year tags are type-frequency markers, not reproductions.** A tag such as **2020 · 2023 · 2024** means this *kind* of question appeared in those years' papers. The numbers here are MLI's own, so you cannot pass by memorising answers — you have to work them.
- **Nothing from the deleted syllabus appears.** No Euclid's Division Lemma, no decimal-expansion classification, no polynomial division algorithm, no cross-multiplication or reducible equations, no area-of-triangle-from-coordinates, no ogives, no Constructions. Old question banks still carry these — skip them.
- **Every figure is an MLI-redrawn schematic** — correct in construction, **not to scale**, not a copy of any textbook figure. Never measure off them.
- **If anything disagrees with your NCERT book or the official CBSE syllabus, those win.** Tell your faculty so this can be corrected. Every numerical has been worked through, but a slip may still remain.

⚠ The three rules that decide your marks in a board paper

1. Method marks are awarded independently of the final answer. A correct formula written down earns its mark even if the arithmetic later goes wrong. So **never** jump straight to the answer — write the formula, then the substitution, then the result, each on its own line.
2. A geometry answer without a figure loses a mark. Draw it, label it, mark the given data on it.
3. Write the units and the concluding sentence. "Hence the height of the tower is 20√3 m" earns the last mark that "20√3" alone does not.

📜 Contents — chapter-wise, with unit weightage

Where the 80 marks sit: Number Systems 6 · Algebra 20 · Coordinate Geometry 6 · Geometry 15 · Trigonometry 12 · Mensuration 10 · Statistics & Probability 11. Algebra alone is a quarter of the paper — Chapters 2, 3, 4 and 5 deserve the most practice time. Internal assessment adds 20 marks.

CHAPTER 1

Real Numbers

Unit I · 6 marks · Almost always one 1-mark and one 3-mark question

📈 What CBSE actually asks from this chapter

Question type	Frequency	Usual marks
HCF & LCM by prime factorisation, then verify HCF × LCM = product	★★★★★	2–3
Prove √2, √3 or √5 is irrational	★★★★★	3
Prove a + b√c is irrational (given √c is irrational)	★★★★	2–3
Word problem on HCF (maximum size / rows) or LCM (bells, laps)	★★★★	3
HCF/LCM given as powers of primes ($a = x^3y^2$ form)	★★★	1

Out of syllabus — do not study: Euclid's Division Lemma / Division Algorithm, and classifying decimal expansions as terminating or non-terminating. Both were removed in the rationalisation. Older PYQ books still print them.

A · HCF and LCM by prime factorisation

Approach. Factorise every number fully into primes using a factor tree — divide repeatedly by the smallest possible prime (2, then 3, then 5, then 7...). **HCF** = product of the **smallest** power of each prime **common** to all the numbers. **LCM** = product of the **highest** power of **every** prime that appears anywhere. Verify with HCF × LCM = product — but only for **two** numbers.

A1 — 2019 · 2023 · 2024 — 3 Marks

Find the HCF and LCM of 96 and 404 by the prime factorisation method, and verify that HCF × LCM equals the product of the two numbers.

Step 1 $96 = 2^5 \times 3 \quad 404 = 2^2 \times 101$ — 1

Step 2 HCF = lowest common power = 2^2 = **4** — ½

Step 3 LCM = $2^5 \times 3 \times 101$ = **9696** — 1

Step 4 Verify: 4 × 9696 = 38784 and 96 × 404 = 38784 ✓ — ½

HCF = 4, LCM = 9696, and the verification holds

Two traps. HCF takes the **lowest** power, LCM the **highest** — students routinely swap them. And the relation HCF × LCM = product works for **two** numbers only; never apply it to three.

A2

2020 · 2024 | 3 Marks

Find the HCF and LCM of 6, 72 and 120 using the prime factorisation method.

Step 1 $6 = 2 \times 3 \quad 72 = 2^3 \times 3^2 \quad 120 = 2^3 \times 3 \times 5$ 1½

Step 2 Primes common to all three: 2 and 3, at lowest powers 2^1 and 3^1. HCF $= 2 \times 3 =$ **6** ¾

Step 3 LCM $= 2^3 \times 3^2 \times 5 = 8 \times 9 \times 5 =$ **360** ¾

HCF = 6 and LCM = 360

Do not verify with HCF × LCM here. $6 \times 360 = 2160$, while $6 \times 72 \times 120 = 51840$. The relation simply does not hold for three numbers — and writing it earns a deduction.

A3

2018 · 2023 | 3 Marks

Find the LCM and HCF of 1296 and 5040 by prime factorisation.

Step 1 $1296 = 2^4 \times 3^4$ 1

Step 2 $5040 = 2^4 \times 3^2 \times 5 \times 7$ 1

Step 3 HCF $= 2^4 \times 3^2 = 16 \times 9 =$ **144** ½

Step 4 LCM $= 2^4 \times 3^4 \times 5 \times 7 = 16 \times 81 \times 35 =$ **45360** ½

HCF = 144 and LCM = 45360

Check yourself. $144 \times 45360 = 6531840$ and $1296 \times 5040 = 6531840$ ✓ With large numbers this check is the only way to catch a factorisation slip.

B · The HCF–LCM relation and word problems

Approach. For **two** numbers, HCF × LCM = product — use it whenever three of the four quantities are known, instead of re-factorising. For word problems: **largest size that divides everything** (maximum capacity, largest tile, greatest number of rows) → **HCF**. **When do they meet again** (bells, laps, traffic lights, least number of...) → **LCM**.

B1

2021 · 2024 | 2 Marks

Can two numbers have 15 as their HCF and 175 as their LCM? Give reasons.

Step 1 The HCF of two numbers must always be a **factor** of their LCM — every common factor divides every common multiple. 1

Step 2 Here 175 ÷ 15 = 11.67..., which is not a whole number, so 15 does not divide 175. 1

No — two numbers cannot have HCF 15 and LCM 175, because the HCF must divide the LCM.

The reason carries the marks, not the yes/no. Simply writing "No" earns nothing. State the property, then show the division fails.

B2

2019 · 2023 · 2025 | 2 Marks

The HCF of two numbers is 23 and their LCM is 1449. If one of the numbers is 161, find the other.

Step 1 For two numbers, HCF × LCM = product of the numbers. ½

Step 2 $23 \times 1449 = 161 \times x \Rightarrow 33327 = 161x$ 1

Step 3 $x = 33327 \div 161 = \mathbf{207}$ ½

The other number is 207

Sanity check. 207 = 9 × 23 and 161 = 7 × 23, so 23 really is their HCF ✓ A ten-second check like this catches an arithmetic slip.

B3

2018 · 2022 · 2024 | 3 Marks

Three bells toll at intervals of 12, 15 and 18 minutes respectively. If they start tolling together, after how much time will they next toll together?

Step 1 "Together again" means a common multiple of all three intervals; the earliest such time is the **LCM**. 1

Step 2 $12 = 2^2 \times 3 \quad 15 = 3 \times 5 \quad 18 = 2 \times 3^2$ 1

Step 3 LCM = $2^2 \times 3^2 \times 5 = 4 \times 9 \times 5 =$ **180 minutes** = 3 hours 1

They will next toll together after 180 minutes, i.e. 3 hours

The word "common" is the trap. It tempts students towards HCF. Ask instead: is the answer bigger or smaller than the given numbers? Bigger → LCM.

B4 2020 · 2023 | 3 Marks

Two tankers contain 850 litres and 680 litres of fuel respectively. Find the maximum capacity of a container which can measure the fuel of either tanker an exact number of times.

Step 1 Maximum capacity that measures both exactly" ⇒ the container size must divide both amounts, and be as large as possible ⇒ **HCF**. 1

Step 2 850 = $2 \times 5^2 \times 17$ 680 = $2^3 \times 5 \times 17$ 1

Step 3 HCF = 2 × 5 × 17 = **170 litres** 1

The maximum capacity of the container is 170 litres

Check: 850 ÷ 170 = 5 and 680 ÷ 170 = 4 — both whole numbers ✓

B5 2022 · 2025 | 3 Marks

Two numbers are in the ratio 4 : 5 and their HCF is 8. Find the two numbers and their LCM.

Step 1 If the ratio is 4 : 5 and the HCF is 8, the numbers are 4 × 8 and 5 × 8 — because 4 and 5 are co-prime, the HCF is exactly the common multiplier. 1½

Step 2 So the numbers are **32 and 40**. ½

Step 3 LCM = (product) ÷ HCF = (32 × 40) ÷ 8 = 1280 ÷ 8 = **160** 1

The numbers are 32 and 40, and their LCM is 160

Why the ratio parts must be co-prime. Had the ratio been 4 : 6, the HCF would not be 8 — 4 and 6 share a factor 2, so the common multiplier would not be the full HCF. Always reduce the ratio first.

C • Composite numbers and the "can it end with 0" type

Approach. To show a number is **composite**, take out a common factor by the distributive law and display it as a product of two factors, each greater than 1. For "can a^n end with 0": a number ends in 0 exactly when it is divisible by 10, i.e. its prime factorisation must contain **both** 2 and 5. Factorise the base and look for the missing prime.

C1

2019 · 2024 | 2 Marks

Check whether 15^n can end with the digit 0 for any natural number n.

Step 1 $15^n = (3 \times 5)^n = 3^n \times 5^n$ ½

Step 2 A number ends in 0 only if it is divisible by $10 = 2 \times 5$, so its prime factorisation must contain **both** 2 and 5. 1

Step 3 The factorisation of 15^n contains only the primes 3 and 5 — the prime **2 never appears**. By the Fundamental Theorem of Arithmetic that factorisation is unique, so no 2 can appear. ½

No — 15^n can never end with the digit 0, for any natural number n

Name the Fundamental Theorem of Arithmetic. It is what guarantees the factorisation is unique, and therefore that the missing prime can never appear. That sentence is the reasoning mark.

C2

2020 · 2023 | 2 Marks

Explain why $3 \times 5 \times 7 + 7$ is a composite number.

Step 1 $3 \times 5 \times 7 + 7 = 7(3 \times 5 + 1)$ *(taking 7 common)* 1

Step 2 $= 7 \times 16 = 112$ ½

Step 3 It is expressible as a product of two factors, 7 and 16, **both greater than 1**, so it has factors other than 1 and itself. ½

Hence $3 \times 5 \times 7 + 7$ is a composite number

The concluding phrase matters. “Both factors greater than 1” is what makes it composite rather than prime — write it out.

D · Irrationality proofs — the most repeated question in the chapter

Approach. Two distinct templates. **(1) Base surd** (√2, √3, √5): assume rational = a/b with a, b **co-prime**, square, show the prime divides both a and b, contradict co-primeness. **(2) Combination** (5 − √3, 2 − 3√5...): assume rational, **isolate the surd**, show it equals a rational expression — contradicting the *known* irrationality of the base surd. Never re-derive the base surd inside a combination proof.

D1 every year · 3 Marks

Prove that √3 is irrational.

Step 1 Assume, to the contrary, that √3 is rational. Then √3 = a/b, where a and b are integers, b ≠ 0, and a, b are **co-prime**. ½

Step 2 Squaring: $3 = a^2/b^2$, so $\mathbf{a^2 = 3b^2}$ ...(i) ½

Step 3 So 3 divides a^2. Since 3 is **prime**, and a prime dividing a^2 must divide a, we get 3 divides a. Write a = 3c. 1

Step 4 Substituting in (i): $9c^2 = 3b^2 \Rightarrow b^2 = 3c^2$. So 3 divides b^2, and hence 3 divides b. ½

Step 5 Now 3 divides both a and b, contradicting that they are co-prime. The assumption must be wrong. ½

Hence √3 is irrational. ■

The mark most often lost is the justification in Step 3 — "3 is prime, and a prime dividing a^2 divides a". Writing only "so 3 divides a" is an unjustified jump. The identical five steps prove √2 and √5; only the number changes.

D2 2019 · 2022 · 2024 · 3 Marks

Prove that 5 − √3 is irrational, given that √3 is irrational.

Step 1 Assume, to the contrary, that 5 − √3 is rational. Then 5 − √3 = a/b with a, b integers, b ≠ 0. 1

Step 2 Isolate the surd: √3 = 5 − a/b = (5b − a)/b 1

Step 3 The right-hand side is a quotient of two integers with non-zero denominator, hence **rational**. So √3 would be rational. ½

Step 4 This contradicts the given fact that √3 is irrational. ½

Hence 5 − √3 is irrational. ■

Do not re-prove √3 irrational. The question hands it to you. Re-deriving it wastes ten minutes and earns nothing.

D3

2020 · 2023 · 2025 | 3 Marks

Prove that $2 - 3\sqrt{5}$ is irrational.

Step 1 Assume $2 - 3\sqrt{5}$ is rational, so $2 - 3\sqrt{5} = a/b$ with a, b integers, $b \neq 0$. 1

Step 2 $3\sqrt{5} = 2 - a/b = (2b - a)/b$ ½

Step 3 $\sqrt{5} = (2b - a)/(3b)$ 1

Step 4 The right side is rational, so $\sqrt{5}$ would be rational — contradicting the known irrationality of $\sqrt{5}$. ½

Hence $2 - 3\sqrt{5}$ is irrational. ■

Divide by the coefficient too. Stopping at $3\sqrt{5}$ = rational is incomplete — you must isolate $\sqrt{5}$ itself before invoking its irrationality.

D4

2021 · 2024 | 3 Marks

Prove that $3\sqrt{2}$ is irrational.

Step 1 Assume $3\sqrt{2}$ is rational, so $3\sqrt{2} = a/b$ with a, b co-prime integers, $b \neq 0$. 1

Step 2 Then $\sqrt{2} = a/(3b)$. 1

Step 3 Since a and 3b are integers with $3b \neq 0$, the right side is rational, so $\sqrt{2}$ would be rational. ½

Step 4 This contradicts the known fact that $\sqrt{2}$ is irrational. ½

Hence $3\sqrt{2}$ is irrational. ■

E • Objective and assertion–reason items

E1

2022 · 2024 | 1 Mark

If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x and y are prime numbers, then HCF(a, b) is

(i) xy (ii) xy^2 (iii) x^3y^3 (iv) x^2y^2

Step 1 HCF takes each common prime to its **lowest** power: x to the power min(3, 1) = 1, and y to the power min(2, 3) = 2.

(ii) xy^2

The companion result: LCM takes the **highest** powers, giving x^3y^3. Note HCF × LCM = x^4y^5 = a × b ✓

E2 2023 · 2025 | 1 Mark

If HCF(a, b) = 12 and $a \times b = 1800$, then LCM(a, b) is
(i) 3600 (ii) 900 (iii) 150 (iv) 90

Step 1 LCM = $(a \times b) \div \text{HCF} = 1800 \div 12$

(iii) 150

E3 2022 · 2024 | 1 Mark

Assertion (A): For any two prime numbers p and q, their HCF is 1 and their LCM is $p + q$.
Reason (R): For any two positive integers, HCF × LCM = product of the two numbers.
(i) Both true, R explains A (ii) Both true, R does not explain A (iii) A false, R true (iv) A true, R false

Step 1 Two distinct primes share no factor, so HCF = 1 — that part of A is right. But their LCM is **$p \times q$**, not $p + q$. So **A is false**.

Step 2 R is a true statement for any two positive integers.

(iii) A is false, R is true

Test A against R. If A were true, HCF × LCM = $1 \times (p + q) = p + q$, which would have to equal pq — false for, say, $p = 2$, $q = 3$ ($5 \neq 6$). One counter-example settles it.

E4 2021 · 2023 | 1 Mark

The product of a non-zero rational number and an irrational number is
(i) always rational (ii) always irrational (iii) rational or irrational (iv) always an integer

Step 1 If a non-zero rational r multiplied an irrational x gave a rational s, then $x = s/r$ would be rational — a contradiction.

(ii) always irrational

The words "non-zero" matter. If the rational number were 0, the product would be 0, which is rational. That single word is the whole question.

CHAPTER 2

Polynomials

Unit II · part of Algebra's 20 marks · Usually one 1-mark and one 2/3-mark question

📈 What CBSE actually asks from this chapter

Question type	Frequency	Usual marks
Find the zeroes of a quadratic polynomial and verify the relation with coefficients	★★★★★	2–3
Form a quadratic polynomial from the given sum and product of zeroes	★★★★★	1–2
Number of zeroes from a given graph	★★★★	1
If α, β are zeroes, find $1/\alpha + 1/\beta$, $\alpha^2 + \beta^2$, $\alpha^3 + \beta^3$ etc.	★★★★	2–3
Find k when one zero, or a relation between zeroes, is given	★★★	2

Out of syllabus — do not study: the division algorithm for polynomials (dividend = divisor × quotient + remainder) and cubic-polynomial zero problems built on it.

The two relations everything rests on. For $ax^2 + bx + c$ with zeroes α and β:
$\alpha + \beta = -b/a$ $\alpha\beta = c/a$
And to build a polynomial from its zeroes: $p(x) = k[x^2 - (\text{sum})x + (\text{product})]$

A · Find the zeroes and verify the relations

Approach. Put the polynomial in standard form $ax^2 + bx + c$ **first**, then split the middle term: find two numbers whose **product is a×c** and whose **sum is b**. Factorise, read off the zeroes, then verify **both** $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$. The verification is worth roughly half the marks — never stop at the zeroes.

A1

2019 · 2023 · 2025 | 3 Marks

Find the zeroes of the quadratic polynomial $x^2 - 3x - 10$ and verify the relationship between the zeroes and the coefficients.

Step 1 $a = 1, b = -3, c = -10$. Product $a \times c = -10$, required sum $= -3 \rightarrow$ the numbers are -5 and $+2$. **1**

Step 2 $x^2 - 5x + 2x - 10 = x(x - 5) + 2(x - 5) = (x - 5)(x + 2)$ **1**

Step 3 Zeroes: $x = 5$ and $x = -2$. **½**

Step 4 Verify: $\alpha + \beta = 5 - 2 = 3$ and $-b/a = 3$ ✓; $\alpha\beta = 5 \times (-2) = -10$ and $c/a = -10$ ✓ **½**

Zeroes are 5 and −2; both relations verified

A2

2020 · 2024 | 3 Marks

Find the zeroes of $2x^2 + 7x + 5$ and verify the relationship between the zeroes and the coefficients.

Step 1 $a = 2, b = 7, c = 5$. Product $= 10$, sum $= 7 \rightarrow$ the numbers are 5 and 2. **1**

Step 2 $2x^2 + 5x + 2x + 5 = x(2x + 5) + 1(2x + 5) = (2x + 5)(x + 1)$ **1**

Step 3 Zeroes: $x = -5/2$ and $x = -1$. **½**

Step 4 Verify: sum $= -5/2 - 1 = -7/2 = -b/a$ ✓; product $= (-5/2)(-1) = 5/2 = c/a$ ✓ **½**

Zeroes are −5/2 and −1; both relations verified

Keep the fraction. Writing the zero of $(2x + 5)$ as -5 instead of $-5/2$ is the standard slip, and it breaks both verifications.

B · Build the polynomial from sum and product

Approach. $p(x) = k[x^2 - (\text{sum})x + (\text{product})]$. Take $k = 1$ for the simplest answer, or $k =$ the common denominator to clear fractions. Always mention k — the answer is **not** unique, and saying so protects a mark.

B1

2021 · 2024 | 2 Marks

Find a quadratic polynomial whose zeroes have sum −3 and product 2. Hence find its zeroes.

Step 1 $p(x) = k[x^2 - (-3)x + 2] = k[x^2 + 3x + 2]$. Take $k = 1$. **1**

Step 2 Factorising: $x^2 + 2x + x + 2 = (x + 2)(x + 1)$, so the zeroes are -2 and -1. **½**

Step 3 Check: sum $= -3$ ✓ and product $= 2$ ✓ **½**

$p(x) = x^2 + 3x + 2$, with zeroes −2 and −1

B2 — 2022 · 2025 — 2 Marks

Find a quadratic polynomial whose zeroes are $2 + \sqrt{3}$ and $2 - \sqrt{3}$.

Step 1 Sum = $(2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$ — the surds cancel. ½

Step 2 Product = $(2 + \sqrt{3})(2 - \sqrt{3}) = 2^2 - (\sqrt{3})^2 = 4 - 3 = 1$ 1

Step 3 $p(x) = x^2 - 4x + 1$ ½

$p(x) = x^2 - 4x + 1$

Conjugate surd zeroes always give rational coefficients. If one zero of a polynomial with rational coefficients is $2 + \sqrt{3}$, the other *must* be $2 - \sqrt{3}$ — a fact CBSE tests directly.

C · Symmetric expressions in the zeroes

Approach. Never find α and β separately. Write down $S = -b/a$ and $P = c/a$, then express what is asked in terms of S and P: $1/\alpha + 1/\beta = S/P$ · $\alpha^2 + \beta^2 = S^2 - 2P$ · $\alpha^3 + \beta^3 = S^3 - 3PS$ · $(\alpha - \beta)^2 = S^2 - 4P$.

C1 — 2018 · 2023 · 2024 — 3 Marks

If α and β are the zeroes of $x^2 - 6x + 8$, find (a) $1/\alpha + 1/\beta$, (b) $\alpha^2 + \beta^2$, (c) $\alpha^3 + \beta^3$.

Step 1 $S = \alpha + \beta = 6$ and $P = \alpha\beta = 8$. ½

Step 2 **(a)** $1/\alpha + 1/\beta = (\alpha + \beta)/(\alpha\beta) = 6/8 =$ **3/4** ¾

Step 3 **(b)** $\alpha^2 + \beta^2 = S^2 - 2P = 36 - 16 =$ **20** ¾

Step 4 **(c)** $\alpha^3 + \beta^3 = S^3 - 3PS = 216 - 3(8)(6) = 216 - 144 =$ **72** 1

(a) 3/4 (b) 20 (c) 72

Verify quickly. Here the zeroes happen to be 4 and 2: $1/4 + 1/2 = 3/4$ ✓, $16 + 4 = 20$ ✓, $64 + 8 = 72$ ✓. But with irrational zeroes only the S–P route is practical, so learn it.

D · Find k from a given condition

Approach. Two routes. If a **zero is given**, substitute it: p(that number) = 0. If a **relation between the zeroes** is given (sum = product, one zero is twice the other, zeroes are reciprocal), write S and P in terms of k and turn the relation into an equation in k.

D1 2019 · 2024 | 2 Marks

If 3 is one zero of the polynomial $x^2 - 5x + k$, find k and the other zero.

Step 1 $x = 3$ is a zero ⇒ $p(3) = 0$: $9 - 15 + k = 0$ **1**

Step 2 $k = 6$. **½**

Step 3 Product of zeroes $= c/a = 6$, so $3 \times \beta = 6 \Rightarrow \beta = 2$. (Check with the sum: $3 + 2 = 5 = -b/a$ ✓) **½**

k = 6 and the other zero is 2

D2 2020 · 2023 · 2025 | 3 Marks

If the sum of the zeroes of the polynomial $kx^2 + 2x + 3k$ is equal to the product of its zeroes, find the value of k.

Step 1 Here $a = k$, $b = 2$, $c = 3k$. Sum $S = -b/a = -2/k$ and product $P = c/a = 3k/k = 3$. **1½**

Step 2 Given $S = P$: $-2/k = 3$ **½**

Step 3 $-2 = 3k \Rightarrow k = \mathbf{-2/3}$ **½**

Step 4 Note $k \neq 0$ is required for the polynomial to be quadratic, and $-2/3$ satisfies that. **½**

k = −2/3

The product simplifies before you use it. $c/a = 3k/k = 3$, a constant — students who leave it as $3k/k$ get a wrong equation. And always state that $k \neq 0$.

D3 2021 · 2024 | 3 Marks

If the sum of the zeroes of $x^2 - (k + 6)x + 2(2k - 1)$ is half their product, find k.

Step 1 $S = -b/a = (k + 6)$ and $P = c/a = 2(2k - 1)$. **1**

Step 2 Given $S = ½P$: $k + 6 = ½ \times 2(2k - 1) = 2k - 1$ **1½**

Step 3 $6 + 1 = 2k - k \Rightarrow k = \mathbf{7}$ **½**

k = 7

Watch the double sign. $b = -(k + 6)$, so $-b/a = +(k + 6)$. Dropping one of the two minus signs is the commonest error in this question.

E · Objective and graph-reading items

E1 2022 · 2024 · 2025 **1 Mark**

If the graph of $y = p(x)$ is a parabola that touches the x-axis at exactly one point and lies above it everywhere else, the number of zeroes of $p(x)$ is

(i) 0 (ii) 1 (iii) 2 (iv) 3

Step 1 Count **points of contact** with the x-axis, not turning points. Touching counts as one zero (a repeated zero).

(ii) 1

Three cases to keep straight. Crossing at two points → 2 zeroes. Touching at one point → 1 zero (repeated). Never meeting the axis → 0 zeroes.

E2 2023 · 2025 **1 Mark**

If α and β are the zeroes of $x^2 + 5x + 6$, then the value of $1/\alpha + 1/\beta$ is

(i) 5/6 (ii) −5/6 (iii) 6/5 (iv) −6/5

Step 1 $S = -5$, $P = 6$, and $1/\alpha + 1/\beta = S/P = -5/6$.

(ii) −5/6

E3 2021 · 2023 **1 Mark**

Assertion (A): $x^2 + 4$ has no real zeroes.

Reason (R): A quadratic polynomial has real zeroes only when its discriminant $b^2 - 4ac$ is non-negative.

(i) Both true, R explains A (ii) Both true, R does not explain A (iii) A true, R false (iv) A false, R true

Step 1 For $x^2 + 4$: $a = 1$, $b = 0$, $c = 4$, so $D = 0 - 16 = -16 < 0$. Hence no real zeroes — A is true.

Step 2 R states the correct general criterion, and it is exactly what settles A.

(i) Both true, and R is the correct explanation of A

CHAPTER 3

Pair of Linear Equations in Two Variables

Unit II · Algebra · Reliably one 3-mark or 5-mark word problem

📈 What CBSE actually asks from this chapter

Question type	Frequency	Usual marks
Word problem solved by substitution or elimination (ages, digits, fractions, boat/stream, fixed + variable charges)	★★★★★	3–5
Consistency test from a_1/a_2, b_1/b_2, c_1/c_2	★★★★★	1–2
Find k for which the pair has a unique / no / infinitely many solutions	★★★★	2
Solve graphically and find the vertices of the triangle formed with an axis	★★★★	5

Out of syllabus — do not study: the **cross-multiplication method**, and equations **reducible** to linear form (the 1/x, 1/y substitutions). Only the graphical, substitution and elimination methods remain.

The consistency test. For $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$:

Condition	Lines	Solutions	Name
$a_1/a_2 \neq b_1/b_2$	Intersecting	Exactly one	Consistent
$a_1/a_2 = b_1/b_2 = c_1/c_2$	Coincident	Infinitely many	Consistent (dependent)
$a_1/a_2 = b_1/b_2 \neq c_1/c_2$	Parallel	None	Inconsistent

A · Solve a pair by substitution or elimination

Approach. Choose **substitution** when some variable already has coefficient 1 or −1; otherwise use **elimination** — multiply to match one variable's coefficients, then add or subtract. Always finish by substituting your answer back into **both** original equations. **Not in syllabus:** the cross-multiplication formula and 1/x, 1/y reducible substitutions — never use them.

A1

2019 · 2023 · 2025 | 3 Marks

Solve for x and y: $2x + 3y = 11$ and $2x - 4y = -24$. Hence find the value of m for which $y = mx + 3$.

Step 1 he coefficients of x are already equal, so subtract straight away: $(2x + 3y) - (2x - 4y) = 11 - (-24)$ 1

Step 2 $y = 35 \Rightarrow y = 5$ ½

Step 3 ubstitute in $2x + 3y = 11$: $2x + 15 = 11 \Rightarrow x = -2$ 1

Step 4 low $y = mx + 3$: $5 = m(-2) + 3 \Rightarrow -2m = 2 \Rightarrow m = -1$ ½

$x = -2$, $y = 5$ and $m = -1$

The last part is a separate mark. Many students solve the pair and stop. Read the word “hence” — it means use the x and y you just found.

A2

2020 · 2024 | 3 Marks

Solve by elimination: $x + 2y = 6$ and $2x + y = 9$.

Step 1 Aultiply the first equation by 2: $2x + 4y = 12$ 1

Step 2 ubtract the second equation: $(2x + 4y) - (2x + y) = 12 - 9 \Rightarrow 3y = 3 \Rightarrow y = 1$ 1

Step 3 ubstitute in $x + 2y = 6$: $x + 2 = 6 \Rightarrow x = 4$ ½

Step 4 heck: $4 + 2(1) = 6$ ✓ and $2(4) + 1 = 9$ ✓ ½

$x = 4$ and $y = 1$

A3

2018 · 2022 | 3 Marks

Solve: $0.2x + 0.3y = 1.3$ and $0.4x + 0.5y = 2.3$.

Step 1 lear the decimals first — multiply both equations by 10: $2x + 3y = 13$ and $4x + 5y = 23$ 1

Step 2 Aultiply the first by 2: $4x + 6y = 26$ ½

Step 3 ubtract the second: $(4x + 6y) - (4x + 5y) = 26 - 23 \Rightarrow y = 3$ 1

Step 4 ubstitute: $2x + 9 = 13 \Rightarrow x = 2$ ½

$x = 2$ and $y = 3$

Clear decimals and fractions before you start. Working with 0.2 and 0.3 through three steps almost guarantees an arithmetic slip.

B · Find k for a given number of solutions

Approach. Write both equations in the form $ax + by + c = 0$ **first**. Then: unique solution → $a_1/a_2 \neq b_1/b_2$ · no solution → $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ · infinitely many → all three ratios equal. For “infinitely many” you must check the **third** ratio as well — a k from the first two may fail it.

B1 2019 · 2023 · 2024 **3 Marks**

Find the value of k for which the pair $3x + y = 1$ and $(2k - 1)x + (k - 1)y = 2k + 1$ has no solution.

Step 1 Standard form: $3x + y - 1 = 0$ and $(2k - 1)x + (k - 1)y - (2k + 1) = 0$ ½

Step 2 No solution ⇒ $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, i.e. $3/(2k - 1) = 1/(k - 1)$ 1

Step 3 $3(k - 1) = 2k - 1 \Rightarrow 3k - 3 = 2k - 1 \Rightarrow k = 2$ 1

Step 4 Check the third ratio with $k = 2$: $a_1/a_2 = 3/3 = 1$ while $c_1/c_2 = -1/-5 = 1/5$. Since $1 \neq 1/5$, the inequality holds and $k = 2$ is valid. ½

$k = 2$

The check is not optional. If the third ratio had also come out equal, the pair would have infinitely many solutions instead of none — and $k = 2$ would have to be rejected.

B2 2020 · 2024 · 2025 **3 Marks**

Find k for which the pair $2x + 3y = 7$ and $(k - 1)x + (k + 2)y = 3k$ has infinitely many solutions.

Step 1 Infinitely many ⇒ $2/(k - 1) = 3/(k + 2) = 7/(3k)$ ½

Step 2 From the first two: $2(k + 2) = 3(k - 1) \Rightarrow 2k + 4 = 3k - 3 \Rightarrow k = 7$ 1½

Step 3 Verify with the third ratio: $2/6 = 1/3$ and $7/21 = 1/3$ ✓ All three ratios agree. 1

$k = 7$

B3 2021 · 2023 **2 Marks**

For what value of k will the pair $kx + 3y = k - 3$ and $12x + ky = k$ have **no solution**?

Step 1 No solution ⇒ $k/12 = 3/k \neq (k - 3)/k$ ½

Step 2 From $k/12 = 3/k$: $k^2 = 36 \Rightarrow k = 6$ or $k = -6$ **1**

Step 3 Test $k = 6$: $a_1/a_2 = 1/2$ and $c_1/c_2 = 3/6 = 1/2$ — all three equal, so this gives **infinitely many** solutions, not none. Reject $k = 6$.

Test $k = -6$: $a_1/a_2 = -1/2$ and $c_1/c_2 = -9/-6 = 3/2$. Unequal ✓ ½

$k = -6$

This is exactly why the third ratio matters. Both $k = \pm 6$ satisfy the first equality, but only one of them actually gives no solution. Stopping at $k^2 = 36$ loses half the marks.

C • Word problems — the guaranteed 3 to 5 marker

Approach. Always begin by writing what x and y stand for — that line carries its own mark. Useful forms: a two-digit number with tens digit x and units digit y is **$10x + y$**, and reversed it is **$10y + x$**. For "fixed charge + rate per unit", let the fixed part be x and the rate be y. For boats, upstream speed is $(x - y)$ and downstream is $(x + y)$.

C1 2018 · 2022 · 2024 **3 Marks**

Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. Find their present ages.

Step 1 Let Nuri's present age be x years and Sonu's be y years. ½

Step 2 Five years ago: $x - 5 = 3(y - 5) \Rightarrow x - 3y = -10$ ...(i) ½

Step 3 Ten years later: $x + 10 = 2(y + 10) \Rightarrow x - 2y = 10$ ...(ii) ½

Step 4 Subtract (i) from (ii): $y = 20$ **1**

Step 5 From (ii): $x = 10 + 40 = 50$. Check: $45 = 3 \times 15$ ✓ and $60 = 2 \times 30$ ✓ ½

Nuri is 50 years old and Sonu is 20 years old

C2

2019 · 2023 · 2025 | 3 Marks

The sum of the digits of a two-digit number is 7. The number obtained by reversing the digits exceeds the original number by 27. Find the number.

Step 1 Let the tens digit be x and the units digit be y, so the number is $10x + y$ and the reversed number is $10y + x$. ½

Step 2 Sum of digits: $x + y = 7$ ...(i) ½

Step 3 Reversed exceeds original by 27: $(10y + x) - (10x + y) = 27 \Rightarrow 9y - 9x = 27 \Rightarrow y - x = 3$ ...(ii) 1

Step 4 Adding (i) and (ii): $2y = 10 \Rightarrow y = 5$, so $x = 2$. ½

Step 5 The number is $10(2) + 5 = 25$. Check: $52 - 25 = 27$ ✓ ½

The number is 25

Answer the number, not the digits. Writing "x = 2, y = 5" and stopping loses the final mark — the question asked for the number.

C3

2020 · 2023 · 2025 | 5 Marks

A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours it can go 40 km upstream and 55 km downstream. Find the speed of the boat in still water and the speed of the stream.

Step 1 Let the boat's speed in still water be x km/h and the stream's be y km/h. Upstream speed = $(x - y)$, downstream = $(x + y)$. 1

Step 2 $30/(x - y) + 44/(x + y) = 10$ ...(i) $40/(x - y) + 55/(x + y) = 13$ ...(ii) 1

Step 3 Multiply (i) by 4 and (ii) by 3, so the first terms match: $120/(x-y) + 176/(x+y) = 40$ and $120/(x-y) + 165/(x+y) = 39$ 1

Step 4 Subtract: $11/(x + y) = 1 \Rightarrow x + y = 11$ 1

Step 5 Put back in (i): $30/(x - y) + 4 = 10 \Rightarrow x - y = 5$. Solving with $x + y = 11$ gives $x = 8$, $y = 3$. 1

Speed of the boat in still water = 8 km/h; speed of the stream = 3 km/h

Note the legal route. Reducible substitutions ($u = 1/(x-y)$) are **out of syllabus**. Scaling the two equations so one fraction cancels, as above, is ordinary elimination and fully allowed.

C4

2021 · 2024 | 3 Marks

A lending library charges a fixed amount for the first three days and an additional charge for each day thereafter. Saritha paid ₹27 for a book kept for seven days, while Susy paid ₹21 for a book kept for five days. Find the fixed charge and the charge per extra day.

Step 1 Let the fixed charge (for the first 3 days) be ₹x and the extra charge per day be ₹y. ½

Step 2 Seven days = 3 fixed + 4 extra: $x + 4y = 27$ ...(i) ½

Step 3 Five days = 3 fixed + 2 extra: $x + 2y = 21$ ...(ii) ½

Step 4 Subtract (ii) from (i): $2y = 6 \Rightarrow y = 3$ 1

Step 5 From (ii): $x + 6 = 21 \Rightarrow x = 15$. Check: $15 + 4(3) = 27$ ✓ ½

Fixed charge = ₹15; extra charge = ₹3 per day

Count the extra days correctly. Seven days means **four** extra days beyond the first three, not seven. Getting this wrong is the whole question.

D · Graphical method and the triangle

Approach. Build a small table of at least two or three points per line, plot with a ruler, and mark the intersection with its coordinates. For the triangle formed with an axis: the two x-intercepts (or y-intercepts) plus the point of intersection are the three vertices. Base = distance between the two intercepts; height = the other coordinate of the apex.

D1

2019 · 2022 · 2024 | 5 Marks

Solve graphically: $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Find the vertices of the triangle formed by these lines and the x-axis, and find its area.

Step 1 For $x - y + 1 = 0$, i.e. $y = x + 1$: the points (0, 1), (2, 3) and (−1, 0). 1

Step 2 For $3x + 2y = 12$: the points (4, 0), (2, 3) and (0, 6). 1

Step 3 Both tables contain (2, 3), so the lines meet there — that is the solution. 1

Step 4 The lines cut the x-axis at (−1, 0) and (4, 0), so the vertices are **(−1, 0), (4, 0) and (2, 3)**. 1

Step 5 Base = 4 − (−1) = 5 units; height = 3 units (the y-coordinate of the apex). Area = ½ × 5 × 3 = 7.5 sq. units 1

$x = 2$, $y = 3$; vertices (−1, 0), (4, 0), (2, 3); area = 7.5 square units

Read the axis carefully. With the **y-axis** instead, the vertices become (0, 1), (0, 6) and (2, 3), base = 5 along the y-axis and height = 2, the x-coordinate of the apex. Using the wrong axis costs three marks.

E · Objective and assertion–reason items

E1

2022 · 2024 | 1 Mark

The pair of equations $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$ represents lines which

(i) intersect at a point (ii) are parallel (iii) are coincident (iv) do not exist

Step 1 $_1/a_2 = 5/7$ and $b_1/b_2 = -4/6 = -2/3$. Since $5/7 \neq -2/3$, the ratios differ.

(i) intersect at a point (a unique solution)

E2

2023 · 2025 | 1 Mark

The pair $x + 2y = 5$ and $3x + 6y = 15$ has

(i) a unique solution (ii) no solution (iii) infinitely many solutions (iv) exactly two solutions

Step 1 $/3 = 2/6 = 5/15 = 1/3$ — all three ratios are equal, so the lines are coincident.

(iii) infinitely many solutions

A linear pair can never have exactly two solutions. Two distinct straight lines meet at most once. The only possibilities are one, none, or infinitely many — so option (iv) is always wrong, whatever the equations.

E3

2021 · 2024 | 1 Mark

Assertion (A): The pair $2x + 3y = 5$ and $4x + 6y = 11$ has no solution.

Reason (R): A pair of linear equations has no solution when $a_1/a_2 = b_1/b_2 \neq c_1/c_2$.

(i) Both true, R explains A (ii) Both true, R does not explain A (iii) A true, R false (iv) A false, R true

Step 1 Ratios: $2/4 = 1/2$, $3/6 = 1/2$, but $5/11 \neq 1/2$. So the condition in R is met and A is true.

Step 2 R states the correct criterion and is precisely what settles A.

(i) Both true, and R is the correct explanation of A

CHAPTER 4

Quadratic Equations

Unit II · Algebra · Usually one 5-mark word problem plus a 1/2-mark discriminant item

📈 What CBSE actually asks from this chapter

Question type	Frequency	Usual marks
Speed–time–distance word problem (train, plane, boat) reducing to a quadratic	★★★★★	5
Nature of roots from the discriminant $D = b^2 - 4ac$	★★★★★	1–2
Find k for equal roots	★★★★★	2–3
Work/time problem (two taps, two pipes, two workers)	★★★★	5
Number problems (two numbers, consecutive integers, reciprocals)	★★★★	3

Discriminant $D = b^2 - 4ac$. $D > 0$ → two distinct real roots · $D = 0$ → two equal real roots · $D < 0$ → no real roots.

Quadratic formula (Shreedharacharya's): $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

A · Is the equation quadratic?

Approach. Expand **both** sides fully, bring everything to one side, and judge only the **final simplified form**. It is quadratic when the highest surviving power of x is exactly 2 with a non-zero coefficient. A cubic-looking equation can simplify to a quadratic (the x^3 terms cancel), and a quadratic-looking one can collapse to a linear equation.

A1 — 2020 · 2023 — 2 Marks

Check whether $(x - 3)(2x + 1) = x(x + 5)$ is a quadratic equation.

Step 1 LHS $= 2x^2 + x - 6x - 3 = 2x^2 - 5x - 3$ ½

Step 2 RHS $= x^2 + 5x$ ½

Step 3 Bring everything to one side: $2x^2 - 5x - 3 - x^2 - 5x = 0 \Rightarrow x^2 - 10x - 3 = 0$ ½

Step 4 The highest power is 2 with coefficient $1 \neq 0$. ½

Yes — it is a quadratic equation, $x^2 - 10x - 3 = 0$

A2

2019 · 2024 | 2 Marks

Check whether $(x - 2)(x + 1) = (x - 1)(x + 3)$ is a quadratic equation.

Step 1 LHS $= x^2 - x - 2$ RHS $= x^2 + 2x - 3$ **1**

Step 2 Subtracting: $x^2 - x - 2 - x^2 - 2x + 3 = 0 \Rightarrow -3x + 1 = 0$ **½**

Step 3 The x^2 terms have cancelled, leaving degree 1. **½**

> **No — it is not quadratic; it reduces to the linear equation $3x = 1$**

> **Never judge by appearance.** Both sides contain x^2, yet the equation is linear. Always simplify to the end before deciding.

B · Solve by factorisation

> **Approach.** Write in the form $ax^2 + bx + c = 0$, then split the middle term: two numbers with **product a×c** and **sum b**. Group in pairs, factorise, set each factor to zero. Keep fractions as fractions — the root of $(2x - 3)$ is $3/2$, not 3.

B1

2019 · 2023 · 2025 | 3 Marks

Solve by factorisation: $6x^2 - x - 2 = 0$.

Step 1 Product $a \times c = 6 \times (-2) = -12$; required sum $= -1 \to$ the numbers are -4 and $+3$. **1**

Step 2 $6x^2 - 4x + 3x - 2 = 2x(3x - 2) + 1(3x - 2) = (3x - 2)(2x + 1)$ **1½**

Step 3 $3x - 2 = 0 \Rightarrow x = 2/3$; $2x + 1 = 0 \Rightarrow x = -1/2$ **½**

> **$x = 2/3$ and $x = -1/2$**

B2

2018 · 2022 · 2024 | 3 Marks

Solve by factorisation: $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$.

Step 1 Product $= \sqrt{2} \times 5\sqrt{2} = 10$; sum $= 7 \to$ the numbers are 5 and 2. **1**

Step 2 $\sqrt{2}\,x^2 + 5x + 2x + 5\sqrt{2} = x(\sqrt{2}\,x + 5) + \sqrt{2}(\sqrt{2}\,x + 5) = (\sqrt{2}\,x + 5)(x + \sqrt{2})$ **1½**

Step 3 Roots: $x = -5/\sqrt{2} = -5\sqrt{2}/2$, and $x = -\sqrt{2}$ **½**

> **$x = -5\sqrt{2}/2$ and $x = -\sqrt{2}$**

> **Rationalise the surd root.** Leaving the answer as $-5/\sqrt{2}$ is not accepted form — multiply numerator and denominator by $\sqrt{2}$.

C · Word problems — the guaranteed 5-marker

Approach. Define the variable in writing, form the equation, solve, then **test both roots against the situation** and reject with a stated reason any that is impossible — negative speed, negative age, negative time, a non-integer count. That rejection sentence carries its own mark. Convert minutes to hours and mixed fractions to improper fractions *before* forming the equation.

C1

2019 · 2023 · 2025 | 5 Marks

A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Step 1 Let the usual speed be x km/h. Usual time = 360/x hours; the faster time = 360/(x + 5) hours. **1**

Step 2 The faster journey is 1 hour shorter: 360/x − 360/(x + 5) = 1 **1**

Step 3 360[(x + 5) − x] = x(x + 5) ⇒ 1800 = x^2 + 5x **1**

Step 4 x^2 + 5x − 1800 = 0. Split: x^2 + 45x − 40x − 1800 = 0 ⇒ (x + 45)(x − 40) = 0 **1**

Step 5 x = 40 or x = −45. **Speed cannot be negative, so x = −45 is rejected.** Check: 360/40 = 9 h and 360/45 = 8 h — a difference of 1 hour ✓ **1**

The speed of the train is 40 km/h

C2

2018 · 2022 · 2024 | 5 Marks

Two water taps together can fill a tank in 9⅛ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Step 1 Let the smaller tap take x hours; the larger then takes (x − 10) hours. Together they take 9⅛ = 75/8 hours. **1**

Step 2 Adding the rates: 1/x + 1/(x − 10) = 8/75 **1**

Step 3 (2x − 10) × 75 = 8x(x − 10) ⇒ 150x − 750 = $8x^2$ − 80x **1**

Step 4 $8x^2$ − 230x + 750 = 0 ⇒ $4x^2$ − 115x + 375 = 0 ⇒ (x − 25)(4x − 15) = 0 **1**

Step 5 x = 25 or x = 15/4 = 3.75. If x = 3.75 the larger tap would take 3.75 − 10 = −6.25 hours, which is impossible, **so it is rejected**. **1**

Smaller tap: 25 hours; larger tap: 15 hours

Convert the mixed fraction first. 9⅛ = 75/8, so the combined rate is 8/75. Using 9.125 as a decimal makes the factorisation impossible.

C3

2020 · 2024 | 3 Marks

Find two consecutive odd positive integers, the sum of whose squares is 394.

Step 1 Let the integers be x and $(x + 2)$, both odd and positive. ½

Step 2 $x^2 + (x + 2)^2 = 394 \Rightarrow 2x^2 + 4x + 4 = 394 \Rightarrow x^2 + 2x - 195 = 0$ 1

Step 3 Split: $x^2 + 15x - 13x - 195 = 0 \Rightarrow (x + 15)(x - 13) = 0$ 1

Step 4 $x = 13$ or $x = -15$. The integers must be **positive**, so $x = -15$ is rejected. Check: $13^2 + 15^2 = 169 + 225 = 394$ ✓ ½

The integers are 13 and 15

Consecutive odd numbers differ by 2, not 1. Using $(x + 1)$ here gives a wrong equation and no valid odd answer.

C4

2021 · 2023 · 2025 | 3 Marks

The sum of a number and its reciprocal is 26/5. Find the number.

Step 1 Let the number be x, so $x + 1/x = 26/5$. ½

Step 2 Multiply throughout by $5x$: $5x^2 + 5 = 26x \Rightarrow 5x^2 - 26x + 5 = 0$ 1

Step 3 Split: $5x^2 - 25x - x + 5 = 0 \Rightarrow 5x(x - 5) - 1(x - 5) = 0 \Rightarrow (x - 5)(5x - 1) = 0$ 1

Step 4 $x = 5$ or $x = 1/5$. **Both are valid** — each is the reciprocal of the other, and nothing in the question forbids either. ½

The number is 5 (or equivalently 1/5)

Do not reject a root out of habit. Reject only when the situation forbids it. Discarding a perfectly valid root also costs a mark.

D · Nature of roots and "find k"

Approach. Always write a, b, c first, then $D = b^2 - 4ac$. $D > 0$ → two distinct real roots · $D = 0$ → two equal real roots · $D < 0$ → no real roots. For "equal roots, find k", set $D = 0$ and solve — then **reject any k that makes a = 0**, since the equation would no longer be quadratic.

D1

2019 · 2022 · 2024 · 2025 | 3 Marks

Find the value of k for which the equation $kx(x - 2) + 6 = 0$ has two equal roots.

Step 1 xpand into standard form: $kx^2 - 2kx + 6 = 0$, so $a = k$, $b = -2k$, $c = 6$. 1

Step 2 qual roots ⇒ $D = 0$: $(-2k)^2 - 4(k)(6) = 0 \Rightarrow 4k^2 - 24k = 0$ 1

Step 3 $k(k - 6) = 0 \Rightarrow k = 0$ or $k = 6$. ½

Step 4 **$= 0$ is rejected**, because then the equation becomes $6 = 0$, which is not a quadratic equation at all. ½

$k = 6$

The rejection of $k = 0$ is examined every single year. Leaving it in the answer loses a mark, and so does rejecting it without giving the reason.

D2

2020 · 2023 | 3 Marks

Find the values of k for which the equation $(k + 4)x^2 + (k + 1)x + 1 = 0$ has equal roots.

Step 1 $= k + 4$, $b = k + 1$, $c = 1$. Equal roots ⇒ $D = 0$: $(k + 1)^2 - 4(k + 4)(1) = 0$ 1

Step 2 $^2 + 2k + 1 - 4k - 16 = 0 \Rightarrow k^2 - 2k - 15 = 0$ 1

Step 3 plit: $(k - 5)(k + 3) = 0 \Rightarrow k = 5$ or $k = -3$. ½

Step 4 oth are acceptable: neither makes $a = k + 4$ equal to zero (that would need $k = -4$). ½

$k = 5$ or $k = -3$

Here both roots survive. Check the $a \neq 0$ condition every time, but reject only if it actually fails — contrast this with D1.

D3

2021 · 2024 | 2 Marks

Find the nature of the roots of $2x^2 - 4x + 3 = 0$. If real roots exist, find them.

Step 1 $= 2$, $b = -4$, $c = 3$. $D = (-4)^2 - 4(2)(3) = 16 - 24 = -8$ 1

Step 2 ince $D < 0$, the equation has **no real roots**, so there is nothing further to find. 1

$D = -8 < 0$, so the equation has no real roots

Stop once $D < 0$. Students who go on to apply the quadratic formula and write $\sqrt{(-8)}$ lose the mark — at this level the square root of a negative number is not defined.

E • "Is it possible" problems and objective items

Approach. These look like ordinary word problems but the real question is whether a solution **exists**. Form the quadratic, compute D, and answer the "is it possible" part **first** from the sign of D — only then solve, and only if $D \geq 0$.

E1

2019 · 2023 · 2025 | 4 Marks

Is the following situation possible? The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48. If so, find their present ages.

Step 1 Let one friend be x years old; the other is $(20 - x)$ years. **1**

Step 2 Four years ago their ages were $(x - 4)$ and $(16 - x)$. Their product: $(x - 4)(16 - x) = 48$ **1**

Step 3 $16x - x^2 - 64 + 4x = 48 \Rightarrow -x^2 + 20x - 112 = 0 \Rightarrow x^2 - 20x + 112 = 0$ **1**

Step 4 $D = (-20)^2 - 4(1)(112) = 400 - 448 = -48 < 0$ **1**

Since $D < 0$ there are no real roots, so the situation is not possible

Answer the question that was asked. The mark is for saying "not possible" and giving $D < 0$ as the reason — not for any age.

E2

2020 · 2024 | 4 Marks

Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2? If so, find its length and breadth.

Step 1 Perimeter $2(l + b) = 80 \Rightarrow l + b = 40$. Let the length be x, so the breadth is $(40 - x)$. **1**

Step 2 Area: $x(40 - x) = 400 \Rightarrow x^2 - 40x + 400 = 0$ **1**

Step 3 $D = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$. Since $D = 0$, real and **equal** roots exist — so yes, it is possible. **1**

Step 4 $x = -b/2a = 40/2 = 20$, so length = breadth = 20 m. **1**

Yes — it is possible; the park is a square of side 20 m

D = 0 means the shape is forced. Equal roots here tell you length and breadth must be the same, i.e. the only such park is a square. Saying so earns the final mark.

E3 2022 · 2024 1 Mark

The quadratic equation $x^2 + 4 = 0$ has

(i) two distinct real roots (ii) two equal real roots (iii) no real roots (iv) exactly one real root

Step 1 $a = 1$, $b = 0$, $c = 4$, so $D = 0 - 16 = -16 < 0$.

(iii) no real roots

E4 2023 · 2025 1 Mark

If one root of the equation $x^2 - 5x + 6 = 0$ is 2, the other root is

(i) 1 (ii) 3 (iii) −3 (iv) 6

Step 1 Sum of roots $= -b/a = 5$, so the other root is $5 - 2 = 3$. (Or: product $= c/a = 6$, so $6 \div 2 = 3$.)

(ii) 3

E5 2021 · 2024 1 Mark

Assertion (A): The equation $2x^2 - 4x + 3 = 0$ has no real roots.

Reason (R): A quadratic equation has real roots only if its discriminant is greater than or equal to zero.

(i) Both true, R explains A (ii) Both true, R does not explain A (iii) A true, R false (iv) A false, R true

Step 1 $D = 16 - 24 = -8 < 0$, so A is true.

Step 2 R states the correct criterion and is exactly what settles A.

(i) Both true, and R is the correct explanation of A

CHAPTER 5

Arithmetic Progressions

Unit II · Algebra · Typically one 2-mark and one 3/5-mark question

📈 What CBSE actually asks from this chapter

Question type	Frequency	Usual marks
Given two terms, find a, d and a required term	★★★★★	2–3
Find the sum of n terms, or how many terms give a stated sum	★★★★★	3–5
Which term of the AP is ... / is a given number a term of the AP	★★★★	2
Real-life AP: salary increments, savings, seats in rows, logs stacked	★★★★	5 / case study
Sum of first n natural numbers, multiples of a number in a range	★★★	3

The three formulas. $a_n = a + (n - 1)d$ $S_n = (n/2)[2a + (n - 1)d]$ $S_n = (n/2)(a + \ell)$ when the last term ℓ is known.

Also useful: $a_n = S_n - S_{n-1}$

A · Two terms given — find a, d and another term

Approach. Write each given term as a + (n − 1)d, giving two equations. **Subtract** them — a disappears and d falls out in one line. Substitute back for a, then answer what was actually asked. A useful shortcut: $a_p - a_q = (p - q)d$.

A1

2019 · 2022 · 2024 | 3 Marks

The 7th term of an AP is −4 and its 13th term is −16. Find the AP and its 20th term.

Step 1 $_7 = a + 6d = -4$ ...(i) $a_{13} = a + 12d = -16$ ...(ii) 1

Step 2 ubtract (i) from (ii): $6d = -12 \Rightarrow d = -2$ 1

Step 3 rom (i): $a = -4 + 12 = 8$. So the AP is 8, 6, 4, 2, ... ½

Step 4 $_{20} = 8 + 19(-2) = 8 - 38 = -30$ ½

The AP is 8, 6, 4, 2, ... and its 20th term is −30

A2

2021 · 2024 | 2 Marks

If the 17th term of an AP exceeds its 10th term by 7, find the common difference.

Step 1 $a_{17} - a_{10} = [a + 16d] - [a + 9d] = 7d$ 1

Step 2 Given this equals 7: $7d = 7 \Rightarrow d = 1$ 1

The common difference is 1

You never need a here. The difference of two terms always kills the first term. Recognising that turns a three-line problem into one line.

B • Which term is it, and is a given number a term at all

Approach. Set $a + (n - 1)d$ equal to the given number and solve for n. If n comes out a **positive integer**, that number is a term and n is its position. If n is fractional or negative, it is **not** a term — and saying so, with the value of n as evidence, is the answer.

B1

2020 · 2023 · 2025 | 3 Marks

For the AP 21, 18, 15, ...

(a) which term is −81? (2)

(b) is 0 a term of this AP? Justify. (1)

Step 1 $a = 21$, $d = 18 - 21 = -3$ ½

Step 2 **(a)** $21 + (n - 1)(-3) = -81 \Rightarrow -3(n - 1) = -102 \Rightarrow n - 1 = 34 \Rightarrow n = 35$ 1½

Step 3 **(b)** $21 + (n - 1)(-3) = 0 \Rightarrow 3(n - 1) = 21 \Rightarrow n = 8$, a positive integer. 1

(a) −81 is the 35th term (b) Yes — 0 is the 8th term

Both outcomes appear in board papers. Here n turns out to be a whole number, so the answer is "yes". If it had come out fractional, the correct answer would be "no" with that value of n quoted as the reason.

B2 2018 · 2022 · 2024 **3 Marks**

How many terms of the AP 9, 17, 25, … must be taken so that their sum is 636?

Step 1 $a = 9$, $d = 8$, $S_n = 636$ ½

Step 2 $636 = (n/2)[18 + 8(n - 1)] \Rightarrow 1272 = n(8n + 10)$ 1

Step 3 $8n^2 + 10n - 1272 = 0 \Rightarrow 4n^2 + 5n - 636 = 0$ ½

Step 4 Split: $4n^2 + 53n - 48n - 636 = 0 \Rightarrow (n - 12)(4n + 53) = 0$ ½

Step 5 $n = 12$ or $n = -53/4$. **A number of terms must be a positive integer**, so $n = 12$. ½

12 terms

C · Sums — multiples in a range, and S_n given as a formula

Approach. For "multiples of k between two limits": find the first and last such multiple, use $a_n = a + (n - 1)d$ to get n, then $S_n = (n/2)(a + \ell)$. When S_n is given as a formula in n, use $a_1 = S_1$ and $a_n = S_n - S_{n-1}$ — do not try to read a and d off the formula by inspection.

C1 2019 · 2023 · 2025 **3 Marks**

Find the sum of all three-digit numbers which are divisible by 7.

Step 1 The smallest three-digit multiple of 7 is 105 and the largest is 994. So $a = 105$, $d = 7$, $\ell = 994$. 1

Step 2 Number of terms: $994 = 105 + (n - 1)7 \Rightarrow 889 = 7(n - 1) \Rightarrow n - 1 = 127 \Rightarrow n = 128$ 1

Step 3 $S_{128} = (128/2)(105 + 994) = 64 \times 1099 = 70336$ 1

70336

Find the true first and last multiple. Starting at 100 or ending at 999 is the standard error — neither is divisible by 7. Divide the limits by 7 and round inward.

C2 2020 · 2024 | 3 Marks

The sum of the first n terms of an AP is given by $S_n = 3n^2 + 5n$. Find its first term, its common difference and its 25th term.

Step 1 ${}_1 = S_1 = 3(1)^2 + 5(1) = \mathbf{8}$ 1

Step 2 ${}_2 = 3(4) + 5(2) = 22$, so $a_2 = S_2 - S_1 = 22 - 8 = 14$ 1

Step 3 $= a_2 - a_1 = 14 - 8 = \mathbf{6}$ ½

Step 4 ${}_{25} = 8 + 24(6) = 8 + 144 = \mathbf{152}$ ½

$a = 8$, $d = 6$ and $a_{25} = 152$

Do not read a and d off the formula. $S_n = 3n^2 + 5n$ does *not* mean $d = 3$ or $a = 5$. Always go through S_1 and S_2.

D • Real-life APs and case studies

Approach. Identify a (the first value) and d (the fixed increase or decrease), then decide whether the question wants a **term** (a single year, a single row) or a **sum** (the total over all of them). Mixing these up is the most expensive error in the chapter.

D1 2021 · 2023 · 2025 | 4 Marks

Case study. A school auditorium has 20 rows of seats. The first row has 18 seats and each row behind it has 2 more seats than the row in front.

(a) Write a and d. (1)

(b) How many seats are in the last row? (1)

(c) How many seats are there in the auditorium altogether? (2)

Step 1 **a)** $a = 18$ and $d = 2$ 1

Step 2 **b)** $a_{20} = 18 + 19(2) = 18 + 38 =$ **56 seats** 1

Step 3 **c)** With the last term known, use $S_n = (n/2)(a + \ell) = (20/2)(18 + 56)$ 1

Step 4 $10 \times 74 =$ **740 seats** 1

(a) $a = 18$, $d = 2$ (b) 56 seats (c) 740 seats

Part (b) is a term, part (c) is a sum. Answering 740 for the last row, or 56 for the total, is the trap this question is built around.

D2 2019 · 2024 | 3 Marks

Subba Rao started work in 1995 at an annual salary of ₹5000 and received an increment of ₹200 every year. In which year did his income reach ₹7000?

Step 1 The salaries form an AP with $a = 5000$ and $d = 200$. **1**

Step 2 $000 = 5000 + (n - 1)(200) \Rightarrow 2000 = 200(n - 1) \Rightarrow n - 1 = 10 \Rightarrow n = 11$ **1½**

Step 3 The 11th year, counting 1995 as the first, is $1995 + 10 =$ **2005**. **½**

His income reached ₹7000 in the year 2005

n = 11 means the 11th year, not 11 years later. Add $(n - 1) = 10$ to 1995, not 11. Answering 2006 is the standard slip.

E · Objective and assertion–reason items

E1 2022 · 2024 | 1 Mark

The common difference of the AP 3, 1, −1, −3, ... is

(i) 2 (ii) −2 (iii) 3 (iv) −3

Step 1 $d = a_2 - a_1 = 1 - 3 = -2$. (Check: $-1 - 1 = -2$ ✓)

(ii) −2

Subtract in the right order. d = later term − earlier term. Doing 3 − 1 gives +2 and the wrong option.

E2 2023 · 2025 | 1 Mark

If the nth term of an AP is 7 − 4n, then its common difference is

(i) 4 (ii) −4 (iii) 7 (iv) −7

Step 1 $a_1 = 7 - 4 = 3$ and $a_2 = 7 - 8 = -1$, so $d = -1 - 3 = -4$.

(ii) −4

Quick rule. When a_n is linear in n, the coefficient of n *is* the common difference. Here that coefficient is −4.

E3 2021 · 2024 1 Mark

Assertion (A): The sum of the first n odd natural numbers is n^2.

Reason (R): The odd natural numbers form an AP with a = 1 and d = 2.

(i) Both true, R explains A (ii) Both true, R does not explain A (iii) A true, R false (iv) A false, R true

Step 1 The odd numbers 1, 3, 5, ... form an AP with a = 1 and d = 2, so R is true.

Step 2 $_n = (n/2)[2(1) + (n - 1)2] = (n/2)(2n) = n^2$, so A is true — and R is exactly what produces it.

(i) Both true, and R is the correct explanation of A

CHAPTER 6

Triangles

Unit IV · Geometry 15 marks · A theorem proof appears almost every year

📈 What CBSE actually asks from this chapter

Question type	Frequency	Usual marks
Prove the Basic Proportionality Theorem (Thales) and use it	★★★★★	5
Numerical on BPT — find x when DE ∥ BC	★★★★★	2–3
Prove two triangles similar (AA, SSS, SAS) and find a side	★★★★★	3–5
Ratio of areas of similar triangles = ratio of squares of corresponding sides	★★★★	2–3
Application: shadows, ladders, poles, mid-point configurations	★★★	3

Note. The **Constructions** chapter (dividing a line segment, constructing similar triangles, tangents) is fully deleted — no construction question will be asked.

A · Basic Proportionality Theorem — proof and numericals

Approach. BPT: if DE ∥ BC then AD/DB = AE/EC. **Converse:** if the two ratios are equal then DE ∥ BC. Use BPT when parallelism is given and a length is wanted; use the converse when lengths are given and parallelism must be proved. Pair the two pieces next to the vertex together, and the two pieces next to the base together — never one from each.

A1

2019 · 2020 · 2023 · 2024 | 5 Marks

Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. (Basic Proportionality Theorem)

Step 1 **Given:** $\triangle ABC$ with $DE \parallel BC$, D on AB and E on AC. **To prove:** $AD/DB = AE/EC$.
Construction: join BE and CD; draw $EM \perp AB$ and $DN \perp AC$. **1**

Step 2 $ar(\triangle ADE) = \frac{1}{2} \times AD \times EM$ and $ar(\triangle DBE) = \frac{1}{2} \times DB \times EM$, so
$ar(\triangle ADE)/ar(\triangle DBE) = AD/DB$ ...(i) **1½**

Step 3 Similarly $ar(\triangle ADE) = \frac{1}{2} \times AE \times DN$ and $ar(\triangle DEC) = \frac{1}{2} \times EC \times DN$, so
$ar(\triangle ADE)/ar(\triangle DEC) = AE/EC$ ...(ii) **1½**

Step 4 $\triangle DBE$ and $\triangle DEC$ stand on the **same base DE** and lie **between the same parallels DE and BC**, so their areas are equal ...(iii) **1**

From (i), (ii) and (iii) the left sides are equal, hence $AD/DB = AE/EC$ ■

Step 4 is the entire proof. Most students write (i) and (ii) correctly and then cannot justify why the two areas are equal. The reason must be written in full — *same base and between the same parallels* — and the figure with both perpendiculars must be drawn.

A2

2022 · 2024 · 2025 | 3 Marks

In $\triangle ABC$, $DE \parallel BC$ with D on AB and E on AC. If $AD = x$ cm, $DB = (x - 2)$ cm, $AE = (x + 2)$ cm and $EC = (x - 1)$ cm, find x.

Step 1 Since $DE \parallel BC$, by BPT: $AD/DB = AE/EC$ **1**

Step 2 $x/(x - 2) = (x + 2)/(x - 1) \Rightarrow x(x - 1) = (x + 2)(x - 2)$ **1**

Step 3 $x^2 - x = x^2 - 4 \Rightarrow x = 4$ **1**

$x = 4$ cm

Check the answer is admissible. With $x = 4$: $AD = 4$, $DB = 2$, $AE = 6$, $EC = 3$, and $4/2 = 6/3 = 2$ ✓ All four lengths are positive, so $x = 4$ is valid.

A3 2021 · 2023 | 2 Marks

In △ABC, DE ∥ BC. If AD = 4 cm, DB = 6 cm and AE = 5 cm, find EC.

Step 1 By BPT: AD/DB = AE/EC ½

Step 2 /6 = 5/EC ⇒ 4 × EC = 30 1

Step 3 C = 7.5 cm ½

EC = 7.5 cm

B · Ratio of areas of similar triangles

Approach. For similar triangles the ratio of areas equals the **square** of the ratio of any pair of corresponding lengths — sides, altitudes, medians, angle bisectors or perimeters alike. So going from areas to lengths means taking a **square root** first; going the other way means squaring.

B1 2019 · 2023 · 2025 | 3 Marks

The areas of two similar triangles are 81 cm^2 and 49 cm^2. If the median of the larger triangle is 13.5 cm, find the corresponding median of the smaller triangle.

Step 1 For similar triangles: ar(larger)/ar(smaller) = (corresponding median of larger / corresponding median of smaller)^2 1

Step 2 1/49 = $(13.5/m)^2$. Take the square root of both sides: 9/7 = 13.5/m 1

Step 3 m = 13.5 × 7/9 = 10.5 cm 1

The corresponding median of the smaller triangle is 10.5 cm

Take the square root before dividing. Using 81/49 = 13.5/m directly gives m = 8.17 — the standard wrong answer in this question type.

B2 2020 · 2024 | 2 Marks

The corresponding sides of two similar triangles are in the ratio 4 : 9. Find the ratio of their areas.

Step 1 Ratio of areas = (ratio of corresponding sides)^2 1

Step 2 ⇒ $(4/9)^2$ = 16/81 1

The ratio of their areas is 16 : 81

C · Proving similarity and real-life applications

Approach. Name the criterion — **AA**, **SSS** or **SAS** — explicitly; that naming carries a mark on its own. Write the similarity statement with the vertices in **matching order**, because every later proportion is read off that order. For shadow and height problems, both objects are vertical and the sun's rays arrive at the same angle, so the two triangles are similar by AA.

C1

2019 · 2022 · 2024 | 3 Marks

A vertical pole 6 m long casts a shadow 4 m long on the ground, and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Step 1 Both the pole and the tower stand vertically, so each makes 90° with the ground; and at the same moment the sun's rays make equal angles with the ground. Hence the two triangles are similar **by AA**. 1

Step 2 So height/shadow is the same for both: $6/4 = h/28$ 1

Step 3 $h = 6 \times 28 \div 4 = 42$ m 1

The height of the tower is 42 m

Keep the ratio consistent. Height over shadow on both sides, or pole over tower on both sides. Mixing them, as in $6/4 = 28/h$, gives 18.67 m — a wrong answer that looks perfectly plausible.

C2

2021 · 2023 · 2025 | 3 Marks

D is a point on the side BC of $\triangle ABC$ such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \cdot CD$.

Step 1 Compare $\triangle ABC$ and $\triangle DAC$. ½

Step 2 $\angle BAC = \angle ADC$ *(given)* and $\angle ACB = \angle DCA$ *(the same angle, common to both)* 1

Step 3 Therefore $\triangle ABC \sim \triangle DAC$ *(by AA)* ½

Step 4 Corresponding sides are proportional, and matching the order $A \leftrightarrow D$, $B \leftrightarrow A$, $C \leftrightarrow C$: $CA/CD = CB/CA$ ½

Step 5 Cross-multiplying: $CA^2 = CB \cdot CD$ ½

Hence $CA^2 = CB \cdot CD$ ■

The vertex order decides the proportion. Writing $\triangle ABC \sim \triangle ACD$ instead of $\triangle ABC \sim \triangle DAC$ pairs the wrong sides and the result will not come out. Match the equal angles first, then write the order.

C3 2020 · 2024 3 Marks

Diagonals AC and BD of a trapezium ABCD with AB ∥ DC intersect at O. Show that OA/OC = OB/OD.

Step 1 Compare △AOB and △COD. ½

Step 2 AB ∥ DC with AC a transversal, so ∠OAB = ∠OCD *(alternate angles)* 1

Step 3 Similarly with BD a transversal, ∠OBA = ∠ODC *(alternate angles)* ½

Step 4 Hence △AOB ~ △COD *(by AA)*, so corresponding sides are proportional: OA/OC = OB/OD 1

Hence OA/OC = OB/OD ■

Name the angle pairs as alternate angles. Saying only "the angles are equal" without the reason loses the mark — the parallel sides are what make them equal.

D · Objective and assertion–reason items

D1 2022 · 2024 1 Mark

If △ABC ~ △PQR with ar(△ABC)/ar(△PQR) = 25/36, then AB : PQ is

(i) 25 : 36 (ii) 5 : 6 (iii) 6 : 5 (iv) 36 : 25

Step 1 Ratio of areas = (ratio of corresponding sides)^2, so AB/PQ = √(25/36) = 5/6.

(ii) 5 : 6

D2 2023 · 2025 1 Mark

All ___ triangles are similar.

(i) isosceles (ii) scalene (iii) equilateral (iv) right-angled

Step 1 Every equilateral triangle has all three angles equal to 60°, so any two of them are equiangular and hence similar by AA.

(iii) equilateral

Why the others fail. Two isosceles triangles can have different apex angles; two right triangles can have different acute angles. Only the equilateral shape is completely fixed.

D3 2021 · 2024 **1 Mark**

Assertion (A): Two congruent triangles are always similar.

Reason (R): Similar figures have the same shape but not necessarily the same size.

(i) Both true, R explains A (ii) Both true, R does not explain A (iii) A true, R false (iv) A false, R true

Step 1 Congruent triangles have equal corresponding angles and sides in the ratio 1 : 1, so they satisfy the definition of similarity — A is true.

Step 2 R correctly defines similarity, and it is precisely why congruence implies similarity (size equal is a special case of size not necessarily equal).

(i) Both true, and R is the correct explanation of A

The converse is false. Similar triangles need not be congruent. "Every congruent pair is similar, but not every similar pair is congruent" is the sentence to remember.

CHAPTER 7

Coordinate Geometry

Unit III · 6 marks · Distance and section formula only

📈 What CBSE actually asks from this chapter

Question type	Frequency	Usual marks
Section formula — find the point dividing a segment in a given ratio	★★★★★	2–3
Find the ratio in which a point or an axis divides a segment	★★★★★	3
Distance formula — prove a triangle isosceles / right-angled, or a quadrilateral a parallelogram, rhombus or square	★★★★★	3–5
Find a point on an axis equidistant from two given points	★★★★	3
Midpoint / find the fourth vertex of a parallelogram	★★★★	2–3

Out of syllabus — do not study: the **area of a triangle from coordinates** formula $\frac{1}{2}|x_1(y_2-y_3) + \ldots|$ and collinearity done through it. Prove collinearity with the **distance formula** instead (AB + BC = AC).

The two formulas that remain.

Distance: $PQ = \sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$

Section (ratio $m_1 : m_2$): $P = \left((m_1x_2+m_2x_1)/(m_1+m_2)\,,\ (m_1y_2+m_2y_1)/(m_1+m_2) \right)$

Midpoint is the special case $m_1 : m_2 = 1 : 1$.

A · Distance formula — naming the figure

Approach. Compute all four sides **and both diagonals** — sides alone never settle the shape. Then read off: all sides equal + diagonals equal → **square**; all sides equal + diagonals unequal → **rhombus**; opposite sides equal + diagonals equal → **rectangle**; only opposite sides equal → **parallelogram**. For collinearity use AB + BC = AC — the area formula is **out of syllabus**.

A1 — 2019 · 2023 · 2025 — **5 Marks**

Show that the points A(1, 7), B(4, 2), C(−1, −1) and D(−4, 4) are the vertices of a square.

Step 1 AB = $\sqrt{(4-1)^2 + (2-7)^2} = \sqrt{9 + 25} = \sqrt{34}$ **1**

Step 2 BC = $\sqrt{(-1-4)^2 + (-1-2)^2} = \sqrt{34}$; CD = $\sqrt{(-4+1)^2 + (4+1)^2} = \sqrt{34}$; DA = $\sqrt{(1+4)^2 + (7-4)^2} = \sqrt{34}$ **2**

Step 3 All four sides are equal, so ABCD is at least a rhombus. **½**

Step 4 Diagonals: AC = $\sqrt{(-1-1)^2 + (-1-7)^2} = \sqrt{68}$ and BD = $\sqrt{(-4-4)^2 + (4-2)^2} = \sqrt{68}$ **1**

Step 5 The diagonals are equal too. A rhombus with equal diagonals is a square. **½**

Hence A, B, C, D are the vertices of a square ■

You must compute the diagonals. Four equal sides alone prove only a rhombus — a rhombus tilted over is not a square. Stopping after the sides costs two marks.

A2 — 2021 · 2024 — **2 Marks**

Find the distance between the points (2, −3) and (−6, 3).

Step 1 d = $\sqrt{(-6 - 2)^2 + (3 - (-3))^2}$ **1**

Step 2 = $\sqrt{(-8)^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10$ units **1**

The distance is 10 units

A3 — 2020 · 2023 — **3 Marks**

Show that the points (1, −1), (5, 2) and (9, 5) are collinear.

Step 1 AB = $\sqrt{(5-1)^2 + (2+1)^2} = \sqrt{16 + 9} = 5$ **1**

Step 2 BC = $\sqrt{(9-5)^2 + (5-2)^2} = \sqrt{16 + 9} = 5$ **½**

Step 3 AC = $\sqrt{(9-1)^2 + (5+1)^2} = \sqrt{64 + 36} = 10$ **1**

Step 4 AB + BC = 5 + 5 = 10 = AC, so B lies on the segment AC. **½**

The three points are collinear ■

Use the distance route only. The area-of-a-triangle formula from coordinates was removed in the rationalisation, so a collinearity proof built on "area = 0" is out of syllabus.

B · Section formula and ratios

Approach. P dividing AB in $m_1 : m_2$ is ($(m_1x_2 + m_2x_1)/(m_1+m_2)$, $(m_1y_2 + m_2y_1)/(m_1+m_2)$) — note m_1 multiplies the **far** point. To **find** a ratio, take it as k : 1, impose the known condition (a coordinate value, or y = 0 on the x-axis, x = 0 on the y-axis), solve for k, then verify with the other coordinate.

B1

2022 · 2025 | 2 Marks

Find the coordinates of the point which divides the line segment joining (4, −3) and (8, 5) in the ratio 3 : 1 internally.

Step 1 $m_1 = 3$, $m_2 = 1$, $(x_1, y_1) = (4, -3)$, $(x_2, y_2) = (8, 5)$ ½

Step 2 $= [3(8) + 1(4)]/4 = 28/4 = 7$ ¾

Step 3 $y = [3(5) + 1(-3)]/4 = 12/4 = 3$ ¾

The point is (7, 3)

m_1 goes with the second point. Writing $[3(4) + 1(8)]/4$ gives (5, −1) — a point that looks perfectly reasonable but is the wrong one.

B2

2019 · 2023 · 2024 | 3 Marks

Find the ratio in which the y-axis divides the line segment joining A(5, −6) and B(−1, −4). Also find the coordinates of the point of division.

Step 1 Let the ratio be k : 1. Every point on the **y-axis** has **x = 0** — that is the condition. 1

Step 2 $= [k(-1) + 1(5)]/(k + 1) = 0 \Rightarrow -k + 5 = 0 \Rightarrow k = 5$, so the ratio is **5 : 1** 1

Step 3 $y = [5(-4) + 1(-6)]/6 = (-20 - 6)/6 = -13/3$ 1

The y-axis divides AB in the ratio 5 : 1, at the point (0, −13/3)

Which coordinate goes to zero. For the **y-axis** set x = 0; for the **x-axis** set y = 0. Swapping these is the commonest error in the whole chapter.

B3 — 2020 · 2024 — 3 Marks

Find the coordinates of the points of trisection of the line segment joining A(2, −2) and B(−7, 4).

Step 1 risection means two points dividing AB in the ratios **1 : 2** and **2 : 1**. 1

Step 2 (1 : 2): x = [1(−7) + 2(2)]/3 = −3/3 = −1; y = [1(4) + 2(−2)]/3 = 0/3 = 0 ⇒ P(−1, 0) 1

Step 3 (2 : 1): x = [2(−7) + 1(2)]/3 = −12/3 = −4; y = [2(4) + 1(−2)]/3 = 6/3 = 2 ⇒ Q(−4, 2) 1

The points of trisection are (−1, 0) and (−4, 2)

Two points, two ratios. Many students find only the 1 : 2 point and stop — trisection always produces **two** points.

C · Midpoint, equidistant points and unknown coordinates

Approach. “Equidistant” always means **square both distances and equate** — squaring first removes the roots and keeps the algebra linear. A point restricted to an axis is written (x, 0) or (0, y) **before** anything else. If a point is the centre of a circle and the segment is a diameter, that centre is the **midpoint**.

C1 — 2021 · 2023 · 2025 — 3 Marks

Find the point on the x-axis which is equidistant from (6, 5) and (−4, 3).

Step 1 point on the x-axis has the form P(k, 0). ½

Step 2 $A^2 = (k-6)^2 + 25$ and $PB^2 = (k+4)^2 + 9$. Equidistant ⇒ $PA^2 = PB^2$. 1

Step 3 $^2 - 12k + 36 + 25 = k^2 + 8k + 16 + 9 \Rightarrow 61 - 12k = 25 + 8k$ 1

Step 4 6 = 20k ⇒ k = 9/5 = 1.8 ½

The point is (9/5, 0), i.e. (1.8, 0)

A fractional answer is fine here. Students who expect a whole number often go back and “correct” a right answer. Check instead: both distances come to √42.64 ✓

C2 — 2022 · 2024 — 2 Marks

AB is a diameter of a circle with centre (−3, 2). If B is (1, 4), find the coordinates of A.

Step 1 ince AB is a diameter, the centre is the **midpoint** of AB. ½

Step 2 et A = (x, y). Then (x + 1)/2 = −3 ⇒ x + 1 = −6 ⇒ x = −7 ¾

Step 3 y + 4)/2 = 2 ⇒ y + 4 = 4 ⇒ y = 0 ¾

A = (−7, 0)

C3

2019 · 2024 | 3 Marks

Find a relation between x and y such that the point P(x, y) is equidistant from A(7, 1) and B(3, 5).

Step 1 $PA^2 = PB^2 \Rightarrow (x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$ 1

Step 2 $^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$ 1

Step 3 $-14x - 2y + 50 = -6x - 10y + 34 \Rightarrow 8y - 8x = -16 \Rightarrow y - x = -2$ 1

The required relation is $x - y = 2$

What the relation means. It is the equation of the perpendicular bisector of AB. Adding that sentence shows understanding and often earns the benefit of the doubt.

D · Objective and assertion–reason items

D1

2022 · 2025 | 1 Mark

The distance of the point (−6, 8) from the origin is

(i) 6 (ii) 8 (iii) 10 (iv) 14

Step 1 Distance from the origin = $\sqrt{x^2 + y^2} = \sqrt{36 + 64} = \sqrt{100} = 10$.

(iii) 10

D2

2023 · 2024 | 1 Mark

The midpoint of the line segment joining (2a, 4) and (−2, 3b) is (1, 2a + 1). The values of a and b are

(i) a = 2, b = 2 (ii) a = 1, b = 2 (iii) a = 2, b = 3 (iv) a = 1, b = 3

Step 1 : $(2a - 2)/2 = 1 \Rightarrow 2a - 2 = 2 \Rightarrow a = 2$.

Step 2 y: $(4 + 3b)/2 = 2a + 1 = 5 \Rightarrow 4 + 3b = 10 \Rightarrow b = 2$.

(i) a = 2, b = 2

Find a first. The y-equation contains a as well, so it cannot be solved until a is known. Doing them in the wrong order stalls the question.

D3 2021 · 2023 1 Mark

Assertion (A): The point (0, 4) lies on the y-axis.

Reason (R): The x-coordinate of every point on the y-axis is zero.

(i) Both true, R explains A (ii) Both true, R does not explain A (iii) A true, R false (iv) A false, R true

Step 1 The point (0, 4) has x-coordinate 0, so it lies on the y-axis — A is true.

Step 2 R states the defining property of the y-axis and is exactly what makes A true.

(i) Both true, and R is the correct explanation of A

CHAPTER 8

Introduction to Trigonometry

Unit V · Trigonometry 12 marks (with Ch 9) · Identity proof nearly every year

📈 What CBSE actually asks from this chapter

Question type	Frequency	Usual marks
Prove a trigonometric identity	★★★★★	3–5
Evaluate an expression at the standard angles 0°, 30°, 45°, 60°, 90°	★★★★★	1–3
Given one ratio, find the others (or evaluate an expression)	★★★★★	2–3
Find the angle from an equation such as 2 sin 2A = √3	★★★★	2
True/false with justification; ratios from a right triangle with a side relation given	★★★	2–4

⚠ **Out of syllabus — do not study: trigonometric ratios of complementary angles**. Every result of the form sin(90° − A) = cos A, tan(90° − A) = cot A, sec(90° − A) = cosec A, and the old Exercise 8.3 built on them, has been **removed** under the rationalised syllabus. Older PYQ books are full of questions like "evaluate sin 18°/cos 72°" or "tan 10° tan 80°" — **skip every one of them**. The chapter now runs: Section 8.2 Trigonometric Ratios, 8.3 Ratios of Specific Angles, 8.4 Trigonometric Identities, with Exercises 8.1, 8.2 and 8.3.

The three identities — all that remains of the theory. $\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \mathrm{cosec}^2\theta$

θ	0°	30°	45°	60°	90°
sin θ	0	1/2	1/√2	√3/2	1
cos θ	1	√3/2	1/√2	1/2	0
tan θ	0	1/√3	1	√3	not defined

The cos row is the sin row reversed; tan = sin ÷ cos; and cosec, sec, cot are the reciprocals of sin, cos, tan.

A · Prove an identity — the certainty of this chapter

Approach. Convert everything to **sin and cos**. Work on the **messier side only** and reduce it to the other — never manipulate both sides at once, and never cross-multiply, because that assumes what you are trying to prove. When a sum or difference sits in a denominator, multiply top and bottom by the **conjugate** and use $1 - \sin^2 = \cos^2$.

A1

2019 · 2023 | 3 Marks

Prove that $(\sin\theta + \cos\theta)/(\sin\theta - \cos\theta) + (\sin\theta - \cos\theta)/(\sin\theta + \cos\theta) = 2/(2\sin^2\theta - 1)$.

Step 1 Take the LHS over the common denominator $(\sin\theta - \cos\theta)(\sin\theta + \cos\theta) = \sin^2\theta - \cos^2\theta$: ½

Step 2 Numerator $= (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2(\sin^2\theta + \cos^2\theta) = 2$ 1

Step 3 Denominator $= \sin^2\theta - \cos^2\theta = \sin^2\theta - (1 - \sin^2\theta) = 2\sin^2\theta - 1$ 1

Step 4 So LHS $= 2/(2\sin^2\theta - 1)$ ½

LHS $= 2/(2\sin^2\theta - 1)$ = RHS ■

Replace $\cos^2\theta$ in the denominator, not $\sin^2\theta$. The RHS is written in sin only, so $\cos^2\theta = 1 - \sin^2\theta$ is the substitution that lands on it.

A2

2018 · 2020 · 2024 | 5 Marks

Prove that $(\tan\theta + \sec\theta - 1)/(\tan\theta - \sec\theta + 1) = (1 + \sin\theta)/\cos\theta$.

Step 1 In the numerator replace the 1 by $\sec^2\theta - \tan^2\theta$ (identity $1 + \tan^2\theta = \sec^2\theta$): 1

Step 2 Numerator $= (\tan\theta + \sec\theta) - (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$ 1

Step 3 $= (\tan\theta + \sec\theta)[1 - \sec\theta + \tan\theta]$ 1

Step 4 The bracket $[1 - \sec\theta + \tan\theta]$ is exactly the denominator $(\tan\theta - \sec\theta + 1)$, so it cancels: LHS $= \tan\theta + \sec\theta$ 1

Step 5 $= \sin\theta/\cos\theta + 1/\cos\theta = (1 + \sin\theta)/\cos\theta$ 1

LHS $= (1 + \sin\theta)/\cos\theta$ = RHS ■

The whole question is the substitution $1 = \sec^2\theta - \tan^2\theta$. Once the numerator is factorised, the matching bracket in the denominator is what you are looking for.

A3

2023 · 2025 | 3 Marks

Prove that $(\sin A + \cos A)(\tan A + \cot A) = \sec A + \operatorname{cosec} A$.

Step 1 $\tan A + \cot A = \sin A/\cos A + \cos A/\sin A = (\sin^2 A + \cos^2 A)/(\sin A\cos A) = 1/(\sin A\cos A)$ 1½

Step 2 LHS $= (\sin A + \cos A)/(\sin A\cos A) = \sin A/(\sin A\cos A) + \cos A/(\sin A\cos A)$ 1

Step 3 $= 1/\cos A + 1/\sin A = \sec A + \operatorname{cosec} A$ ½

LHS $= \sec A + \operatorname{cosec} A$ = RHS ■

A4

2019 · 2023 | 3 Marks

If $1 + \sin^2\theta = 3 \sin\theta \cos\theta$, prove that $\tan\theta = 1$ or $1/2$.

Step 1 Divide both sides by $\cos^2\theta$: $\sec^2\theta + \tan^2\theta = 3 \tan\theta$ **1**

Step 2 Use $\sec^2\theta = 1 + \tan^2\theta$: $1 + 2\tan^2\theta = 3\tan\theta \Rightarrow 2\tan^2\theta - 3\tan\theta + 1 = 0$ **1**

Step 3 $(2\tan\theta - 1)(\tan\theta - 1) = 0 \Rightarrow \tan\theta = 1/2$ or $\tan\theta = 1$ **1**

$\tan\theta = 1$ or $1/2$ ■

Dividing by $\cos^2\theta$ turns it into a quadratic in $\tan\theta$. Note $1/\cos^2\theta = \sec^2\theta$ — students often forget to divide the 1 on the left.

B · Evaluate at the standard angles

Approach. Substitute the table values first, simplify numerator and denominator separately, then rationalise any surd left in the denominator. Look for pairs that cancel before doing arithmetic — cos 30° and sin 60° are the same number, and $\sin^2\theta + \cos^2\theta$ anywhere is simply 1.

B1

2023 · 2025 | 2 Marks

Evaluate: $2\sec^2\theta + 3\,\mathrm{cosec}^2\theta - 2\sin\theta\cos\theta$, if $\theta = 45°$.

Step 1 $\sec 45° = \mathrm{cosec}\, 45° = \sqrt{2}$, $\sin 45° = \cos 45° = 1/\sqrt{2}$ **½**

Step 2 $= 2(2) + 3(2) - 2(1/\sqrt{2})(1/\sqrt{2}) = 4 + 6 - 1 = 9$ **1½**

9

B2

2019 · 2024 | 2 Marks

Evaluate: $4/\cot^2 30° + 1/\sin^2 60° - \cos^2 45°$.

Step 1 $\cot 30° = \sqrt{3}$, $\sin 60° = \sqrt{3}/2$, $\cos 45° = 1/\sqrt{2}$ **½**

Step 2 $= 4/3 + 4/3 - 1/2 = (8 + 8 - 3)/6 = 13/6$ **1½**

13/6

$1/\sin^2 60° = 1 \div (3/4) = 4/3$, not 3/4. Invert after squaring.

B3 2019 · 2022 | 3 Marks

If $\sin(A + B) = 1$ and $\cos(A - B) = \sqrt{3}/2$, where $0° < A + B \le 90°$ and $A > B$, find A and B.

Step 1 in$(A + B) = 1 = \sin 90° \Rightarrow A + B = 90°$ ...(i) **1**

Step 2 os$(A - B) = \sqrt{3}/2 = \cos 30° \Rightarrow A - B = 30°$ ...(ii) **1**

Step 3 dding: $2A = 120° \Rightarrow A = 60°$. Subtracting: $2B = 60° \Rightarrow B = 30°$ **1**

$A = 60°$, $B = 30°$

B4 2020 · 2024 | 2 Marks

If $A = 60°$ and $B = 30°$, verify that $\sin(A - B) = \sin A \cos B - \cos A \sin B$.

Step 1 HS $= \sin(60° - 30°) = \sin 30° = 1/2$ **½**

Step 2 HS $= (\sqrt{3}/2)(\sqrt{3}/2) - (1/2)(1/2) = 3/4 - 1/4 = 1/2$ **1**

Step 3 HS = RHS **½**

Verified: both sides equal 1/2

$\sin(A - B) \ne \sin A - \sin B$. Here $\sin 60° - \sin 30° = (\sqrt{3} - 1)/2 \ne 1/2$.

C • One ratio given — find the rest

Approach. Label two sides with a common factor k, find the third by Pythagoras, then read off every ratio. **Draw the triangle every time** — ten seconds spent there removes the opposite/adjacent confusion that costs most of the lost marks. When the expression is homogeneous in sin and cos, dividing throughout by cos turns it into tan and the answer falls out in one line.

C1 2019 · 2024 | 3 Marks

If $\tan A = 4/3$, find the value of $(\sin A + \cos A)/(\sin A - \cos A)$.

Step 1 ivide numerator and denominator by cos A — the expression is homogeneous, so this is legitimate. **1**

Step 2 $(\tan A + 1)/(\tan A - 1)$ **1**

Step 3 $(4/3 + 1)/(4/3 - 1) = (7/3)/(1/3) = 7$ **1**

7

The 3–4–5 triangle also works ($\sin A = 4/5$, $\cos A = 3/5$, giving $(7/5)/(1/5) = 7$) but takes longer. Learn the divide-by-cos move — it is what the question is testing.

C2

2019 · 2023 | 3 Marks

If cot θ = 15/8, evaluate [(2 + 2 sin θ)(1 − sin θ)] / [(1 + cos θ)(2 − 2 cos θ)].

Step 1 ake 2 common from top and bottom: = [2(1 + sin θ)(1 − sin θ)] / [2(1 + cos θ)(1 − cos θ)] ½

Step 2 = $(1 - \sin^2\theta)/(1 - \cos^2\theta) = \cos^2\theta/\sin^2\theta = \cot^2\theta$ 1½

Step 3 = $(15/8)^2 = 225/64$ 1

225/64

Simplify before substituting. The expression collapses to $\cot^2\theta$, so sin θ and cos θ never need to be found.

C3

2020 · 2024 | 3 Marks

If 4 tan θ = 3, evaluate (4 sin θ − cos θ + 1)/(4 sin θ + cos θ − 1).

Step 1 an θ = 3/4: opposite = 3k, adjacent = 4k, hypotenuse = $\sqrt{9k^2 + 16k^2} = 5k$ 1

Step 2 in θ = 3/5, cos θ = 4/5 ½

Step 3 umerator = 12/5 − 4/5 + 1 = 13/5; Denominator = 12/5 + 4/5 − 1 = 11/5 1

Step 4 alue = (13/5)/(11/5) = 13/11 ½

13/11

The divide-by-cos trick does not work here — the constant 1 makes the expression non-homogeneous. Find sin θ and cos θ first.

D · Objective and true/false items

D1

2022 · 2024 | 1 Mark

If sin A = 1/2, then the value of cot A is

(i) √3 (ii) 1/√3 (iii) √3/2 (iv) 1

Step 1 in A = 1/2 ⇒ A = 30°, and cot 30° = √3.

(i) √3

D2 2023 · 2025 1 Mark

The value of $\sin^2\theta + 1/(1 + \tan^2\theta)$ is

(i) 0 (ii) 1 (iii) 2 (iv) $\sin^2\theta$

Step 1 $/(1 + \tan^2\theta) = 1/\sec^2\theta = \cos^2\theta$, so the value is $\sin^2\theta + \cos^2\theta = 1$.

(ii) 1

D3 2021 · 2024 1 Mark

Assertion (A): cot A is not defined for A = 0°.

Reason (R): cot A = cos A / sin A, and sin 0° = 0.

(i) Both true, R explains A (ii) Both true, R does not explain A (iii) A true, R false (iv) A false, R true

Step 1 ot A = cos A / sin A, and at A = 0° the denominator sin 0° = 0, so the quotient is undefined — A is true.

Step 2 gives exactly that reason, so it explains A.

(i) Both true, and R is the correct explanation of A

CHAPTER 9

Some Applications of Trigonometry

Unit V · Heights and Distances · A 5-mark question is near-certain

📈 What CBSE actually asks from this chapter

Question type	Frequency	Usual marks
Two angles of elevation/depression from one point — find the distance between two objects	★★★★★	5
Observer moves towards/away and the angle changes — find the height	★★★★★	5
Broken tree / ladder against a wall	★★★★	3
Angle of depression from a lighthouse / aeroplane to two ships	★★★★	5
Tower and building, each seeing the other	★★★	5

Q1 · Lighthouse and two ships (same side)

2017 · 2020 · 2023 · 2025 **5 Marks**

From the top of a 100 m high lighthouse, the angles of depression of two ships on the same side are 30° and 45°. If one ship is exactly behind the other, find the distance between the two ships. (Take $\sqrt{3} = 1.732$)

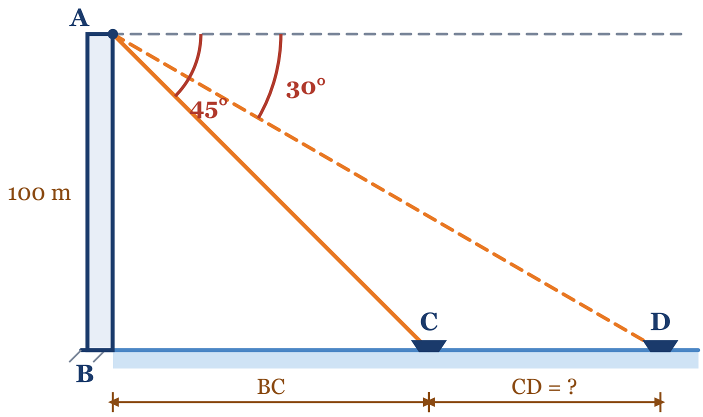

Fig. 9.A — Depression at A equals elevation at each ship (alternate angles with the horizontal).

Note: *MLI-redrawn schematic — angles drawn true, lengths to scale where practical. Never measure off the figure; work only from the given data.*

Step 1 et AB = 100 m be the lighthouse, C the nearer ship (45°) and D the farther one (30°). The angle of depression at A equals the angle of elevation at the ship (alternate angles with the horizontal). **1**

Step 2 n △ABC: $\tan 45° = AB/BC \Rightarrow 1 = 100/BC \Rightarrow BC = 100$ m **1½**

Step 3 n △ABD: $\tan 30° = AB/BD \Rightarrow 1/\sqrt{3} = 100/BD \Rightarrow BD = 100\sqrt{3}$ m **1½**

Step 4 D = $BD - BC = 100\sqrt{3} - 100 = 100(\sqrt{3} - 1) = 100 \times 0.732 = 73.2$ m **1**

The distance between the ships is $100(\sqrt{3} - 1)$ m ≈ 73.2 m

The smaller angle gives the larger distance. The ship seen at 30° is farther than the one at 45°. A negative answer means the two have been swapped.

Q2 · Observer walks towards the tower 2018 · 2022 · 2024 **5 Marks**

The angle of elevation of the top of a tower from a point on the ground is 30°. On walking 40 m towards the tower, the angle of elevation becomes 60°. Find the height of the tower.

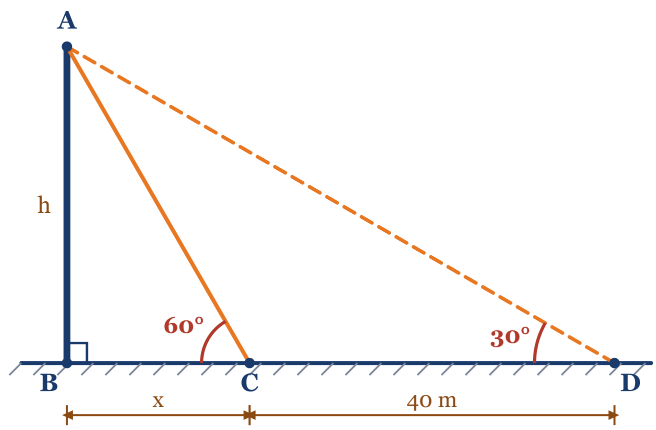

Fig. 9.B — C is the nearer point (60°); D is 40 m further away (30°).

Note: *MLI-redrawn schematic — angles drawn true, lengths to scale where practical. Never measure off the figure; work only from the given data.*

Step 1 et AB = h be the tower, C the nearer point (60°), D the farther point (30°), CD = 40 m and BC = x. **1**

Step 2 n △ABC: tan 60° = h/x ⇒ h = √3 x ...(i) **1½**

Step 3 n △ABD: tan 30° = h/(x + 40) ⇒ x + 40 = √3 h ...(ii) **1½**

Step 4 ut (i) in (ii): x + 40 = 3x ⇒ x = 20 m, so h = 20√3 m ≈ 34.64 m **1**

The height of the tower is 20√3 m ≈ 34.64 m

Put the unknown at the nearer point. BC = x and BD = x + 40 keeps the algebra to two lines.

Q3 · Shadow 40 m longer

2019 · 2023 · 2025 | 3 Marks

The shadow of a tower standing on level ground is found to be 40 m longer when the Sun's altitude is 30° than when it is 60°. Find the height of the tower.

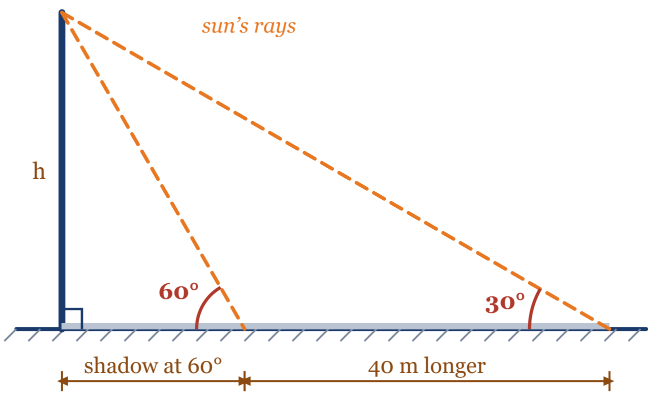

Fig. 9.C — The Sun's altitude is the angle the ray makes with the ground at the tip of the shadow.

Note: *MLI-redrawn schematic — angles drawn true, lengths to scale where practical. Never measure off the figure; work only from the given data.*

Step 1 et the height be h and the shorter shadow (Sun at 60°) be x; the longer shadow is x + 40. **½**

Step 2 an 60° = h/x ⇒ h = √3 x ...(i) **1**

Step 3 an 30° = h/(x + 40) ⇒ x + 40 = √3 h = 3x ⇒ x = 20 m **1**

Step 4 = 20√3 m ≈ 34.64 m **½**

The height of the tower is 20√3 m

The Sun's altitude is measured at the tip of the shadow, where the ray meets the ground — not at the top of the tower.

Q4 · Flagstaff on a building

2019 · 2022 | 3 Marks

From a point P on the ground, the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top of the building, and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff and the distance of the building from P. (Take $\sqrt{3} = 1.732$)

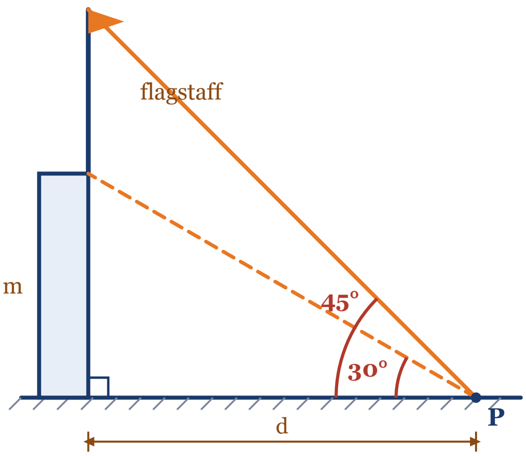

Fig. 9.D — 30° to the top of the building, 45° to the top of the flagstaff, both from P.

Note: *MLI-redrawn schematic — angles drawn true, lengths to scale where practical. Never measure off the figure; work only from the given data.*

Step 1 Let the distance of the building from P be d. $\tan 30° = 10/d \Rightarrow d = 10\sqrt{3}$ m ≈ 17.32 m **1**

Step 2 Let the flagstaff be ℓ. $\tan 45° = (10 + \ell)/d \Rightarrow 10 + \ell = 10\sqrt{3}$ **1**

Step 3 $\ell = 10\sqrt{3} - 10 = 10(\sqrt{3} - 1) = 7.32$ m **1**

Flagstaff = 7.32 m; distance of the building from P = 17.32 m

Q5 · Two men on opposite sides of a lighthouse

2020 · 2024 | 3 Marks

Two men on either side of a 100 m high lighthouse, in line with its base, find that the angles of elevation of the top of the lighthouse from the two men are 45° and 30°. Find the distance between the two men. (Take $\sqrt{3} = 1.732$)

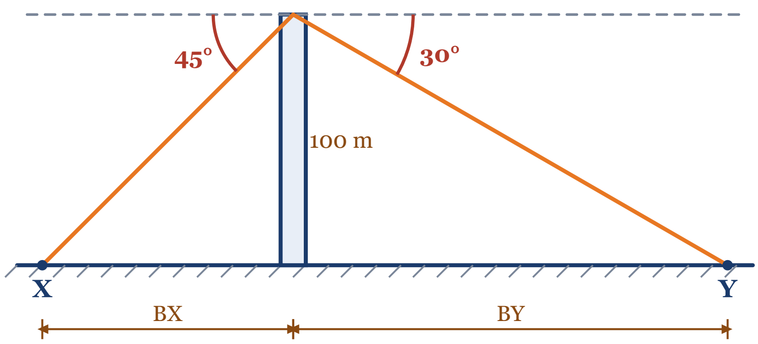

Fig. 9.E — The two men are on opposite sides of the lighthouse, so their distances are added.

Note: *MLI-redrawn schematic — angles drawn true, lengths to scale where practical. Never measure off the figure; work only from the given data.*

Step 1 Let the base be B and the men X (45°) and Y (30°), on opposite sides of B. ½

Step 2 $\tan 45° = 100/BX \Rightarrow BX = 100$ m 1

Step 3 $\tan 30° = 100/BY \Rightarrow BY = 100\sqrt{3}$ m 1

Step 4 The men are on opposite sides, so $XY = BX + BY = 100(1 + \sqrt{3}) = 273.2$ m ½

The distance between the two men is $100(\sqrt{3} + 1)$ m ≈ 273.2 m

Opposite sides ⇒ add; same side ⇒ subtract. Compare with Q1: the same two angles give 73.2 m there and 273.2 m here.

Q6 · Speed of an aeroplane

2018 · 2023 | 5 Marks

The angle of elevation of an aeroplane from a point O on the ground is 60°. After a flight of 30 seconds, the angle of elevation becomes 30°. If the aeroplane is flying at a constant height of $3600\sqrt{3}$ m, find its speed in km/h.

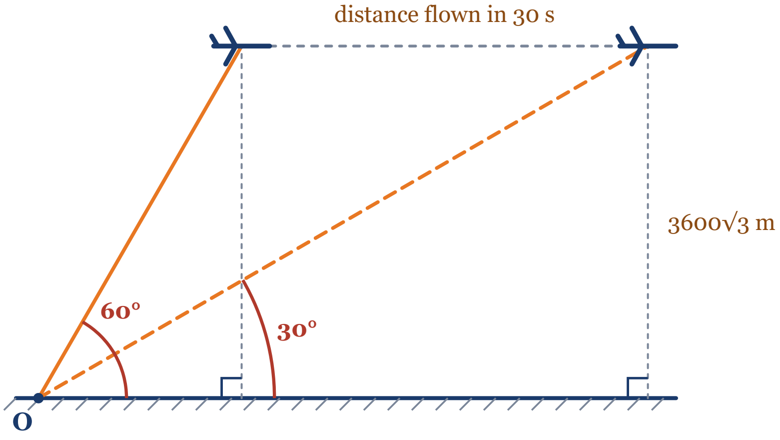

Fig. 9.F — The aeroplane flies horizontally at a constant height of $3600\sqrt{3}$ m.

Note: *MLI-redrawn schematic — angles drawn true, lengths to scale where practical. Never measure off the figure; work only from the given data.*

Step 1 Let the two positions be at horizontal distances d_1 (60°) and d_2 (30°) from O. ½

Step 2 $\tan 60° = 3600\sqrt{3}/d_1 \Rightarrow d_1 = 3600$ m 1

Step 3 $\tan 30° = 3600\sqrt{3}/d_2 \Rightarrow d_2 = 3600\sqrt{3} \times \sqrt{3} = 10800$ m 1

Step 4 Distance flown $= d_2 - d_1 = 7200$ m in 30 s 1

Step 5 Speed $= 7200/30 = 240$ m/s $= 240 \times 18/5 = 864$ km/h 1½

The speed of the aeroplane is 864 km/h (240 m/s)

Convert units at the end. 1 m/s = 18/5 km/h. Leaving the answer in m/s when km/h is asked costs the last mark.

Q7 · Aeroplane and two ships **2019 · 2024** **5 Marks**

An aeroplane at an altitude of 1200 m finds that two ships are sailing towards it in the same direction. The angles of depression of the ships as observed from the aeroplane are 60° and 30° respectively. Find the distance between the two ships. (Take √3 = 1.732)

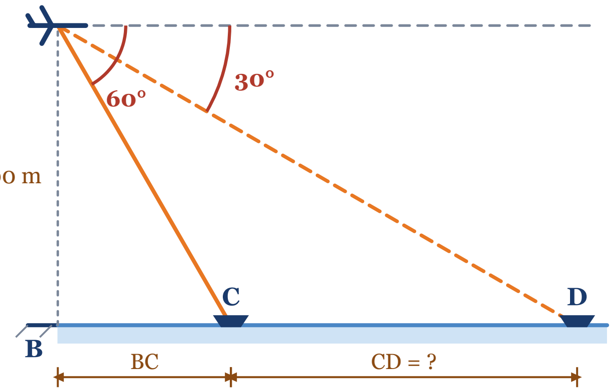

Fig. 9.G — Both angles of depression are measured below the aeroplane's horizontal.

Note: *MLI-redrawn schematic — angles drawn true, lengths to scale where practical. Never measure off the figure; work only from the given data.*

Step 1 et A be the aeroplane, B the point directly below it (AB = 1200 m), C the nearer ship (60°) and D the farther ship (30°). Depression at A = elevation at each ship. **1**

Step 2 n △ABC: $\tan 60° = 1200/BC \Rightarrow BC = 1200/\sqrt{3} = 400\sqrt{3}$ m **1½**

Step 3 n △ABD: $\tan 30° = 1200/BD \Rightarrow BD = 1200\sqrt{3}$ m **1½**

Step 4 D = BD − BC = $1200\sqrt{3} - 400\sqrt{3} = 800\sqrt{3} = 1385.6$ m **1**

The distance between the ships is $800\sqrt{3}$ m ≈ 1385.6 m

Rationalise first: $1200/\sqrt{3} = 400\sqrt{3}$. Then both distances are multiples of √3 and subtract in one line.

Q8 · Two observation points, one above the other

2020 · 2024 **5 Marks**

The angle of elevation of the top of a vertical tower from a point A on the ground is 60°. From a point B, 10 m vertically above A, the angle of elevation of the top is 45°. Find the height of the tower. (Take $\sqrt{3} = 1.732$)

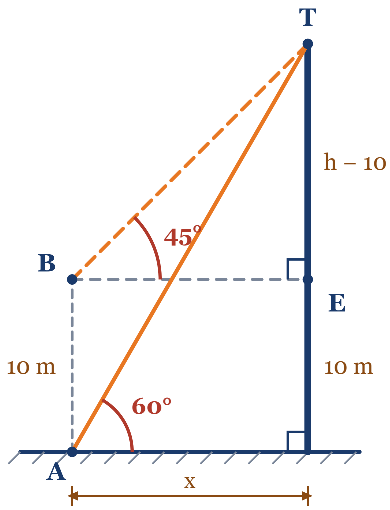

Fig. 9.H — B is 10 m vertically above A; the horizontal BE meets the tower 10 m above its foot.

Note: *MLI-redrawn schematic — angles drawn true, lengths to scale where practical. Never measure off the figure; work only from the given data.*

Step 1 et the tower be of height h at horizontal distance x from A. Draw the horizontal BE from B to the tower; then E is 10 m above the foot, and $TE = h - 10$. **1**

Step 2 rom A: $\tan 60° = h/x \Rightarrow h = \sqrt{3}\,x$ ...(i) **1**

Step 3 rom B: $\tan 45° = (h - 10)/x \Rightarrow h - 10 = x$ ...(ii) **1**

Step 4 rom (i) and (ii): $\sqrt{3}\,x - x = 10 \Rightarrow x = 10/(\sqrt{3} - 1) = 5(\sqrt{3} + 1)$ m **1**

Step 5 $= \sqrt{3} \times 5(\sqrt{3} + 1) = 5(3 + \sqrt{3}) = 15 + 8.66 = 23.66$ m **1**

The height of the tower is $5(3 + \sqrt{3})$ m ≈ 23.66 m

The horizontal from B is essential. The 45° is measured from BE, not from the ground, so the opposite side is $h - 10$, not h.

Q9 · Cloud and its reflection in a lake

2019 · 2023 · 2025 | 5 Marks

The angle of elevation of a cloud from a point 60 m above the surface of a lake is 30°, and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud above the lake.

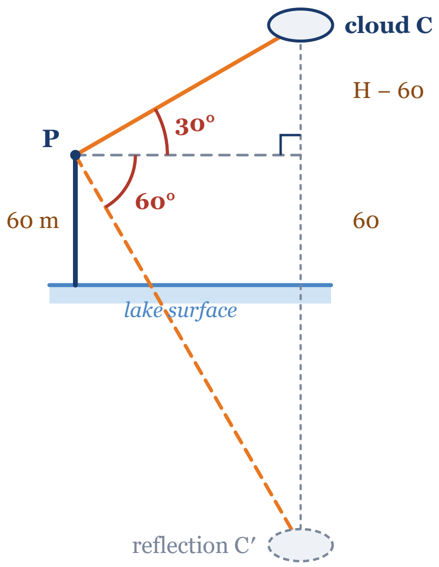

Fig. 9.I — The reflection C′ is as far below the lake surface as the cloud C is above it.

Note: *MLI-redrawn schematic — angles drawn true, lengths to scale where practical. Never measure off the figure; work only from the given data.*

Step 1 Let the cloud C be H m above the lake and its reflection C′ be H m below the surface. P is 60 m above the lake; the horizontal from P meets the vertical through C at M, with PM = x. **1**

Step 2 Then CM = H − 60 and C′M = H + 60. **½**

Step 3 tan 30° = (H − 60)/x ⇒ x = √3(H − 60) ...(i) **1**

Step 4 tan 60° = (H + 60)/x ⇒ H + 60 = √3 x ...(ii) **1**

Step 5 Put (i) in (ii): H + 60 = 3(H − 60) ⇒ 2H = 240 ⇒ H = 120 m **1½**

The cloud is 120 m above the lake

C′M is H + 60, not H. The reflection is H below the surface, and P is a further 60 m above it.

Q10 · Objective 2022 · 2024 1 Mark

A pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground. The Sun's angle of elevation is

(i) 30° (ii) 45° (iii) 60° (iv) 90°

Step 1 an $\theta = 6/(2\sqrt{3}) = 3/\sqrt{3} = \sqrt{3} \Rightarrow \theta = 60°$

(iii) 60°

Q11 · Objective 2023 · 2025 1 Mark

The ratio of the length of a rod to its shadow is $1 : \sqrt{3}$. The angle of elevation of the Sun is

(i) 30° (ii) 45° (iii) 60° (iv) 90°

Step 1 an θ = rod/shadow = $1/\sqrt{3} \Rightarrow \theta = 30°$

(i) 30°

CHAPTER 10

Circles

Unit IV · Geometry · One tangent theorem proof appears almost every year

📈 What CBSE actually asks from this chapter

Question type	Frequency	Usual marks
Prove: the lengths of tangents from an external point are equal	★★★★★	3–5
Prove: the tangent at any point is perpendicular to the radius	★★★★★	3
Numerical using OT ⊥ tangent and Pythagoras	★★★★★	2–3
Quadrilateral circumscribing a circle — AB + CD = BC + AD	★★★★	3–5
Angle between two tangents and the angle at the centre	★★★★	2–3

Note. Constructing tangents is deleted along with the whole Constructions chapter. You still need tangent **theorems** and numericals.

Q1 · Equal tangents theorem

2018 · 2020 · 2023 · 2025 | **3 Marks**

Prove that the lengths of tangents drawn from an external point to a circle are equal.

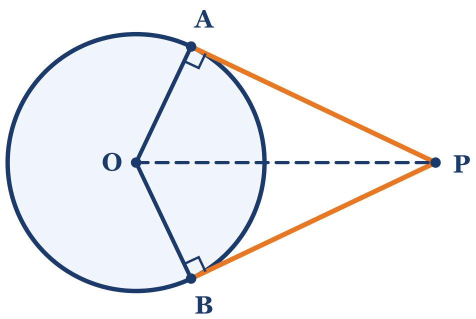

Fig. 10.A — Join OA, OB, OP. Right angles at A and B; OA = OB; OP common — RHS.

Note: *MLI-redrawn schematic, constructed from the given data (tangents truly tangent, right angles true). Never measure off the figure; work only from the given data.*

Step 1 **Given:** a circle with centre O, an external point P, and tangents PA and PB touching the circle at A and B. **To prove:** PA = PB. **Construction:** join OA, OB and OP. ½

Step 2 ∠OAP = ∠OBP = 90° (the tangent is perpendicular to the radius at the point of contact) ½

Step 3 In right triangles OAP and OBP: OA = OB (radii) and OP = OP (common hypotenuse) 1

Step 4 △OAP ≅ △OBP (RHS), and hence PA = PB (CPCT) 1

PA = PB ■

Name every reason. “RHS” and “CPCT” carry the marks, and the figure must show OA, OB and OP joined.

Q2 · Tangent is perpendicular to the radius 2019 · 2022 · 2024 | 3 Marks

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

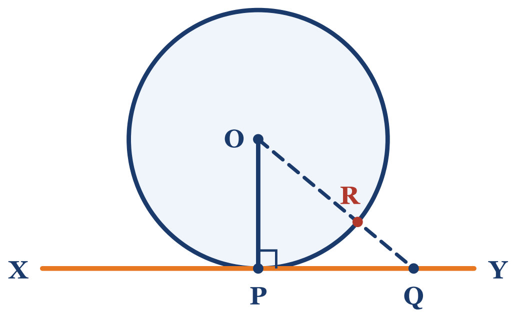

Fig. 10.B — Q is any other point on XY; OQ meets the circle at R, so OQ > OR = OP.

Note: *MLI-redrawn schematic, constructed from the given data (tangents truly tangent, right angles true). Never measure off the figure; work only from the given data.*

Step 1 **iven:** a circle with centre O and a tangent XY at P. **To prove:** OP ⊥ XY. ½

Step 2 ake any point Q on XY other than P. Q lies outside the circle (a tangent meets the circle only at P). Join OQ, meeting the circle at R. ½

Step 3 ince Q is outside the circle, OQ > OR = OP (radius). So OQ > OP for every point Q of XY other than P. 1

Step 4 lence OP is the shortest distance from O to the line XY, and the shortest segment from a point to a line is the perpendicular. ∴ OP ⊥ XY. 1

OP ⊥ XY ■

The key sentence is "of all segments from a point to a line, the perpendicular is the shortest". Without it, the proof is incomplete.

Q3 · Angle at the centre from a base angle

2019 · 2023 | 2 Marks

From an external point P, tangents PA and PB are drawn to a circle with centre O. If ∠PAB = 50°, find ∠AOB.

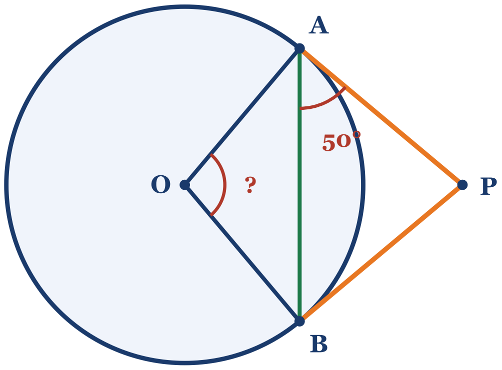

Fig. 10.C — PA = PB, so ∠PAB = ∠PBA = 50° and ∠APB = 80°.

Note: *MLI-redrawn schematic, constructed from the given data (tangents truly tangent, right angles true). Never measure off the figure; work only from the given data.*

Step 1 PA = PB (tangents from an external point), so △PAB is isosceles and ∠PBA = ∠PAB = 50°. Hence ∠APB = 180° − 100° = 80° **1**

Step 2 ∠AOB + ∠APB = 180° (OAPB has right angles at A and B) ⇒ ∠AOB = 100° **1**

∠AOB = 100°

∠PAB is not ∠OAP. ∠OAP is 90°; ∠PAB is the angle between the tangent and the chord AB.

Q4 · ∠PTQ = 2∠OPQ

2020 · 2023 · 2025 | 3 Marks

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2∠OPQ.

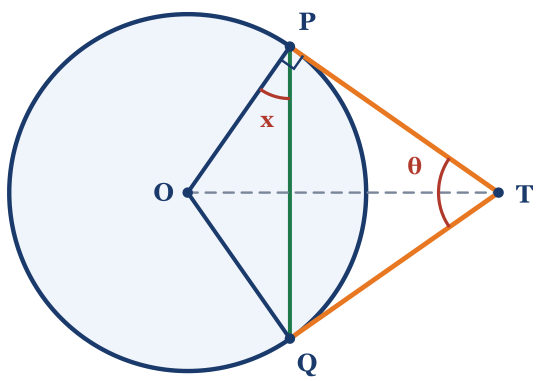

Fig. 10.D — θ = ∠PTQ and x = ∠OPQ. TP = TQ makes △TPQ isosceles; OP ⊥ TP.

Note: *MLI-redrawn schematic, constructed from the given data (tangents truly tangent, right angles true). Never measure off the figure; work only from the given data.*

Step 1 Let ∠PTQ = θ. TP = TQ (tangents from T), so △TPQ is isosceles and ∠TPQ = ∠TQP. **1**

Step 2 ∠TPQ = (180° − θ)/2 = 90° − θ/2 **½**

Step 3 ∠OPT = 90° (radius ⊥ tangent), so ∠OPQ = ∠OPT − ∠TPQ = 90° − (90° − θ/2) = θ/2 **1**

Step 4 Hence ∠PTQ = θ = 2∠OPQ **½**

∠PTQ = 2∠OPQ ■

Q5 · Incircle of a triangle 2019 · 2022 · 2024 **3 Marks**

A circle is inscribed in △ABC touching AB, BC and CA at D, E and F respectively. If AB = 10 cm, BC = 8 cm and CA = 12 cm, find AD, BE and CF.

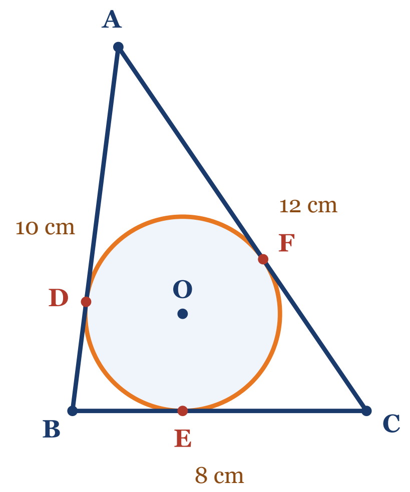

Fig. 10.E — D, E, F are the points of contact on AB, BC, CA.

Note: *MLI-redrawn schematic, constructed from the given data (tangents truly tangent, right angles true). Never measure off the figure; work only from the given data.*

Step 1 Tangents from a point are equal: AD = AF = x, BD = BE = y, CE = CF = z. **½**

Step 2 AB: x + y = 10; BC: y + z = 8; CA: z + x = 12. **1**

Step 3 Adding: 2(x + y + z) = 30 ⇒ x + y + z = 15. **½**

Step 4 = 15 − 8 = 7, y = 15 − 12 = 3, z = 15 − 10 = 5. **1**

AD = 7 cm, BE = 3 cm, CF = 5 cm

Check: AD + DB = 7 + 3 = 10 ✓, BE + EC = 3 + 5 = 8 ✓, CF + FA = 5 + 7 = 12 ✓.

Q6 · Missing side of a circumscribed quadrilateral

2020 · 2024 | 2 Marks

A quadrilateral ABCD is drawn to circumscribe a circle. If AB = 6 cm, BC = 7 cm and CD = 4 cm, find AD.

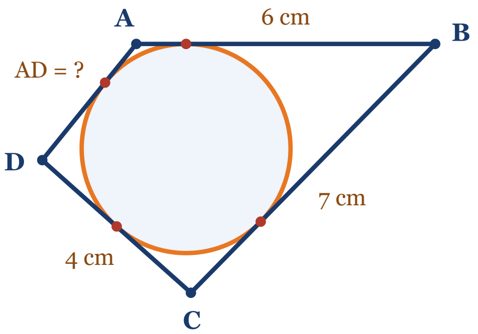

Fig. 10.F — Drawn to scale: a quadrilateral with AB = 6, BC = 7, CD = 4 cm circumscribing a circle forces AD = 3 cm.

Note: *MLI-redrawn schematic, constructed from the given data (tangents truly tangent, right angles true). Never measure off the figure; work only from the given data.*

Step 1 For a quadrilateral circumscribing a circle, AB + CD = BC + AD (equal tangents from each vertex). 1

Step 2 + 4 = 7 + AD ⇒ AD = 3 cm 1

AD = 3 cm

Pair the opposite sides. AB goes with CD, and BC with AD. Adding adjacent sides gives a wrong answer.

Q7 · Tangents at the ends of a chord 2020 · 2023 · 2025 **5 Marks**

PQ is a chord of length 8 cm of a circle of radius 5 cm with centre O. The tangents at P and Q intersect at T. Find the length TP.

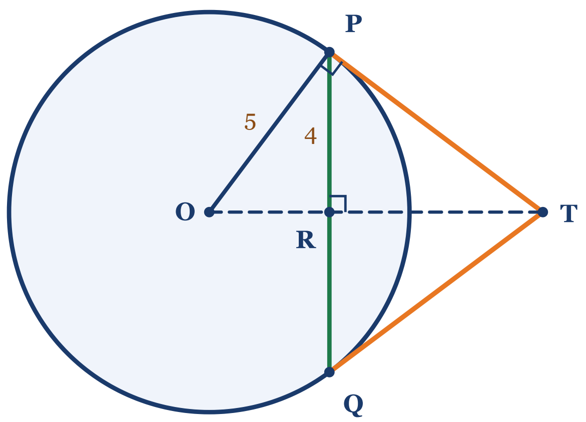

Fig. 10.G — OT is the perpendicular bisector of PQ, so PR = 4 cm and OR = 3 cm.

Note: *MLI-redrawn schematic, constructed from the given data (tangents truly tangent, right angles true). Never measure off the figure; work only from the given data.*

Step 1 Join OT, meeting PQ at R. $\triangle OPT \cong \triangle OQT$ (SSS), so OT bisects $\angle POQ$; then OT is the perpendicular bisector of PQ. So PR = 4 cm and $\angle ORP = 90°$. **1**

Step 2 In $\triangle ORP$: $OR = \sqrt{(OP^2 - PR^2)} = \sqrt{(25 - 16)} = 3$ cm **1**

Step 3 Let TR = y. In $\triangle TRP$: $TP^2 = y^2 + 16$ ...(i). In $\triangle OPT$ (right-angled at P): $OT^2 = TP^2 + OP^2 \Rightarrow (y + 3)^2 = TP^2 + 25$ ...(ii) **1**

Step 4 Put (i) in (ii): $y^2 + 6y + 9 = y^2 + 16 + 25 \Rightarrow 6y = 32 \Rightarrow y = 16/3$ cm **1**

Step 5 $TP^2 = (16/3)^2 + 16 = 256/9 + 144/9 = 400/9 \Rightarrow TP = 20/3$ cm **1**

TP = 20/3 cm ≈ 6.67 cm

Two right triangles share TR. One equation from $\triangle TRP$ and one from $\triangle OPT$; the y^2 terms cancel, leaving a linear equation.

Q8 · Tangents at the ends of a chord make equal angles

2021 · 2023 | 2 Marks

Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

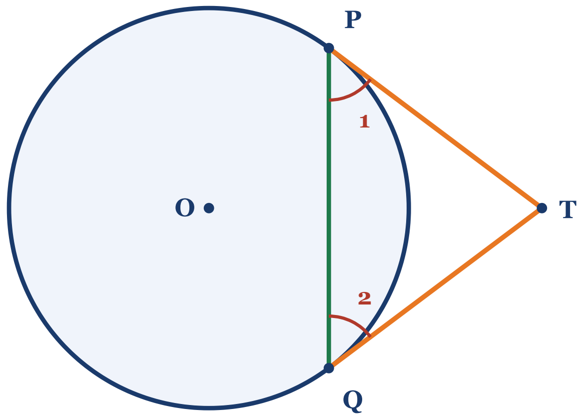

Fig. 10.H — The tangents at P and Q meet at T. ∠1 = ∠TPQ, ∠2 = ∠TQP; to show ∠1 = ∠2.

Note: *MLI-redrawn schematic, constructed from the given data (tangents truly tangent, right angles true). Never measure off the figure; work only from the given data.*

Step 1 Let the tangents at the ends P and Q of chord PQ meet at T. Then TP = TQ (tangents from an external point). **1**

Step 2 So △TPQ is isosceles, and the angles opposite the equal sides are equal: ∠TPQ = ∠TQP. **1**

The tangents make equal angles with the chord ■

Q9 · OT is the perpendicular bisector of PQ

2022 · 2024 | 3 Marks

From an external point T, tangents TP and TQ are drawn to a circle with centre O. Prove that OT is the perpendicular bisector of the line segment PQ.

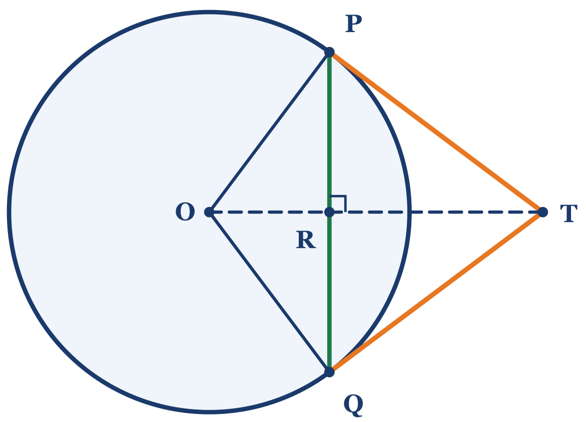

Fig. 10.I — To prove: PR = RQ and ∠PRT = 90°, i.e. OT is the perpendicular bisector of PQ.

Note: *MLI-redrawn schematic, constructed from the given data (tangents truly tangent, right angles true). Never measure off the figure; work only from the given data.*

Step 1 △OPT ≅ △OQT (OP = OQ radii, TP = TQ tangents, OT common — SSS), so ∠PTO = ∠QTO. **1**

Step 2 Let OT meet PQ at R. In △PTR and △QTR: TP = TQ, ∠PTR = ∠QTR, TR common ⇒ △PTR ≅ △QTR (SAS). **1**

Step 3 Hence PR = RQ and ∠PRT = ∠QRT (CPCT). These are a linear pair, so each is 90°. **1**

OT ⊥ PQ and bisects it ■

Two congruences, in order. The first gives the equal angles at T; only then can the second (SAS) be written.

Q10 · Objective

2022 · 2025 | 1 Mark

If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then the length of each tangent is

(a) 3√3/2 cm (b) 6 cm (c) 3 cm (d) 3√3 cm

Step 1 OP bisects the 60°, so in right △OAP: tan 30° = OA/PA ⇒ PA = 3 ÷ (1/√3) = 3√3 cm.

(d) 3√3 cm

Q11 · Objective 2023 · 2024 1 Mark

The number of tangents that can be drawn to a circle from a point lying inside it is

(a) 0 (b) 1 (c) 2 (d) infinitely many

Point on the circle: 1 tangent. Point outside: 2 tangents.

Step 1 Every line through an interior point cuts the circle in two points, so none of them is a tangent.

(a) 0

tip

CHAPTER 11

Areas Related to Circles

Unit VI · Mensuration 10 marks (with Ch 12) · Sector and segment every year

📈 What CBSE actually asks from this chapter

Question type	Frequency	Usual marks
Area of a sector and length of its arc	★★★★★	2–3
Area of a **segment** (sector minus triangle)	★★★★★	3–5
Grazing area of a tied animal, corner sectors of a square field	★★★★	3
Area swept by the hands of a clock / a windscreen wiper	★★★★	3
Shaded-region figures built from circles and squares	★★★★	3–5

The sector formulas. For a sector of angle θ° in a circle of radius r:

Area = $\boxed{(\theta/360) \times \pi r^2}$ Arc length = $\boxed{(\theta/360) \times 2\pi r}$ Perimeter of a sector = $\boxed{\text{arc} + 2r}$

Area of a segment = area of the sector − area of the triangle formed by the two radii and the chord.

Q1 · Angle and area from the arc

2020 · 2023 · 2025 | **3 Marks**

The length of an arc of a sector of a circle of radius 14 cm is 22 cm. Find the angle of the sector and its area. (Take $\pi = 22/7$)

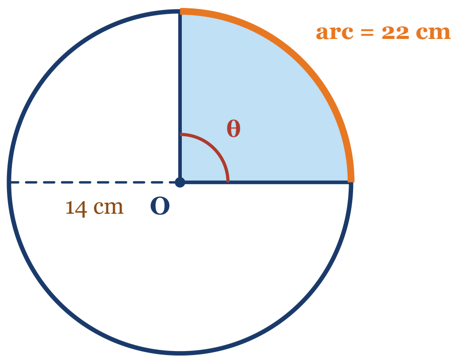

Fig. 11.A — The arc (orange) is 22 cm long; θ is to be found.

Note: *MLI-redrawn figure, constructed from the given data (angles true, lengths to scale). Work only from the given data, never by measuring the figure.*

Step 1 Arc $= (\theta/360) \times 2\pi r \Rightarrow 22 = (\theta/360) \times 2 \times (22/7) \times 14 = (\theta/360) \times 88$ **1**

Step 2 $\theta = 22 \times 360/88 = 90°$ **½**

Step 3 Area $= (90/360) \times (22/7) \times 14 \times 14 = (1/4) \times 616 = 154\ cm^2$ **1**

Step 4 Check with Area $= ½ \times arc \times r = ½ \times 22 \times 14 = 154\ cm^2$ ✓ **½**

$\theta = 90°$, area $= 154\ cm^2$

Area of a sector = ½ × arc length × radius. It gives the area directly, without finding θ first.

Q2 · Minor segment (right angle) 2018 · 2022 · 2024 **3 Marks**

Find the area of the minor segment of a circle of radius 14 cm when the corresponding chord subtends a right angle at the centre. (Take π = 22/7)

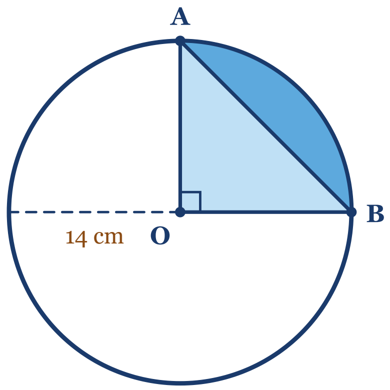

Fig. 11.B — Chord AB subtends 90°; dark = minor segment = sector − right triangle OAB.

Note: *MLI-redrawn figure, constructed from the given data (angles true, lengths to scale). Work only from the given data, never by measuring the figure.*

Step 1 Area of sector OAB = (90/360) × (22/7) × 14 × 14 = 154 cm^2 **1**

Step 2 △OAB is right-angled at O with legs 14 cm: area = ½ × 14 × 14 = 98 cm^2 **1**

Step 3 Minor segment = 154 − 98 = 56 cm^2 **1**

Area of the minor segment = 56 cm^2

Segment = sector − triangle. Stopping at 154 cm^2 (the sector) is the commonest loss of marks.

Q3 · Square with quadrants at the corners 2019 · 2023 · 2025 **3 Marks**

ABCD is a square of side 14 cm. With centres A, B, C and D, four quadrants of radius 7 cm are drawn inside the square. Find the area of the region of the square not covered by the quadrants. (Take π = 22/7)

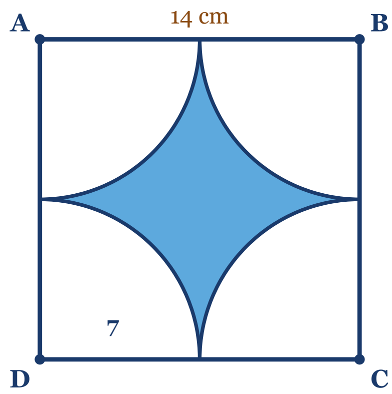

Fig. 11.C — Four quadrants of radius 7 cm at the corners; the shaded part is what remains of the square.

Note: *MLI-redrawn figure, constructed from the given data (angles true, lengths to scale). Work only from the given data, never by measuring the figure.*

Step 1 Area of the square = 14 × 14 = 196 cm^2 ½

Step 2 Four quadrants of equal radius together make one full circle of radius 7 cm: area = (22/7) × 7 × 7 = 154 cm^2 1½

Step 3 Shaded area = 196 − 154 = 42 cm^2 1

Area of the shaded region = 42 cm^2

4 quadrants = 1 circle. Each corner angle of a square is 90°, so four quarter-circles of the same radius add up to exactly one circle.

Q4 · Petals from four semicircles

2019 · 2024 | 5 Marks

ABCD is a square of side 14 cm. Semicircles are drawn inside the square on each of its four sides as diameter. Find the area of the region common to the semicircles (the four "petals"). (Take $\pi = 22/7$)

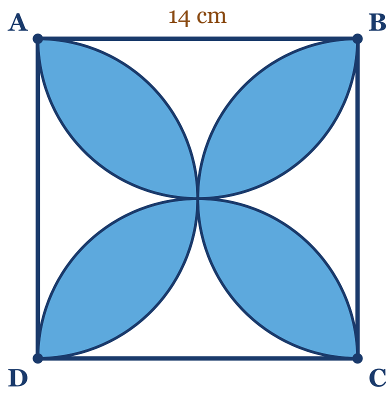

Fig. 11.D — Semicircles (dashed) on all four sides, drawn inwards; their overlaps form the four shaded petals.

Note: *MLI-redrawn figure, constructed from the given data (angles true, lengths to scale). Work only from the given data, never by measuring the figure.*

Step 1 Each semicircle has radius 7 cm. Total area of the four semicircles $= 4 \times \frac{1}{2} \times (22/7) \times 7 \times 7 = 308$ cm² **1**

Step 2 The four semicircles together cover the square completely, and each petal is covered twice (by two semicircles). **1**

Step 3 So (sum of semicircle areas) = (area of square) + (area of petals) **1**

Step 4 $308 = 196 +$ petals **1**

Step 5 Area of the petals $= 308 - 196 = 112$ cm² **1**

Area of the shaded petals = 112 cm²

Check by segments: each petal $= 2 \times$ (quadrant − right triangle) $= 2 \times (38.5 - 24.5) = 28$ cm², and $4 \times 28 = 112$ cm² ✓

Q5 · Minor segment (60°)

2020 · 2023 | 3 Marks

A chord of a circle of radius 14 cm subtends an angle of 60° at the centre. Find the area of the corresponding minor segment. (Take $\pi = 22/7$ and $\sqrt{3} = 1.73$)

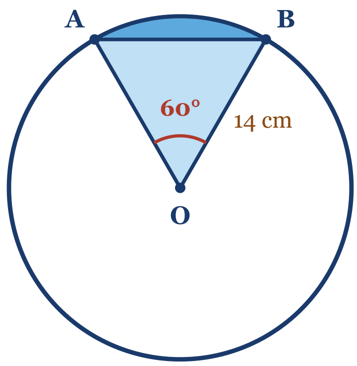

Fig. 11.E — Chord AB subtends 60°; triangle OAB is equilateral. Dark: minor segment.

Note: *MLI-redrawn figure, constructed from the given data (angles true, lengths to scale). Work only from the given data, never by measuring the figure.*

Step 1 Area of sector = $(60/360) \times (22/7) \times 14 \times 14 = 308/3 = 102.67$ cm^2 **1**

Step 2 OA = OB = 14 cm and ∠AOB = 60°, so △OAB is equilateral: area = $(\sqrt{3}/4) \times 14^2 = 49\sqrt{3} = 49 \times 1.73 = 84.77$ cm^2 **1**

Step 3 Minor segment = $102.67 - 84.77 = 17.90$ cm^2 **1**

Area of the minor segment ≈ 17.90 cm^2

60° means equilateral. Two radii with 60° between them force the third side to equal the radius too.

Q6 · Circular race track

2019 · 2022 | 3 Marks

A race track is in the form of a ring whose inner circumference is 352 m and outer circumference is 396 m. Find the width of the track and its area. (Take $\pi = 22/7$)

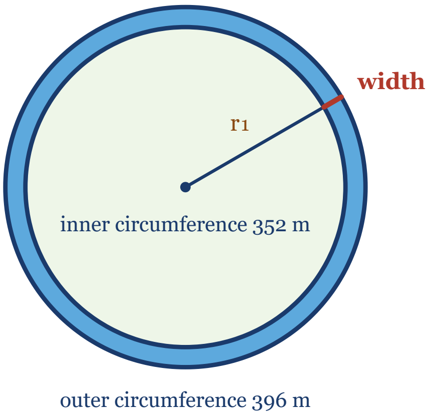

Fig. 11.F — The track is the ring between the inner (352 m) and outer (396 m) circles.

Note: *MLI-redrawn figure, constructed from the given data (angles true, lengths to scale). Work only from the given data, never by measuring the figure.*

Step 1 $2\pi r_1 = 352 \Rightarrow r_1 = 352 \times 7/44 = 56$ m; $2\pi r_2 = 396 \Rightarrow r_2 = 396 \times 7/44 = 63$ m **1**

Step 2 Width $= r_2 - r_1 = 63 - 56 = 7$ m **½**

Step 3 Area of the track $= \pi(r_2^2 - r_1^2) = (22/7)(63 + 56)(63 - 56) = (22/7) \times 119 \times 7$ **1**

Step 4 $= 22 \times 119 = 2618$ m^2 **½**

Width = 7 m; area of the track = 2618 m^2

Use $a^2 - b^2 = (a + b)(a - b)$. It turns $63^2 - 56^2$ into 119×7, and the 7 cancels with $\pi = 22/7$.

Q7 · Swinging pendulum

2020 · 2024 | 2 Marks

A pendulum swings through an angle of 30° and describes an arc 8.8 cm in length. Find the length of the pendulum. (Take π = 22/7)

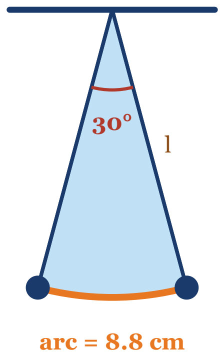

Fig. 11.G — The bob moves along an arc of 8.8 cm while the string turns through 30°.

Note: *MLI-redrawn figure, constructed from the given data (angles true, lengths to scale). Work only from the given data, never by measuring the figure.*

Step 1 The bob moves along an arc of radius l (the length of the pendulum): 8.8 = (30/360) × 2 × (22/7) × l **1**

Step 2 l = 8.8 × 360 × 7/(30 × 44) = 16.8 cm **1**

Length of the pendulum = 16.8 cm

Q8 · Square inscribed in a circle

2021 · 2023 | 3 Marks

A square ABCD is inscribed in a circle of radius 7 cm. Find the area of the region inside the circle but outside the square. (Take $\pi = 22/7$)

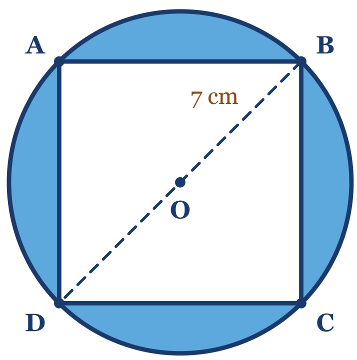

Fig. 11.H — Square ABCD inscribed in the circle; its diagonal is the diameter (14 cm). Shaded: the four segments.

Note: *MLI-redrawn figure, constructed from the given data (angles true, lengths to scale). Work only from the given data, never by measuring the figure.*

Step 1 Area of the circle = $(22/7) \times 7 \times 7 = 154$ cm^2 **1**

Step 2 The diagonal of the square is a diameter = 14 cm, so area of square = $\frac{1}{2} \times d^2 = \frac{1}{2} \times 196 = 98$ cm^2 **1**

Step 3 Required area = $154 - 98 = 56$ cm^2 **1**

Area of the shaded region = 56 cm^2

The diagonal, not the side, is 14 cm. Taking the side as 14 cm gives a square larger than the circle — an impossible answer.

Q9 · Objective

2022 · 2024 | 1 Mark

If the circumference and the area of a circle are numerically equal, then the radius of the circle is

(a) 1 unit (b) 2 units (c) 4 units (d) π units

Step 1 $2\pi r = \pi r^2 \Rightarrow r = 2$ $(r \neq 0)$.

(b) 2 units

Q10 · Objective

2023 · 2025 | 1 Mark

The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. The area of the sector is

(a) 31.2 cm^2 (b) 15.6 cm^2 (c) 7.8 cm^2 (d) 26 cm^2

Perimeter of a sector = arc + 2r. Subtract both radii before using the arc.

Step 1 Arc = 16.4 − 2 × 5.2 = 6 cm; area = ½ × 6 × 5.2 = 15.6 cm^2.

(b) 15.6 cm^2

tip

CHAPTER 12

Surface Areas and Volumes

Unit VI · Mensuration · A combination-of-solids question is near-certain

📈 What CBSE actually asks from this chapter

Question type	Frequency	Usual marks
Combination of solids — cone on hemisphere, cylinder with hemispherical ends	★★★★★	5
Cost of painting / plastering a combined solid	★★★★	5
Hollow solids — pipes, cylindrical vessels	★★★	3

Out of syllabus — do not study: the **frustum of a cone**, and **conversion of one solid into another** (melting and recasting, water flowing through a pipe into a tank). Only combinations of two solids remain.

Solid	Curved / Lateral surface	Total surface	Volume
Cylinder	$2\pi rh$	$2\pi r(r + h)$	$\pi r^2 h$
Cone	$\pi r\ell$	$\pi r(r + \ell)$	$\frac{1}{3}\pi r^2 h$
Sphere	$4\pi r^2$	$4\pi r^2$	$\frac{2}{3}\pi r^3 \times 2 = (4/3)\pi r^3$
Hemisphere	$2\pi r^2$	$3\pi r^2$	$(2/3)\pi r^3$

Slant height of a cone: $\ell = \sqrt{(r^2 + h^2)}$

Q1 · Cost of painting a tank

2019 · 2023 | 3 Marks

A cylindrical water tank of diameter 14 m and height 5 m is to be painted on its curved surface and its top. Find the cost of painting at ₹25 per m^2. (Take $\pi = 22/7$)

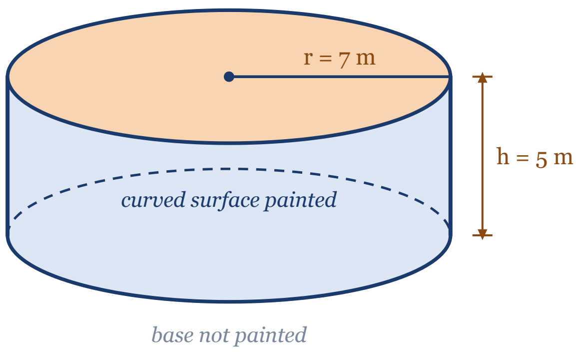

Fig. 12.A — Painted: curved surface and the top (orange); the base is not painted.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Step 1 $= 7$ m, $h = 5$ m. Curved surface $= 2\pi rh = 2 \times (22/7) \times 7 \times 5 = 220$ m^2 **1**

Step 2 Top $= \pi r^2 = (22/7) \times 49 = 154$ m^2; area to be painted $= 220 + 154 = 374$ m^2 **1**

Step 3 Cost $= 374 \times 25 =$ ₹9,350 **1**

The cost of painting is ₹9,350

Read which faces are included. "Curved surface and top" excludes the base; adding it is the standard error.

Q2 · Volume of a toy (cone on hemisphere)

2019 · 2023 · 2025 | 3 Marks

A solid toy is in the form of a hemisphere surmounted by a right circular cone of the same radius. The height of the cone is 10 cm and the radius of its base is 7 cm. Find the volume of the toy. (Take π = 22/7)

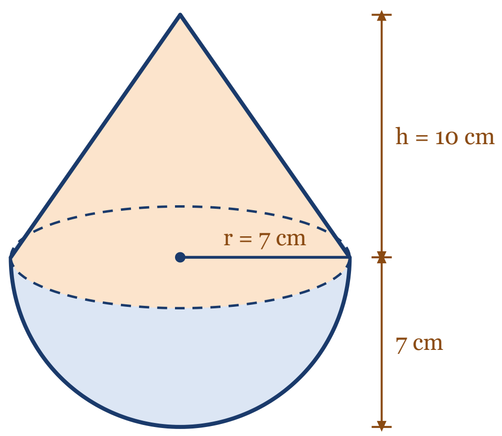

Fig. 12.B — Cone (height 10 cm) on a hemisphere, both of radius 7 cm.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Step 1 Volume = volume of cone + volume of hemisphere = $\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$ ½

Step 2 = ⅓ × (22/7) × 49 × 10 + ⅔ × (22/7) × 343 1

Step 3 = (22/7) × (490 + 686)/3 = (22/7) × 392 = 1232 cm^3 1½

Volume of the toy = 1232 cm^3

Volumes simply add. Unlike surface area, nothing is lost at the joint.

Q3 · Volume of a capsule-shaped solid

2020 · 2024 | 3 Marks

A solid is in the shape of a cylinder with two hemispherical ends. Its total length is 20 cm and its diameter is 7 cm. Find its volume. (Take $\pi = 22/7$)

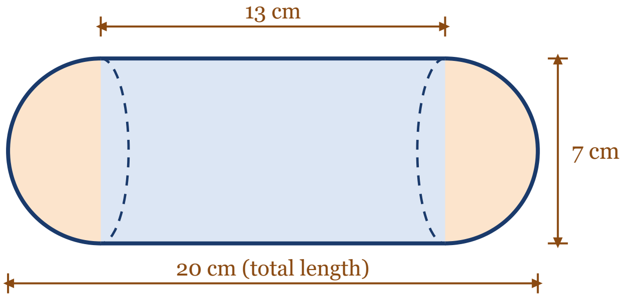

Fig. 12.C — A cylinder with a hemisphere at each end: radius 3.5 cm, cylindrical part 13 cm.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Step 1 = 3.5 cm; length of the cylindrical part = 20 − 2 × 3.5 = 13 cm **1**

Step 2 wo hemispheres make one sphere: Volume = $\pi r^2 h + (4/3)\pi r^3$ **½**

Step 3 = (22/7)(12.25)(13) + (4/3)(22/7)(42.875) = 500.5 + 179.67 **1**

Step 4 = 680.17 cm^3 (approx.) **½**

Volume ≈ 680.17 cm^3

Subtract 2r, not r. There is a hemisphere at each end.

Q4 · Cylinder with two conical holes

2019 · 2022 | 3 Marks

From a solid cylinder of height 14 cm and base diameter 7 cm, two equal conical holes, each of radius 2.1 cm and height 4 cm, are cut off (one from each end). Find the volume of the remaining solid. (Take $\pi = 22/7$)

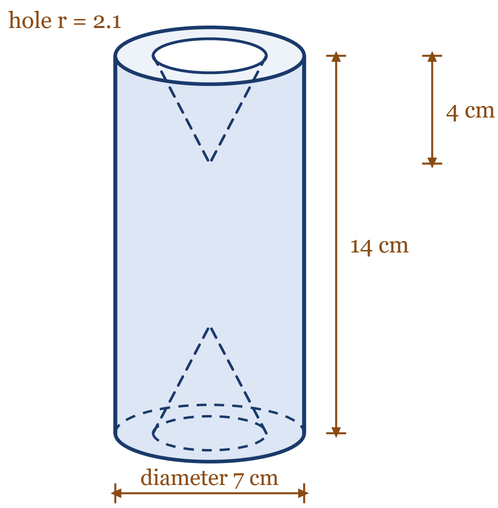

Fig. 12.D — Two equal conical holes (radius 2.1 cm, depth 4 cm, dashed), one from each end.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Step 1 Volume of cylinder = $(22/7) \times 3.5 \times 3.5 \times 14 = 539 \text{ cm}^3$ 1

Step 2 Volume of one cone = $\frac{1}{3} \times (22/7) \times 2.1 \times 2.1 \times 4 = 18.48 \text{ cm}^3$; two cones = 36.96 cm^3 1

Step 3 Remaining volume = $539 - 36.96 = 502.04 \text{ cm}^3$ 1

Volume of the remaining solid = 502.04 cm^3

Q5 · Canvas for a circus tent 2018 · 2023 **3 Marks**

A circus tent is cylindrical to a height of 3 m and conical above it. If its base radius is 52.5 m and the slant height of the conical part is 53 m, find the area of the canvas needed. (Take π = 22/7)

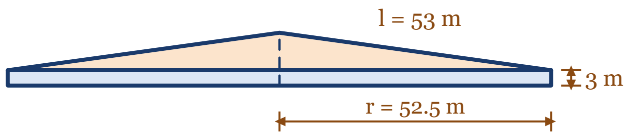

Fig. 12.E — Front view (to scale): a 3 m cylindrical wall under a very flat cone of slant height 53 m.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Step 1 Canvas = CSA of cylinder + CSA of cone = $2\pi rh + \pi rl = \pi r(2h + l)$ **1**

Step 2 $= (22/7) \times 52.5 \times (6 + 53) = 165 \times 59$ **1**

Step 3 $= 9735\ \text{m}^2$ **1**

Canvas needed = $9735\ \text{m}^2$

No base, no joint. The ground is not covered, and the circle where the cone meets the cylinder is inside.

Q6 · Apparent and actual capacity of a glass

2023 · 2024 | 3 Marks

A juice-seller serves juice in a cylindrical glass of inner diameter 5 cm and height 10 cm, but the bottom of the glass has a hemispherical raised portion. Find the apparent capacity of the glass and its actual capacity. (Use $\pi = 3.14$)

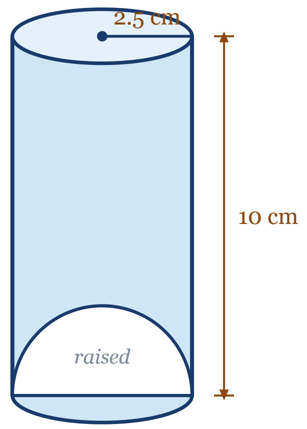

Fig. 12.F — A glass with a hemispherical raised bottom (white): the true capacity excludes the hemisphere.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Step 1 Apparent capacity = $\pi r^2 h = 3.14 \times 2.5 \times 2.5 \times 10 = 196.25\ \text{cm}^3$ **1**

Step 2 Volume of the raised hemisphere = $\frac{2}{3}\pi r^3 = \frac{2}{3} \times 3.14 \times 15.625 = 32.71\ \text{cm}^3$ **1**

Step 3 Actual capacity = $196.25 - 32.71 = 163.54\ \text{cm}^3$ **1**

Apparent capacity = $196.25\ \text{cm}^3$; actual capacity ≈ $163.54\ \text{cm}^3$

The raised part holds no juice, so its volume is subtracted from what the glass appears to hold.

Q7 · Three solids combined

2020 · 2024 | 5 Marks

A solid is in the form of a right circular cylinder with a hemisphere at one end and a cone at the other end. Their common radius is 7 cm, and the heights of the cylindrical and conical parts are 10 cm and 6 cm respectively. Find the total surface area of the solid. (Take $\pi = 22/7$ and $\sqrt{85} \approx 9.22$)

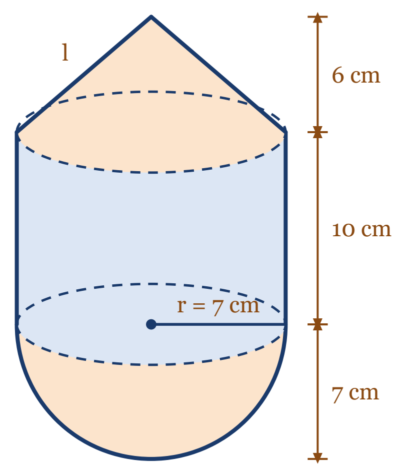

Fig. 12.G — Cone (6 cm) on a cylinder (10 cm) on a hemisphere, all of radius 7 cm.

Note: *MLI-redrawn figure, drawn to scale from the given data (dashed lines = hidden edges). Work only from the given data, never by measuring the figure.*

Step 1 Only curved surfaces are exposed: TSA = CSA of cylinder + CSA of hemisphere + CSA of cone **1**

Step 2 CSA of cylinder = $2\pi rh = 2 \times (22/7) \times 7 \times 10 = 440$ cm^2 **1**

Step 3 CSA of hemisphere = $2\pi r^2 = 2 \times (22/7) \times 49 = 308$ cm^2 **1**

Step 4 Slant height $l = \sqrt{(7^2 + 6^2)} = \sqrt{85} \approx 9.22$ cm; CSA of cone = $\pi rl = 22 \times 9.22 = 202.84$ cm^2 **1**

Step 5 TSA = 440 + 308 + 202.84 = 950.84 cm^2 **1**

Total surface area ≈ 950.84 cm^2

Use l, not h, for the cone. Writing $\pi rh = 22 \times 6 = 132$ is the commonest slip here.

Q8 · Objective

2022 · 2025 | 1 Mark

The total surface area of a solid hemisphere of radius 7 cm is

(a) 154 cm^2 (b) 308 cm^2 (c) 462 cm^2 (d) 616 cm^2

Step 1 TSA = curved $2\pi r^2$ + flat $\pi r^2 = 3\pi r^2 = 3 \times (22/7) \times 49 = 462$ cm^2.

(c) 462 cm^2

Q9 · Objective 2023 · 2024 1 Mark

A cylinder and a cone have equal radii and equal heights. The ratio of the volume of the cylinder to that of the cone is

(a) 1 : 3 (b) 3 : 1 (c) 2 : 1 (d) 1 : 2

Step 1 $\pi r^2h : \frac{1}{3}\pi r^2h = 3 : 1$.

(b) 3 : 1

CHAPTER 13

Statistics

Unit VII · Statistics & Probability 11 marks · Mean, median or mode every year

📈 What CBSE actually asks from this chapter

Question type	Frequency	Usual marks
Mean of grouped data (direct, assumed-mean or step-deviation)	★★★★★	3
Median of grouped data	★★★★★	3
Mode of grouped data	★★★★★	3
Find missing frequencies x, y when the mean or median is given	★★★★★	5
Use the empirical relation 3 Median = Mode + 2 Mean	★★★	1–2

Out of syllabus — do not study: ogives and the graphical determination of the median (less-than and more-than curves). Cumulative frequency **tables** are still needed for the median — only the **graph** is gone.

The three formulas.

Mean (step-deviation): $\boxed{\bar{x} = a + h \times (\Sigma f_i u_i \,/\, \Sigma f_i)}$ where $u_i = (x_i - a)/h$

Median: $\boxed{M = l + [(n/2 - cf)/f] \times h}$

Mode: $\boxed{Z = l + [(f_1 - f_0)/(2f_1 - f_0 - f_2)] \times h}$

Empirical relation: $\boxed{3\ \text{Median} = \text{Mode} + 2\ \text{Mean}}$

Q1 · Median of grouped data

2020 · 2023 · 2025 **3 Marks**

Find the median of the following data:

Class	0–10	10–20	20–30	30–40	40–50
Frequency	2	12	22	8	6

Step 1 Cumulative frequency table:

Class	f	cf
0–10	2	2
10–20	12	14
20–30	22	36
30–40	8	44
40–50	6	50
Total	**n = 50**	**—**

1

Step 2 n/2 = 25. The cf just greater than 25 is 36, so the median class is 20–30: l = 20, cf = 14, f = 22, h = 10.

½

Step 3 Median = l + [(n/2 − cf)/f] × h = 20 + [(25 − 14)/22] × 10 = 20 + 5 = 25 **1½**

Median = 25

cf is that of the class *before* the median class (14 here), not of the median class itself (36).

Q2 · Missing frequencies from the median

2018 · 2022 · 2024 | 5 Marks

The median of the following data is 525 and the total frequency is 100. Find x and y.

Class	0–100	100–200	200–300	300–400	400–500	500–600	600–700	700–800	800–900	900–1000
Frequency	2	5	x	12	17	20	y	9	7	4

Step 1 otal: $2 + 5 + x + 12 + 17 + 20 + y + 9 + 7 + 4 = 100 \Rightarrow 76 + x + y = 100 \Rightarrow x + y = 24$ ...(i) **1**

Step 2 Median 525 lies in 500–600, so $l = 500$, $f = 20$, $h = 100$ and $n/2 = 50$. **1**

Step 3 f before the median class $= 2 + 5 + x + 12 + 17 = 36 + x$ **1**

Step 4 $25 = 500 + [(50 - 36 - x)/20] \times 100 \Rightarrow 25 = 5(14 - x) \Rightarrow x = 9$ **1**

Step 5 rom (i): $y = 24 - 9 = 15$ **1**

x = 9 and y = 15

Two unknowns need two equations: the total gives one, the median formula the other. Write the total equation first — it is a free mark.

Q3 · Mode of grouped data

2020 · 2023 · 2024 | 3 Marks

Find the mode of the following data:

Class	0–20	20–40	40–60	60–80	80–100	100–120
Frequency	8	10	10	16	12	6

Step 1 Highest frequency = 16, so the modal class is 60–80: $l = 60$, $f_1 = 16$, $f_0 = 10$ (class before), $f_2 = 12$ (class after), $h = 20$. **1**

Step 2 Mode $= l + [(f_1 - f_0)/(2f_1 - f_0 - f_2)] \times h = 60 + [6/(32 - 10 - 12)] \times 20$ **1**

Step 3 $= 60 + (6/10) \times 20 = 72$ **1**

Mode = 72

Both f_0 and f_2 are subtracted in the denominator: $2f_1 - f_0 - f_2$.

Q4 · Mean by step-deviation 2019 · 2022 · 2025 **3 Marks**

Find the mean of the following distribution:

Class	10–25	25–40	40–55	55–70	70–85	85–100
Frequency	2	3	7	6	6	6

Step 1 Take $a = 47.5$ and $h = 15$, $u_i = (x_i - 47.5)/15$:

Class	f_i	x_i	u_i	f_iu_i
10–25	2	17.5	-2	-4
25–40	3	32.5	-1	-3
40–55	7	47.5	0	0
55–70	6	62.5	1	6
70–85	6	77.5	2	12
85–100	6	92.5	3	18
Total	**30**			**29**

1½

Step 2 Mean $= a + h(\Sigma f_iu_i/\Sigma f_i) = 47.5 + 15 \times (29/30) = 47.5 + 14.5 = 62$ 1½

Mean = 62

Show the table. Marks are given for the x_i, u_i and f_iu_i columns even if the final arithmetic slips.

Q5 · Missing frequency from the mean

2019 · 2023 | 3 Marks

The mean of the following distribution is 50. Find the value of p.

Class	0–20	20–40	40–60	60–80	80–100
Frequency	17	28	32	p	19

Step 1 Working table (direct method):

Class	f_i	x_i	$f_i x_i$
0–20	17	10	170
20–40	28	30	840
40–60	32	50	1600
60–80	p	70	70p
80–100	19	90	1710
Total	**96 + p**		**4320 + 70p**

1

Step 2 Mean $= \Sigma f_i x_i / \Sigma f_i \Rightarrow 50 = (4320 + 70p)/(96 + p)$ ½

Step 3 $4800 + 50p = 4320 + 70p \Rightarrow 20p = 480 \Rightarrow p = 24$ 1½

p = 24

Check: $\Sigma f = 120$, $\Sigma fx = 4320 + 1680 = 6000$, and $6000/120 = 50$ ✓

Q6 · Empirical relation

2021 · 2024 | 1 Mark

If the mean and median of a distribution are 28 and 26 respectively, find the mode.

Step 1 3 Median = Mode + 2 Mean ⇒ Mode $= 3(26) - 2(28) = 78 - 56 = 22$

Mode = 22

Q7 · Objective

2023 · 2025 | 1 Mark

For a distribution, mode = 24 and mean = 60. Its median is

(a) 36 (b) 42 (c) 48 (d) 52

Step 1 3 Median $= 24 + 2(60) = 144 \Rightarrow$ Median = 48.

(c) 48

CHAPTER 14

Probability

Unit VII · The most scoring chapter in the paper — treat it as free marks

📈 What CBSE actually asks from this chapter

Question type	Frequency	Usual marks
Cards from a well-shuffled deck of 52	★★★★★	2–3
Two dice thrown together — sum, product, doublets	★★★★★	3
Balls drawn from a bag, with a condition fixing an unknown count	★★★★★	3
Numbered cards / tickets — primes, perfect squares, divisibility	★★★★	2–3
P(not E) = 1 − P(E) as a one-mark item	★★★★	1

Note. Only **theoretical** probability of single events is in the syllabus. The old set-theory / conditional-probability material is not.

P(E) = (number of favourable outcomes) ÷ (total number of outcomes), with $0 \le P(E) \le 1$ and $P(\text{not } E) = 1 - P(E)$.
Deck of 52: 26 red + 26 black · four suits of 13 (hearts and diamonds red; spades and clubs black) · 12 face cards (4 each of king, queen, jack) · 4 aces. An ace is **not** a face card.
Two dice: always 36 outcomes, never 21 — (2, 3) and (3, 2) are different.

Q1 · Cards

2019 · 2022 · 2024 | 3 Marks

One card is drawn at random from a well-shuffled deck of 52 cards. Find the probability that the card is (a) a king, (b) a red card that is not a face card, (c) neither a heart nor a king.

Step 1 Total outcomes = 52. ½

Step 2 (a) There are 4 kings: P = 4/52 = 1/13 ½

Step 3 (b) Red cards = 26, of which 6 are face cards (K, Q, J of hearts and of diamonds). Red non-face cards = 20: P = 20/52 = 5/13 1

Step 4 (c) Hearts = 13, plus the 3 kings that are not hearts = 16 cards excluded. Remaining = 36: P = 36/52 = 9/13 1

(a) 1/13 (b) 5/13 (c) 9/13

Count the overlap once. In (c), 13 + 4 = 17 is wrong: the king of hearts is in both groups.

Q2 · Two dice

2020 · 2023 · 2025 **3 Marks**

Two dice are thrown together. Find the probability of getting (a) a sum of 9, (b) a doublet, (c) a sum less than or equal to 4.

first die

second die	1	2	3	4	5	6
1	**2**	**3**	**4**	5	6	7
2	**3**	**4**	5	6	7	8
3	**4**	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

orange: sum 9 (4 cells) · blue: doublets (6) · bold: sum ≤ 4 (6)

Fig. 14.A — Each cell shows the sum on the two dice; all 36 cells are equally likely.

Note: *MLI-drawn table of all 36 equally likely outcomes.*

Step 1 Total outcomes = 6 × 6 = 36 (see the table). ½

Step 2 (a) Sum 9: (3,6), (4,5), (5,4), (6,3) — 4 outcomes: P = 4/36 = 1/9 1

Step 3 (b) Doublets: (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) — 6 outcomes: P = 6/36 = 1/6 ½

Step 4 (c) Sum ≤ 4: (1,1), (1,2), (2,1), (1,3), (3,1), (2,2) — 6 outcomes: P = 6/36 = 1/6 1

(a) 1/9 (b) 1/6 (c) 1/6

(3, 6) and (6, 3) are different outcomes. The denominator is always 36, never 21.

Q3 · Bag of balls with an unknown

2018 · 2023 · 2024 | 3 Marks

A bag contains 12 balls, of which x are white. If 6 more white balls are put in the bag, the probability of drawing a white ball becomes double what it was before. Find x.

Step 1 Originally P(white) = x/12. 1

Step 2 After adding 6 white balls: total = 18, white = x + 6, so P = (x + 6)/18. 1

Step 3 (x + 6)/18 = 2 × x/12 = x/6 ⇒ 6x + 36 = 18x ⇒ x = 3 1

x = 3

The total changes too: after adding 6 balls the denominator is 18, not 12.

Q4 · Numbered cards

2021 · 2024 | 3 Marks

Cards numbered 1 to 30 are put in a box and one is drawn at random. Find the probability that the number on the card is (a) a prime number, (b) a perfect square, (c) divisible by both 3 and 5.

Step 1 Total outcomes = 30. ½

Step 2 (a) Primes: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 — 10 numbers: P = 10/30 = 1/3 1

Step 3 (b) Perfect squares: 1, 4, 9, 16, 25 — 5 numbers: P = 5/30 = 1/6 ½

Step 4 (c) Divisible by both 3 and 5 means divisible by 15: 15, 30 — 2 numbers: P = 2/30 = 1/15 1

(a) 1/3 (b) 1/6 (c) 1/15

1 is not prime, but 1 is a perfect square. Both facts are tested on purpose.

Q5 · Product on two dice

2022 · 2024 | 2 Marks

Two dice are thrown at the same time. Find the probability that the product of the numbers on them is a prime number.

Step 1 A product is prime only if one die shows 1 and the other a prime: (1,2), (2,1), (1,3), (3,1), (1,5), (5,1) — 6 outcomes. 1

Step 2 P = 6/36 = 1/6 1

P = 1/6

(1, 1) gives product 1, which is not prime. Any product with both numbers greater than 1 is composite.

Q6 · Three coins

2020 · 2023 | 2 Marks

Three unbiased coins are tossed together. Find the probability of getting at least two heads.

Step 1 Sample space: HHH, HHT, HTH, THH, HTT, THT, TTH, TTT — 8 outcomes. 1

Step 2 At least two heads: HHH, HHT, HTH, THH — 4 outcomes. P = 4/8 = 1/2 1

P = 1/2

"At least two" includes three. Leaving out HHH gives 3/8, the usual wrong answer.

Q7 · Objective

2023 · 2025 | 1 Mark

A letter is chosen at random from the letters of the word PROBABILITY. The probability that it is a vowel is

(a) 3/11 (b) 4/11 (c) 5/11 (d) 7/11

Step 1 11 letters in all; vowels are O, A, I, I — 4 of them (count every I).

(b) 4/11