MATH LOVE

Education as a Service (EaaS)

LIMITS AND DERIVATIVES

Class 11 CBSE - Complete Study Material

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1. INTRODUCTION TO LIMITS

The concept of limit is fundamental to calculus. It describes the behavior of a function as the input approaches a particular value. Limits help us understand continuity, derivatives, and integrals.

Definition of Limit:

Let f(x) be a function defined on an open interval around point 'a' (except possibly at 'a' itself). We say the limit of f(x) as x approaches 'a' is L, written as:

$$\lim_{x \to a} f(x) = L$$

This means: As x gets closer and closer to 'a' from both sides, f(x) gets closer and closer to L.

1.1 Left Hand Limit (LHL) and Right Hand Limit (RHL)

Left Hand Limit (LHL): Limit as x approaches 'a' from left (values less than 'a')

$$\lim_{x \to a^-} f(x) \text{ or } \lim_{h \to 0} f(a - h)$$

Right Hand Limit (RHL): Limit as x approaches 'a' from right (values greater than 'a')

$$\lim_{x \to a^+} f(x) \text{ or } \lim_{h \to 0} f(a + h)$$

Existence of Limit:

$$\lim_{x \to a} f(x) \text{ exists} \Leftrightarrow \text{LHL} = \text{RHL}$$

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$$

2. FUNDAMENTAL THEOREMS ON LIMITS

Let $\lim_{x \to a} f(x) = l$ and $\lim_{x \to a} g(x) = m$, then:

1. Sum Rule:

$$\lim_{x \to a} [f(x) + g(x)] = l + m$$

2. Difference Rule:

$$\lim_{x \to a} [f(x) - g(x)] = l - m$$

3. Product Rule:

$$\lim_{x \to a} [f(x) \times g(x)] = l \times m$$

4. Quotient Rule:

$$\lim_{x \to a} [f(x)/g(x)] = l/m \text{ (provided } m \neq 0\text{)}$$

5. Constant Multiple:

$$\lim_{x \to a} [k \cdot f(x)] = k \cdot l \text{ (where k is a constant)}$$

6. **Power Rule:**

$$\lim_{x \to a} [f(x)]^n = l^n$$

3. STANDARD LIMITS - FORMULAE

These are the most important limit formulas you MUST memorize:

1. **Polynomial Limit:**

$$\lim_{x \to a} x^n = a^n$$

2. **Constant Limit:**

$$\lim_{x \to a} k = k \text{ (where k is any constant)}$$

3. **Most Important - Trigonometric Limits:**

$$\lim_{x \to 0} (\sin x)/x = 1$$

$$\lim_{x \to 0} (\tan x)/x = 1$$

$$\lim_{x \to 0} (1 - \cos x)/x = 0$$

$$\lim_{x \to 0} (1 - \cos x)/x^2 = 1/2$$

4. **Exponential Limit:**

$$\lim_{x \to 0} (e^x - 1)/x = 1$$

$$\lim_{x \to 0} (a^x - 1)/x = \log_e a = \ln a$$

5. **Logarithmic Limit:**

$$\lim_{x \to 0} \log(1 + x)/x = 1$$

$$\lim_{x \to 0} \log_a(1 + x)/x = \log_a e = 1/\ln a$$

6. **The Number 'e' Limit:**

$$\lim_{x \to \infty} (1 + 1/x)^x = e \approx 2.718$$

$$\lim_{x \to 0} (1 + x)^{1/x} = e$$

⚠️ Indeterminate Forms:

When evaluating limits, if you get any of these forms, you CANNOT directly conclude the limit:

- 0/0 (Most common)
- ∞/∞
- 0 × ∞
- ∞ - ∞
- 0^0, 1^∞, ∞^0

These forms require special techniques like factorization, rationalization, or using standard limits.

4. METHODS TO EVALUATE LIMITS

4.1 Direct Substitution Method

If direct substitution doesn't give 0/0 or ∞/∞, simply substitute the value.

Example 1:

Evaluate: $\lim_{x \to 2} (x^2 + 3x + 1)$

Solution: Direct substitution:

$= 2^2 + 3(2) + 1 = 4 + 6 + 1 =$ **11**

4.2 Factorization Method

Used when we get 0/0 form. Factor numerator and denominator, cancel common factors.

Example 2:

Evaluate: $\lim_{x\to3} (x^2 - 9)/(x - 3)$

Solution:

Direct substitution gives 0/0 (indeterminate)

$= \lim_{x\to3} (x^2 - 3^2)/(x - 3)$

$= \lim_{x\to3} (x - 3)(x + 3)/(x - 3)$

$= \lim_{x\to3} (x + 3)$ [Cancel (x - 3)]

= 3 + 3 = **6**

4.3 Rationalization Method

Used when square roots are involved. Multiply by conjugate.

Example 3:

Evaluate: $\lim_{x\to0} (\sqrt{(1 + x)} - 1)/x$

Solution:

Multiply numerator and denominator by conjugate $\sqrt{(1 + x)} + 1$:

$= \lim_{x\to0} [(\sqrt{(1 + x)} - 1)(\sqrt{(1 + x)} + 1)] / [x(\sqrt{(1 + x)} + 1)]$

$= \lim_{x\to0} [(1 + x) - 1] / [x(\sqrt{(1 + x)} + 1)]$

$= \lim_{x\to0} x / [x(\sqrt{(1 + x)} + 1)]$

$= \lim_{x\to0} 1 / (\sqrt{(1 + x)} + 1)$

= $1/(\sqrt{1} + 1) = 1/2 =$ **0.5**

4.4 Using Standard Limits

Convert the given limit to standard form and apply known results.

Example 4:

Evaluate: $\lim_{x\to 0} (\sin 5x)/(3x)$

Solution:

$= \lim_{x\to 0} (\sin 5x)/(5x) \times (5x)/(3x) \times 5/3$

$= \lim_{x\to 0} (\sin 5x)/(5x) \times 5/3$

Let $u = 5x$, as $x\to 0$, $u\to 0$

$= \lim_{u\to 0} (\sin u)/u \times 5/3$

$= 1 \times 5/3 =$ **5/3**

5. LIMITS AT INFINITY

Important Results for Limits at Infinity:

1. Polynomial at infinity:

$$\lim_{x\to\infty} (ax^n + bx^{n-1} + \ldots + k) = \infty \text{ (if } a > 0, n > 0)$$

2. Rational function at infinity:

For $\lim_{x\to\infty} P(x)/Q(x)$, where $P(x)$ and $Q(x)$ are polynomials:

- If degree of P > degree of Q: Limit = ∞

- If degree of P < degree of Q: Limit = 0
- If degree of P = degree of Q: Limit = ratio of leading coefficients

3. **Important limits:**

$$\lim_{x\to\infty} 1/x = 0$$

$$\lim_{x\to\infty} 1/x^n = 0 \text{ (for } n > 0)$$

Example 5:

Evaluate: $\lim_{x\to\infty} (3x^2 + 2x + 1)/(5x^2 - 4x + 3)$

Solution:

Degree of numerator = degree of denominator = 2

Limit = ratio of leading coefficients = 3/5 = **0.6**

6. INTRODUCTION TO DERIVATIVES

The derivative measures the rate at which a function changes. It represents the slope of the tangent line to the curve at any point.

Definition of Derivative (First Principles):

The derivative of function f(x) at x = a is defined as:

$$f'(a) = \lim_{h\to 0} [f(a + h) - f(a)]/h$$

Alternatively, if we let x = a + h, then as $h\to 0$, $x\to a$:

$$f'(a) = \lim_{x\to a} [f(x) - f(a)]/(x - a)$$

For general x (not specific point a):

$$f'(x) = \lim_{h\to 0} [f(x + h) - f(x)]/h$$

📝 Notations for Derivatives:

If y = f(x), the derivative can be written as:

- f'(x) (read as "f prime of x")
- dy/dx (read as "dy by dx")
- d/dx [f(x)]
- Df(x)
- y' or y_1

7. DERIVATIVES OF STANDARD FUNCTIONS

🔥 MUST MEMORIZE - Basic Derivatives:

Function f(x)	Derivative f'(x)
k (constant)	0
x	1
x^n	nx^{n-1}
√x or $x^{1/2}$	1/(2√x)
1/x or x^{-1}	$-1/x^2$
sin x	cos x
cos x	-sin x

Function f(x)	Derivative f'(x)
tan x	$\sec^2 x$
cot x	$-\text{cosec}^2 x$ or $-\csc^2 x$
sec x	sec x tan x
cosec x or csc x	-cosec x cot x or -csc x cot x
e^x	e^x
a^x	$a^x \ln a$
ln x or $\log_e x$	1/x
$\log_a x$	1/(x ln a)

8. RULES OF DIFFERENTIATION

Let u = u(x) and v = v(x) be two differentiable functions:

1. Constant Multiple Rule:

d/dx [k·f(x)] = k·f'(x) where k is constant

2. Sum Rule:

d/dx [u + v] = du/dx + dv/dx = u' + v'

3. Difference Rule:

d/dx [u - v] = du/dx - dv/dx = u' - v'

4. Product Rule:

$$d/dx\,[u\cdot v] = u\cdot(dv/dx) + v\cdot(du/dx) = u\cdot v' + v\cdot u'$$

5. Quotient Rule:

$$d/dx\,[u/v] = [v\cdot(du/dx) - u\cdot(dv/dx)]/v^2 = (v\cdot u' - u\cdot v')/v^2$$

6. Power Rule (General):

$$d/dx\,[f(x)]^n = n\cdot[f(x)]^{n-1}\cdot f'(x)$$

7. Chain Rule:

If $y = f(u)$ and $u = g(x)$, then $dy/dx = (dy/du)\cdot(du/dx)$

Derivation 1: Derivative of x^n using First Principles

Given: $f(x) = x^n$
To Find: $f'(x)$

Step 1: Apply first principles definition:

$$f'(x) = \lim_{h\to 0} [(x + h)^n - x^n]/h$$

Step 2: Expand $(x + h)^n$ using binomial theorem:

$$(x + h)^n = x^n + nx^{n-1}h + [n(n-1)/2!]x^{n-2}h^2 + \ldots$$

Step 3: Substitute in limit:

$$f'(x) = \lim_{h\to 0} [x^n + nx^{n-1}h + h^2(\ldots) - x^n]/h$$

$$= \lim_{h\to 0} [nx^{n-1}h + h^2(\ldots)]/h$$

Step 4: Cancel h:

$$= \lim_{h\to 0} [nx^{n-1} + h(\ldots)]$$

Step 5: As $h \to 0$, terms with h vanish:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

9. DERIVATIVES FROM FIRST PRINCIPLES - EXAMPLES

Example 6: Find derivative of $f(x) = 3x^2$ from first principles

Solution:

$f'(x) = \lim_{h \to 0} [f(x + h) - f(x)]/h$

$= \lim_{h \to 0} [3(x + h)^2 - 3x^2]/h$

$= \lim_{h \to 0} [3(x^2 + 2xh + h^2) - 3x^2]/h$

$= \lim_{h \to 0} [3x^2 + 6xh + 3h^2 - 3x^2]/h$

$= \lim_{h \to 0} [6xh + 3h^2]/h$

$= \lim_{h \to 0} [6x + 3h]$

$= \mathbf{6x}$

Example 7: Find derivative of $f(x) = 1/x$ from first principles

Solution:

$f'(x) = \lim_{h \to 0} [f(x + h) - f(x)]/h$

$= \lim_{h \to 0} [1/(x + h) - 1/x]/h$

$= \lim_{h \to 0} [x - (x + h)]/[hx(x + h)]$

$= \lim_{h\to 0} [-h]/[hx(x + h)]$

$= \lim_{h\to 0} [-1]/[x(x + h)]$

$= -1/x^2 =$ **$-x^{-2}$**

10. PRACTICE QUESTIONS - TYPE WISE

Type 1: Direct Substitution

Q1. Evaluate: $\lim_{x\to 3} (2x^2 - 5x + 1)$

Answer: $= 2(9) - 5(3) + 1 = 18 - 15 + 1 =$ **4**

Q2. Evaluate: $\lim_{x\to 0} (x^3 + 2x^2 + 3x + 4)$

Answer: $= 0 + 0 + 0 + 4 =$ **4**

Type 2: Factorization (0/0 Form)

Q3. Evaluate: $\lim_{x\to 2} (x^3 - 8)/(x - 2)$

Solution:

$= \lim_{x\to 2} (x^3 - 2^3)/(x - 2)$

$= \lim_{x\to 2} (x - 2)(x^2 + 2x + 4)/(x - 2)$

$= \lim_{x\to 2} (x^2 + 2x + 4)$

$= 4 + 4 + 4 =$ **12**

Q4. Evaluate: $\lim_{x\to-1} (x^2 + 3x + 2)/(x^2 - 1)$

Solution:

$= \lim_{x\to-1} (x + 1)(x + 2)/[(x - 1)(x + 1)]$

$= \lim_{x\to-1} (x + 2)/(x - 1)$

$= 1/(-2) =$ **-1/2**

Type 3: Rationalization

Q5. Evaluate: $\lim_{x\to4} (\sqrt{x} - 2)/(x - 4)$

Solution:

Multiply by conjugate $(\sqrt{x} + 2)/(\sqrt{x} + 2)$:

$= \lim_{x\to4} (x - 4)/[(x - 4)(\sqrt{x} + 2)]$

$= \lim_{x\to4} 1/(\sqrt{x} + 2)$

$= 1/(2 + 2) =$ **1/4**

Q6. Evaluate: $\lim_{x\to0} (\sqrt{(1 + x)} - \sqrt{(1 - x)})/x$

Solution:

Rationalize: multiply by $[\sqrt{(1 + x)} + \sqrt{(1 - x)}]$

$= \lim_{x\to0} [(1 + x) - (1 - x)]/[x(\sqrt{(1 + x)} + \sqrt{(1 - x)})]$

$= \lim_{x\to0} 2x/[x(\sqrt{(1 + x)} + \sqrt{(1 - x)})]$

$= \lim_{x\to0} 2/(\sqrt{(1 + x)} + \sqrt{(1 - x)})$

$= 2/(1 + 1) =$ **1**

Type 4: Using Standard Trigonometric Limits

Q7. Evaluate: $\lim_{x \to 0}$ (sin 7x)/(sin 3x)

Solution:

= $\lim_{x \to 0}$ [(sin 7x)/7x] × (7x/x) × (x/3x) × [3x/(sin 3x)]

= $\lim_{x \to 0}$ [(sin 7x)/(7x)] × 7 × (1/3) × [1/(sin 3x)/(3x)]

= 1 × 7 × (1/3) × (1/1) = **7/3**

Q8. Evaluate: $\lim_{x \to 0}$ (1 - cos 2x)/x^2

Solution:

Using identity 1 - cos 2x = 2$\sin^2$ x:

= $\lim_{x \to 0}$ (2$\sin^2$ x)/x^2

= 2 × $\lim_{x \to 0}$ $(\sin x/x)^2$

= 2 × 1^2 = **2**

Type 5: Limits at Infinity

Q9. Evaluate: $\lim_{x \to \infty}$ ($4x^3 + 2x^2 - 1$)/($2x^3 - 5x + 3$)

Solution:

Degree of numerator = degree of denominator = 3

Limit = ratio of coefficients of x^3 = 4/2 = **2**

Q10. Evaluate: $\lim_{x \to \infty}$ ($5x^2 + 3x + 1$)/($2x^3 + x - 4$)

Solution:

Degree of numerator (2) < degree of denominator (3)

Limit = **0**

Type 6: Derivatives Using Rules

Q11. Find dy/dx if $y = x^5 + 3x^3 - 2x + 7$

Solution:

$dy/dx = 5x^4 + 9x^2 - 2 + 0$

= $\mathbf{5x^4 + 9x^2 - 2}$

Q12. Find dy/dx if $y = (x^2 + 1)(x^3 - 2)$

Solution: Using product rule u·v' + v·u'

$u = x^2 + 1$, $u' = 2x$

$v = x^3 - 2$, $v' = 3x^2$

$dy/dx = (x^2 + 1)(3x^2) + (x^3 - 2)(2x)$

$= 3x^4 + 3x^2 + 2x^4 - 4x$

= $\mathbf{5x^4 + 3x^2 - 4x}$

Type 7: Quotient Rule

Q13. Find dy/dx if $y = (x^2 + 1)/(x - 2)$

Solution: Using quotient rule $(v \cdot u' - u \cdot v')/v^2$

$u = x^2 + 1$, $u' = 2x$

$v = x - 2$, $v' = 1$

$dy/dx = [(x - 2)(2x) - (x^2 + 1)(1)]/(x - 2)^2$

$= [2x^2 - 4x - x^2 - 1]/(x - 2)^2$

$=$ **$(x^2 - 4x - 1)/(x - 2)^2$**

Type 8: Trigonometric Derivatives

Q14. Find dy/dx if $y = \sin x + \cos x + \tan x$

Solution:

$dy/dx = \cos x + (-\sin x) + \sec^2 x$

$=$ **$\cos x - \sin x + \sec^2 x$**

Q15. Find dy/dx if $y = \sin x \cdot \cos x$

Solution: Using product rule

$= \sin x \cdot (-\sin x) + \cos x \cdot (\cos x)$

$= -\sin^2 x + \cos^2 x$

$=$ **$\cos 2x$** (using identity $\cos^2 x - \sin^2 x = \cos 2x$)

11. CASE STUDIES WITH QUESTIONS

Case Study 1: Motion of a Particle

The position of a particle moving along a straight line is given by $s(t) = t^3 - 6t^2 + 9t + 2$, where s is in meters and t is in seconds.

Question 1: Find the velocity of the particle at time t = 2 seconds.

(a) -3 m/s **(b)** 0 m/s **(c)** 3 m/s **(d)** 9 m/s

Question 2: At what time does the particle come to rest (velocity = 0)?

(a) t = 1 s only **(b)** t = 3 s only
(c) t = 1 s and t = 3 s **(d)** Never

Question 3: The acceleration of the particle at t = 1 second is:

(a) -6 m/s^2 **(b)** 0 m/s^2 **(c)** 6 m/s^2 **(d)** 12 m/s^2

Solutions - Case Study 1:

Answer 1: (c) 3 m/s

Velocity $v(t) = ds/dt = 3t^2 - 12t + 9$

At t = 2: v(2) = 3(4) - 12(2) + 9 = 12 - 24 + 9 = -3 m/s

Wait! Let me recalculate: v(2) = 3(4) - 12(2) + 9 = 12 - 24 + 9 = -3 m/s. So answer should be (a) -3 m/s

Answer 2: (c) t = 1 s and t = 3 s

$v(t) = 0 \implies 3t^2 - 12t + 9 = 0$

$\implies t^2 - 4t + 3 = 0$

$\implies (t - 1)(t - 3) = 0$

$\implies t = 1$ s or $t = 3$ s

Answer 3: (a) -6 m/s^2

Acceleration $a(t) = dv/dt = d^2s/dt^2 = 6t - 12$

At t = 1: a(1) = 6(1) - 12 = -6 m/s^2

Case Study 2: Temperature Change

The temperature T (in °C) of a chemical reaction at time t minutes is given by $T(t) = t^3 - 9t^2 + 15t + 25$, for $0 \le t \le 6$.

Question 4: The rate of change of temperature at t = 1 minute is:

(a) 0 °C/min **(b)** 3 °C/min **(c)** 6 °C/min **(d)** 9 °C/min

Question 5: At what time(s) is the rate of temperature change zero?

(a) t = 1 min **(b)** t = 5 min **(c)** t = 1 and t = 5 min **(d)** Never

Solutions - Case Study 2:

Answer 4: (d) 9 °C/min

$dT/dt = 3t^2 - 18t + 15$

At t = 1: dT/dt = 3(1) - 18(1) + 15 = 3 - 18 + 15 = 0 °C/min

Correction: Answer should be (a) 0 °C/min

Answer 5: (c) t = 1 and t = 5 min

$dT/dt = 0 \Longrightarrow 3t^2 - 18t + 15 = 0$

$\Longrightarrow t^2 - 6t + 5 = 0$

$\Longrightarrow (t - 1)(t - 5) = 0$

$\Longrightarrow$ t = 1 min or t = 5 min

12. IMPORTANT EXAM QUESTIONS - BOARD PATTERN

VERY SHORT ANSWER TYPE (1 Mark Each)

Q16. Evaluate: $\lim_{x\to0}$ (sin 3x)/x

Answer: = $\lim_{x\to0}$ (sin 3x)/(3x) × 3 = 1 × 3 = **3**

Q17. Find dy/dx if y = x^4

Answer: dy/dx = **$4x^3$**

Q18. Find the derivative of cos x with respect to x.

Answer: d/dx(cos x) = **-sin x**

Q19. Evaluate: $\lim_{x\to2}$ (x + 3)

Answer: = 2 + 3 = **5**

Q20. If f(x) = 5, find f'(x).

Answer: Derivative of constant = **0**

SHORT ANSWER TYPE-I (2 Marks Each)

Q21. Evaluate: $\lim_{x\to0}$ (tan 2x)/(sin 3x)

Solution:

= $\lim_{x\to0}$ [(tan 2x)/(2x)] × (2x/x) × (x/3x) × [3x/(sin 3x)]

= 1 × 2 × (1/3) × 1 = **2/3**

Q22. Find dy/dx if y = x^2 · sin x

Solution: Using product rule

= x^2 · (cos x) + sin x · (2x)

= $\mathbf{x^2 \cos x + 2x \sin x}$

Q23. Evaluate: $\lim_{x\to5} (x^2 - 25)/(x - 5)$

Solution:

$= \lim_{x\to5} (x - 5)(x + 5)/(x - 5)$

$= \lim_{x\to5} (x + 5) = 5 + 5 =$ **10**

SHORT ANSWER TYPE-II (3 Marks Each)

Q24. Find the derivative of x^3 from first principles.

Solution:

$f(x) = x^3$, $f(x + h) = (x + h)^3$

$f'(x) = \lim_{h\to0} [(x + h)^3 - x^3]/h$

$= \lim_{h\to0} [x^3 + 3x^2h + 3xh^2 + h^3 - x^3]/h$

$= \lim_{h\to0} [3x^2h + 3xh^2 + h^3]/h$

$= \lim_{h\to0} [3x^2 + 3xh + h^2]$

= $\mathbf{3x^2}$

Q25. Find dy/dx if $y = (2x + 3)/(x^2 + 1)$

Solution: Using quotient rule

$u = 2x + 3$, $u' = 2$

$v = x^2 + 1$, $v' = 2x$

$dy/dx = [(x^2 + 1)(2) - (2x + 3)(2x)]/(x^2 + 1)^2$

$= [2x^2 + 2 - 4x^2 - 6x]/(x^2 + 1)^2$

= **$(-2x^2 - 6x + 2)/(x^2 + 1)^2$**

LONG ANSWER TYPE (5 Marks Each)

Q26. Evaluate: $\lim_{x\to0}$ [(√(x + 9) - 3)/x]

Solution:

Multiply by conjugate [√(x + 9) + 3]:

= $\lim_{x\to0}$ [(x + 9) - 9]/[x(√(x + 9) + 3)]

= $\lim_{x\to0}$ x/[x(√(x + 9) + 3)]

= $\lim_{x\to0}$ 1/(√(x + 9) + 3)

= 1/(√9 + 3) = 1/(3 + 3) = **1/6**

Q27. Find the derivative of sin x from first principles and verify d/dx(sin x) = cos x.

Solution:

f(x) = sin x

f'(x) = $\lim_{h\to0}$ [sin(x + h) - sin x]/h

Using sin A - sin B = 2cos[(A + B)/2]sin[(A - B)/2]:

= $\lim_{h\to0}$ [2cos(x + h/2)sin(h/2)]/h

= $\lim_{h\to0}$ [cos(x + h/2)] × [sin(h/2)/(h/2)]

= cos x × 1 = **cos x** ✓ Verified

13. QUICK REVISION - FORMULA SHEET

Topic	Formula/Result	Remarks
Definition of Limit	$\lim_{x\to a} f(x) = L$	LHL = RHL = L
Standard Limit 1	$\lim_{x\to 0} (\sin x)/x = 1$	Most important!
Standard Limit 2	$\lim_{x\to 0} (\tan x)/x = 1$	Use tan x = sin x / cos x
Standard Limit 3	$\lim_{x\to 0} (1 - \cos x)/x^2 = 1/2$	Use $1 - \cos x = 2\sin^2(x/2)$
Exponential Limit	$\lim_{x\to 0} (e^x - 1)/x = 1$	Base = e
Logarithmic Limit	$\lim_{x\to 0} \log(1 + x)/x = 1$	Natural log (ln)
Number e	$\lim_{x\to\infty} (1 + 1/x)^x = e$	$e \approx 2.718$
Derivative Definition	$f'(x) = \lim_{h\to 0} [f(x+h) - f(x)]/h$	First principles
Power Rule	$d/dx(x^n) = nx^{n-1}$	For any real n
Constant Rule	$d/dx(k) = 0$	k is constant
Sum Rule	$d/dx[u + v] = u' + v'$	Derivative of sum
Product Rule	$d/dx[uv] = uv' + vu'$	Derivative of product
Quotient Rule	$d/dx[u/v] = (vu' - uv')/v^2$	$v \neq 0$
d/dx(sin x)	cos x	Trigonometric
d/dx(cos x)	-sin x	Note the negative sign
d/dx(tan x)	$\sec^2 x$	$= 1 + \tan^2 x$
d/dx(cot x)	$-\text{cosec}^2 x$	$= -(1 + \cot^2 x)$
d/dx(sec x)	sec x tan x	Product of sec and tan
d/dx(cosec x)	-cosec x cot x	Negative sign important

Topic	Formula/Result	Remarks
$d/dx(e^x)$	e^x	Unique property!
$d/dx(\ln x)$	$1/x$	Natural logarithm
$d/dx(a^x)$	$a^x \ln a$	Any constant base a
$d/dx(\log_a x)$	$1/(x \ln a)$	Base a logarithm

14. IMPORTANT TIPS FOR EXAMS

✅ Do's:

- **MEMORIZE all standard limits** - especially sin x/x, tan x/x, and $(1 - \cos x)/x^2$
- **Learn ALL derivative formulas** by heart - no exceptions!
- Always check if direct substitution works first
- For 0/0 form, try factorization before other methods
- For square roots, think rationalization immediately
- Write all steps clearly in exam - partial marks matter!
- Verify your answer by substituting back when possible
- Practice product and quotient rules extensively
- Remember: derivative of constant = 0

❌ Don'ts:

- Don't divide by zero - if denominator becomes 0, it's indeterminate!
- Don't forget negative signs (especially in cos x derivative)

- Don't confuse product rule with chain rule
- Never write derivative without showing method in board exams
- Don't skip rationalization step when √ is present
- Don't assume limit exists without checking LHL = RHL
- Don't use L'Hospital's rule in Class 11 (not in syllabus)

15. COMMON MISTAKES TO AVOID

⚠️ Mistake 1: Direct substitution without checking

Wrong: $\lim_{x\to 2} (x^2 - 4)/(x - 2) = 0/0$ ❌

Right: Getting 0/0 means you need to factor/rationalize/use standard limits ✓

⚠️ Mistake 2: Forgetting negative sign

Wrong: d/dx(cos x) = sin x ❌

Right: d/dx(cos x) = **-sin x** ✓

⚠️ Mistake 3: Product rule confusion

Wrong: d/dx[uv] = u'v' ❌

Right: d/dx[uv] = **uv' + vu'** ✓

⚠️ Mistake 4: Quotient rule sign error

Wrong: $d/dx[u/v] = (vu' + uv')/v^2$ ❌

Right: $d/dx[u/v] =$ **$(vu' - uv')/v^2$** ✓ (It's MINUS not plus!)

16. MOST EXPECTED HOME EXAM QUESTIONS (AVERAGE TO ABOVE AVERAGE LEVEL)

LEVEL 1: AVERAGE DIFFICULTY (Expected in Every Exam)

LIMITS - Average Level Questions

Q28. Evaluate: $\lim_{x\to 0} (\sqrt{(1 + x + x^2)} - 1)/x$

Solution:

Multiply by conjugate $[\sqrt{(1 + x + x^2)} + 1]$:

$= \lim_{x\to 0} [(1 + x + x^2) - 1]/[x(\sqrt{(1 + x + x^2)} + 1)]$

$= \lim_{x\to 0} (x + x^2)/[x(\sqrt{(1 + x + x^2)} + 1)]$

$= \lim_{x\to 0} (1 + x)/(\sqrt{(1 + x + x^2)} + 1)$

$= (1 + 0)/(\sqrt{1} + 1) =$ **$1/2$**

Q29. Evaluate: $\lim_{x\to 0} (\cos 2x - \cos 3x)/x^2$

Solution:

Using $\cos C - \cos D = -2\sin[(C+D)/2]\sin[(C-D)/2]$:

$= \lim_{x\to 0} [-2\sin(5x/2)\sin(-x/2)]/x^2$

$= \lim_{x\to 0} [2\sin(5x/2)\sin(x/2)]/x^2$

$= \lim_{x\to0} [\sin(5x/2)/(5x/2)] \times (5/2) \times [\sin(x/2)/(x/2)] \times (1/2)$

$= 1 \times (5/2) \times 1 \times (1/2) =$ **5/4**

Q30. Evaluate: $\lim_{x\to1} (x^4 - 1)/(x - 1)$

Solution:

$= \lim_{x\to1} (x^2 - 1)(x^2 + 1)/(x - 1)$

$= \lim_{x\to1} (x - 1)(x + 1)(x^2 + 1)/(x - 1)$

$= \lim_{x\to1} (x + 1)(x^2 + 1)$

$= 2 \times 2 =$ **4**

Q31. Evaluate: $\lim_{x\to0} (2x)/\sqrt{(1 + x)} - \sqrt{(1 - x)}$

Solution:

Rationalize by multiplying $[\sqrt{(1 + x)} + \sqrt{(1 - x)}]$:

$= \lim_{x\to0} 2x[\sqrt{(1 + x)} + \sqrt{(1 - x)}]/[(1 + x) - (1 - x)]$

$= \lim_{x\to0} 2x[\sqrt{(1 + x)} + \sqrt{(1 - x)}]/2x$

$= \lim_{x\to0} [\sqrt{(1 + x)} + \sqrt{(1 - x)}]$

$= \sqrt{1} + \sqrt{1} =$ **2**

Q32. Evaluate: $\lim_{x\to\pi/4} (1 - \tan x)/(1 - \sqrt{2} \sin x)$

Solution:

At $x = \pi/4$: $\tan(\pi/4) = 1$, $\sin(\pi/4) = 1/\sqrt{2}$

Direct substitution gives 0/0 form

Using L'Hospital or substitution method:

Let $x = \pi/4 + h$, as $x\to\pi/4$, $h\to0$

After simplification: = **√2/2**

DERIVATIVES - Average Level Questions

Q33. Find dy/dx if $y = (3x^2 + 2x + 1)(x^3 - 5)$

Solution: Using product rule u·v' + v·u'

$u = 3x^2 + 2x + 1, u' = 6x + 2$

$v = x^3 - 5, v' = 3x^2$

$dy/dx = (3x^2 + 2x + 1)(3x^2) + (x^3 - 5)(6x + 2)$

$= 9x^4 + 6x^3 + 3x^2 + 6x^4 + 2x^3 - 30x - 10$

= $\mathbf{15x^4 + 8x^3 + 3x^2 - 30x - 10}$

Q34. Find dy/dx if $y = x^2/(x^2 + 1)$

Solution: Using quotient rule $(v·u' - u·v')/v^2$

$u = x^2, u' = 2x$

$v = x^2 + 1, v' = 2x$

$dy/dx = [(x^2 + 1)(2x) - x^2(2x)]/(x^2 + 1)^2$

$= [2x^3 + 2x - 2x^3]/(x^2 + 1)^2$

= $\mathbf{2x/(x^2 + 1)^2}$

Q35. Find derivative of $\sqrt{(1 + x^2)}$ from first principles

Solution:

$f(x) = \sqrt{(1 + x^2)}$

$f'(x) = \lim_{h\to 0} [\sqrt{(1 + (x+h)^2)} - \sqrt{(1 + x^2)}]/h$

Rationalize by multiplying conjugate:

$= \lim_{h\to 0} [(1 + x^2 + 2xh + h^2) - (1 + x^2)]/[h(\sqrt{(1 + (x+h)^2)} + \sqrt{(1 + x^2)})]$

$= \lim_{h\to 0} (2xh + h^2)/[h(\sqrt{(1 + (x+h)^2)} + \sqrt{(1 + x^2)})]$

$= \lim_{h\to 0} (2x + h)/(\sqrt{(1 + (x+h)^2)} + \sqrt{(1 + x^2)})$

$= 2x/(2\sqrt{(1 + x^2)}) =$ **$x/\sqrt{(1 + x^2)}$**

LEVEL 2: ABOVE AVERAGE DIFFICULTY (Scoring Questions)

LIMITS - Above Average Level

Q36. Evaluate: $\lim_{x\to 0} (1 - \cos x\sqrt{\cos 2x})/x^2$

Solution:

Using $1 - \cos \theta \approx \theta^2/2$ for small θ and $\sqrt{(1-\theta)} \approx 1 - \theta/2$:

$\cos 2x \approx 1 - (2x)^2/2 = 1 - 2x^2$

$\sqrt{\cos 2x} \approx \sqrt{(1 - 2x^2)} \approx 1 - x^2$

$\cos x\sqrt{\cos 2x} \approx (1 - x^2/2)(1 - x^2) \approx 1 - 3x^2/2$

$= \lim_{x\to 0} (1 - 1 + 3x^2/2)/x^2 =$ **3/2**

Q37. Evaluate: $\lim_{x\to a} (x \sin a - a \sin x)/(x - a)$

Solution:

$= \lim_{x\to a} [x \sin a - a \sin a + a \sin a - a \sin x]/(x - a)$

$= \lim_{x\to a} [\sin a(x - a) + a(\sin a - \sin x)]/(x - a)$

$= \lim_{x\to a} [\sin a + a(\sin a - \sin x)/(x - a)]$

Using $\sin A - \sin B = 2\cos[(A+B)/2]\sin[(A-B)/2]$:

$= \sin a + a \times \lim_{x\to a} [2\cos((a+x)/2)\sin((a-x)/2)]/(x - a)$

$= \sin a + a \times \cos a \times (-1)$

= **sin a - a cos a**

Q38. Evaluate: $\lim_{x\to 0} (e^x - e^{-x} - 2x)/(x - \sin x)$

Solution:

Using expansions: $e^x \approx 1 + x + x^2/2 + x^3/6$, $e^{-x} \approx 1 - x + x^2/2 - x^3/6$

$e^x - e^{-x} \approx 2x + x^3/3$

Numerator: $2x + x^3/3 - 2x = x^3/3$

$\sin x \approx x - x^3/6$

Denominator: $x - (x - x^3/6) = x^3/6$

$= \lim_{x\to 0} (x^3/3)/(x^3/6) = (1/3)/(1/6) =$ **2**

Q39. Evaluate: $\lim_{x\to\infty} x[\sqrt{(x^2 + 1)} - x]$

Solution:

Multiply by conjugate $[\sqrt{(x^2 + 1)} + x]$:

$= \lim_{x\to\infty} x[(x^2 + 1) - x^2]/[\sqrt{(x^2 + 1)} + x]$

$= \lim_{x\to\infty} x/[\sqrt{(x^2 + 1)} + x]$

Divide numerator and denominator by x:

$= \lim_{x\to\infty} 1/[\sqrt{(1 + 1/x^2)} + 1]$

$= 1/(\sqrt{1} + 1) =$ **1/2**

Q40. Evaluate: $\lim_{x\to 0} (\tan x - \sin x)/x^3$

Solution:

$\tan x - \sin x = \sin x/\cos x - \sin x = \sin x(1 - \cos x)/\cos x$

$= \lim_{x\to 0} [\sin x(1 - \cos x)]/(x^3 \cos x)$

$= \lim_{x\to 0} (\sin x/x) \times [(1 - \cos x)/x^2] \times (1/\cos x)$

Using standard limits: $\sin x/x \to 1$, $(1 - \cos x)/x^2 \to 1/2$, $\cos x \to 1$

$= 1 \times (1/2) \times 1 =$ **1/2**

DERIVATIVES - Above Average Level

Q41. Find dy/dx if $y = (ax^2 + b)/(cx^2 + d)$

Solution: Using quotient rule

$u = ax^2 + b, u' = 2ax$

$v = cx^2 + d, v' = 2cx$

$dy/dx = [(cx^2 + d)(2ax) - (ax^2 + b)(2cx)]/(cx^2 + d)^2$

$= [2acx^3 + 2adx - 2acx^3 - 2bcx]/(cx^2 + d)^2$

$= [2adx - 2bcx]/(cx^2 + d)^2$

= $\mathbf{2x(ad - bc)/(cx^2 + d)^2}$

Q42. Find dy/dx if $y = (x^2 + 1)(x^3 + 1)(x^4 + 1)$

Solution: Using product rule for three functions

Let $u = x^2 + 1$, $v = x^3 + 1$, $w = x^4 + 1$

$d/dx[uvw] = vw \cdot u' + uw \cdot v' + uv \cdot w'$

$= (x^3 + 1)(x^4 + 1)(2x) + (x^2 + 1)(x^4 + 1)(3x^2) + (x^2 + 1)(x^3 + 1)(4x^3)$

= $\mathbf{2x(x^3 + 1)(x^4 + 1) + 3x^2(x^2 + 1)(x^4 + 1) + 4x^3(x^2 + 1)(x^3 + 1)}$

Q43. If $y = (x + \sqrt{(x^2 + 1)})^n$, prove that $dy/dx = n(x + \sqrt{(x^2 + 1)})^{n-1}(1 + x/\sqrt{(x^2 + 1)})$

Solution:

Using power rule: $dy/dx = n(x + \sqrt{(x^2 + 1)})^{n-1} \times d/dx[x + \sqrt{(x^2 + 1)}]$

$d/dx[x + \sqrt{(x^2 + 1)}] = 1 + x/\sqrt{(x^2 + 1)}$

Therefore: $dy/dx = n(x + \sqrt{(x^2 + 1)})^{n-1}(1 + x/\sqrt{(x^2 + 1)})$ ✓ Proved

Q44. Find dy/dx if $y = \sin x \cdot \cos x \cdot \tan x$

Solution: Using product rule for three functions

$d/dx[uvw] = vw \cdot u' + uw \cdot v' + uv \cdot w'$

$= (\cos x)(\tan x)(\cos x) + (\sin x)(\tan x)(-\sin x) + (\sin x)(\cos x)(\sec^2 x)$

$= \cos^2 x \tan x - \sin^2 x \tan x + \sin x \cos x \sec^2 x$

$= (\cos^2 x - \sin^2 x) \tan x + \sin x/\cos x$

$= \cos 2x \cdot \tan x + \tan x$

= **$\tan x(1 + \cos 2x)$**

LEVEL 3: CHALLENGING (Most Expected Tricky Questions)

LIMITS - Challenging Level

Q45. Evaluate: $\lim_{x\to0} [(1 + x)^1/^x - e]/x$

Solution: (Very Important - Often Asked)

Using expansion $(1 + x)^1/^x = e(1 - x/2 + 11x^2/24 + ...)$

$= \lim_{x\to0} [e(1 - x/2 + ...) - e]/x$

$= \lim_{x\to0} [e(-x/2 + ...)]/x$

$= e \times \lim_{x\to0} (-1/2 + ...)$

= **$-e/2$**

Q46. Evaluate: $\lim_{x\to0} (a^x - b^x)/x$, where $a, b > 0$

Solution:

$= \lim_{x\to0} [(a^x - 1)/x - (b^x - 1)/x]$

Using standard limit: $\lim_{x\to0} (a^x - 1)/x = \ln a$

$= \ln a - \ln b$

= **$\ln(a/b)$**

Q47. Evaluate: $\lim_{x\to 0}$ [sin(π $\cos^2$ x)]/x^2

Solution:

$\cos^2 x = (1 + \cos 2x)/2 \approx (1 + 1 - 2x^2)/2 = 1 - x^2$

$\pi \cos^2 x \approx \pi(1 - x^2) = \pi - \pi x^2$

$\sin(\pi \cos^2 x) = \sin(\pi - \pi x^2) = \sin(\pi x^2)$

$= \lim_{x\to 0} \sin(\pi x^2)/x^2$

$= \lim_{x\to 0} [\sin(\pi x^2)/(\pi x^2)] \times \pi$

$= 1 \times \pi =$ **π**

Q48. If $\lim_{x\to 1} (x^4 - 1)/(x - 1) = \lim_{x\to k} (x^3 - k^3)/(x^2 - k^2)$, find k

Solution:

Left side: $\lim_{x\to 1} (x^4 - 1)/(x - 1) = \lim_{x\to 1} (x - 1)(x^3 + x^2 + x + 1)/(x - 1) = 4$

Right side: $\lim_{x\to k} (x^3 - k^3)/(x^2 - k^2) = \lim_{x\to k} (x - k)(x^2 + xk + k^2)/[(x - k)(x + k)]$

$= (k^2 + k^2 + k^2)/(2k) = 3k^2/2k = 3k/2$

Therefore: $3k/2 = 4 \Longrightarrow k =$ **8/3**

DERIVATIVES - Challenging Level

Q49. If f(x) = |x - 1| + |x - 2|, discuss the differentiability at x = 1 and x = 2

Solution:

At x = 1:

$\text{LHD} = \lim_{h\to 0^-} [f(1+h) - f(1)]/h = \lim_{h\to 0^-} [|h| + |h - 1| - 1]/h = -2$

$\text{RHD} = \lim_{h\to 0^+} [f(1+h) - f(1)]/h = \lim_{h\to 0^+} [h + |h - 1| - 1]/h = 0$

Since LHD ≠ RHD, **f(x) is NOT differentiable at x = 1**

Similarly at $x = 2$: **NOT differentiable**

Q50. If $y = x/(x - 1)$, find d^2y/dx^2 (Second derivative)

Solution:

First find dy/dx using quotient rule:

$dy/dx = [(x - 1)(1) - x(1)]/(x - 1)^2 = -1/(x - 1)^2$

Now find second derivative:

$d^2y/dx^2 = d/dx[-1/(x - 1)^2] = d/dx[-(x - 1)^{-2}]$

$= -(-2)(x - 1)^{-3} \times 1$

= $\mathbf{2/(x - 1)^3}$

Q51. Find dy/dx if $x\sqrt{(1 + y)} + y\sqrt{(1 + x)} = 0$

Solution: (Implicit Differentiation)

Differentiating both sides with respect to x:

$\sqrt{(1 + y)} + x\cdot(1/(2\sqrt{(1 + y)}))\cdot dy/dx + (dy/dx)\sqrt{(1 + x)} + y\cdot(1/(2\sqrt{(1 + x)})) = 0$

Rearranging terms with dy/dx:

$dy/dx[x/(2\sqrt{(1 + y)}) + \sqrt{(1 + x)}] = -\sqrt{(1 + y)} - y/(2\sqrt{(1 + x)})$

dy/dx = $\mathbf{-[\sqrt{(1 + y)} + y/(2\sqrt{(1 + x)})]/[x/(2\sqrt{(1 + y)}) + \sqrt{(1 + x)}]}$

MIXED CONCEPT QUESTIONS (Very Important for Home Exams)

Questions Combining Limits and Derivatives

Q52. If $f(x) = x^2$ for $x \geq 0$ and $f(x) = -x^2$ for $x < 0$, find f'(0) if it exists

Solution:

LHD at $x = 0$: $\lim_{h \to 0^-} [f(0+h) - f(0)]/h = \lim_{h \to 0^-} [-h^2 - 0]/h = 0$

RHD at x = 0: $\lim_{h\to0^+}$ [f(0+h) - f(0)]/h = $\lim_{h\to0^+}$ [h^2 - 0]/h = 0

Since LHD = RHD = 0, **f'(0) = 0**

Q53. Evaluate: $\lim_{x\to a}$ [f(x) - f(a)]/(x - a) if f(x) = x^3

Solution:

This is the definition of derivative f'(a)!

f'(x) = $3x^2$

Therefore: $\lim_{x\to a}$ [f(x) - f(a)]/(x - a) = f'(a) = $\mathbf{3a^2}$

Q54. If $\lim_{x\to0}$ [sin 2x + a sin x]/[x^3] exists and is finite, find 'a' and the value of limit

Solution:

For limit to exist and be finite, numerator must have x^3 as factor

Using sin x ≈ x - x^3/6 for small x:

sin 2x ≈ 2x - $8x^3$/6 = 2x - $4x^3$/3

a sin x ≈ ax - ax^3/6

Numerator: 2x + ax - (4/3 + a/6)x^3 + ...

For x to cancel: 2 + a = 0 ⟹ **a = -2**

Then limit = -(4/3 - 2/6) = -(4/3 - 1/3) = **-1**

APPLICATION-BASED QUESTIONS (Most Expected)

Application 1: Rate of Change in Real Life

The radius of a circular oil spill on water is increasing at the rate of 2 m/s. Find the rate at which the area is increasing when radius is 10 m.

Solution:

Given: $dr/dt = 2$ m/s, $r = 10$ m

Area $A = \pi r^2$

$dA/dt = d/dt(\pi r^2) = \pi \times 2r \times dr/dt$

At $r = 10$: $dA/dt = \pi \times 2(10) \times 2 =$ **40π m²/s**

Application 2: Velocity and Acceleration

A particle moves along x-axis such that its position at time t is given by $x = t^3 - 9t^2 + 24t$. Find:

(a) Velocity at t = 2 sec

(b) When is the particle at rest?

(c) Acceleration when velocity is zero

Solution:

(a) $v = dx/dt = 3t^2 - 18t + 24$

At $t = 2$: $v = 3(4) - 18(2) + 24 = 12 - 36 + 24 =$ **0 m/s**

(b) Particle at rest when $v = 0$:

$3t^2 - 18t + 24 = 0 \Longrightarrow t^2 - 6t + 8 = 0$

$(t - 2)(t - 4) = 0 \Longrightarrow$ **$t = 2$ sec and $t = 4$ sec**

(c) $a = dv/dt = 6t - 18$

At $t = 2$: $a = 6(2) - 18 =$ **-6 m/s²**

At $t = 4$: $a = 6(4) - 18 =$ **6 m/s²**

🎯 TOP 10 MOST EXPECTED QUESTION PATTERNS:

1. $\lim_{x \to 0}$ (sin ax)/(sin bx) type - **ALWAYS comes!**
2. Rationalization with nested square roots - **Very common**
3. Product of three functions derivative - **Scoring question**
4. $(1 + x)^{1/x}$ related limits - **Above average level**
5. Quotient rule with polynomials - **100% expected**
6. First principles for √x or trigonometric functions - **5 marks question**
7. $(a^x - 1)/x$ type limits - **Important formula based**
8. Motion problems (velocity/acceleration) - **Application question**
9. Limits at infinity with surds - **Tricky but scoring**
10. Implicit differentiation - **New pattern question**

17. COMPREHENSIVE PRACTICE SET FOR HOME EXAM

Section A: Limits (50 Questions - All Levels)

BASIC LEVEL (Q1-Q15):

1. $\lim_{x \to 3} (x^2 + 2x - 3)$
2. $\lim_{x \to 0} (5x + 3)$
3. $\lim_{x \to 1} (x^3 - 1)/(x - 1)$
4. $\lim_{x \to 2} (x^2 - 4)/(x^2 - 3x + 2)$
5. $\lim_{x \to 0} (\sin 4x)/x$
6. $\lim_{x \to 0} (\tan 5x)/(2x)$
7. $\lim_{x \to 0} (\sin 3x)/(\sin 7x)$
8. $\lim_{x \to 0} (1 - \cos 4x)/x^2$
9. $\lim_{x \to 9} (\sqrt{x} - 3)/(x - 9)$
10. $\lim_{x \to 0} (\sqrt{4 + x} - 2)/x$

11. $\lim_{x \to 0} (e^{2x} - 1)/x$

12. $\lim_{x \to 0} \log(1 + 3x)/x$

13. $\lim_{x \to \infty} (3x^2 + 2x)/(x^2 + 5)$

14. $\lim_{x \to \infty} (2x + 1)/(x^2 - 3)$

15. $\lim_{x \to 4} (x - 4)/(\sqrt{x} - 2)$

AVERAGE LEVEL (Q16-Q35):

16. $\lim_{x \to 0} (\sin 2x + \sin 3x)/(x)$

17. $\lim_{x \to 0} (\tan x - \sin x)/x^3$

18. $\lim_{x \to 0} (\sqrt{(1 + x + x^2)} - 1)/x$

19. $\lim_{x \to \pi/6} (2\sin x - 1)/(6x - \pi)$

20. $\lim_{x \to a} (\sqrt{x} - \sqrt{a})/(x - a)$

21. $\lim_{x \to 0} (1 - \cos 2x)/(\sin^2 x)$

22. $\lim_{x \to 0} x \tan x/(1 - \cos x)$

23. $\lim_{x \to 1} (x^7 - 2x^5 + 1)/(x^3 - 3x^2 + 2)$

24. $\lim_{x \to 0} (e^{3x} - e^{2x})/x$

25. $\lim_{x \to 0} [\log(1 + 2x) - \log(1 + x)]/x$

26. $\lim_{x \to \infty} \sqrt{(x^2 + x + 1)} - \sqrt{(x^2 - x + 1)}$

27. $\lim_{x \to 0} (\sqrt{(1 + x)} - \sqrt{(1 - x)})/x$

28. $\lim_{x \to 0} (\cos 2x - \cos 3x)/x^2$

29. $\lim_{x \to 2} (x^3 - 8)/(x^2 - 4)$

30. $\lim_{x \to 0} (2x)/[\sqrt{(1 + x)} - \sqrt{(1 - x)}]$

31. $\lim_{x \to 0} (a^x - b^x)/x$

32. $\lim_{x \to 0} (\sin x - \tan x)/(x^3)$

33. $\lim_{x \to 16} (x - 16)/(\sqrt{x} - 4)$

34. $\lim_{x \to \infty} (3x + 5)/(2x - 7)$

35. $\lim_{x \to 0} [(1 + x)^n - 1]/x$

ABOVE AVERAGE LEVEL (Q36-Q50):

36. $\lim_{x\to 0} (1 - \cos x\sqrt{\cos 2x})/x^2$
37. $\lim_{x\to a} (x \sin a - a \sin x)/(x - a)$
38. $\lim_{x\to 0} (e^x - e^{-x} - 2x)/(x - \sin x)$
39. $\lim_{x\to \infty} x[\sqrt{(x^2 + 1)} - x]$
40. $\lim_{x\to 0} [(1 + x)^{1/x} - e]/x$
41. $\lim_{x\to 0} [\sin(\pi \cos^2 x)]/x^2$
42. $\lim_{x\to 0} (\sin x - x \cos x)/x^3$
43. $\lim_{x\to 0} (\tan x - x)/(x - \sin x)$
44. $\lim_{x\to 0} [\sqrt{(1 + x^2)} - \sqrt{(1 - x^2)}]/x$
45. $\lim_{x\to \infty} [\sqrt{(x^2 + ax)} - \sqrt{(x^2 + bx)}]$
46. $\lim_{x\to 0} (\sin x - 2\sin 2x + \sin 3x)/x^3$
47. $\lim_{x\to 0} (e^{\sin x} - 1 - x)/x^2$
48. $\lim_{x\to 0} [(1 + 2x)^{1/x} - e^2]/x$
49. $\lim_{x\to \pi/4} (\sin x - \cos x)/(x - \pi/4)$
50. $\lim_{x\to 0} [\log(1 + x + x^2) + \log(1 - x + x^2)]/[\sec x - \cos x]$

Section B: Derivatives (50 Questions - All Levels)

BASIC LEVEL - Direct Application (Q51-Q65):

51. $y = x^7$
52. $y = 5x^3 + 2x - 7$
53. $y = \sqrt{x} + 1/\sqrt{x}$
54. $y = 1/x^3$
55. $y = \sin x + \cos x$
56. $y = \tan x - \cot x$

57. $y = e^x + \ln x$

58. $y = x^4 - 3x^2 + 5x - 2$

59. y = sec x + cosec x

60. $y = 5x^2 + 3/x + 7$

61. y = (x + 1)(x + 2)

62. y = sin x cos x

63. $y = x^2/3 + 2x$

64. $y = \sqrt{(x^2 + 1)}$ [hint: use chain rule]

65. $y = 1/(x^2 + 1)$

AVERAGE LEVEL - Product & Quotient Rules (Q66-Q85):

66. $y = x^2 \sin x$ (Product rule)

67. y = x cos x (Product rule)

68. y = sin x · tan x (Product rule)

69. $y = e^x \cdot x^2$ (Product rule)

70. y = (x + 1)/(x - 1) (Quotient rule)

71. $y = (x^2 + 1)/(x^2 - 1)$ (Quotient rule)

72. y = sin x/x (Quotient rule)

73. y = (ax + b)/(cx + d) (Quotient rule)

74. $y = x^2 \tan x$ (Product rule)

75. $y = (3x^2 + 2)(2x^3 - 5)$ (Product rule)

76. $y = x/(x^2 + 1)$ (Quotient rule)

77. y = (x - 1)(x - 2)(x - 3) (Product of 3)

78. y = tan x/x (Quotient rule)

79. $y = x^2 e^x$ (Product rule)

80. y = (sin x + cos x)/(sin x - cos x) (Quotient rule)

81. $y = x\sqrt{x}$ (Product rule or simplify first)

82. $y = (x^2 + a^2)/(x^2 - a^2)$ (Quotient rule)

83. $y = (2x + 3)/(x^2 + 1)$ (Quotient rule)

84. $y = x \sin x \cos x$ (Product of 3)

85. $y = (x^3 - 1)/(x + 1)$ (Quotient rule)

ABOVE AVERAGE - First Principles & Advanced (Q86-Q100):

86. Find derivative of x^4 using first principles

87. Find derivative of $\sqrt{x}$ using first principles

88. Find derivative of $1/x$ using first principles

89. Find derivative of $\sqrt{(x^2 + 1)}$ using first principles

90. Find derivative of $\sin x$ from first principles

91. $y = (x^2 + 1)(x^3 + 1)(x^4 + 1)$

92. $y = (ax^2 + b)/(cx^2 + d)$

93. If $y = x/(x - 1)$, find d^2y/dx^2

94. If $x^2y^2 = 1$, find dy/dx (Implicit)

95. If $x + y = xy$, find dy/dx (Implicit)

96. $y = |x^3 - 3x^2|$, discuss differentiability

97. $y = (\sin x)(\cos x)(\tan x)$

98. Prove: If $y = (x + \sqrt{(x^2 + 1)})^n$, then $(x^2 + 1)dy/dx = ny\sqrt{(x^2 + 1)}$

99. $y = \sqrt{[(x - 1)/(x + 1)]}$

100. If $x\sqrt{(1 + y)} + y\sqrt{(1 + x)} = 0$, find dy/dx

Section C: Mixed & Application Questions (20 Questions)

WORD PROBLEMS & APPLICATIONS (Q101-Q120):

101. The radius of a circle is increasing at 3 cm/s. Find rate of increase of area when r = 5 cm.

102. A stone is dropped into a pond. Circular ripples spread at 4 m/s. Find rate of area increase when radius is 10 m.

103. A particle moves with $s(t) = t^3 - 6t^2 + 9t$. Find when velocity is zero.

104. Position $s = 2t^3 - 9t^2 + 12t - 3$. Find velocity at $t = 2$ and acceleration at $t = 1$.
105. The volume V of a cube of side x is $V = x^3$. Find rate of change of V with respect to x when $x = 5$.
106. A balloon is being inflated. If radius increases at 2 cm/min, find rate of volume increase when $r = 7$ cm.
107. If $f(x) = |x - 2|$, find $f'(x)$ for $x \neq 2$. Discuss differentiability at $x = 2$.
108. A man 2m tall walks at 3 m/s toward a lamp post 6m high. How fast is his shadow shortening?
109. Find points on $y = x^2$ where tangent is parallel to line $4x - 2y + 3 = 0$.
110. The area of a square is increasing at 8 cm^2/s. Find rate of side increase when side is 4 cm.
111. If $\lim_{x \to 2} [x^2 + ax + b]/(x - 2)$ exists, find a and b.
112. Find derivative of $|x|$ at $x = 0$. Does it exist?
113. A particle's position: $s = t^4 - 4t^3 + 4t^2$. When does it reverse direction?
114. Evaluate: $\lim_{x \to 0} [f(x) - f(0)]/x$ if $f(x) = x^2 \sin(1/x)$ for $x \neq 0$, $f(0) = 0$
115. The diagonal of a square is increasing at 2 cm/s. Find rate of area increase when diagonal is 10 cm.
116. If $y = x^x$, find dy/dx (Use logarithmic differentiation)
117. Water flows into a conical tank at 2 m^3/min. If radius:height = 1:2, find rate of height increase when $h = 4$m.
118. Find $\lim_{x \to \infty} [(x + a)/(x + b)]^x$
119. A ladder 10m long leans against a wall. If bottom slides at 2 m/s, how fast is top sliding down when bottom is 6m from wall?
120. If $f(x) = x|x|$, find $f'(0)$ using first principles.

📝 ANSWER KEY - QUICK REFERENCE

LIMITS (Selected Answers):

Q1: 12 | Q2: 3 | Q3: 3 | Q4: 4 | Q5: 4 | Q6: 5/2 | Q7: 3/7 | Q8: 8 | Q9: 1/6 | Q10: 1/4

Q11: 2 | Q12: 3 | Q13: 3 | Q14: 0 | Q15: 4 | Q20: n | Q28: 1/2 | Q29: 5/4 | Q36: 3/2

Q37: sin a - a cos a | Q39: 1/2 | Q40: 1/2 | Q46: ln(a/b) | Q47: π

DERIVATIVES (Selected Answers):

Q51: $7x^6$ | Q52: $15x^2 + 2$ | Q53: 1/(2√x) - 1/(2x^(3/2)) | Q55: cos x - sin x

Q66: $x^2 \cos x + 2x \sin x$ | Q70: $-1/(x - 1)^2$ | Q71: $-2x/(x^2 - 1)^2$

APPLICATIONS (Selected Answers):

Q101: 30π cm^2/s | Q103: t = 1s, t = 3s | Q105: 75 units | Q106: 196π cm^3/min

⏰ TIME MANAGEMENT FOR HOME EXAM:

- **1 Mark Questions:** 2-3 minutes each (Direct substitution/formula)
- **2 Mark Questions:** 4-5 minutes each (One method/rule application)
- **3 Mark Questions:** 6-8 minutes each (Detailed working required)
- **5 Mark Questions:** 10-12 minutes each (Complete derivation/proof)
- **Case Studies:** 8-10 minutes for 3-4 MCQs
- **Review Time:** Last 10-15 minutes for checking

Pro Tip: Solve in order: Easy → Average → Difficult. Don't get stuck on one question!

🎯 FINAL PREPARATION CHECKLIST (2 Days Before Exam):

- ✓ Revise ALL standard limit formulas (sin x/x, tan x/x, etc.)
- ✓ Memorize ALL derivative formulas (trigonometric, exponential, logarithmic)

- ✓ Practice 10 factorization questions
- ✓ Practice 10 rationalization questions
- ✓ Solve 5 product rule questions
- ✓ Solve 5 quotient rule questions
- ✓ Do 2 first principles derivations completely
- ✓ Solve 2 application problems (velocity/rate of change)
- ✓ Review common mistakes section
- ✓ Sleep well night before exam - Fresh mind = Better performance!

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